Project: IEEE P802.15 Working Group for Wireless Personal Area Networks (WPANs)

Submission Title: Effects of Phase Shift Errors on the Antenna Directivity of Phased Arrays in Indoor Terahertz Communications

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Abstract: This contribution derives the expectation of the antenna directivity in the presence of random phase shift errors occurring at phased array antennas. The theoretical estimate is validated with the simulation result and offers a simple way to determine the precision requirement of the phase shifter for a given directivity.

Purpose: Information of IG THz

Effects of Phase Shift Errors on the Antenna Directivity of Phased Arrays in Indoor Terahertz Communications

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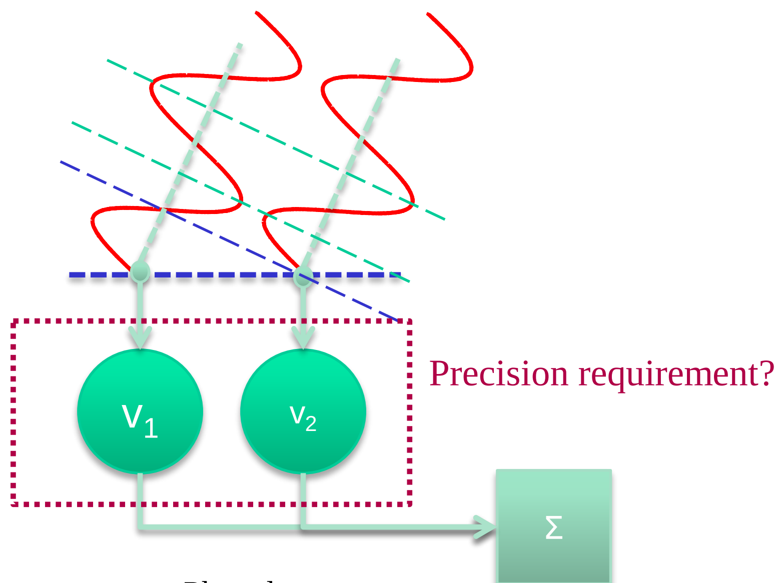
The results presented in this contribution are based on [1]

Motivation

- According to the Friis law, the path loss is extremely high at the THz frequency and must be compensated by a high antenna directivity.
- With the phased array, we can adjust the main lobe direction with the beamforming vector to realise a high antenna directivity.
- However, the precision of the hardware realisation is a challenge for the manufacturer especially for the extremely high frequency.



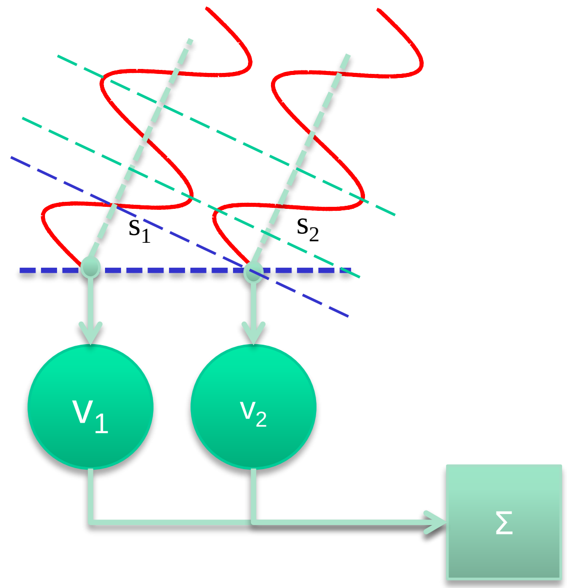
Phased array

Derivation of Directivity in Phased Arrays

- The radiation intensity in calculated as:

$$P(\theta,\phi) = p(\theta,\phi)\left|\mathbf{s}^T(\theta,\phi)\mathbf{v}\right|^2$$

  - p(θ,φ): antenna directivity of an antenna element
  - **s**(θ,φ): steering vector (signal delay)
  - **v**: beamforming vector (phase shift)



- The normalization factor is defined as:

$$F = \int_0^{\pi}\int_0^{2\pi} P(\theta,\phi)\sin\theta\, d\phi\, d\theta$$

- The directivity is the ratio of the radiation intensity in the desired direction and the normalization factor:

$$D(\theta,\phi) = \frac{4\pi P(\theta,\phi)}{F} = \frac{4\pi P(\theta,\phi)}{\int_0^{\pi}\int_0^{2\pi} P(\theta,\phi)\sin\theta\, d\phi\, d\theta}$$

Assumptions for the Phase Errors

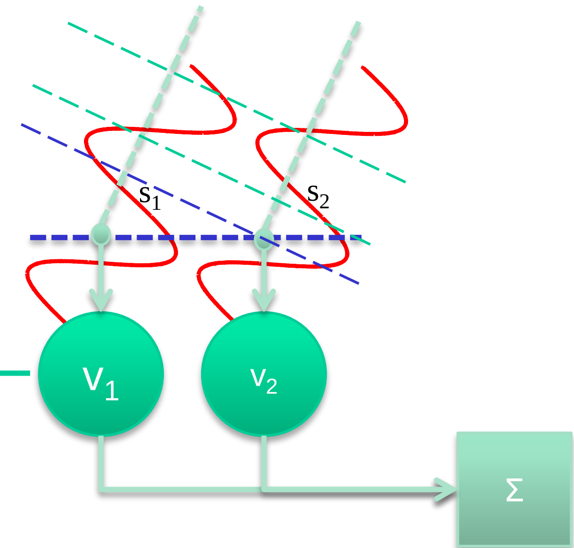
- Random phase errors on v_i affect both **denominator** and **numerator** of the antenna directivity.

Uncertainty

$$P(\theta,\phi) = p(\theta,\phi)\left|\mathbf{s}^T(\theta,\phi)\mathbf{v}\right|^2$$

$$D(\theta,\phi) = \frac{4\pi P(\theta,\phi)}{F} = \frac{4\pi P(\theta,\phi)}{\int_0^{\pi}\int_0^{2\pi} P(\theta,\phi)\sin\theta\, d\phi\, d\theta}$$

Phase error



- Assumptions for the phase errors (v_i):
  - The expectation is 0.
  - The phase error is characterized by the variation σ^2.
  - The phase errors on 2 antenna elements are uncorrelated.
  - The moments of the phase errors of order higher than 2 are negligible.

Effect of Phase Errors on Radiation Intensity and Normalization Factor (1/2)

- We approximate the complex number with the **Taylor series**:

$$e^{j\alpha} = \sum_{i=0}^{\infty} e^{j\alpha_0} \frac{(j\Delta\alpha)^i}{i!} \approx e^{j\alpha_0} \left(1 + j\Delta\alpha - \frac{\Delta\alpha^2}{2!}\right)$$

- The expectation of the radiation intensity with phase errors is:

$$E(P') = p(\theta, \phi) \cdot (L^2 - L^2\sigma^2 + L\sigma^2)$$

  - p(θ,φ): antenna directivity of an antenna element
  - L: number of antenna elements
  - σ: standard deviation of random errors

Effect of Phase Errors on Radiation Intensity and Normalization Factor (2/2)

- The expectation of the normalization intensity with phase errors is:

$$E(F') = \quad F - \sigma^2 F + F_e L \sigma^2$$

  - F: normalization factor without phase errors
  - F_e: normalization factor of one antenna element without phase errors

Impact of Phase Errors on Antenna Directivity

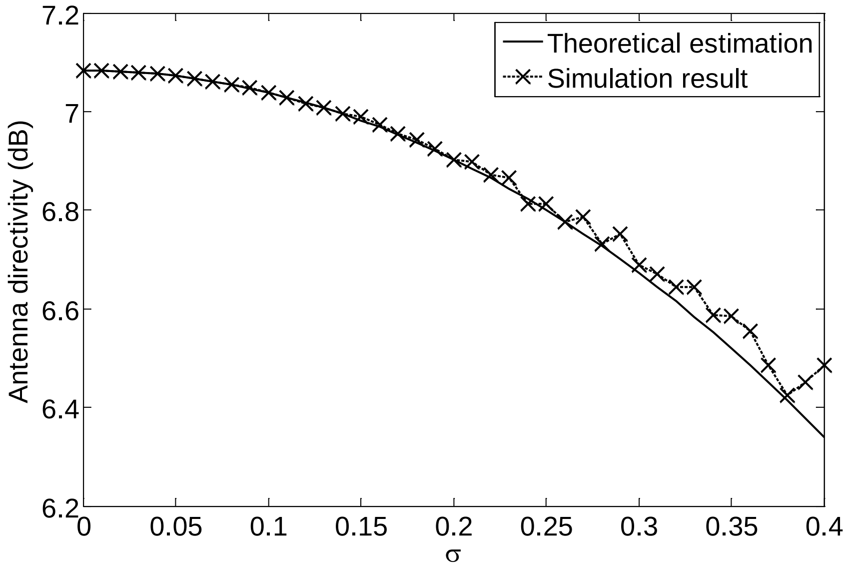
- As the normalization factor F is much larger than the radiation intensity P, the expectation of the antenna directivity can be approximated as the ratio of the expectations of the radiation intensity and the normalization factor:

$$E[D'(\theta,\phi)] = \frac{E[P'(\theta,\phi)]}{E(F')} = \frac{4\pi p(\theta,\phi)\cdot(L^2 - L^2\sigma^2 + L\sigma^2)}{F - \sigma^2 F + F_e L\sigma^2}$$

Parameters causing a decrease in antenna directivity in case of phase shift errors are:
large error standard deviations σ
large array sizes L
small normalization factors F

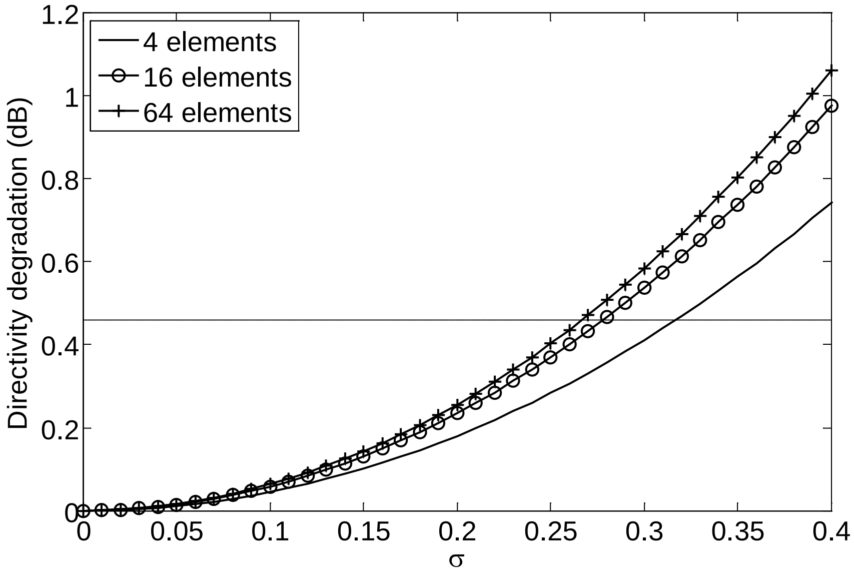
Verification of the Method by Monte Carlo Simulation

- We run 500 simulations for every standard deviation and take the mean value.
- The analytical estimate matches the simulation results especially well for small standard deviation values because the approximations result in smaller deviations.
- The analytical estimate is much more convinient to use than the Mont-Carlo simulations.



Directivity Degradation for Different Array Sizes

- Larger array size increases the directivity degradation, but the increment rate decreases.
- The table below lists the admissible standard deviations for 10% degradation (of the directivity in lnear units).
- The current hardware technique can not meet such high precision requirements. A calibration is necessary after the manufacture.



Array size	Admissible standard deviation
4	0.32
16	0.28
64	0.27

Conclusions

- This contribution presents an analytical estimate of the phased array directivity degradation at the presence of the random phase shift errors.
- The estimate method shows that with larger array size and smaller normalization factor, the degradation is higher.
- The analytical estimate is verified by the simulation results.

References

[1] B. Peng, S. Priebe, T. Kürner, Effects of Phase Shift Errors on the Antenna Directivity of Phased Arrays in Indoor Terahertz Communications, accepted for publication in Proc. The Eleventh International Symposium on Wireless Communication Systems, Barcelona/Spain, August 26-29, 2014