Project: IEEE P802.15 Working Group for Wireless Personal Area Networks (WPANs)

Submission Title: LOS Link Budget
Date Submitted: July 2015
Source: Rick Roberts [Intel]
Address
Voice: 503-712-5012, E-Mail: richard.d.roberts@intel.com

Re:

Abstract:

Purpose:

This document is a revision of work originally presented to IEEE802.15.7 in September 2009.

Project: IEEE P802.15 Working Group for Wireless Personal Area Networks (WPANs)

Submission Title: Update on VLC Link Budget Work
Date Submitted: September 2009

Source: Rick Roberts [Intel], Zhengyuan Xu [University of California, Riverside]
Address
Voice: 503-712-5012, E-Mail: richard.d.roberts@intel.com, dxu@ee.ucr.edu

Re:

Abstract: Update on the VLC link budget work. The one remaining issue is the calculation of the noise density.

Purpose:

This contribution is in response to the call for channel models and address the line-of-sight OCC channel.

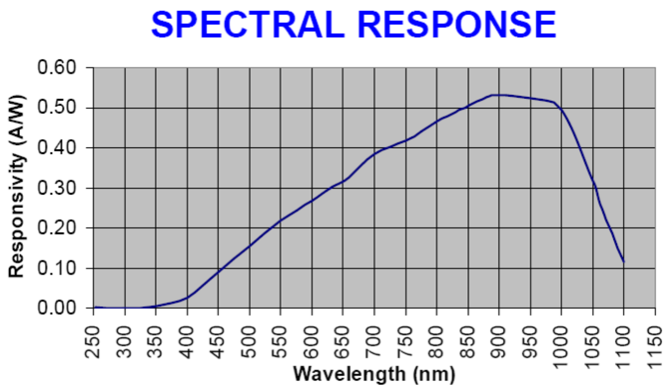
Intel believes that for many OCC usages, a channel model is not needed because there is no implied guaranteed quality of service since the light source is a signal of opportunity. If performance is not adequate then the user needs to move closer to the source to improve the signal-to-noise ratio. However, the automotive use case is an exception (has a QoS requirement) and will be given additional emphasis in the presentation.

ToC

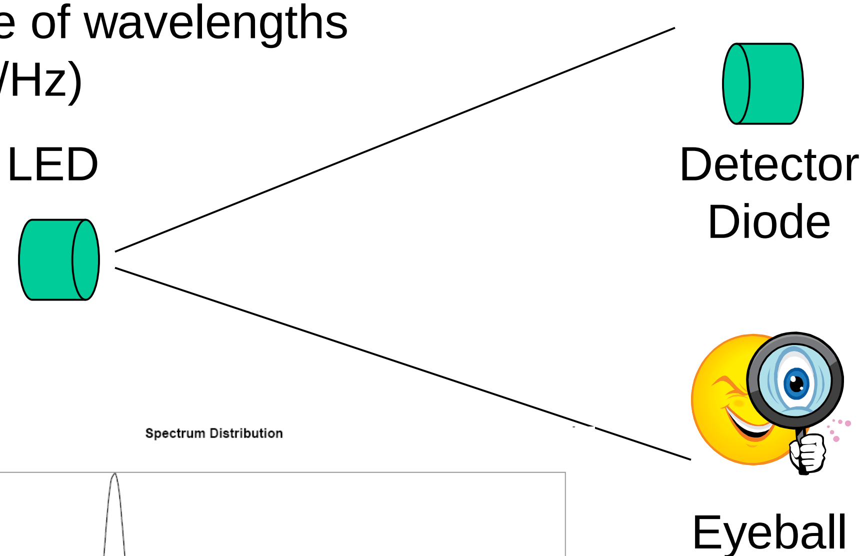
- Radiometric (Physical) vs. Photometric (Visual)
- Path loss due to line-of-sight (LOS) light propagation
- Beam Divergence
- Atmospheric Attenuation Due to Fog
- Propagation Path Loss
- 850 nm NIR specific analysis
- Appendix A: Ascertaining the LED parameters of interest
- Appendix B: Calculating integrated spectral flex density
- Appendix C: Receiver noise density calculations
- Appendix D: Solid angle path loss model
- Appendix E: RX aperture and magnification factor
- Appendix F: Fog diffusion ‘glow’

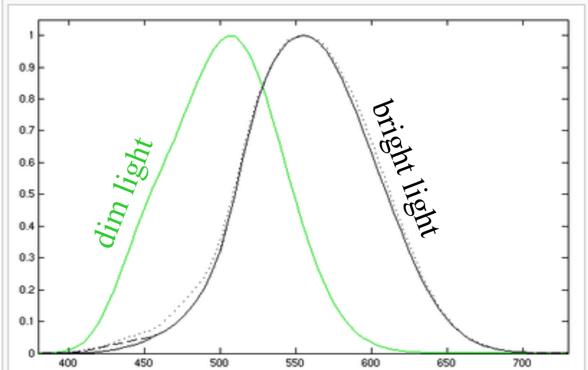
Radiometric (Physical) vs. Photometric (Visual)

The human eye and the detector diode have different frequency responses and hence perceive the same LED source differently.

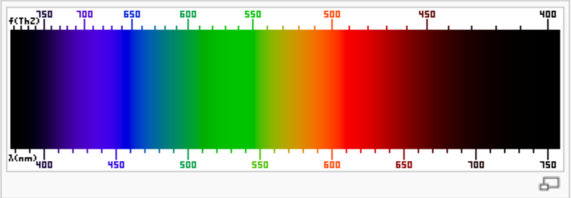


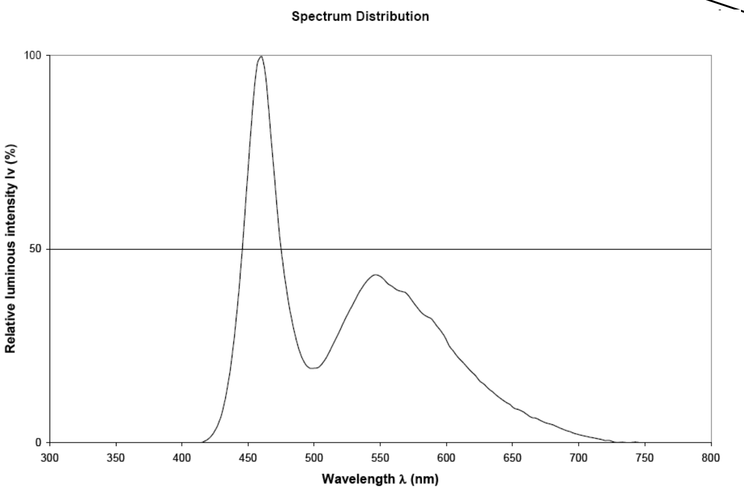
A white LED spews optical power across a range of wavelengths (mW/Hz)





Photopic (black) and scotopic [1] (green) luminosity functions. The photopic includes the CIE 1931 standard [2] (solid), the Judd-Vos 1978 modified data [3] (dashed), and the Sharpe, Stockman, Jagla & Jägle 2005 data [4] (dotted). The horizontal axis is wavelength in nm.





Units

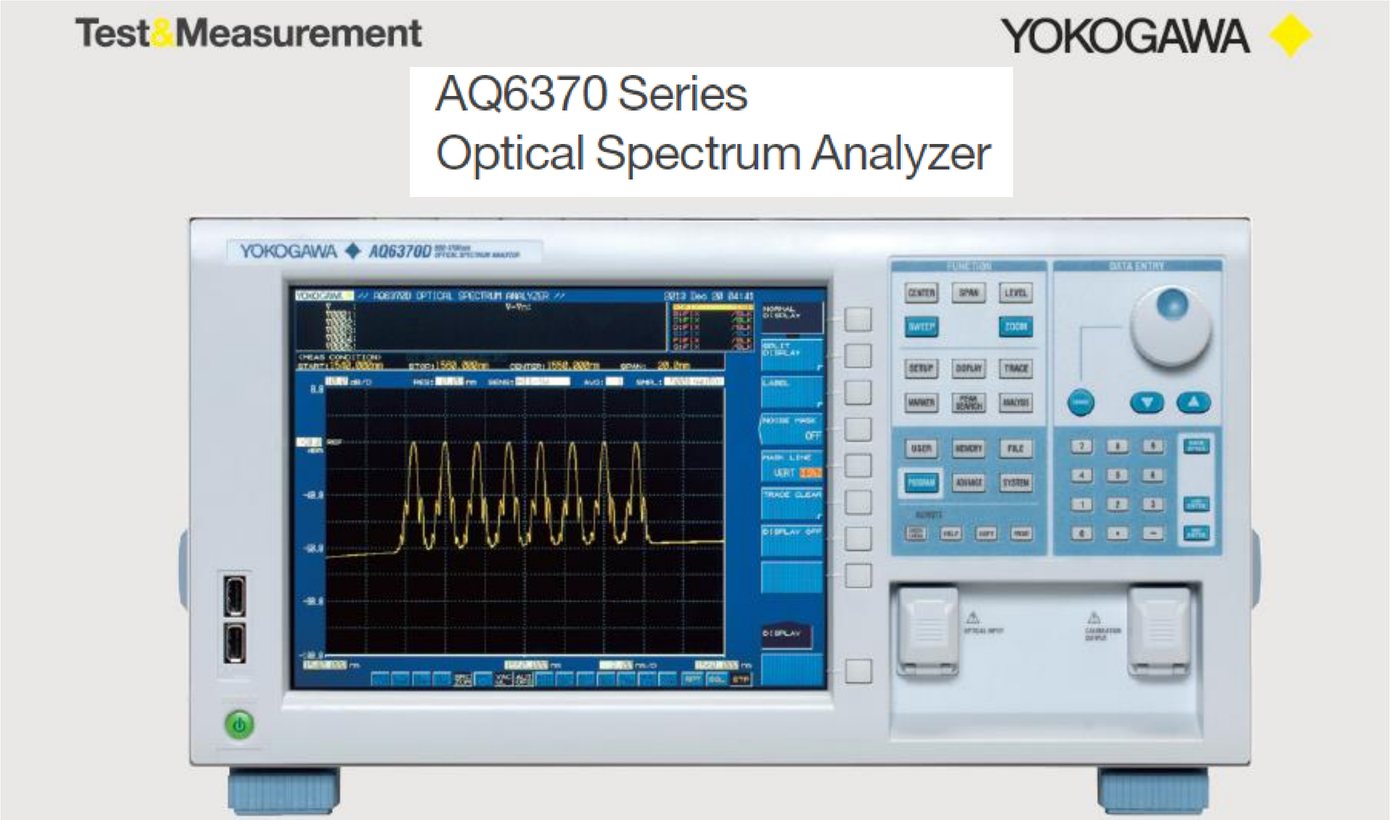
	Radiometric (Physical)	**Photometric (Visual)**
Total Flux	Watts (W)	lumens (lm)
Flux Density	W/cm^2	lm/cm^2
Source Intensity	W/sr	candela = lm/sr
Illuminance		Lux (lx) = lm/m^2
Irradiance	W/m^2	

For data link budgets we want to use **Radiometric units**

For illumination applications we want to use **Photometric units** (which include the frequency response of the human eye)

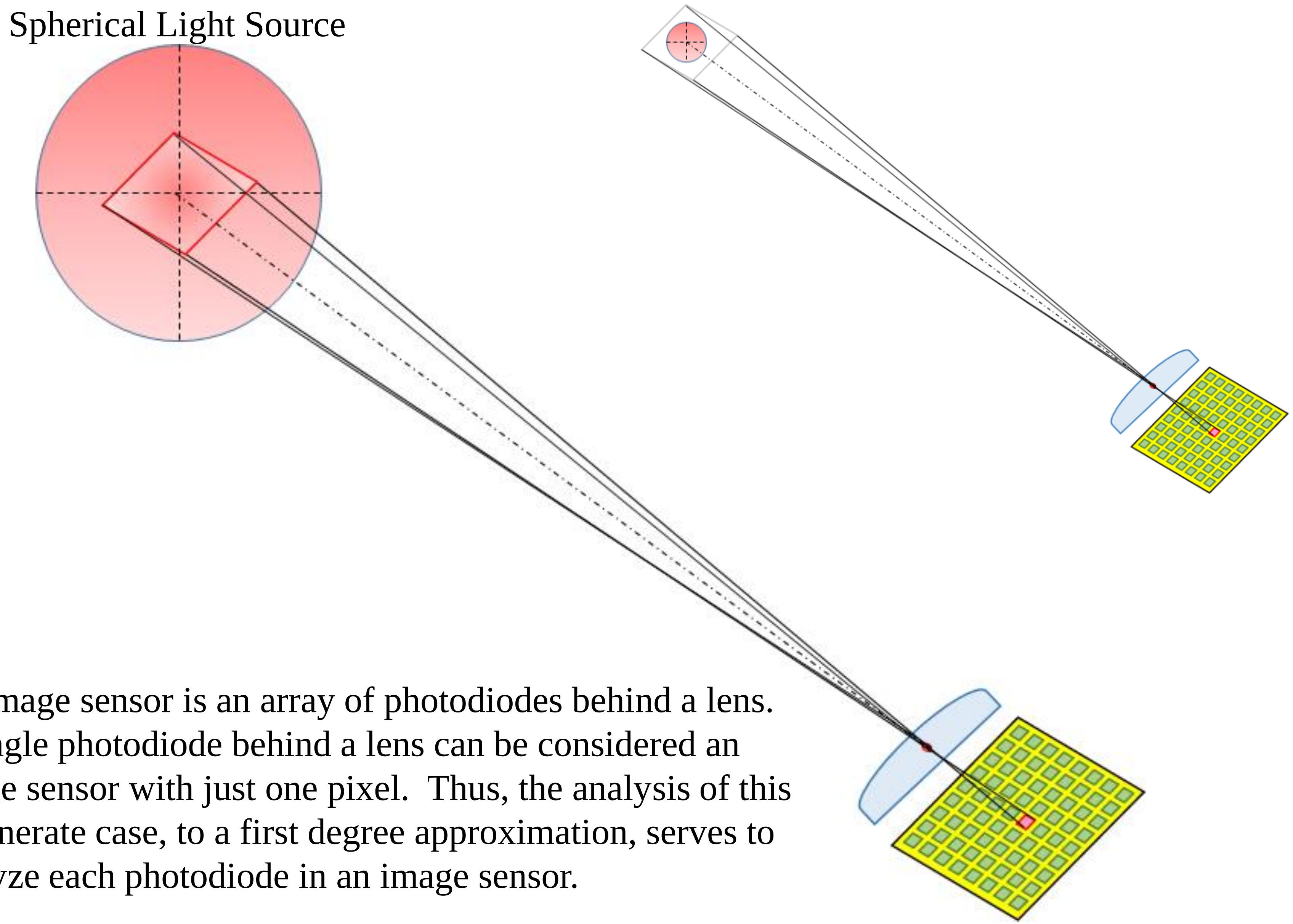
Most VIS LED vendors generally only provide Photometric data since illumination is the market today and the use of VIS LEDs for data is an obscure usage.

Often IR LED vendors will provide radiometric data for LEDs intended for communications. Appendix A discusses a method to convert photometric units to radiometric units, but it is not recommended by the author. It is felt that the better method is to obtain radiometric data by measurement.

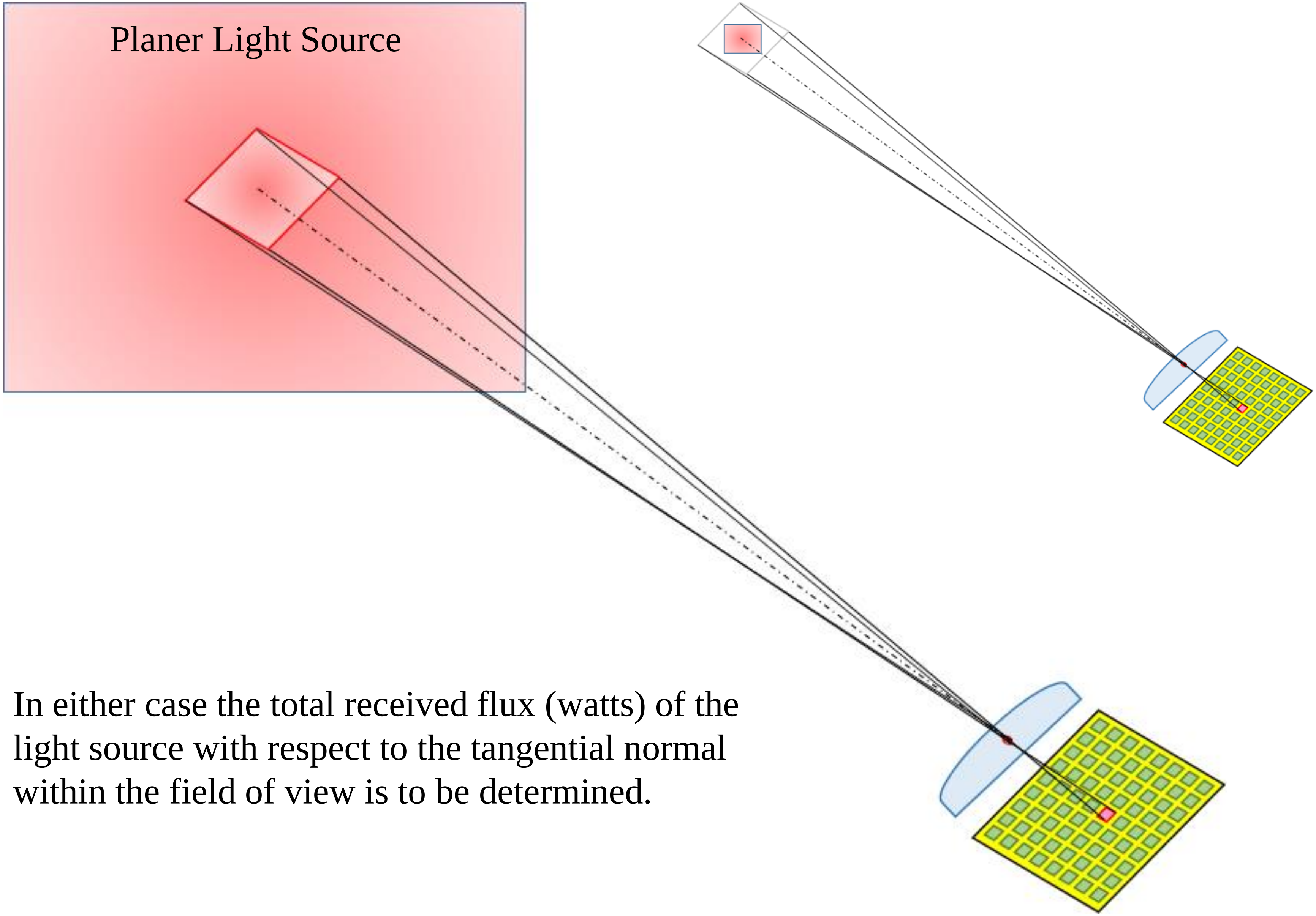


Best way to determine optical power and spectrum
… measure it using a known aperture sensor!

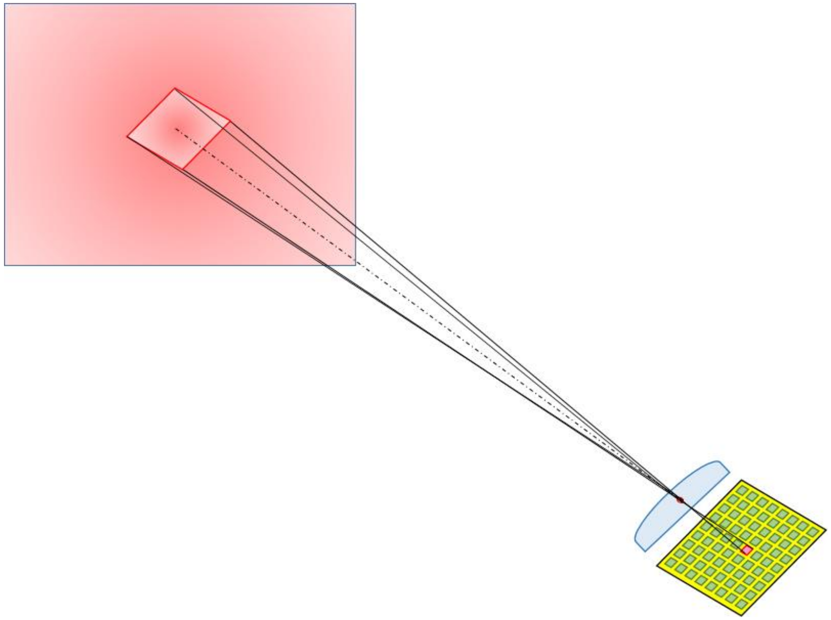
Path loss due to line-of-sight (LOS) light propagation

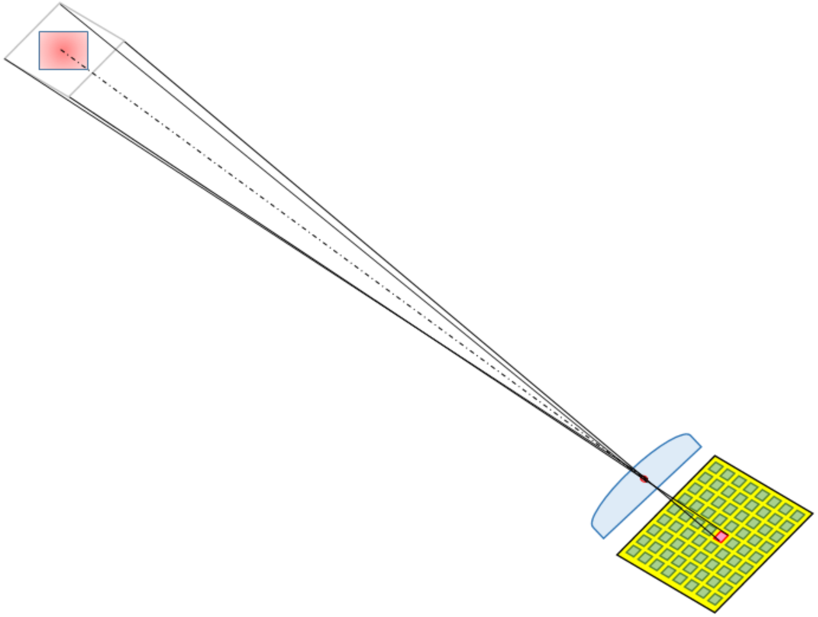


An image sensor is an array of photodiodes behind a lens. A single photodiode behind a lens can be considered an image sensor with just one pixel. Thus, the analysis of this degenerate case, to a first degree approximation, serves to analyze each photodiode in an image sensor.



In either case the total received flux (watts) of the light source with respect to the tangential normal within the field of view is to be determined.

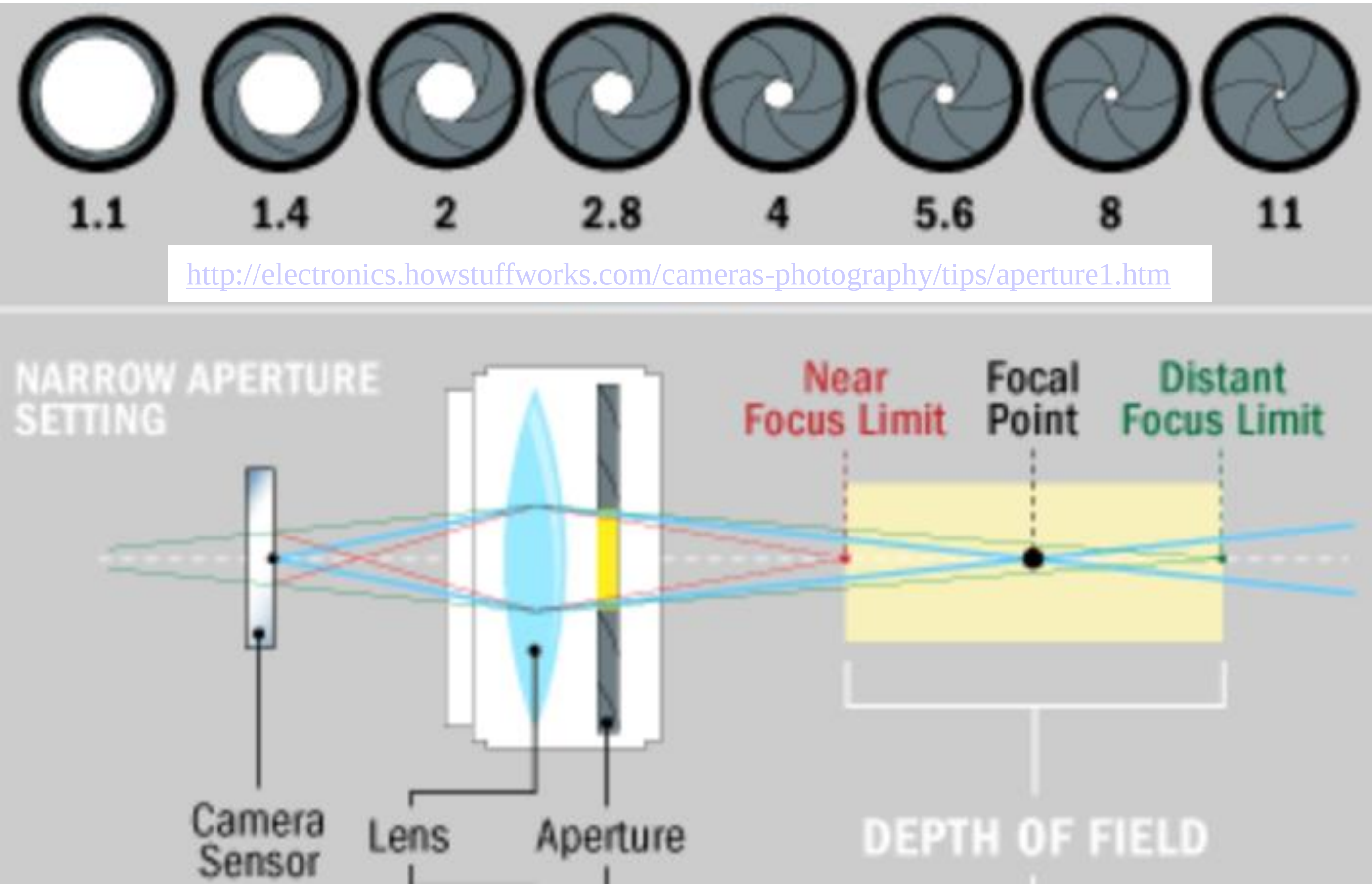




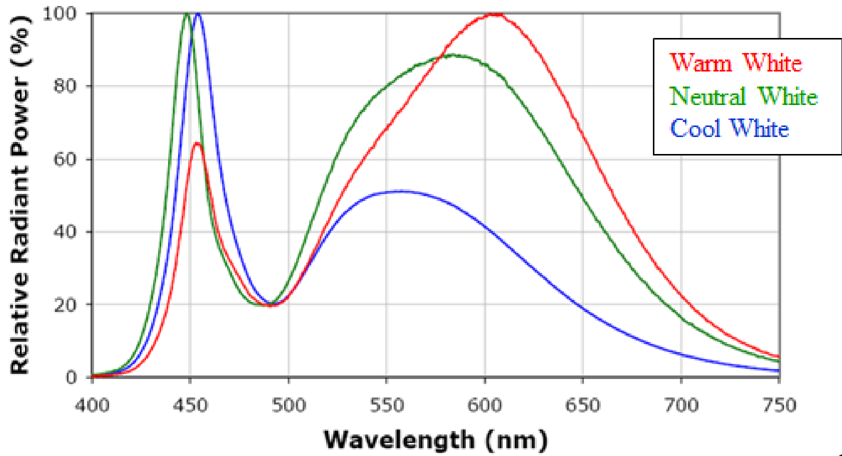
When determining the tangential normal transmitted spectral flux density (watts per unit area per wavelength $W/m^2 \cdot nm$), there are obviously two cases to consider: 1) area of the source exceeds the FOV; 2) FOV exceeds the area of the source.

Case 1: Use the average flex density within the FOV.

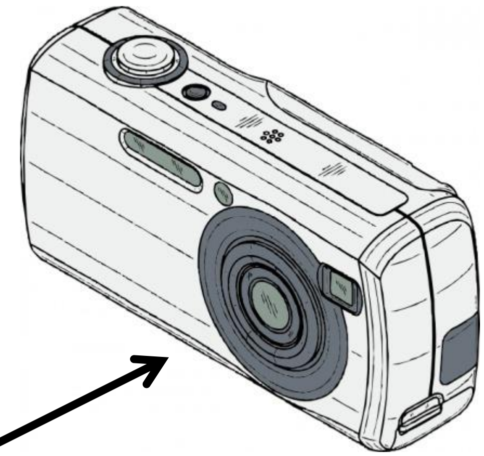
Case 2: Use the total available flex since the total source area is within the FOV.



The area of the aperture opening determines the amount of light entering the camera. The total flex entering the camera is the product of the aperture area and the irradiance flex density.



White

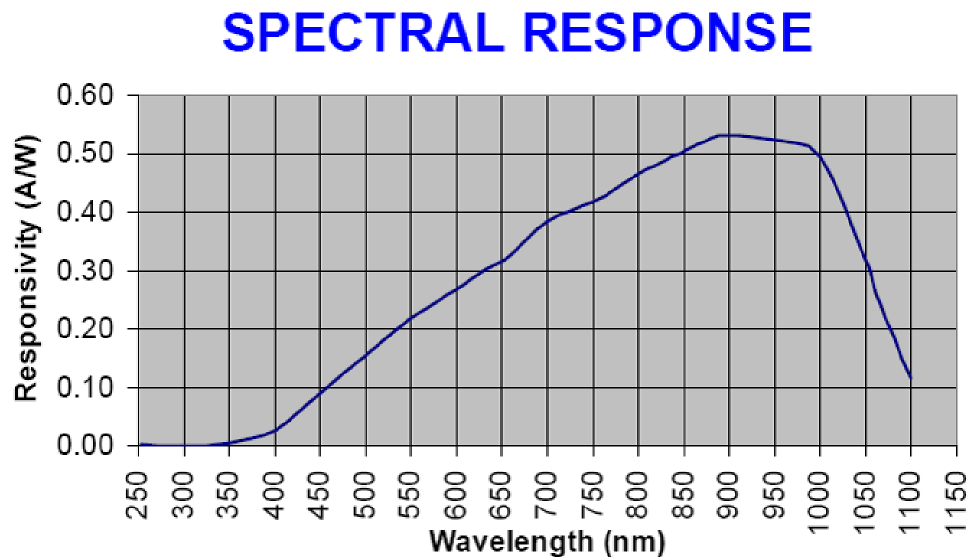


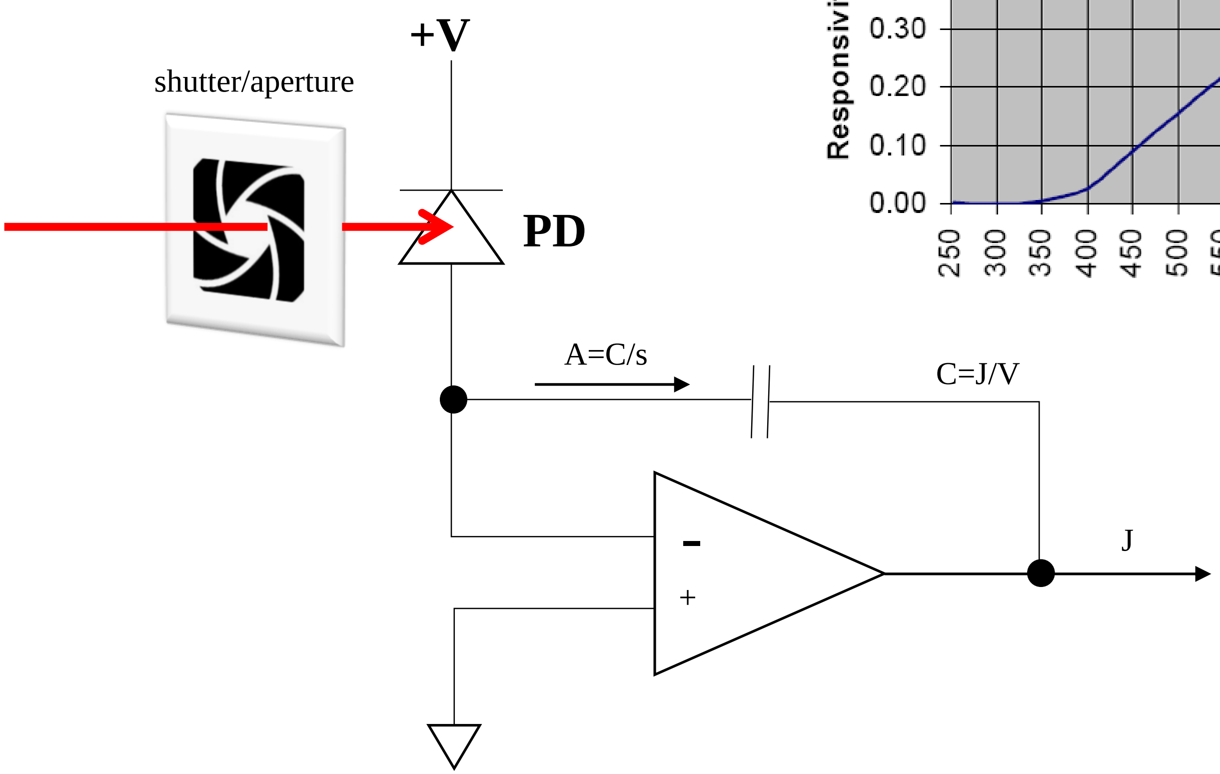
Received Spectral Irradiance $\frac{Watts}{m^2 \cdot nm}$

Transmitted Spectral Flux Density $\frac{Watts}{m^2 \cdot nm}$

Integration across the wavelengths of interest converts spectral flux density and spectral irradiance to spectral flux and irradiance $\frac{Watts}{m^2}$.

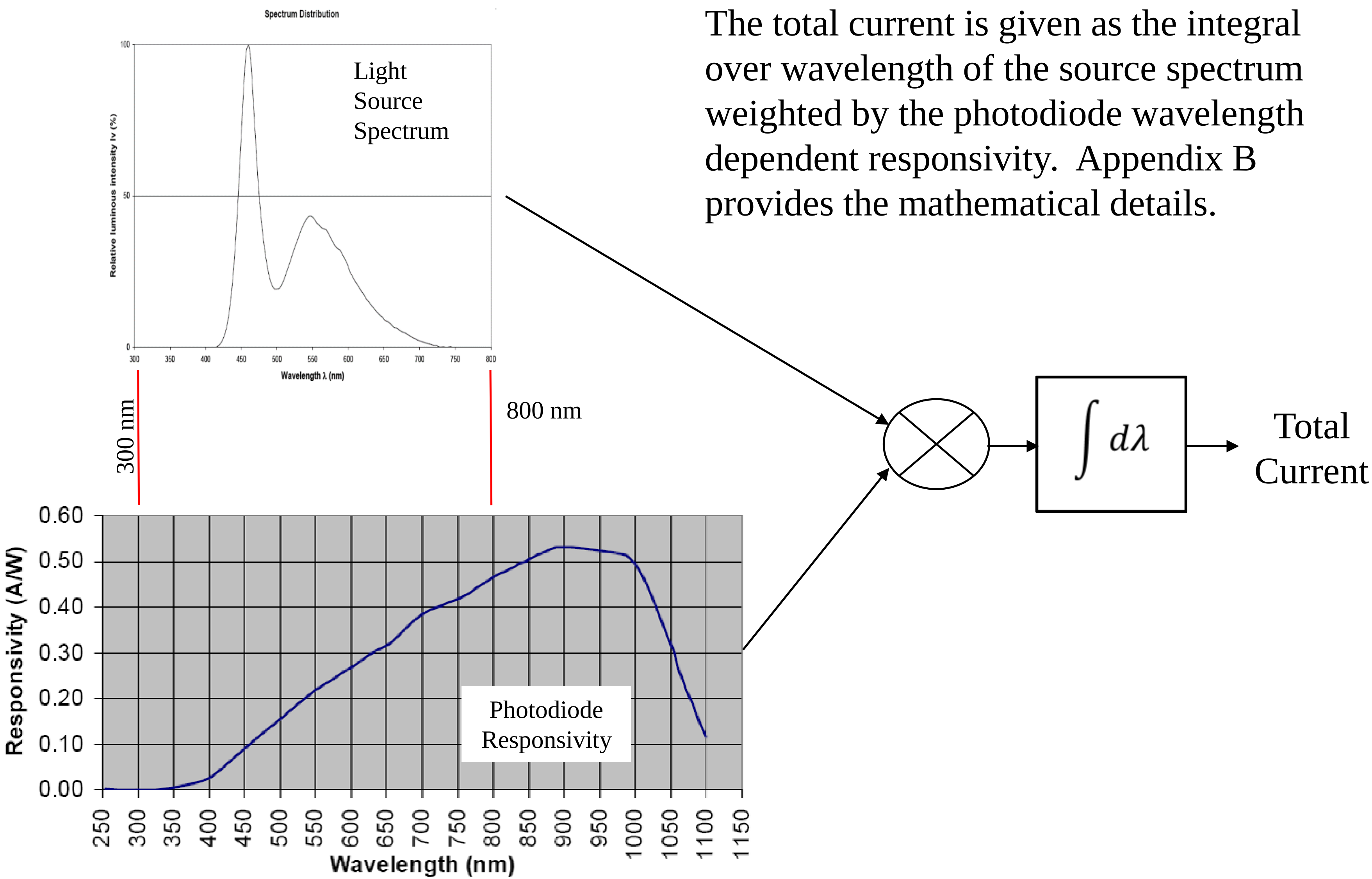
The detector diode vendors provide the spectral response $\frac{A}{W}$ information.





The detector diode current is integrated to provide the energy per bit in Joules.

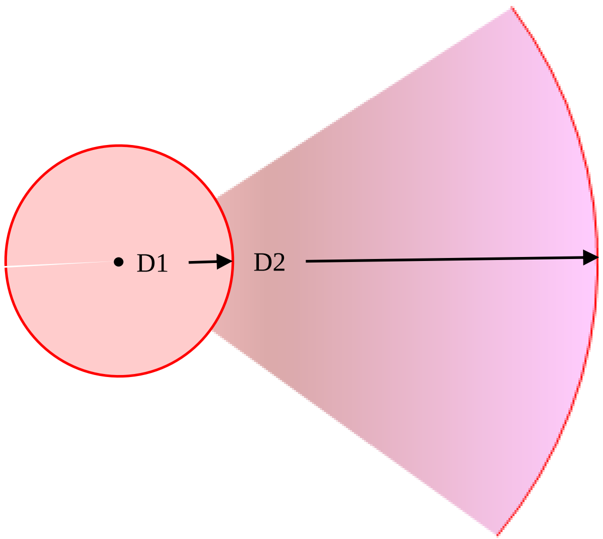
The total current is given as the integral over wavelength of the source spectrum weighted by the photodiode wavelength dependent responsivity. Appendix B provides the mathematical details.



Beam Divergence

Beam Divergence

Spherical Source



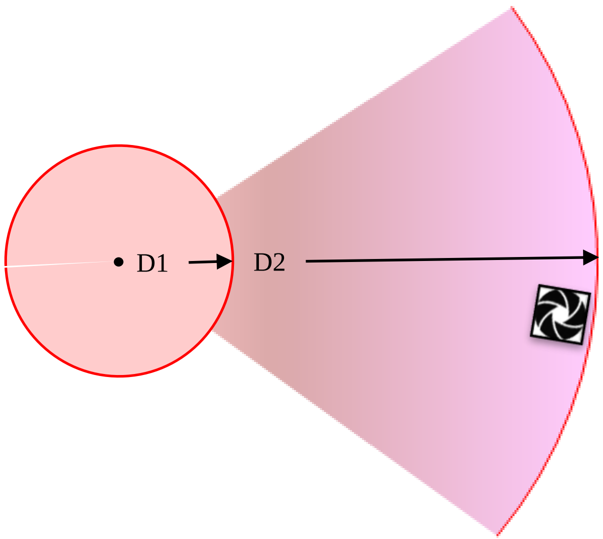
Surface power density is inversely proportional to the surface area increase: see slide 20 for details.

Planar Source

Preferably, the light intensity divergence should be supplied by the vendor. The alternative is to measure the divergence and then use the estimates presented on slide 22.

Camera aperture on diverged surface power density

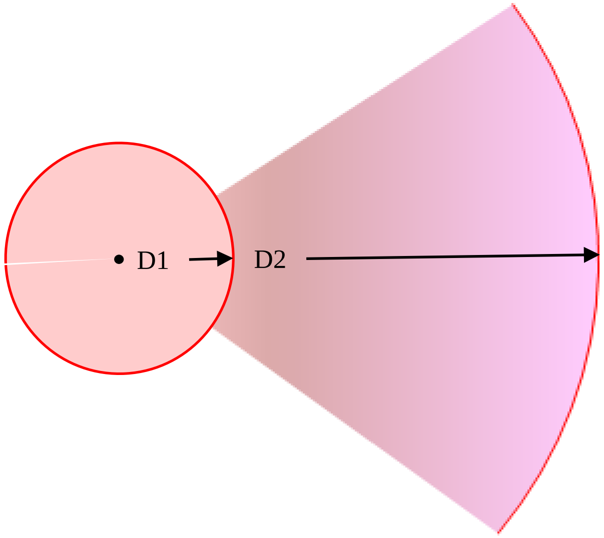
Spherical Source



Planar Source

The camera aperture is projected on to the increased surface area, of which the surface area has reduced power density.

Spherical Bulb Beam Divergence

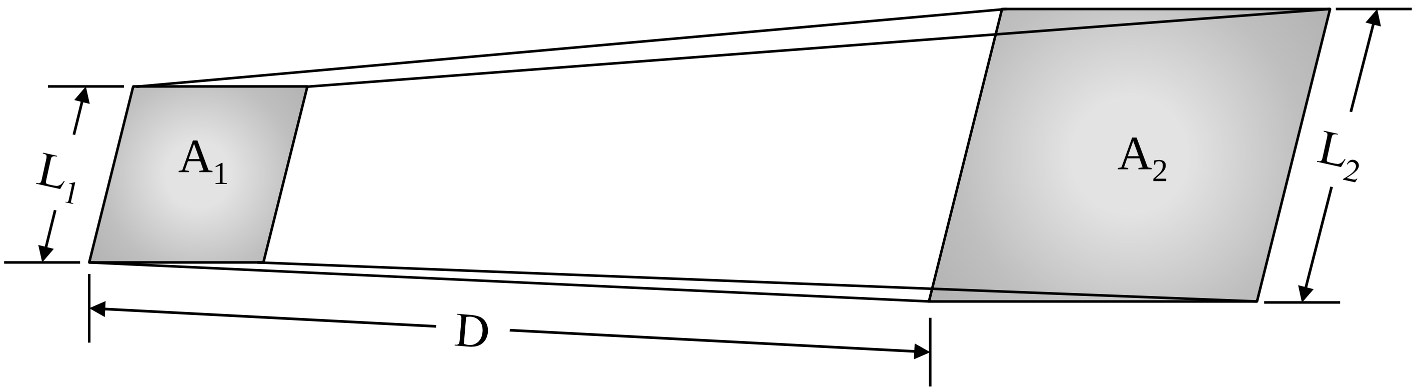


$$Dispersion\ Loss = 10 \cdot log_{10}\left(\frac{A_2}{A_1}\right) = 20 \cdot log_{10}\left(\frac{D_2}{D_1}\right)$$

For a spherical bulb, the dispersion loss is proportional to the increase in distance squared (similar to RF with an isotropic radiator).

For a spherical source, all illuminated pixels are orthonormal since they fall within the field of view.

Light Panel Beam Divergence



Divergence $\theta = 2\ \text{arctan}\left(\frac{L_2 - L_1}{2 \cdot D}\right)$

$$L_2 = 2 \cdot D \cdot tan\left(\frac{\theta}{2}\right) + L_1$$

Squaring both sides

$$A_2 = 4 \cdot D^2 \cdot tan^2\left(\frac{\theta}{2}\right) + 2 \cdot D \cdot L_1 \cdot tan\left(\frac{\theta}{2}\right) + A_1$$

$$\frac{A_2}{A_1} = \frac{4 \cdot D^2 \cdot tan^2\left(\frac{\theta}{2}\right)}{A_1} + \frac{2 \cdot D \cdot L_1 \cdot tan\left(\frac{\theta}{2}\right)}{A_1} + 1$$

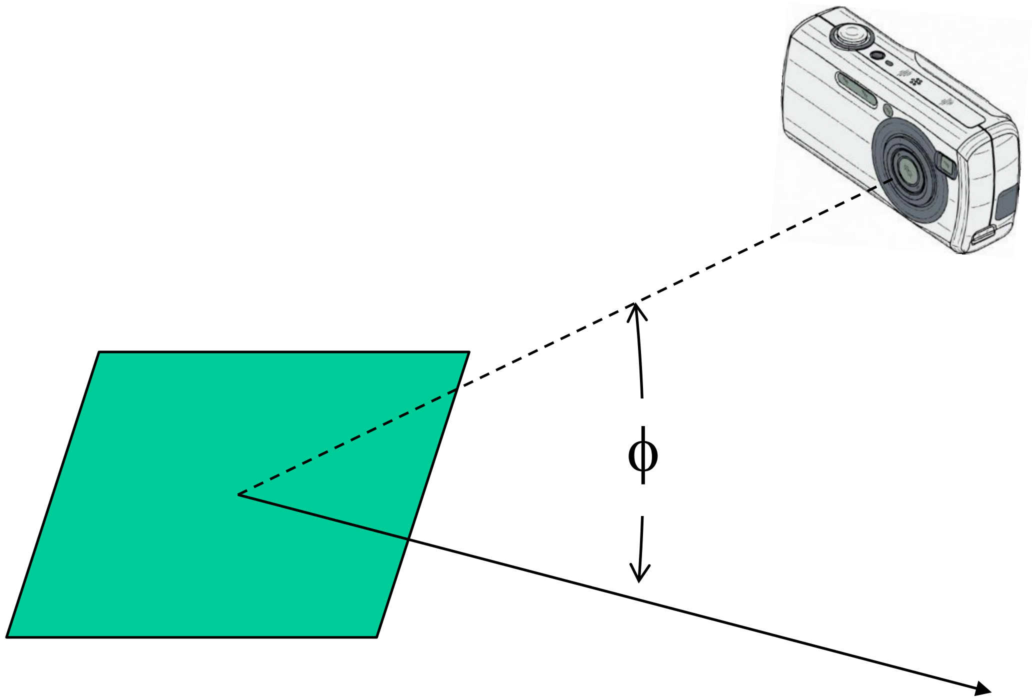
$$\frac{A_2}{A_1} \propto \frac{4 \cdot D^2 \cdot tan^2\left(\frac{\theta}{2}\right)}{A_1}$$

Observations on panel light dispersion:

1. dispersion is proportional to dispersion angle
2. dispersion increases as distance squared
3. dispersion is inversely proportional to the size of the panel
4. the area ratio ≥ 1.0

$$Dispersion\ Loss = 10 \cdot log_{10}\left(\frac{A_2}{A_1}\right)$$

The light panel source case can also experience additional loss due to camera viewing angle.



A precise analysis would require vendor data on the light panel angular radiation. Lacking such data, an approximation can be made as

$$\text{Angular Loss} \approx 10 \cdot log_{10}\{cos(\emptyset)\}.$$

Atmospheric Attenuation Due to Fog

From the paper by Kim, et. al. …

$$\Lambda\left({}^{dB}/_{km}\right) = 10 \cdot log_{10}\left(e^{\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\ nm}\right]^{-q}\right\}}\right)$$

where V = visibility in km

λ = wavelength in nm

q = the size distribution of the scattering particles

= 1.6 for high visibility (V > 50 km)

= 1.3 for average visibility (6 km < V < 50 km)

= 0.16 V + 0.34 for haze visibility (1 km < V < 6 km)

= V – 0.5 for mist visibility (0.5 km < V < 1 km)

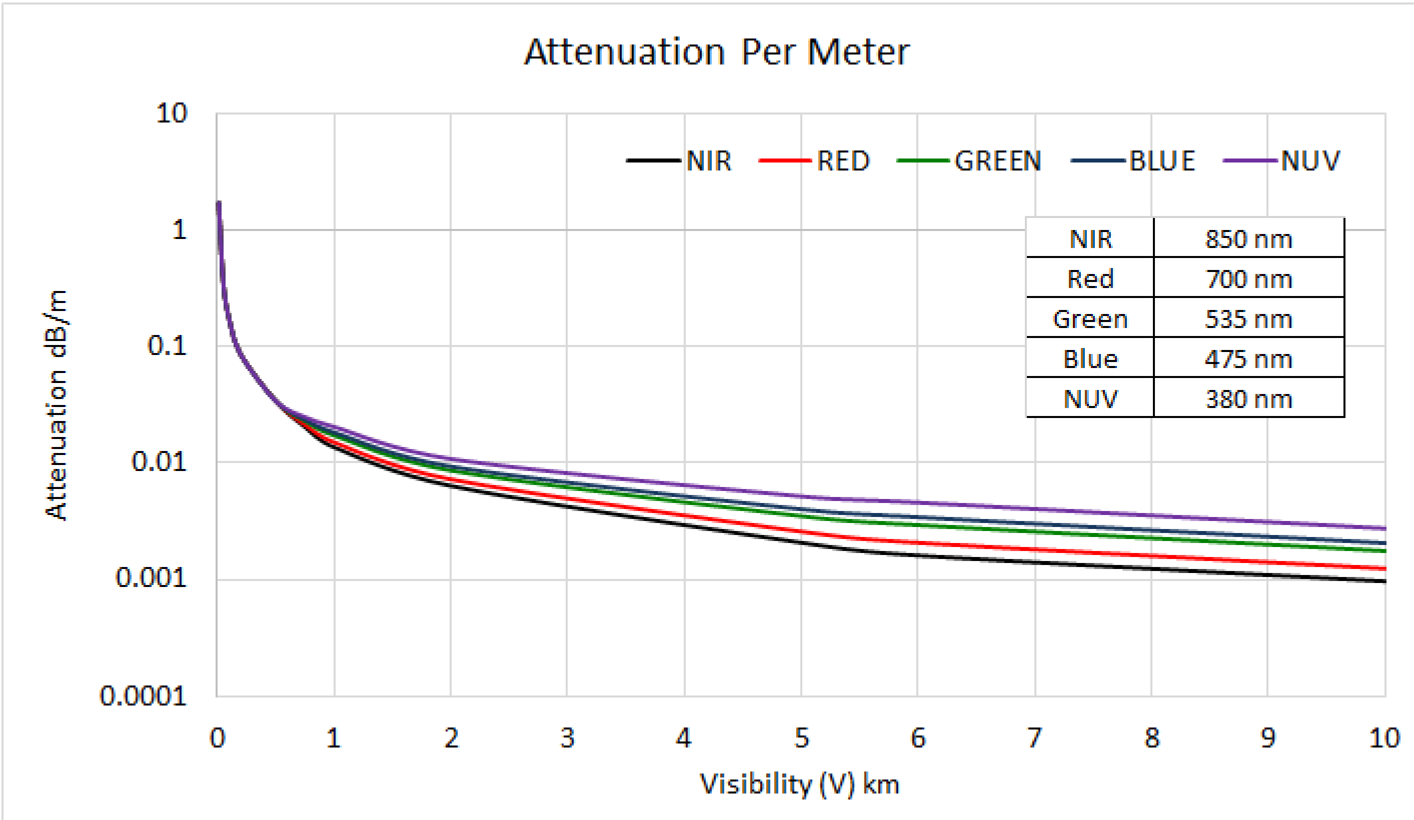
= 0 for fog visibility (V < 0.5 km)

For the most part, the distances of interest are much less than a kilometer so we can express this on a per meter basis as

$$\Lambda\left({}^{dB}/_{m}\right) = 0.01 \cdot log_{10}\left(e^{\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\ nm}\right]^{-q}\right\}}\right)$$

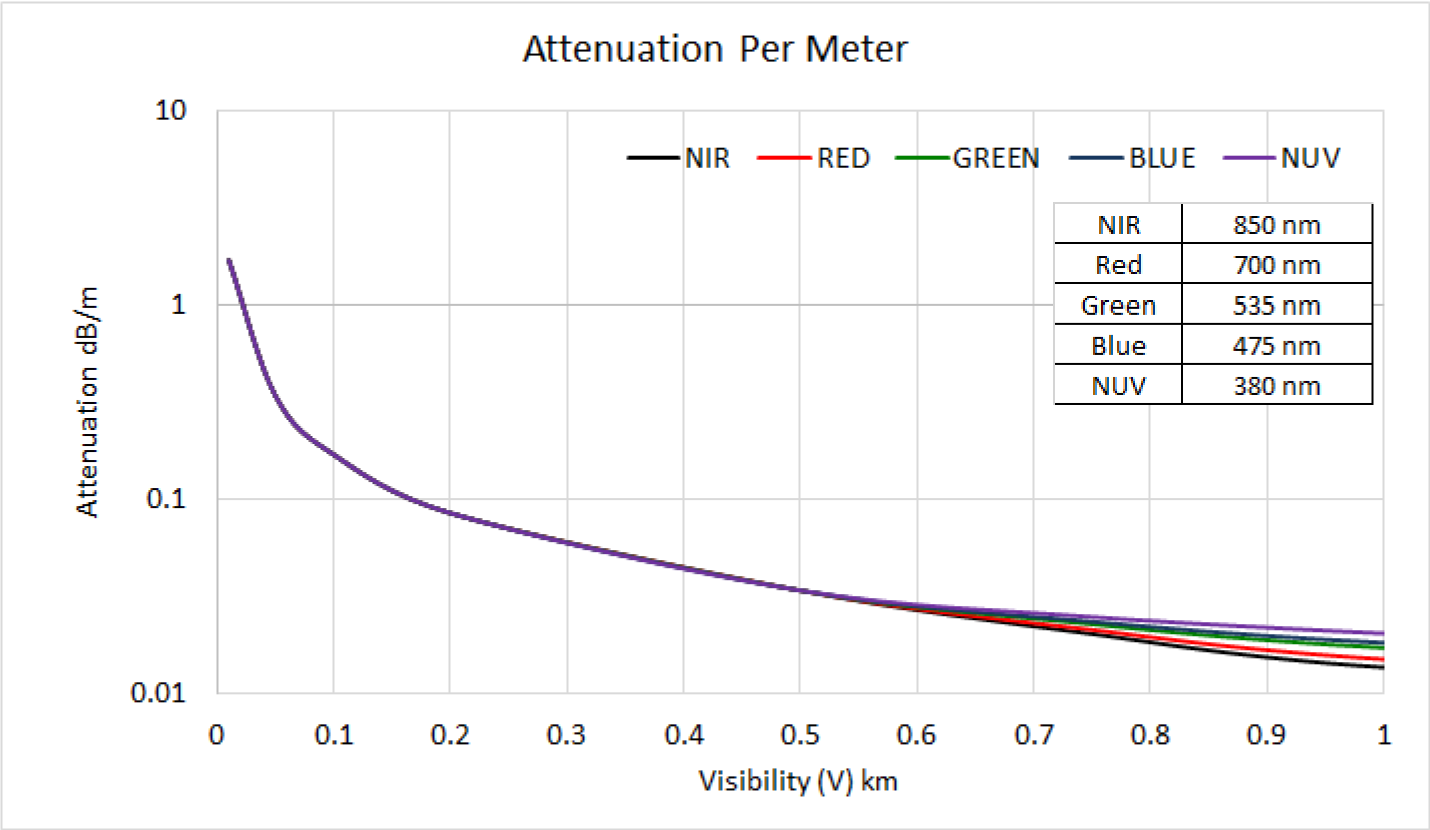
Comparison of laser beam propagation at 785 nm and 1550 nm in fog and haze for optical wireless communications; Isaac I. Kim, Bruce McArthur, and Eric Korevaar; www.ece.mcmaster.ca/~hranilovic/woc/resources/local/spie2000b.pdf

Fog and Haze Attenuation by Wavelength



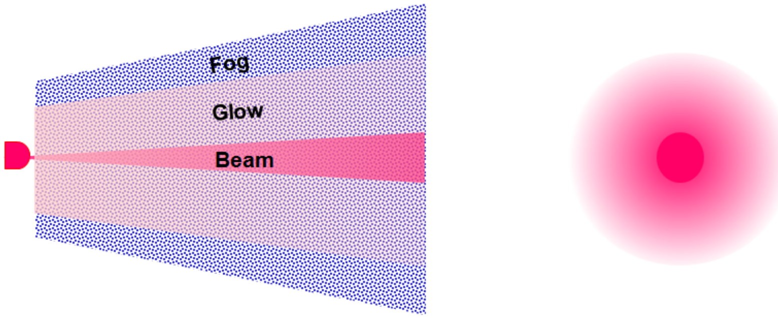
fog **haze** **clear**

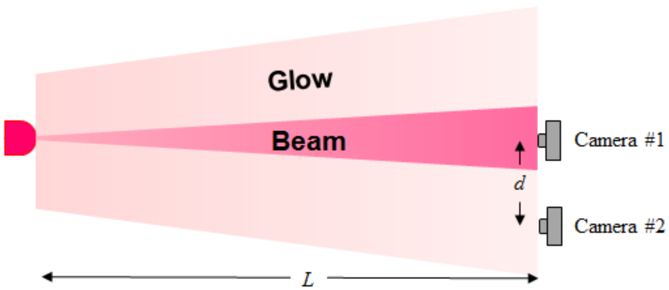
Fog and Haze Attenuation by Wavelength

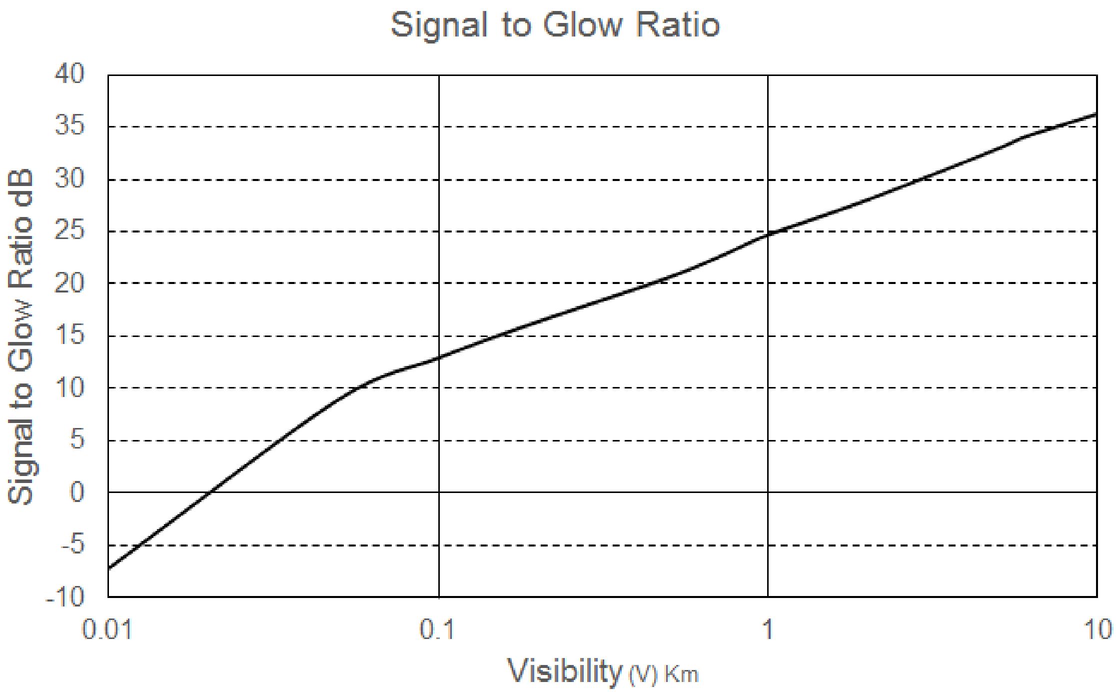


heavy fog **light fog**

Fog causes light scattering in all directions causing a "glow" about the main beam. Appendix F provides an estimate of the impact of the fog diffusion glow in regards to multi-camera operation.







Example assumptions …

- Wavelength: 850 nm
- Beam Radius: 1m
- Off beam distance "*d*": 3m
- Standoff length "*L*": 10m

Propagation Path Loss

$$\text{Propagation Loss (dB)} = \text{Dispersion Loss (dB)} + \text{Angular Loss (dB)} + \Lambda\text{(dB/m)} \cdot \text{distance (m)}$$

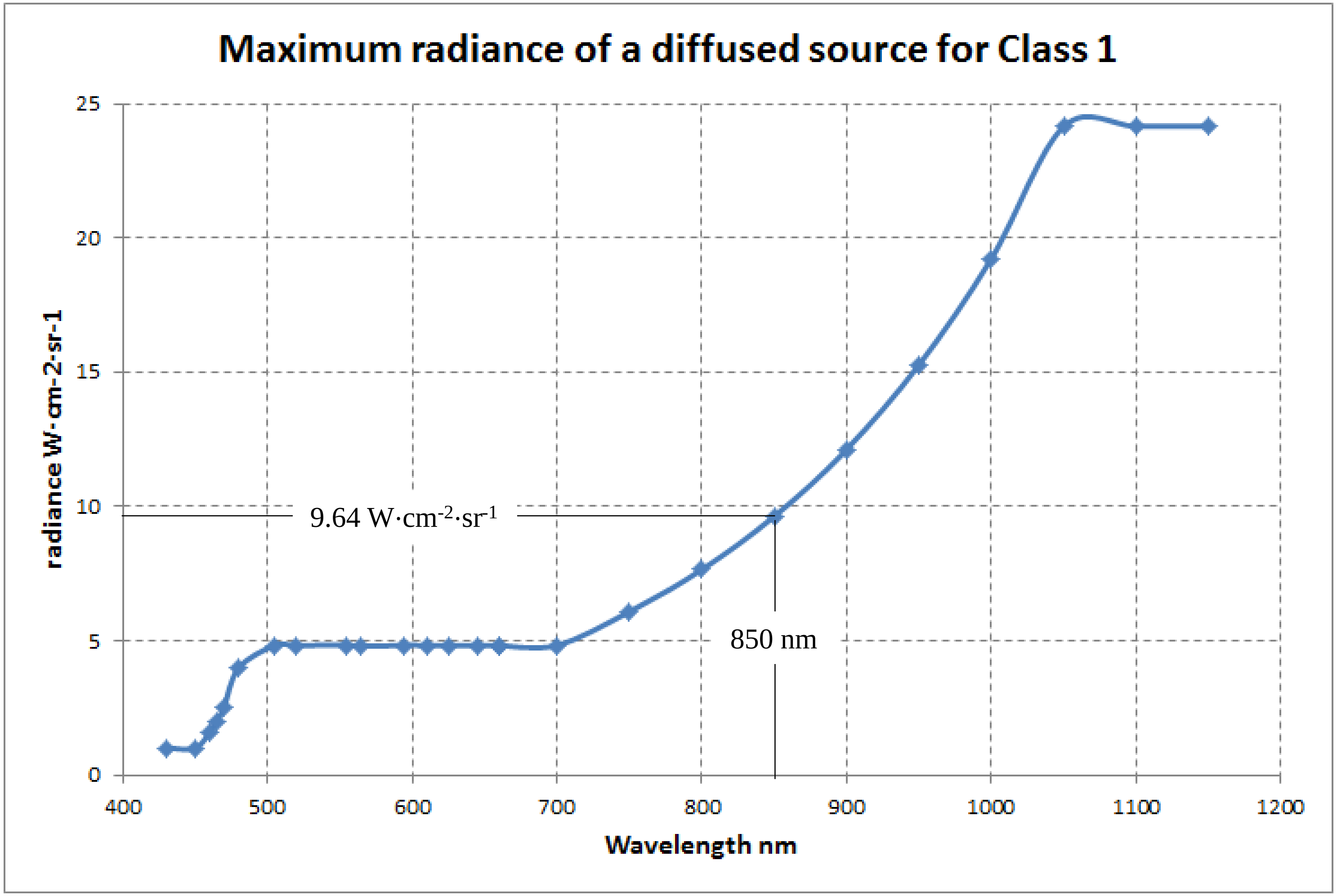
$$\text{Ingested Flex (W)} = \text{Source Flex Density} \cdot \text{Propagation Loss (ratio)}$$

The signal-to-noise ratio (SNR) is defined as the ratio of the ingested flex to the receiver noise. The receiver noise can be either calculated or measured. Given real hardware, it is probably easier to measure the noise than calculate it since such calculations would require extensive knowledge of the receiver structure. Nevertheless, appendix C outlines a method of doing the calculations.

It should be noted that when calculating the propagation loss, the magnification factor can cause a lower spatial power density at the image sensor (approximated in appendix E) that is inversely proportional to the square of the magnification factor. The impact is application and modem scheme dependent and needs to be evaluated on a per case basis.

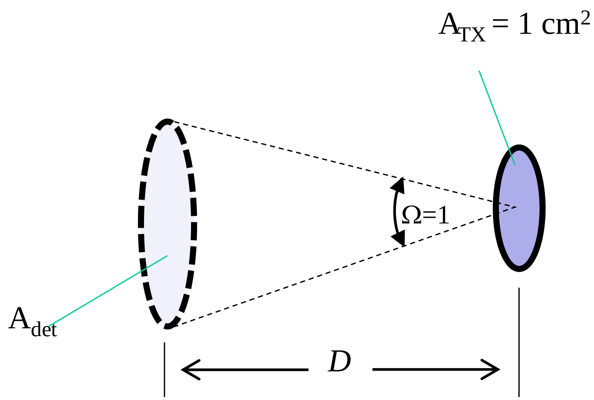
850 nm NIR specific analysis

IEC60825 NIR Safety Limits



Example of Class 1 laser device – e.g. laser pointer	*Note: 9.64 W·cm^{-2}·sr^{-1} is bright … but SUN radiance is 2.4 kW·cm^{-2}·sr^{-1}*

Interpretation of IEC60825 NIR Safety Limits



Interpretation for a A_{TX} = 1 cm²

$$\Omega_{\det,TX} = \frac{A_{\det}}{D^2} = 1 \quad D = \sqrt{A_{\det}}$$

See appendix D for more discussion on source intensity using solid angles.

To a first order approximation …
… how much power is ingested by the detector?

$$P_{\det} = A_{TX}\, cm^2 \cdot 9.6 \frac{W}{cm^2 \cdot sr} \cdot \Omega_{\det,TX}\, sr = (9.6 \cdot A_{TX}) \cdot \left(\frac{A_{\det}}{D^2}\right) W$$

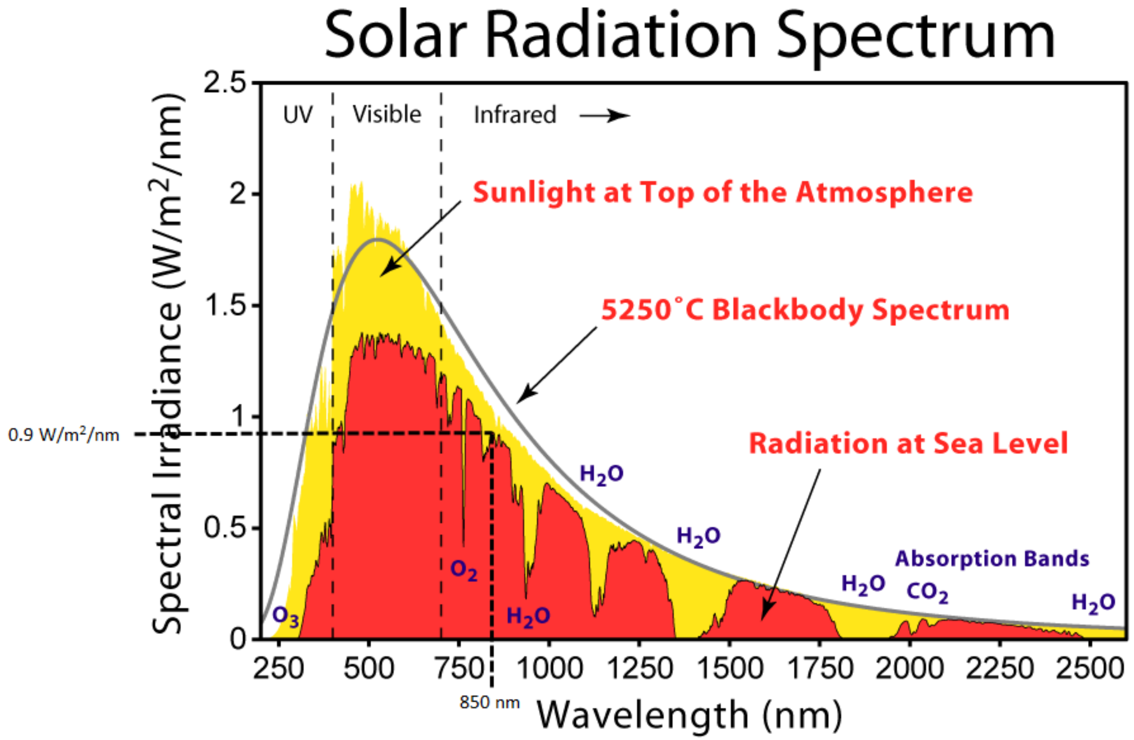
$$P_{TX} = (9.6 \cdot A_{TX}) W \qquad L_{channel} = \frac{A_{\det}}{D^2}$$

Receiver Noise

Receiver noise floor is a mixture of thermal noise and shot noise.

$$S_{total}(f) = S_{shot}(f) + S_{thermal}(f)$$

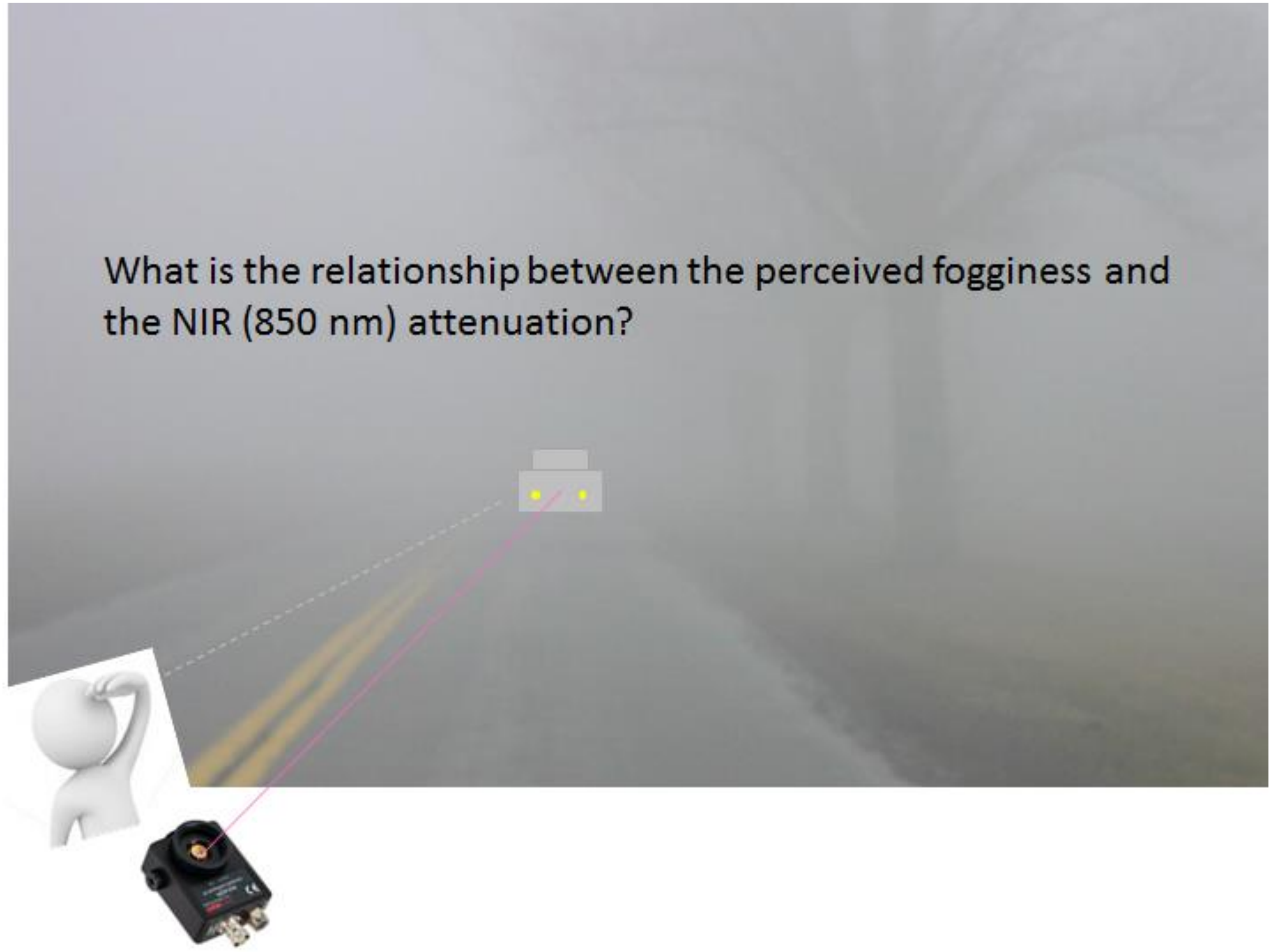
The biggest contributor to shot noise is the ambient solar spectral irradiance on a bright sunny day.

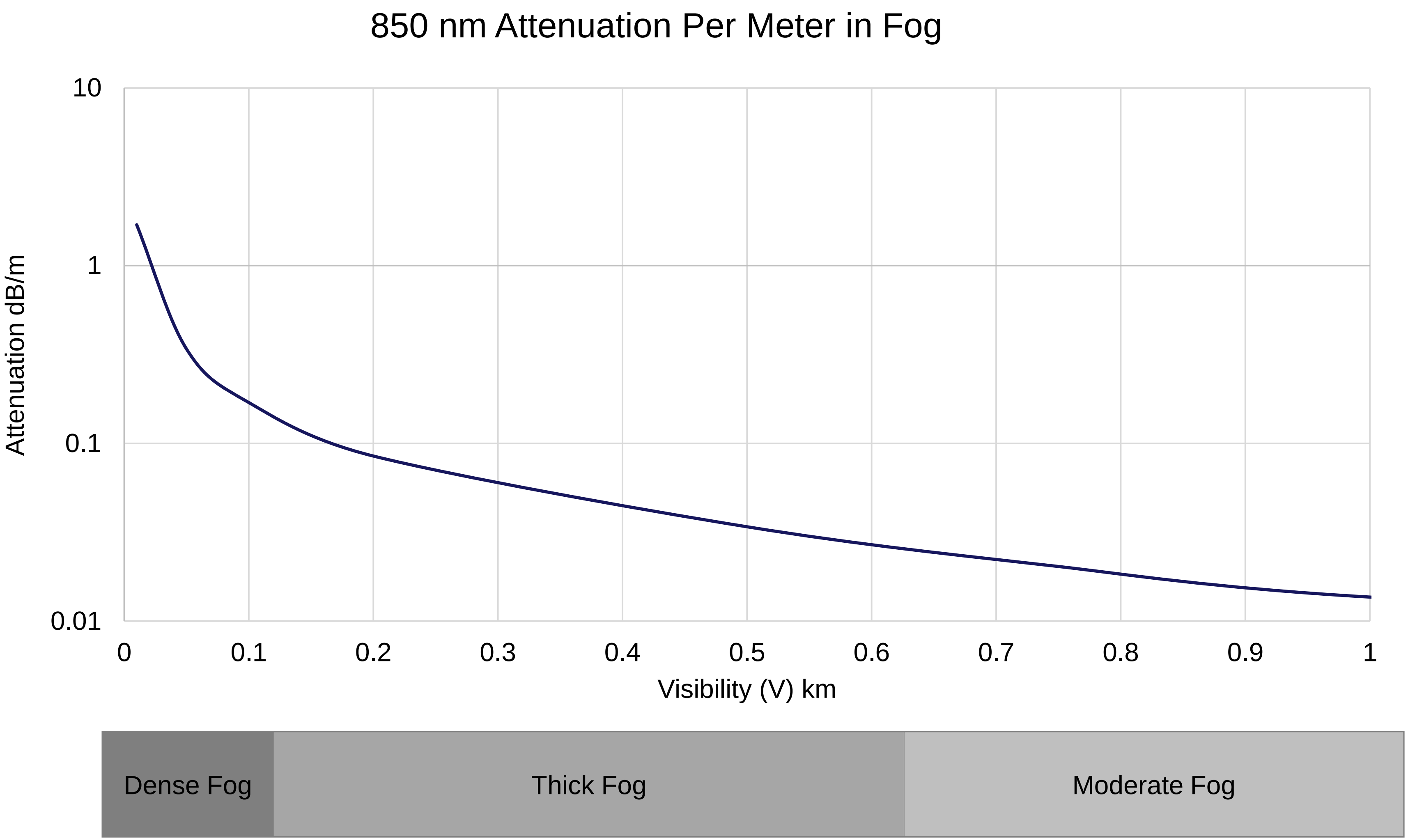


Sun spectral irradiance at 850 nm (sea level): 90 uW/cm²/nm

Short noise is caused by the random arrival time of photons from a light source.

Impact of Fog Attenuation





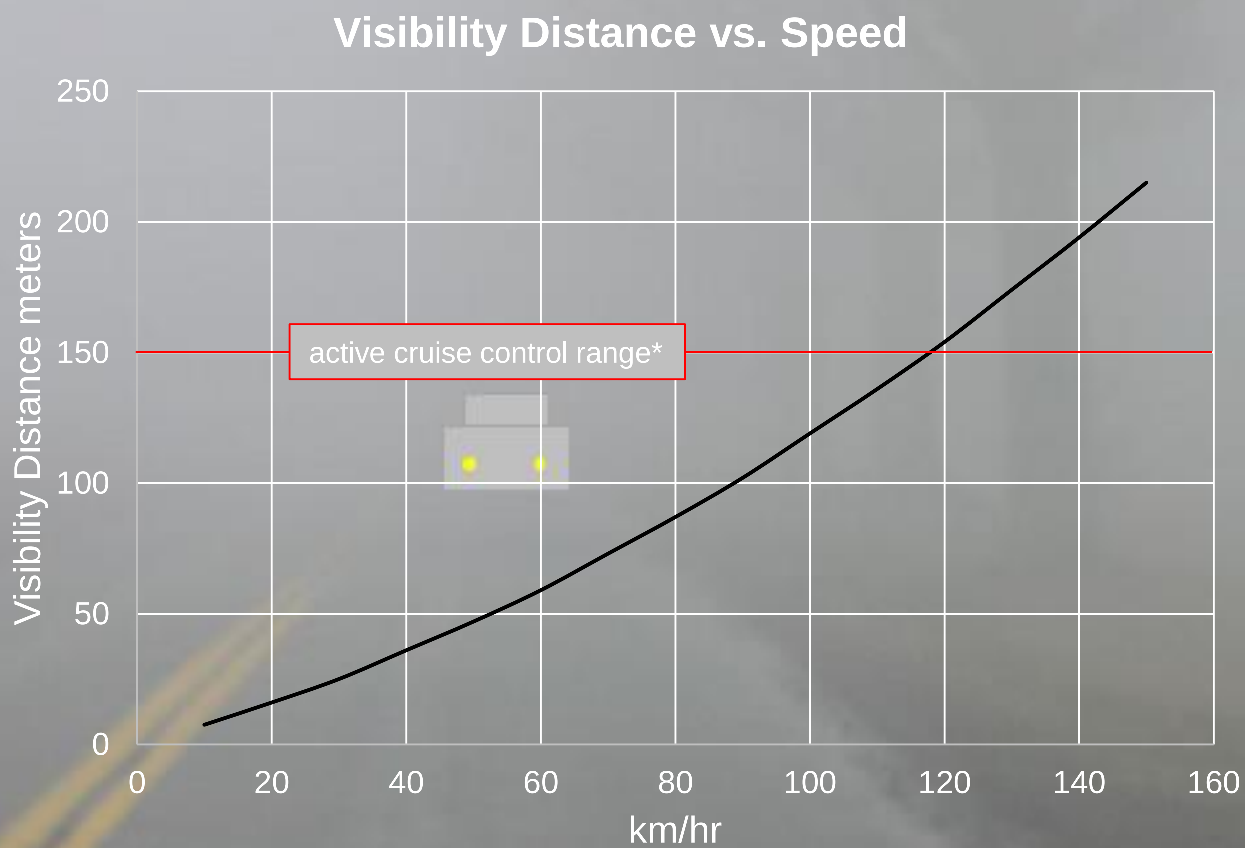
$$L_{total}\Big|_{dB} = L_{channel}\Big|_{dB} + L_{fog}\Big|_{dB} = -10\log\left(\frac{A_{\det}}{D^2}\right) + \alpha_m \cdot D$$

Reaction Time and Stopping Distance Implications

http://easycalculation.com/engineering/civil/vehicle-stopping-distance.php

Assumptions:

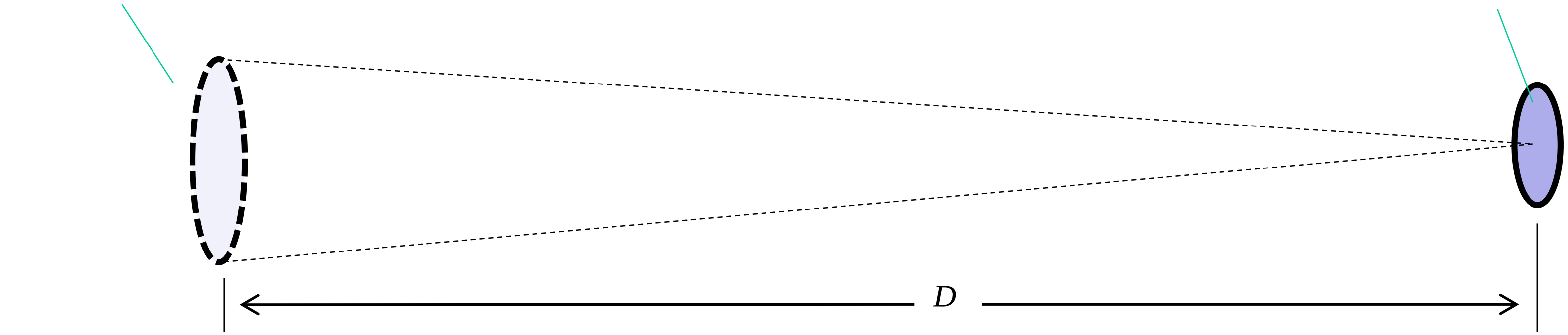
- **road coefficient of friction: 0.8**
- **reaction time: 2.5 secs (typical)**
- **road grade: level**



Automotive Industry Dilemma: what to do when the sensor technology out performs the human eye? Do we encourage blind driving?

*http://www.bmw.com/com/en/insights/technology/technology_guide/articles/active_cruise_control_stop_go.html

Example Link Analysis



A detailed 850 nm link budget will be presented as part of the Intel proposal to IEEE802.15.7r1.

Appendices

Appendix A

Ascertaining the LED parameters of interest

On the following pages, equation (2.2.1) and Figure 2.1.1 are from the book *Introduction to Solid-State Lighting* by A. Zukauskas, et.al. The equation relates the power spectral distribution $S(\lambda)$ (W/nm) to luminous flux Φ_v (lm).

Find transmitted power and spectral density

The LED total luminous flux F_t (lumens) is given as $F_t = 683 \int_{380nm}^{780nm} S_t(\lambda) V(\lambda) d\lambda$ (2.2.1)

V(λ) is the relative luminous efficiency function defined by CIE and given in the table (from internet) and curve (from the book Fig 2.1.1)

Wavelength (nm)	Photoptic Luminous Efficiency V(λ)	Wavelength (nm)	Photoptic Luminous Efficiency V(λ)
380	0.00004	580	0.870
390	0.00012	590	0.757
400	0.0004	600	0.361
410	0.0012	610	0.503
420	0.0040	620	0.381
430	0.0116	630	0.265
440	0.023	640	0.175
450	0.038	650	0.107
460	0.060	660	0.061
470	0.091	670	0.032
480	0.139	680	0.017
490	0.208	690	0.0082
500	0.323	700	0.0041
510	0.503	710	0.0021
520	0.710	720	0.00105
530	0.862	730	0.00052
540	0.954	740	0.00025
550	0.995	750	0.00012
560	0.995	770	0.00003
570	0.952		

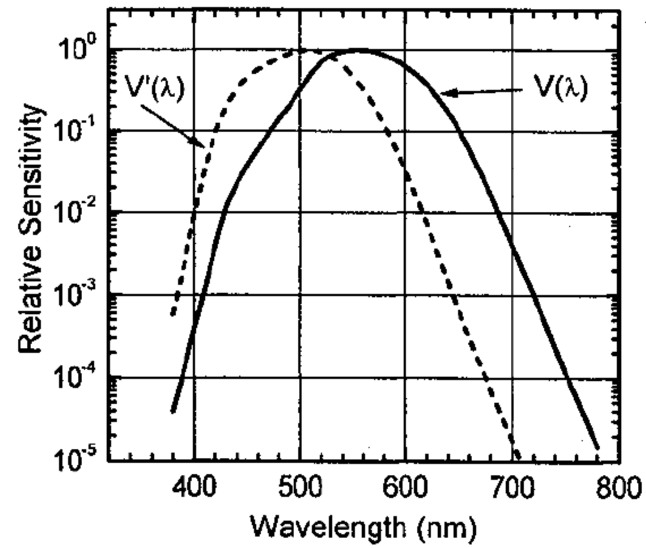


Fig 2.1.1

Sometimes it is convenient to use a Gaussian curve fitting for *V(λ)* (from internet)

$$V(\lambda) \cong 1.019 e^{-285.4(\lambda - 0.559)^2}, \quad \lambda : \text{in } \mu m$$

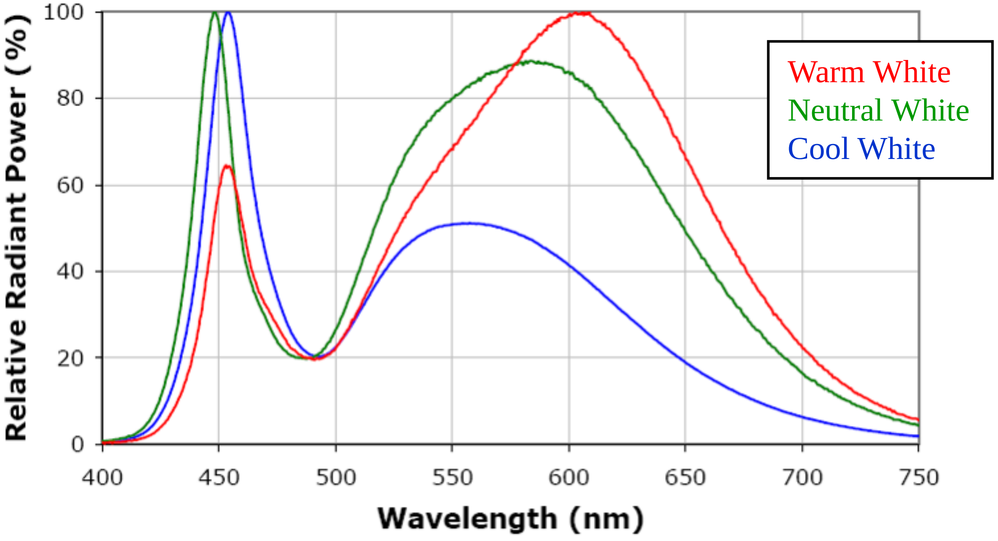
Typically we only know a normalized spectral curve $S_t'(\lambda)$ instead of $S_t(\lambda)$ in (2.2.1). Denote their relation as $S_t(\lambda)=c_t\, S_t'(\lambda)$ with an unknown scaling factor c_t that can be found from

$$F_t = c_t \cdot 683 \int_{380nm}^{780nm} S_t^{'}(\lambda)V(\lambda)d\lambda \longrightarrow c_t = \frac{F_t}{683 \int_{380nm}^{780nm} S_t^{'}(\lambda)V(\lambda)d\lambda}$$

$$S_t(\lambda) = c_t\ S_t^{'}(\lambda)$$

$$P_t = \int_{\lambda_L}^{\lambda_H} S_t(\lambda)d\lambda$$

Remark:
The above step to find $S_t(\lambda)$ can be skipped if $S_t(\lambda)$ can be either measured using a spectrometer or supplied by the LED vendor.



White

Find transmitter luminous spatial intensity distribution $I_0 g_t(\theta)$

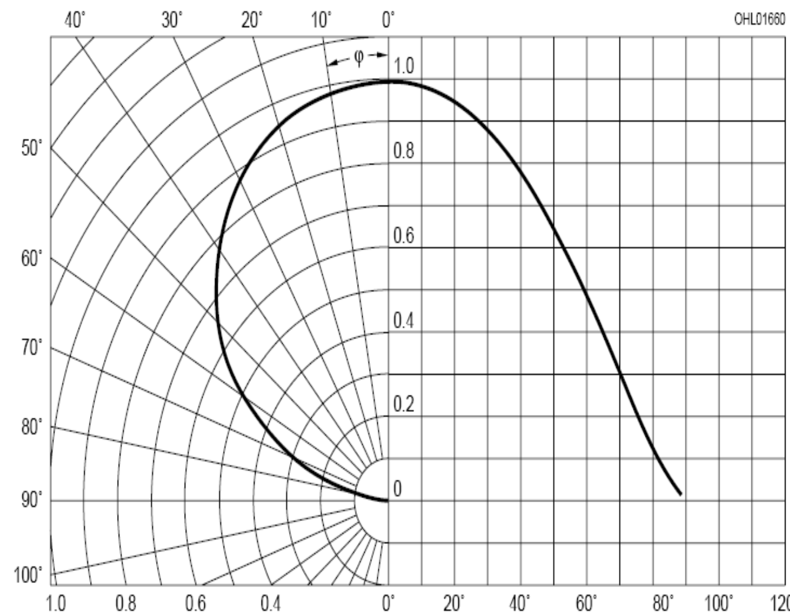
A normalized spatial luminous intensity distribution $g_t(\theta)$ is provided by a vendor. We need to find the axial intensity I_0 that is defined as the luminous intensity (candelas) on the axis of the source (zero solid angle). Since the luminous flux F_t is also a spatial integral of spatial luminous intensity in addition to spectral integral we used before, we have the following relation

$$F_t = \int_0^{\Omega_{\max}} I_0 * g_t(\theta) d\Omega = I_0 \int_0^{\theta_{\max}} 2\pi g_t(\theta) \sin\theta d\theta$$

$$\longrightarrow \quad I_0 = \frac{F_t}{\int_0^{\theta_{\max}} 2\pi g_t(\theta) \sin\theta d\theta}$$

where $\Omega_{\max}$ and $\theta_{\max}$ are the source beam solid angle and maximum half angle respectively and $\Omega_{\max} = 2\pi(1-\cos\theta_{\max})$.

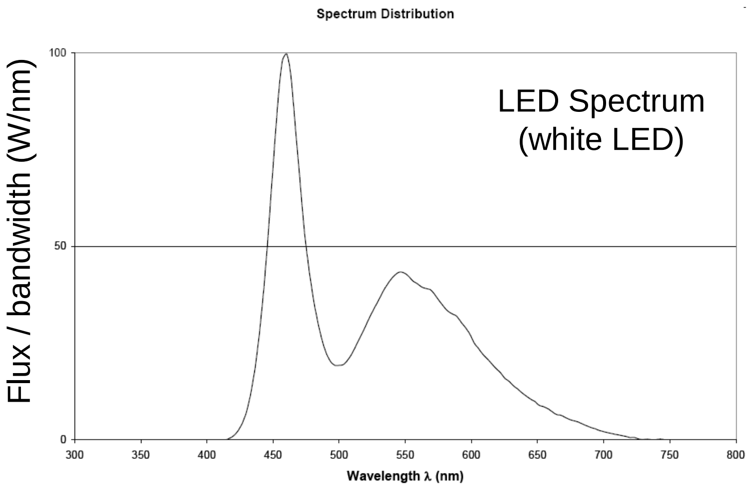
Note: if the axial intensity is provided by the vendor then one only need convert the intensity from candelas to watts/sr.



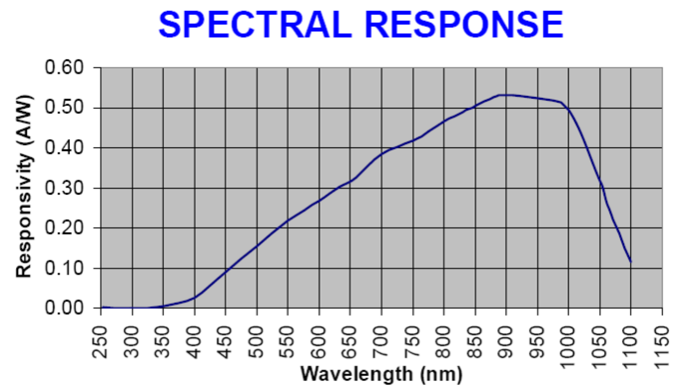
normalized spatial luminous intensity distribution

Appendix B

Calculating Integrated Spectral Flex Density



This is the information we need from the LED vendor



The detector diode vendors are giving us the info we need

For best performance we want the detector spectral responsivity to be "matched" to the LED spectral density. In general this is hard to due, especially for white LEDs.

$$P_{RX} \propto \left[\int_{\lambda=250\,nm}^{\lambda=1150\,nm} S(\lambda)\cdot R(\lambda)\cdot L(\lambda) \quad d\lambda \right]^2 R_2$$

Where $T(\lambda)$ is the transmitter power spectral density (W/nm)
$R(\lambda)$ is the detector responsivity (A/W at λ)
$L(\lambda)$ is the propagation loss (loss at λ)

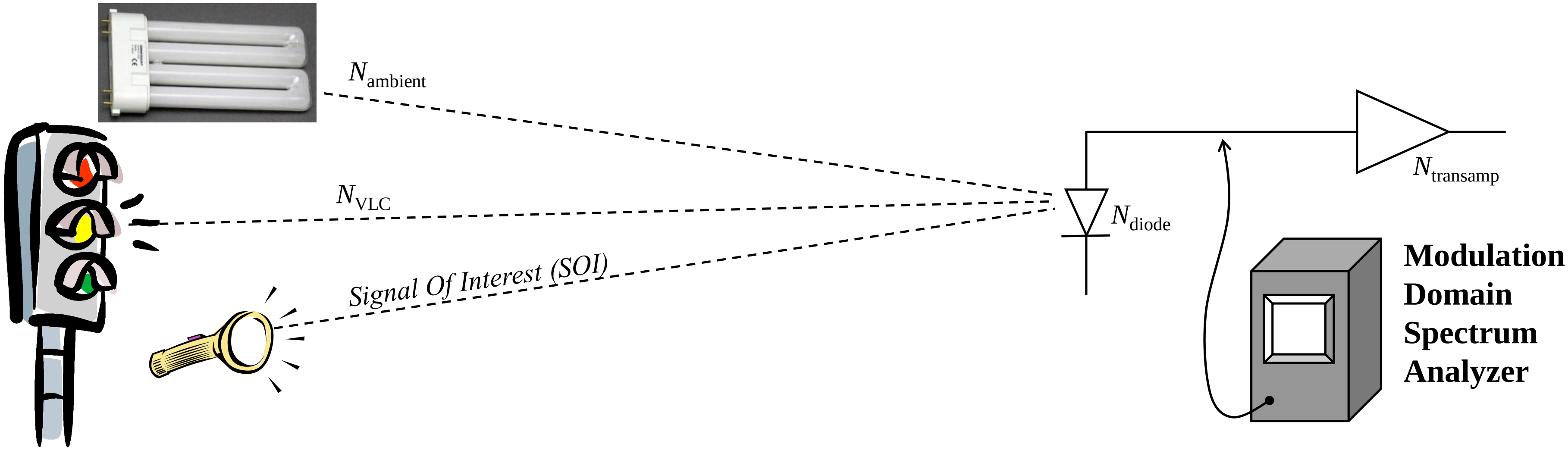
Appendix C

Receiver Noise Density Calculations

Determining the noise density *No*

What are the sources that contribute to the noise density?

- Photodetector Noise
- Transimpedance Amplifier Noise
- Ambient "in-band" noise
- Interference from other VLC sources
- Others?



Modulation Domain Spectrum

Ambient "In-Band" Noise Floor

This probably has to be empirically measured for many different environments

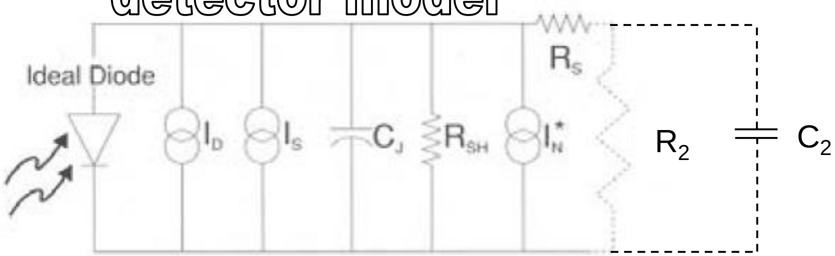
Interference from other VLC sources

For OCC this source of noise is unlikely because of the angle-of-arrival mapping of the lens because the interferer would have to be on the same angular vector as the desired noise source. In general, an interfering source will form an image elsewhere on the image sensor and can be spatially filtered out.

The detector itself contributes a noise density N_{diode} (W/ √ Hz)

$$i_{npd} = \sqrt{i_{shot}^2 + i_{thermal}^2} = \sqrt{2q(I_D + I_S + I_B) + 4kT/R_{SH}} \quad \text{(A/√ Hz)}$$

detector model



I_D = Dark current, Amps
I_S = Light signal current, Amps, (I_S=RP_O)
R = Photodiode responsivity at a wavelength of irradiance, Amps/Watt
P_O = Light power incident on photodiode active area, Watts
R_{SH} = Shunt resistance, Ohms
I_N^* = Noise Current, Amps rms
C_J = Junction Capacitance, Farads
R_S = Series resistance, Ohms
R_2 = Load resistance, Ohms

q is the electron charge (1.6e-19 coulombs)
I_D is the dark current
I_S is the signal current
I_B is the background light induced current
B is the bandwidth (B=1 Hz for N_0)
k is Boltzmann's constant (1.38e-23 J/K)
T is the Kelvin temperature (~290° K)
R_{SH} is the shunt resistance

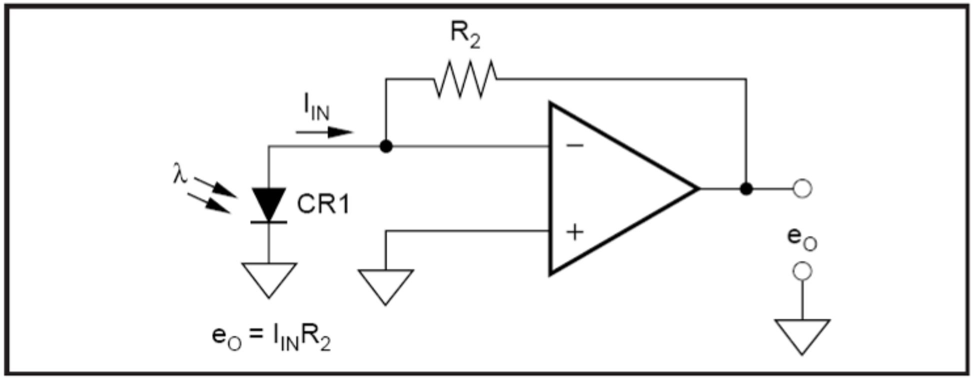


FIGURE 3. Pin Photo Diode Application.

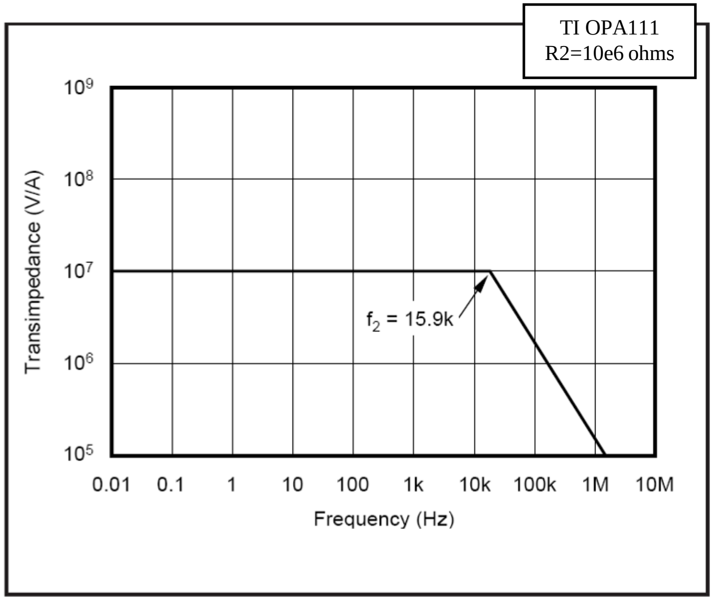


FIGURE 6. Transimpedance.

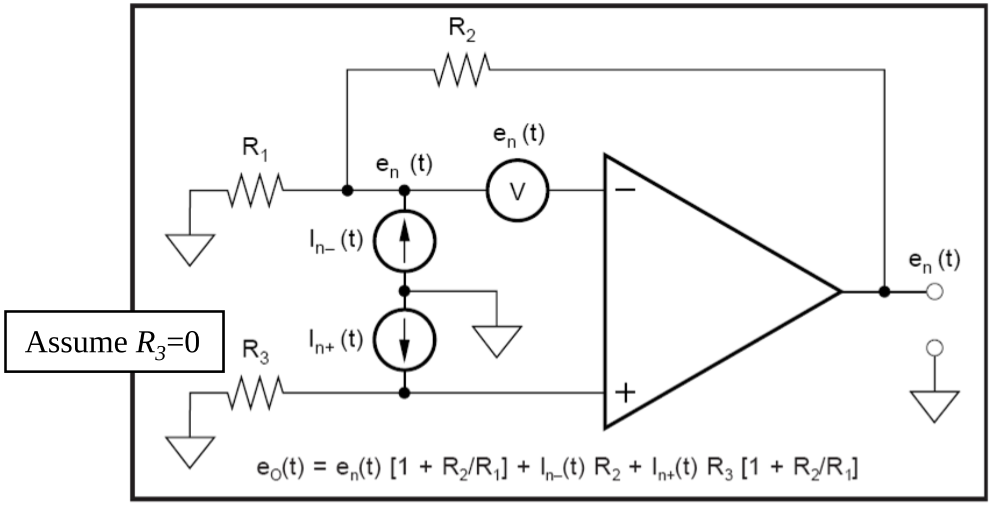


FIGURE 2. Circuit With Error Sources.

Transimpedance Amplifier Noise Analysis

Ref. TI/Burr-Brown Application Bulletin SBOA060
“Noise Anaysis of FET Transimpedance Amplifiers”

http://focus.ti.com/lit/an/sboa060/sboa060.pdf

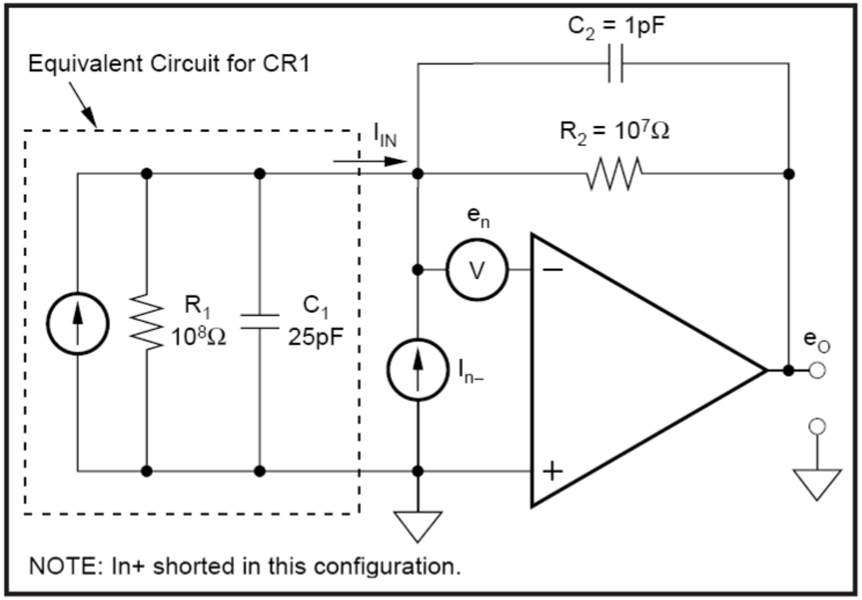


FIGURE 4. Noise Model of Photodiode Application.

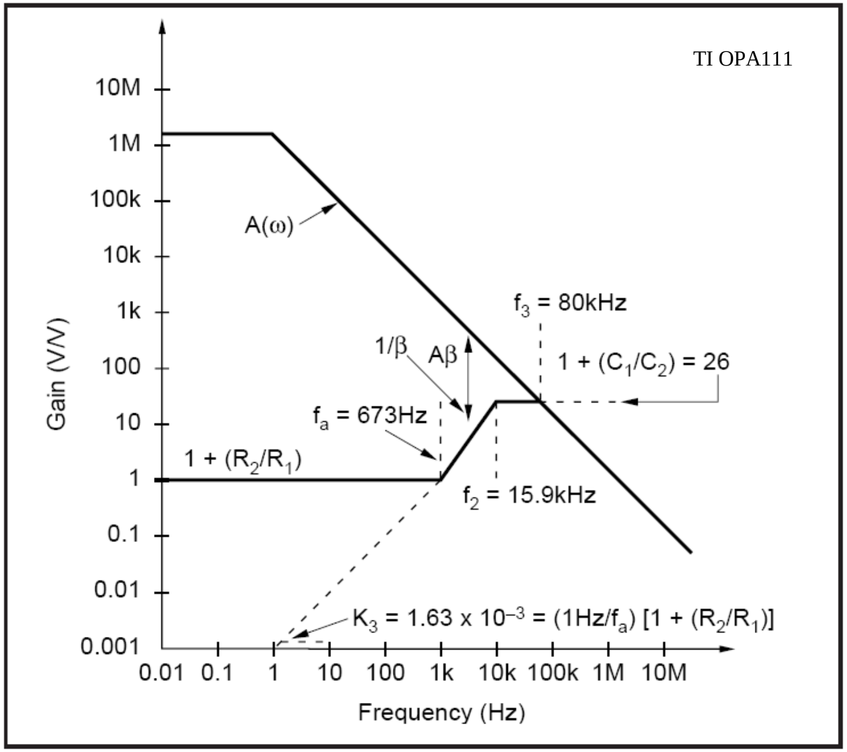


FIGURE 7. Noise Voltage Gain.

The resistors and capacitors form critical corner frequencies as shown below:

$$f_2 = \frac{1}{2\pi R_2 C_2}$$

$$f_a = \frac{1}{2\pi\left(R_1 \parallel R_2\right)\left(C_1 \parallel C_2\right)}$$

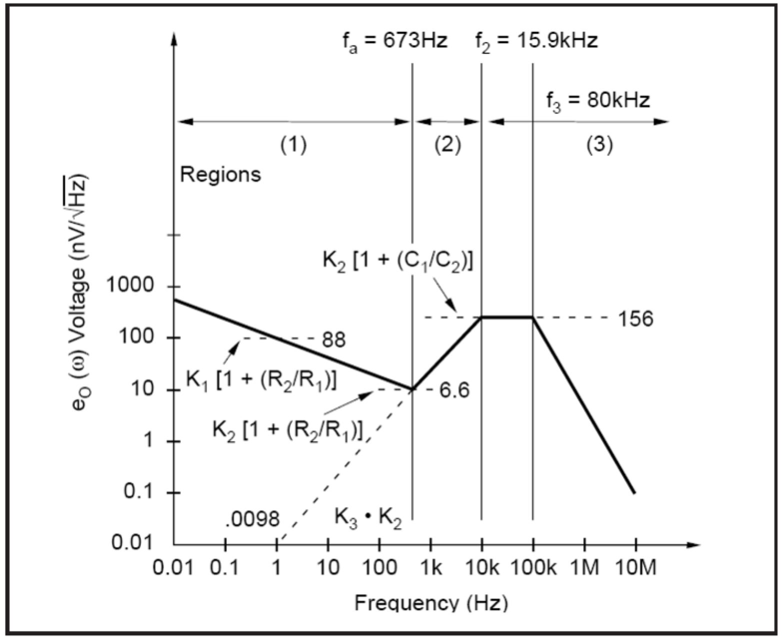


FIGURE 8. Output Voltage Noise Spectral Density.

Typically op-amps have three noise regions … the above noise regions are for the TI OPA111 op-amp. It is anticipated most outdoor VLC implementations will be bandpass systems operating in noise region 3.

NOISE								
Input Voltage Noise, f = 0.1Hz to 10Hz			90			*		nVp-p
	Noise Regions for the TI OPA228		15			*		nVrms
Input Voltage Noise Density, f = 10Hz e_n			3.5			*		nV/√Hz
f = 100Hz			3			*		nV/√Hz
f = 1kHz			3			*		nV/√Hz
Current Noise Density, f = 1kHz i_n			0.4			*		pA/√Hz

The approximation output noise is given by

$$N_0 = N_0^{n3} + N_0^{R} + N_0^{i_n}$$

where

$$N_0^{n3} = K_3^2 \cdot \left(1 + \frac{C_1}{C_2}\right)^2$$

$$N_0^{R} = 4kTR_2$$

$$N_0^{i_n} = \left(i_{nop}^2 + i_{npd}^2\right)R_2^2 = \left(i_{nop}^2 + 2q(I_D + I_S + I_B) + 4kT/R_{SH}\right)R_2^2$$

The signal current is given as

$$I_{sig} = \int_{\lambda=\lambda_{rL}}^{\lambda=\lambda_{rH}} S_r(\lambda) \cdot R_f(\lambda) \cdot R_D(\lambda) \quad d\lambda$$

Then electrical SNR is

$$\frac{E_b}{N_0} = \frac{\left(R_2 \int_{\lambda=\lambda_{rL}}^{\lambda=\lambda_{rH}} S(\lambda) \cdot R(\lambda) \cdot L(\lambda) \quad d\lambda \right)^2 / Rate}{K_3^2 \left(1 + \frac{C_1}{C_2}\right)^2 + 4kTR_2 + \left(i_{nop}^2 + 2q(I_D + I_S + I_B) + 4kT/R_{SH}\right) R_2^2}$$

Eb/No Dimensional Analysis

$$W = \frac{J}{s} = V \cdot A \qquad \Omega = \frac{V}{A} = \frac{J}{A^2 s}$$

$$V = \frac{W}{A} = \frac{J}{A \cdot s} \qquad Hz = \frac{1}{s} \qquad A = \frac{C}{s}$$

$q \rightarrow C(coulombs)$

$k \rightarrow J / K$

$T \rightarrow K$

$$\left(\frac{V}{A} \cdot W \cdot \frac{A}{W}\right)^2 \cdot \frac{1}{1/s} = \frac{V^2}{Hz}$$

$$\frac{E_b}{N_0} = \frac{\left(R_2 \int_{\lambda=380\,nm}^{\lambda=780\,nm} S(\lambda) \cdot R(\lambda) \cdot L(\lambda) \quad d\lambda\right)^2 / Rate}{K_3^2\left(1+\frac{C_1}{C_2}\right)^2 + 4kTR_2 + \left(i_{nop}^2 + 2q(I_D + I_S + I_B) + 4kT/R_{SH}\right)R_2^2}$$

$$\left(\frac{V}{\sqrt{Hz}}\right)^2 = \frac{V^2}{Hz}$$

$$\frac{J}{K} \cdot K \cdot \frac{V}{A} = \frac{J}{A} \cdot V \cdot \frac{s}{s} = \frac{J}{A \cdot s} \cdot V \cdot s = \frac{V^2}{Hz}$$

$$\frac{A^2}{Hz} \cdot \left(\frac{V}{A}\right)^2 = \frac{V^2}{Hz}$$

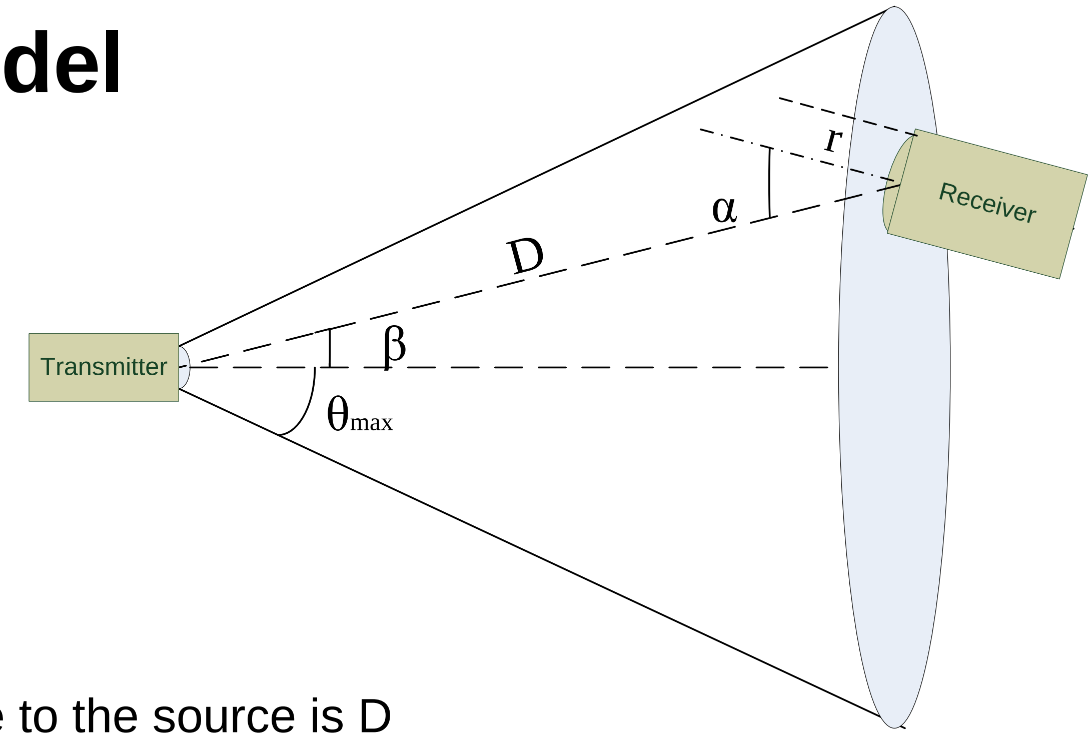
$$C \cdot A \cdot \frac{V^2}{A^2} = \frac{C}{A} V^2 = \frac{C}{C//s} V^2 = V^2 s = \frac{V^2}{Hz}$$

$$\frac{J}{K} \cdot K \cdot \frac{\left[V/A\right]^2}{V/A} = \frac{V^2}{Hz}$$

Appendix D

Solid angle path loss model

LOS Link Model



- The receiver distance to the source is D
- The receiver aperture radius is *r* and receiver area is A_r
- The angle between receiver normal and source-receiver line is α
- The angle between source beam axis and source-receiver line (viewing angle) is β
- The solid angle of the receiver seen by the source is Ω_r

The luminous angular intensity of the source at the receiver direction is $I_0 g_t(\beta)$, and therefore the receiver ingested luminous flux $F_r = I_0 g_t(\beta)\Omega_r$.

The luminous path loss can be represented as

$$L_L = \frac{F_r}{F_t} = \frac{I_0 g_t(\beta)\Omega_r}{I_0 \int_0^{\theta_{\max}} 2\pi g_t(\theta)\sin\theta d\theta} = \frac{g_t(\beta)\Omega_r}{\int_0^{\theta_{\max}} 2\pi g_t(\theta)\sin\theta d\theta} \approx \frac{g_t(\beta)A_r \cos\alpha}{D^2 \int_0^{\theta_{\max}} 2\pi g_t(\theta)\sin\theta d\theta}$$

where Ω_r is the receiver solid angle which satisfies $A_r\cos(\alpha) \approx D^2\Omega_r$.

Power path loss L_p can be proven equal to luminous path loss L_L as follows:

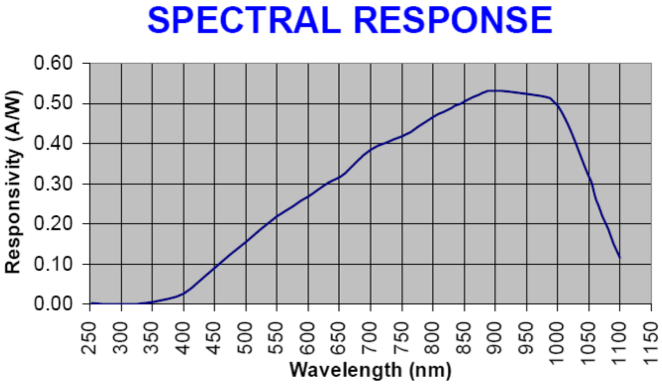
- Optical power can be written as $P = \int_{\lambda_L}^{\lambda_H} S(\lambda)d\lambda$
- In LOS free space propagation, path loss is assumed independent of wavelength. Power path loss can be represented as $L_p = S_2(\lambda)/S_1(\lambda) = P_2/P_1$.
- Luminous flux is related to $S(\lambda)$ as $F = 683\ lm/W \times \int_{\lambda_L}^{\lambda_H} S(\lambda)V(\lambda)d\lambda$, which is linear with $S(\lambda)$

Therefore, $L_L = F_2/F_1 = L_p$.

Find received optical and electrical power

Now that we have known the optical power loss due to LOS propagation, we can obtain the received optical spectral density from transmitter optical spectral density as $S_r(\lambda) = L_p S_t(\lambda) = L_L S_t(\lambda)$

Received optical power $P_{r,o} = \int_{\lambda=\lambda_{rL}}^{\lambda=\lambda_{rH}} S_r(\lambda) \cdot R_f(\lambda)\, d\lambda$



Suppose we use a photodiode detector to receive the signal light. We can obtain the electrical power of the signal as

$$P_{r,e} = \left[\int_{\lambda=250nm}^{\lambda=1150nm} S_r(\lambda) \cdot R_f(\lambda) \cdot R_D(\lambda) \quad d\lambda \right]^2 R_L$$

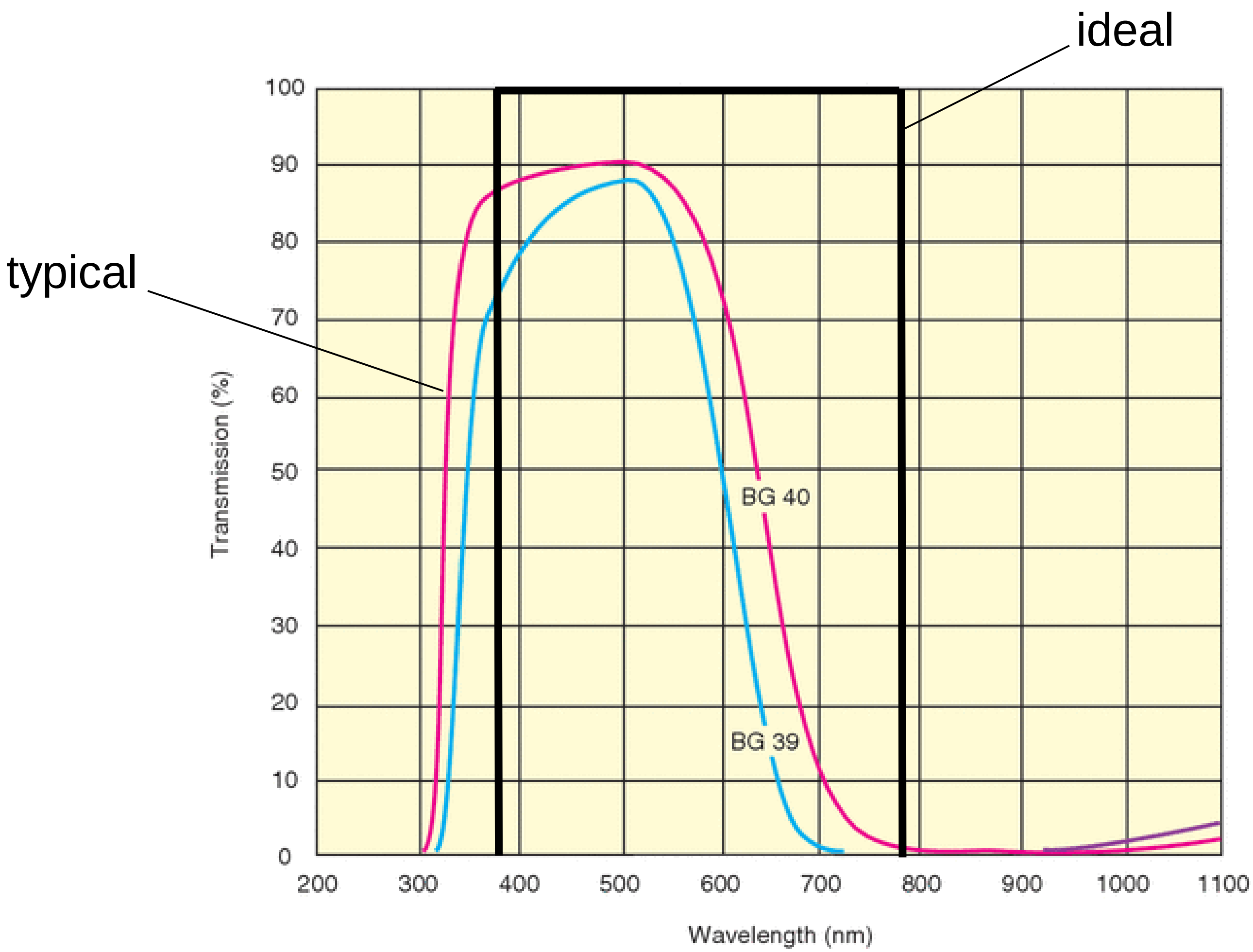
The detector diode vendors are giving us the info we need

where $S_r(\lambda)$ is the received light power spectral density (W/nm)

$R_r(\lambda)$ is the receiver filter spectral response

$R_D(\lambda)$ is the detector responsivity (A/W at λ) $R_D(\lambda) = \dfrac{\eta q \lambda}{hc}$

Exemplary Optical Filter Response



http://www.newport.com/images/webclickthru-EN/images/2226.gif

Summary of key steps to obtain received optical power and electrical power

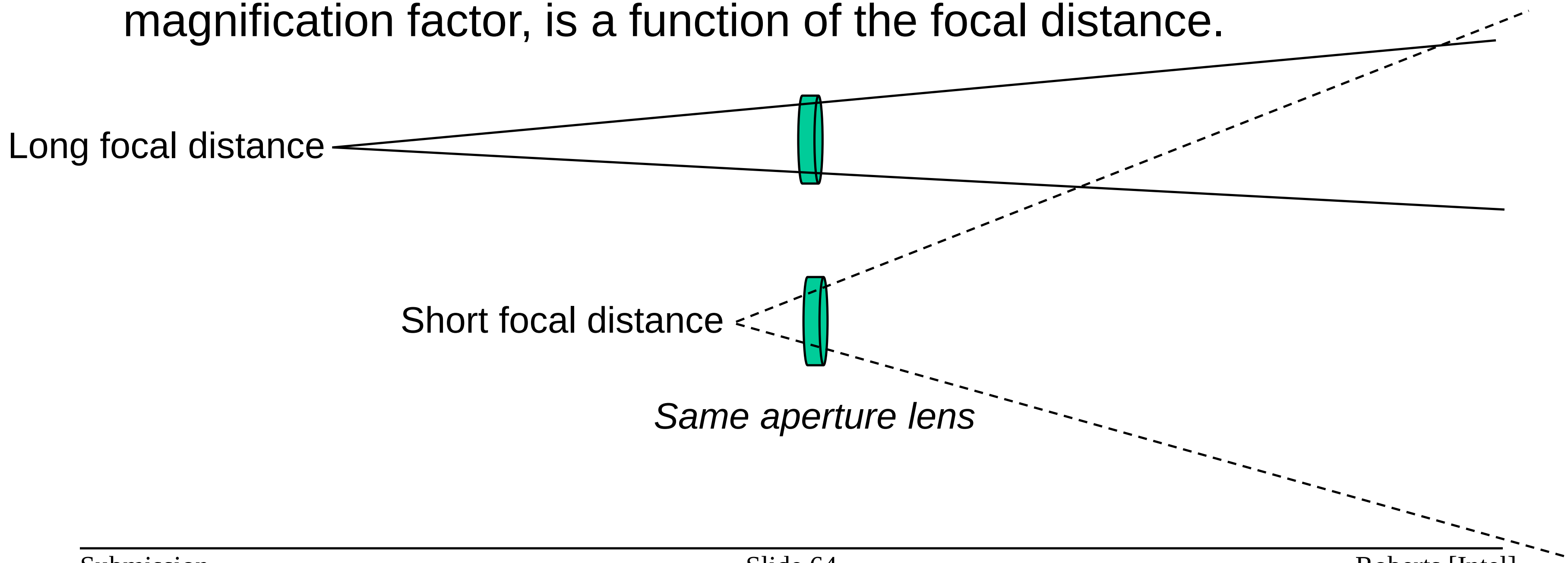
- Calculate transmitter (source) optical power from given luminous flux F_t (lumens) and normalized spectral curve $S_t'(\lambda)$
- Find the transmitter axial intensity I_0 from given luminous flux F_t and luminous spatial intensity distribution $g_t(\theta)$
- Find receiver ingested luminous flux F_r from receiver solid angle and transmitter luminous spatial intensity distribution $I_0\, g_t(\theta)$
- Find the luminous path loss L_L from F_t and F_r
- Prove power path loss L_p is equal to luminous path loss L_L from which to find received optical power spectral curve $S_r(\lambda)$
- Calculate received optical power and electrical power

Appendix E

RX aperture and magnification factor

Comment on RX aperture vs. magnification factor

It is the author’s opinion that the RX aperture determines the “brightness” of an observed object and that the field of view determines the magnification factor of an observed object. That is, for a given aperture the magnification factor determines how big an object appears but not how bright an object appears. The observed brightness is solely a function of the aperture size. It should be noted that the field of view, and hence the magnification factor, is a function of the focal distance.



Magnification factor and power density

When viewing a light source, the magnification factor impacts the spatial power density projected onto the image sensor. As mentioned previously, the aperture determines how much power is ingested, but the magnification factor (which is a function of the focal length) determines the image power density.

Case 1 … less magnification

- shorter focal length
- image appears smaller
- higher power density

Assumptions

- constant aperture
- ingested power is constant

Case 2 … more magnification

- longer focal length
- image appears larger
- lower power density

The change in power density ΔP_D is related to magnification factor *M* as

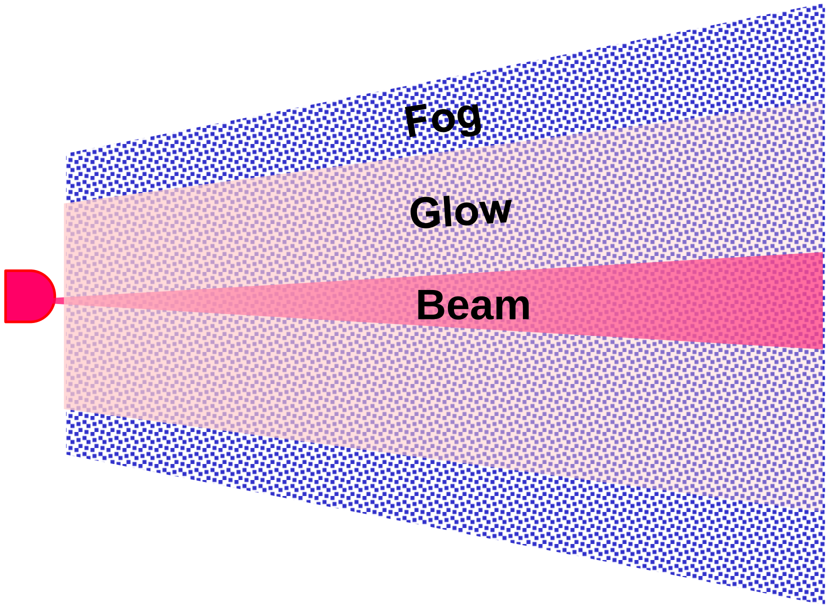
$$\Delta P_D = \frac{P/(\pi \cdot \{Mr\}^2)}{P/(\pi \cdot r^2)} = \frac{1}{M^2}$$

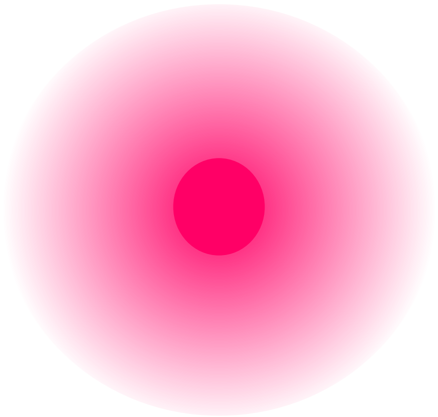
The change in power density is inversely proportional to the square of the magnification factor.

Appendix F

Fog diffusion ‘glow’

“





Fog causes light scattering in all directions. Forward scattering is realized when the light initially reflects backwards then reflects one or more times towards the forward direction. The net result is the light appears to have a radial glow about an intense inner beam that is attenuated as per Kim's equation. The intensity of the radial glow is inversely proportional to the Kim attenuation; that is, the inner core attenuation is due to diffused light causing the radial glow.

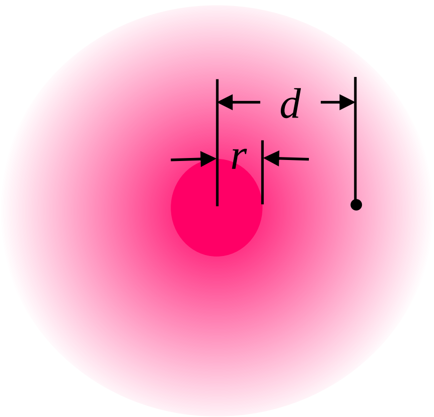
Inner Core Attenuation as per Kin's modified equation (negative gain ratio) …

$$A(km)_{beam} = e^{-\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\,nm}\right]^{-q}\right\}}$$

Proportional total glow intensity approximation (ratio) …

$$A(km)_{glow} = 1 - e^{-\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\,nm}\right]^{-q}\right\}}$$

The intensity of the glow off the main beam is proportional to the ratio squared of the beam radius *r* to the distance *d*.



The glow at the point of interest is approximated as:

Ratio: $A(km)_{point\_glow} = \left(\frac{r}{d}\right)^2 \left\{1 - e^{-\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\,nm}\right]^{-q}\right\}}\right\}$

dB: $A(dB/km)_{point\_glow} = 10 * log_{10}\left[\left(\frac{r}{d}\right)^2 \left\{1 - e^{-\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\,nm}\right]^{-q}\right\}}\right\}\right]$

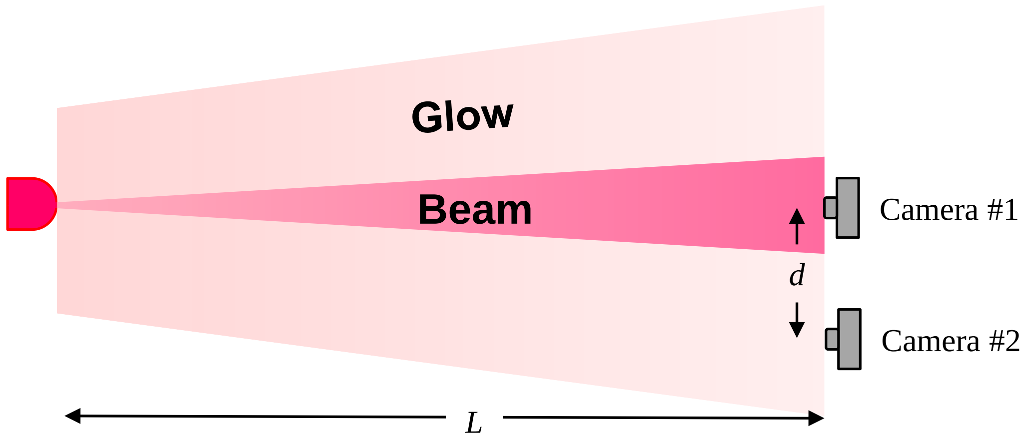
The signal to glow ratio (i.e. ratio between a point in the main beam with radius *r* to a point off the main beam at distance *d*) is given as (for 1 km distance) …

$$R(km)_{SGR} = \frac{e^{-\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\,nm}\right]^{-q}\right\}}}{\left(\frac{r}{d}\right)^2\left\{1-e^{-\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\,nm}\right]^{-q}\right\}}\right\}} = \left(\frac{d}{r}\right)^2 \frac{e^{-\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\,nm}\right]^{-q}\right\}}}{\left\{1-e^{-\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\,nm}\right]^{-q}\right\}}\right\}}$$

$$R_{dB}(km)_{SGR} = 10 * log_{10}\left[\left(\frac{d}{r}\right)^2 \frac{e^{-\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\,nm}\right]^{-q}\right\}}}{\left\{1-e^{-\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\,nm}\right]^{-q}\right\}}\right\}}\right].$$

These results need to be scaled for arbitrary (presumably less) distance.

The SGR results (signal to glow ratio) are currently for 1 km distance but needs to be scaled for more practical shorter distances, and the results should also include the directivity introduced by the camera's field of view. We can realize the former by introducing an exponential scaling term based upon the operational distance *L* and the latter by approximating the field of view via an inverse cosine scaling term.

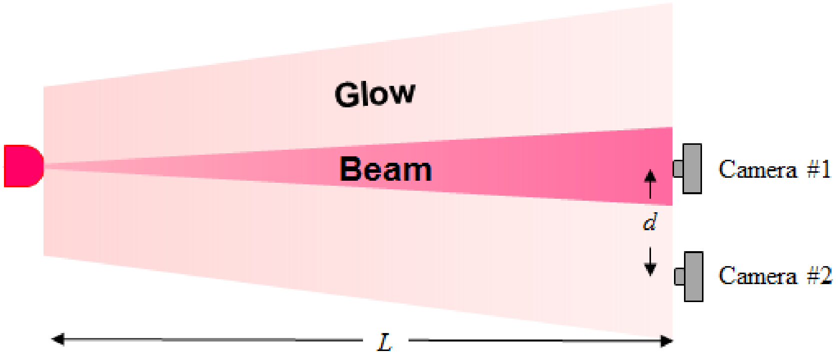


The modified SGR results are then given by

$$R(km)_{SGR} = \left(\frac{d}{r}\right)^2 \cdot \frac{e^{-\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\,nm}\right]^{-q}\right\}^{L/1000}}}{\left\{1 - e^{-\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\,nm}\right]^{-q}\right\}^{L/1000}}\right\}} \cdot \frac{1}{cos\left(tan^{-1}\left(\frac{d}{L}\right)\right)}.$$

The results are expressed in dBs as

$$R_{dB}(km)_{SGR} = 10 * log_{10}\left[\left(\frac{d}{r}\right)^2 \cdot \frac{e^{-\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\,nm}\right]^{-q}\right\}^{L/1000}}}{\left\{1 - e^{-\left\{\frac{3.91}{V}\left[\frac{\lambda}{550\,nm}\right]^{-q}\right\}^{L/1000}}\right\}} \cdot \frac{1}{cos\left(tan^{-1}\left(\frac{d}{L}\right)\right)}\right].$$



Example assumptions …

- Wavelength: 850 nm
- Beam Radius: 1m
- Off beam distance "*d*": 3m
- Standoff length "*L*": 10m