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Re: []

Abstract: [Beamforming]

Purpose: [OFDM & SC Beamforming]

Content

- Beamforming
  - Beamforming Terminology
  - Beamforming Protocol
    - Two-level Training Mechanism
    - Optional Tracking
- On-Demand Beamforming
- Pro-Active Beamforming
- Superframe Structure

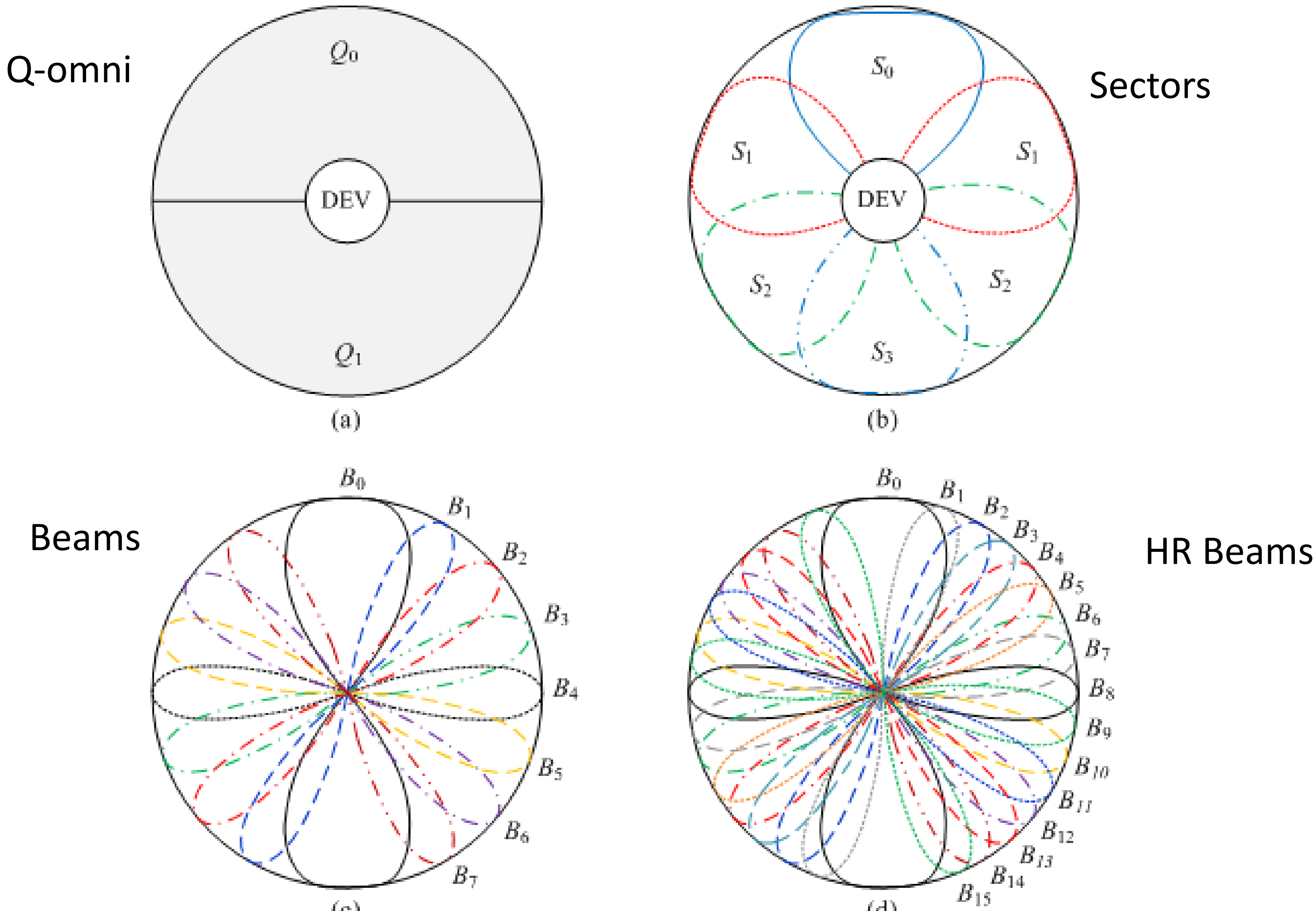
Beamforming

- Two protocols are specified:
  - On-demand beamforming
    - CTA based between two DEVs or PNC and a DEV
  - Pro-active beamforming
    - PNC is the source of data to one or multiple DEVs
    - Sector training in beacon and remainder in CTA
- Optimality criterion
  - BST (mandatory): Beam Switching (Steering) & Tracking
  - PET (optional): Pattern Estimation & Tracking (Appendix I)

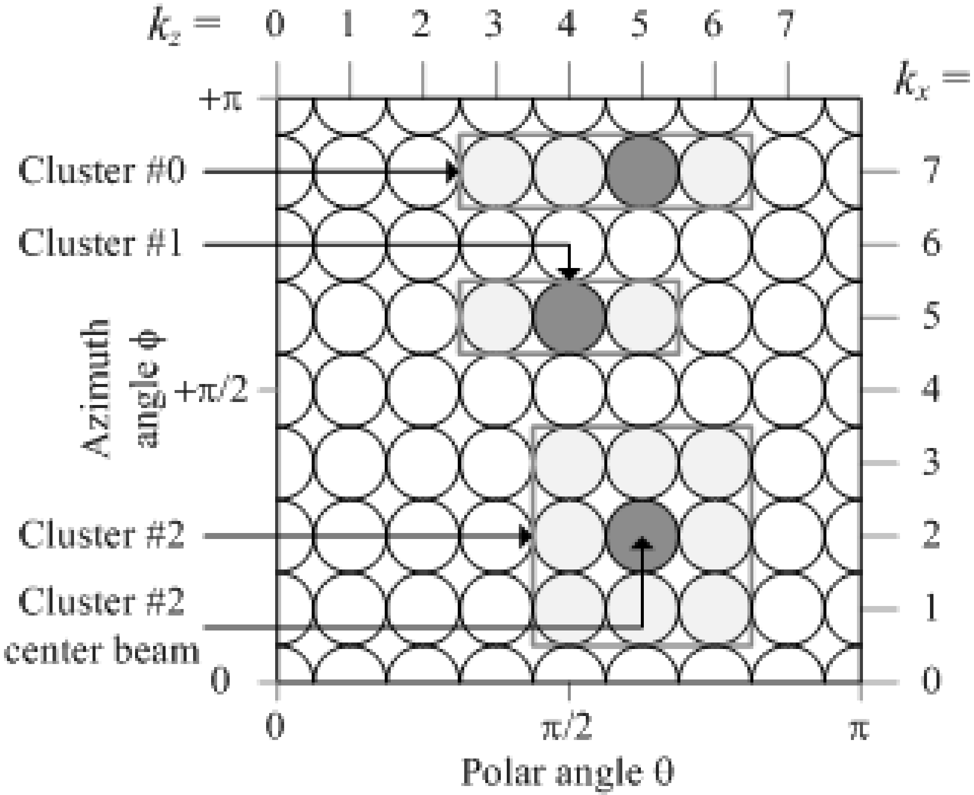
Beamforming Protocol

- On-demand and Pro-active beamforming protocols are implemented using a Two-level training mechanism followed by an optional tracking phase
- Two-level training mechanism
  - Sector (coarse) training level
  - Beam (fine) training level
- Optional tracking phase
  - HR (High Resolution) beam tracking

Beamforming Terminology



Beamforming Terminology



Beamforming Terminology

- Antenna elements:
  - $M^{(d,t)}$ & $M^{(d,r)}$: # of transmit & receive antenna elements
- Q-omni patterns: $Q^{(d,t)}$ & $Q^{(d,r)}$:
  - $I^{(d,t)}$ & $I^{(d,r)}$: #of transmit & receive Q-omni patterns
  - $i^{(d,t)}$ & $i^{(d,r)}$: best pair of transmit & receive patterns
- Sectors: $S^{(d,t)}$ & $S^{(d,r)}$:
  - $J^{(d,t)}$ & $J^{(d,r)}$: # of transmit & receive Q-omni patterns
  - $j^{(d,t)}$ & $j^{(d,r)}$: : best pair of transmit & receive patterns
- Beams: $B^{(d,t)}$ & $B^{(d,r)}$:
  - $K^{(d,t)}$ & $K^{(d,r)}$: # of transmit & receive Q-omni patterns
  - $k^{(d,t)}$ & $k^{(d,r)}$: best pair of transmit & receive patterns

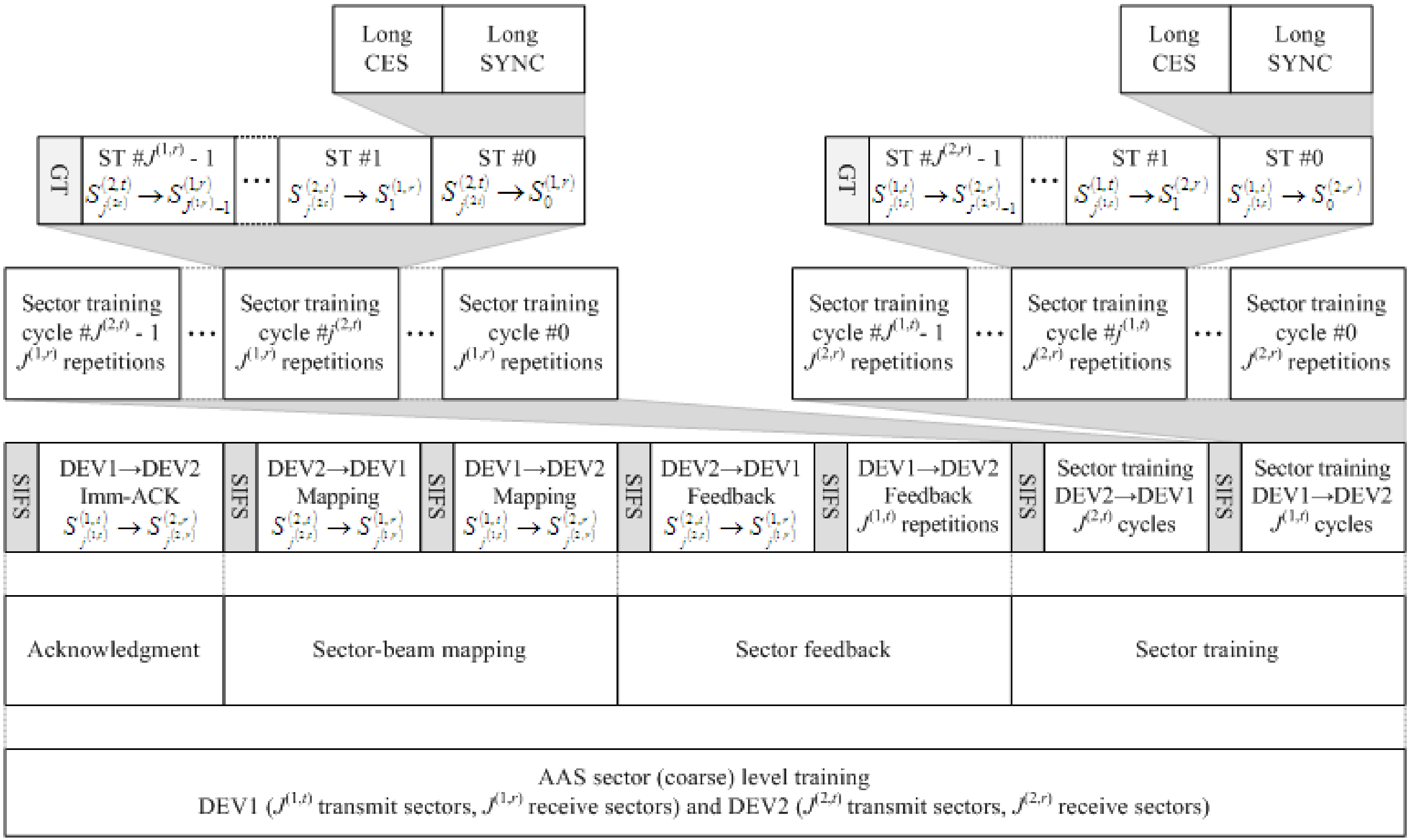
Two-level training mechanism

- Sector level training:
  - Objective is to find the best pair of sectors (in terms of LQI) from DEV1 → DEV2 and from DEV2 → DEV1
  - Second best pair is reported as well and the LQIs
- Beam level training
  - Best sector and second best are sliced into beams
  - BST: the objective is to find the best beam pair from DEV1 → DEV2 and from DEV2 → DEV1;
  - BST: the 2nd best beam should be part of a different cluster, i.e. not adjacent to the best beam
  - PET: find the optimal beamformer and combiner vector (weights) from DEV1 → DEV2 and from DEV2 → DEV1

Sector Level Training

- The sector level training consists of four stages:
  - Sector training: DEV1 → DEV2 and from DEV2 → DEV1
    - DEV1 → DEV2 $J^{(1,t)}$ cycles, each cycle = $J^{(2,r)}$ repetitions of a ST
    - DEV2 → DEV1 $J^{(2,t)}$ cycles, each cycle = $J^{(1,r)}$ repetitions of a ST
    - ST: Sector Training sequence = long preamble
  - Feedback
    - DEV1 informs DEV2 of its best Tx sector, $j^{(2,t)}$, second best and corresponding LQIs, DEV2 does the same
  - Mapping
    - DEV1 informs DEV2 of its number of Tx/RX beams it is planning to use in the next phase (beam training), and how it is clustering them if needed
    - DEV2 provides DEV1 with similar info
  - Acknowledgment (completes the sector level)

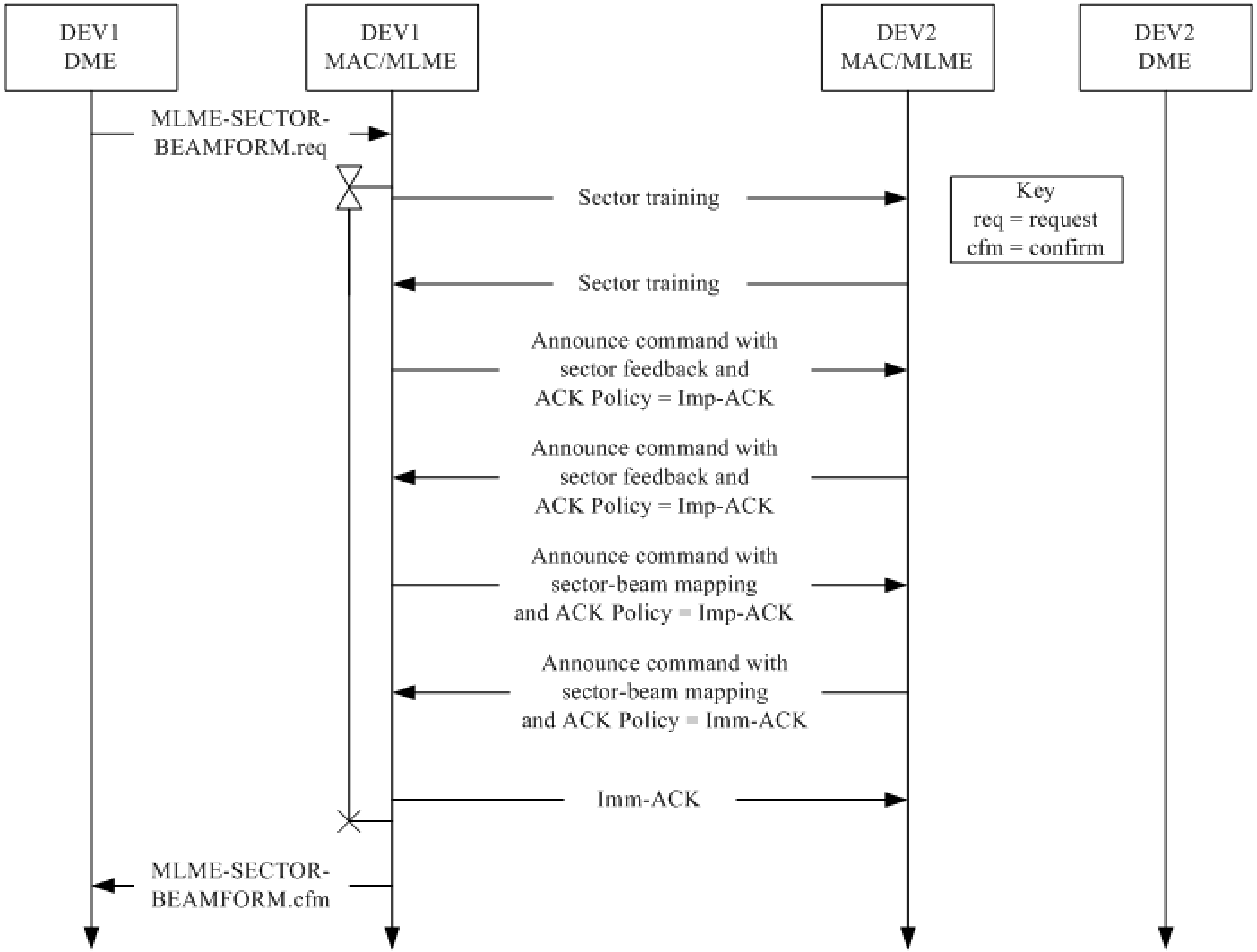
Sector training level



Sector Level IEs

- Feedback IE
  - Index of best sector, 2nd best sector and corresponding LQIs
- Mapping Phase
  - Mapping IE
    - Number of Tx & Rx beams to be used in “Beam Level Training” and training sequence mode (long, medium, or short preamble)
  - Clustering IE (optional)
    - Number of Tx & Rx Clusters, number of beams in each cluster, and clusters encoding for PET criterion
  - PET IE (optional)
    - Antenna elements info and codebooks to be used in “Beam Level Training”

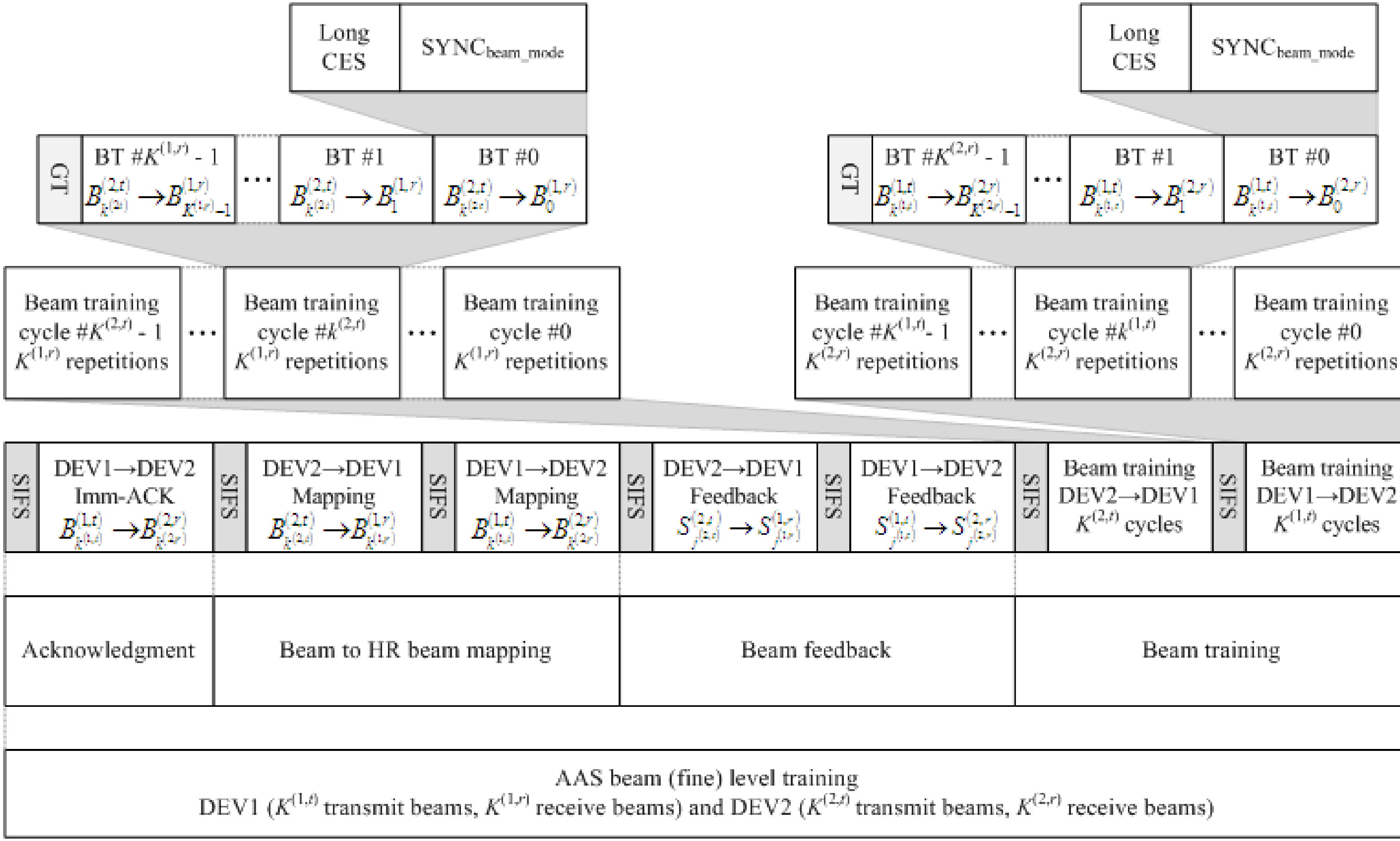
Sector Level Process



Beam Level Training

- The beam level training consists of four stages:
  - Beam training: DEV1 $\longrightarrow$ DEV2 and from DEV2 $\longrightarrow$ DEV1
    - DEV1 $\longrightarrow$ DEV2 $K^{(1,t)}$ cycles, each cycle = $K^{(2,r)}$ repetitions of a BT
    - DEV2 $\longrightarrow$ DEV1 $K^{(2,t)}$ cycles, each cycle = $K^{(1,r)}$ repetitions of a BT
    - BT: Beam Training sequence = long , medium, or short preamble
  - Feedback
    - DEV1 informs DEV2 of its best Tx beam, $k^{(2,t)}$, second best and corresponding LQIs, DEV2 provides DEV1 with similar info
  - Mapping
    - DEV1 informs DEV2 of its number of Tx/RX HR beams it is planning to use in the next phase (tracking), and how it is clustering them if needed
    - The best and 2nd best clusters tracking frequencies if required
    - DEV2 provides DEV1 with similar info
  - Acknowledgment (completes the beam level)

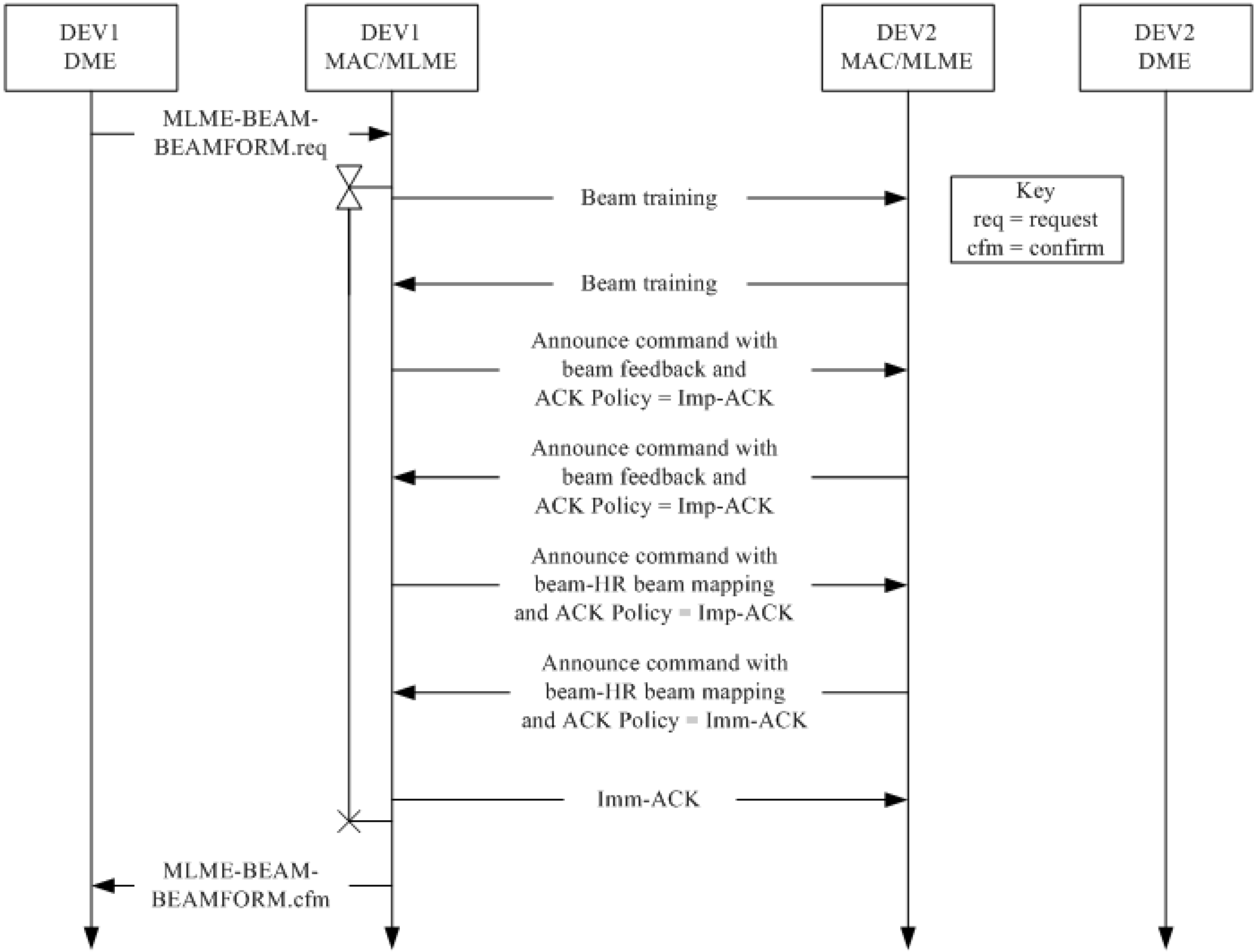
Beam Level Training



Beam Level IEs

- Feedback IE
  - Feedback IE: Index of best beam, 2nd best and LQIs
  - Phase & amplitude IEs: PET only IEs with information on feedback antenna weights
- Mapping Phase
  - Mapping IE
    - Number of Tx & Rx HR beams to be used in “Tracking” , training sequence mode (long, medium, or short preamble), and best and 2nd best clusters tracking frequencies
  - Clustering IE (optional)
    - Number of Tx & Rx Clusters, number of HR beams in each cluster, and clusters encoding for PET criterion
  - HR PET IE (optional)
    - Codebooks to be used in “Tracking”

Beam Level Process



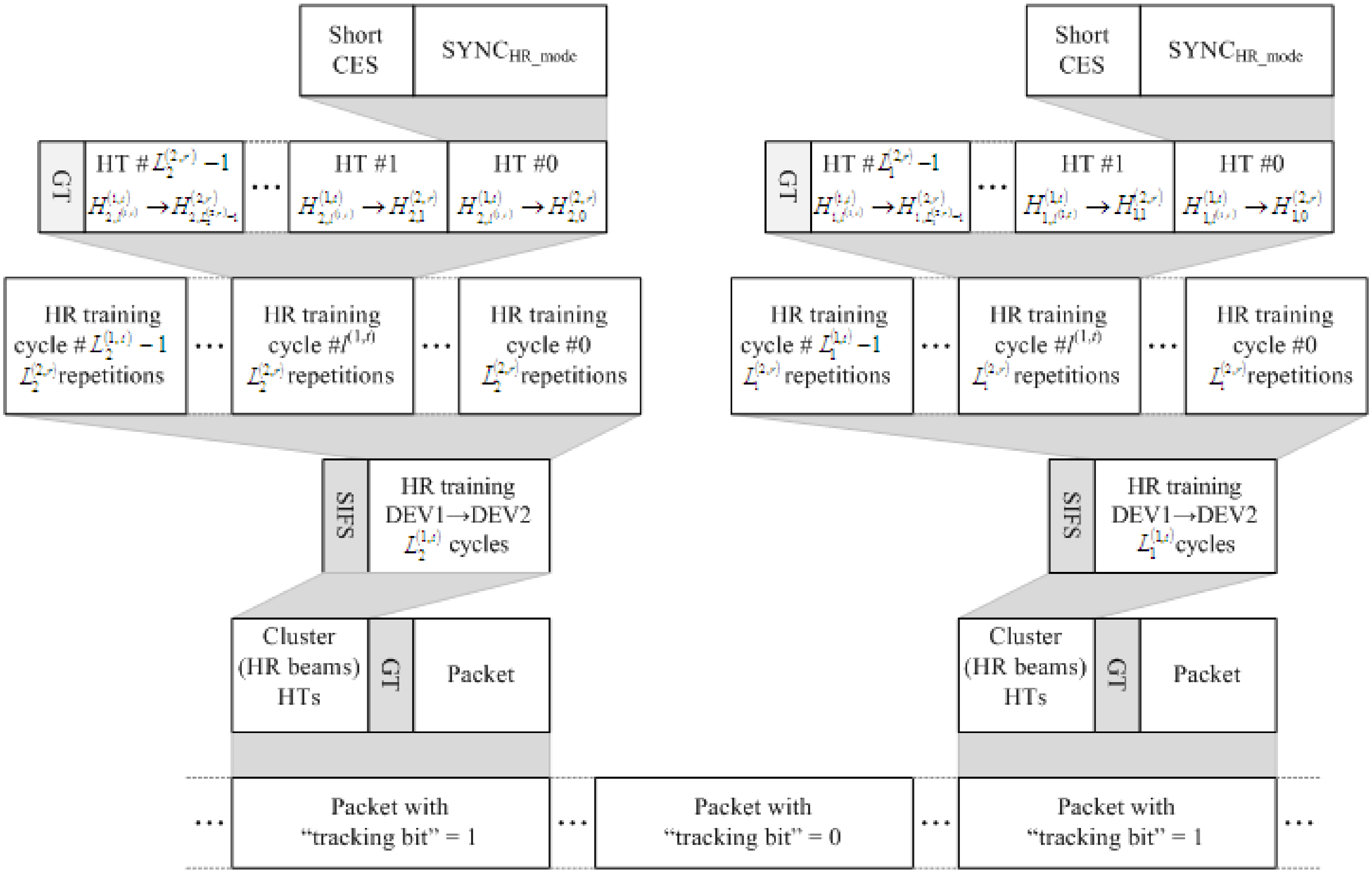
Tracking Phase

- Cluster: set of adjacent beams identified by center beam
- Clusters are paired, i.e. associate a cluster from DEV1 to a cluster from DEV2
- Track center beam in each cluster (re-clustering)
- Tracking packets are used to enable distributed tracking
- A tracking packet with “tracking bit” set to 1 is followed by one or more BTs (Beam Training sequence)
- Low-latency uses one BT per tracking packet, otherwise a cluster follows a tracking packet

Tracking Frequencies

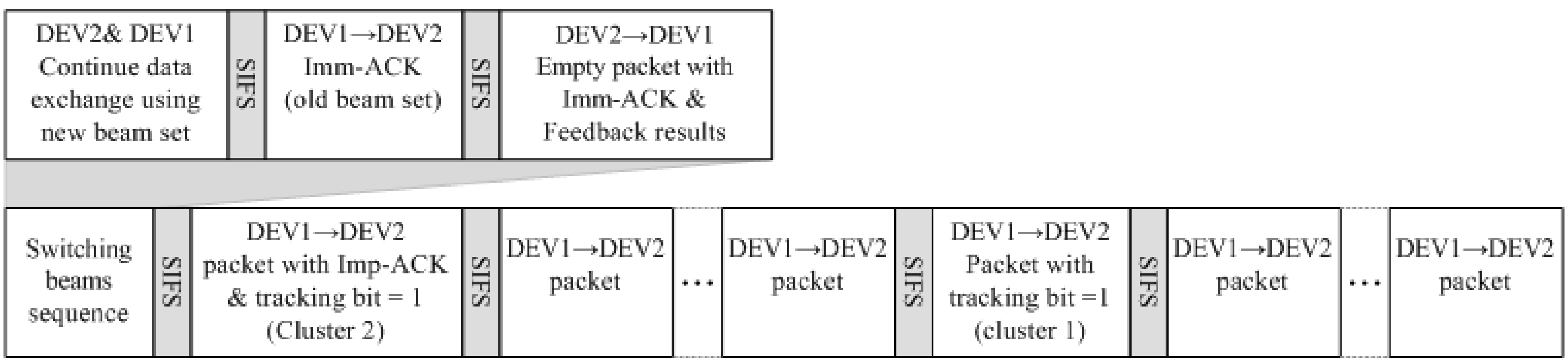
- Upon completion of beam level training DEV1 (PNC) and DEV2 have knowledge of tracking frequencies for best and 2nd best clusters
- Best cluster frequency tracking ≥ 2nd cluster frequency tracking
- Frequency tracking is chosen by each device independently
- The frequency resolution is flexible: range (1μs to 1ms)

Tracking Phase (Normal Mode)



Tracking Process

- After the completion of the 2nd cluster, DEV1 request feedback from DEV2
- DEV2 replies with feedback results in an empty data packet and ACK policy = Imm-ACK
- DEV1 Acknowledges reception before switching to new beam



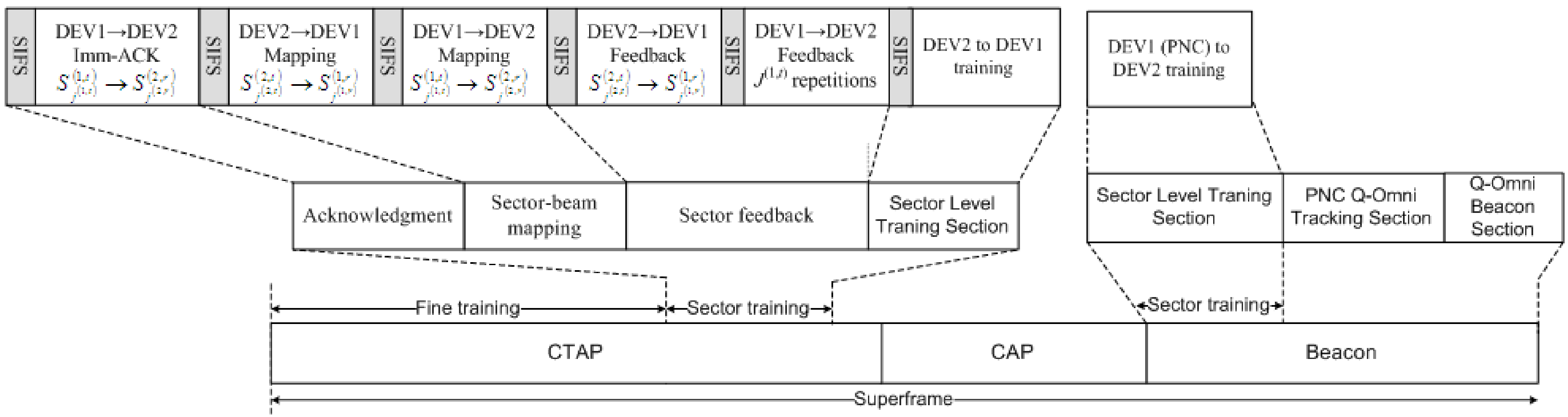
On-Demand Beamforming

- It takes place in a CTA between two devices: DEV1 (source) & DEV2(destination);
- DEV1 reserves a CTA for the special purpose of beamforming
- Two-level training (Sector & Beam) as described before
- Optional tracking (HR beams) takes place in the data CTA

Pro-Active Beamforming

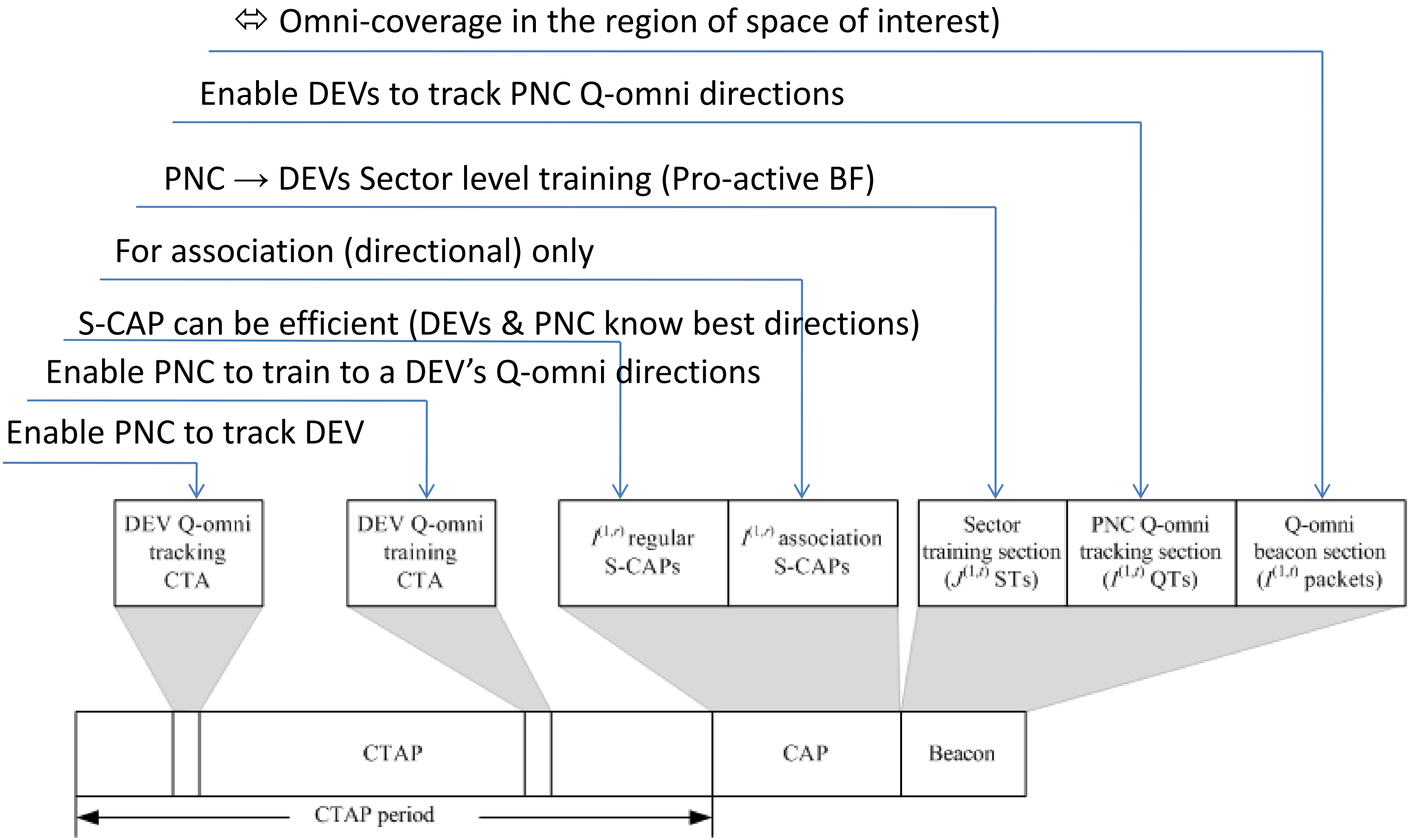
- Sector training sequences takes place in the beacon and can be used by all DEVs;
- Each DEV reserves a CTA for the special purpose of beam level training;
- Sector Feedback & beam training takes place between PNC and a single DEV;
- Tracking takes place in the data CTA allocated to PNC & DEV

Pro-Active Beamforming



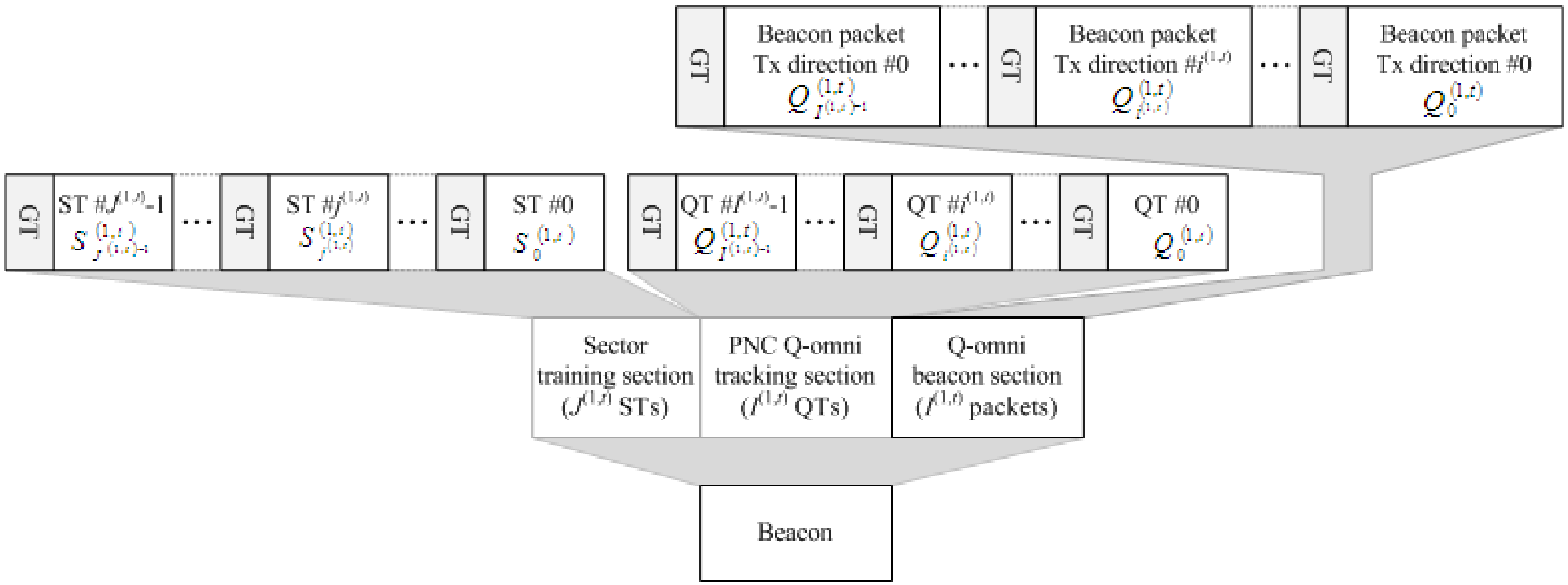
Superframe Structure

Superframe Structure



The Beacon

- Beacon packet is repeated $I^{(1,t)}$ times, each repetition is sent in a different direction
- The set of directions (possibly overlapping) provide ~ omni coverage of the region of space of interest

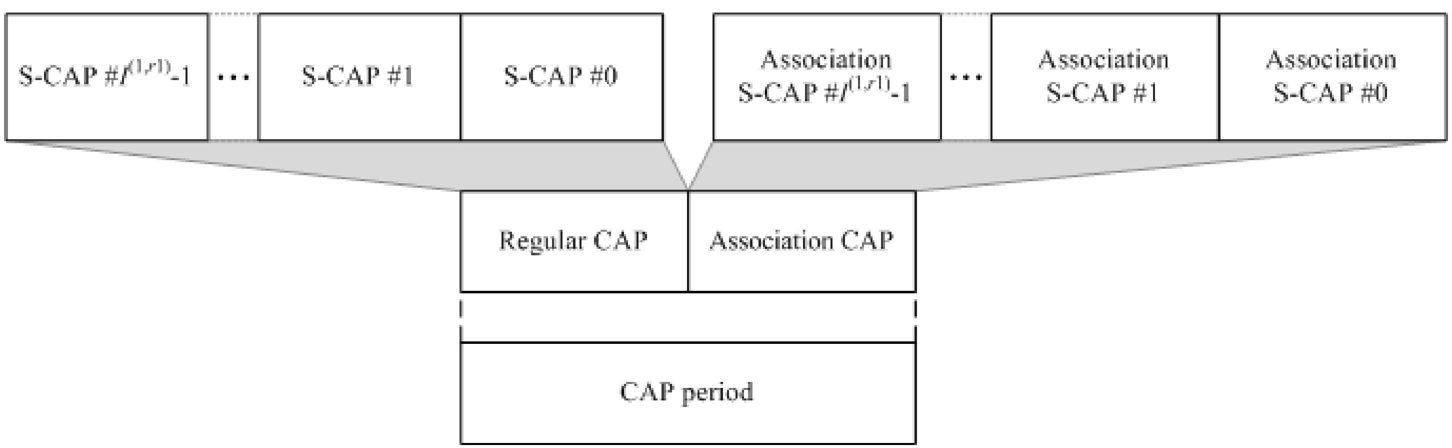


The Beacon

- The optional PNC Q-omni tracking section consists of $I^{(1,t)}$ QT (Q-omni Training) sequences transmitted in the $I^{(1,t)}$ PNC different QT directions
- This section enables all DEVs to track the PNC Q-omni directions, and track best and 2nd best Q-omni directions
- The sector training section consists of $J^{(1,t)}$ ST (Sector Training) sequences transmitted in the $J^{(1,t)}$ PNC different ST directions
- The sector training section enables all DEVs to find and track PNC best sector and 2nd best and is used as the 1st step in the pro-active beamforming

The CAP

- The CAP is divided into two sections:
  - Association CAP: used for association only
  - Regular CAP: used for all other command frames and data frames
- This allow a more efficient usage of the regular CAP since after association PNC and DEV know each other directions
- The associations and S-CAPs can be distributed over multiple superframes. Figure below is an example



PNC Discovery

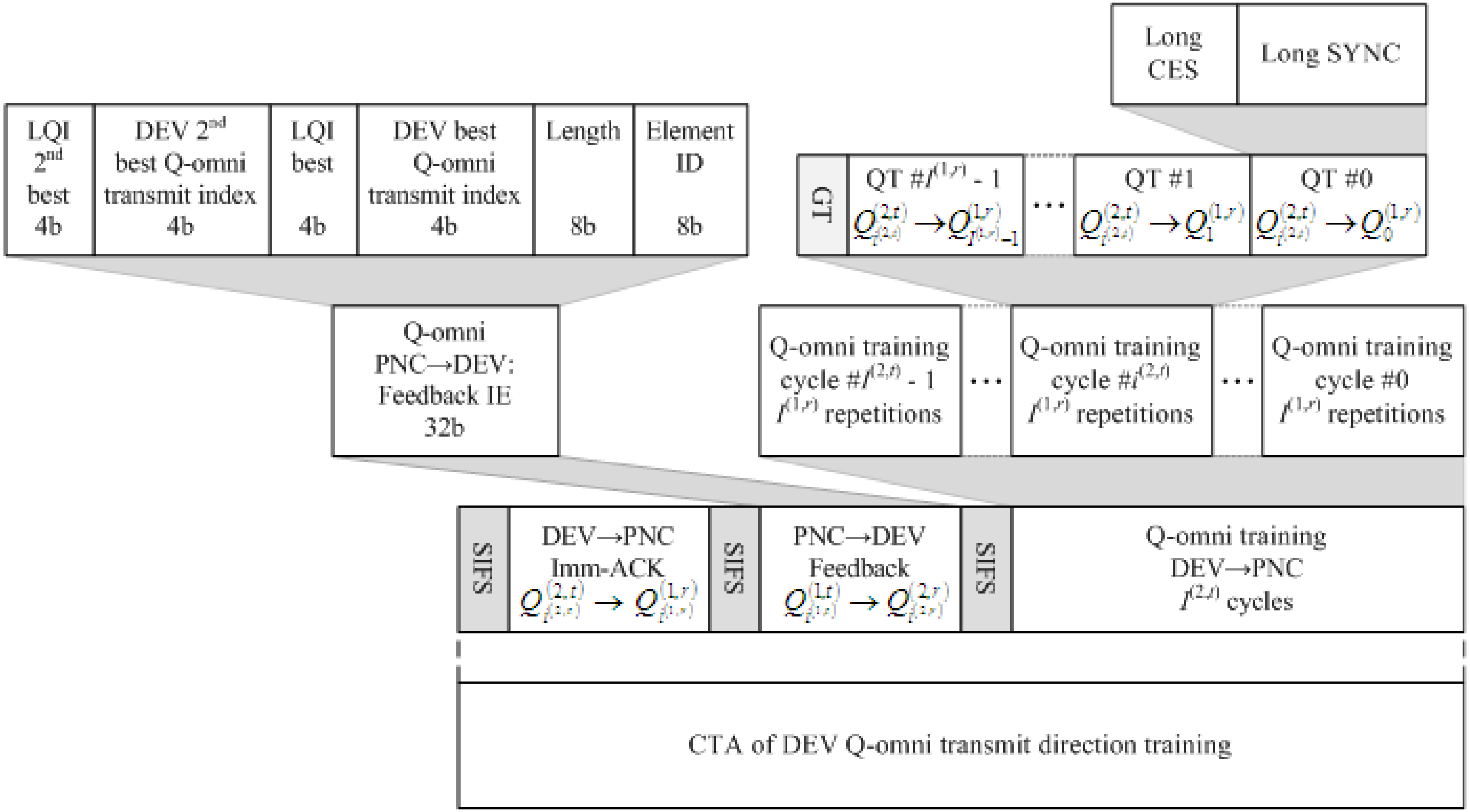
- DEV searches beacons at all its *I*(2,r) Q-omni directions, to find best and 2nd best PNC Tx Q-omni directions
- Once the best beacon direction is identified, DEV will acquire knowledge of:
  - PNC Tx Q-omni direction identifier
  - Number of PNC Tx & Rx Q-omni directions
  - Number of S-CAPs and S-Associations in current superframe
- The best PNC Tx direction will be used for further communications with the PNC.

Association

- DEV transmits an Association Request command during each association S-CAP and in each Q-omni Tx direction until it receives a response. DEV informs PNC of best PNC Tx direction
- Upon successful association, DEV and PNC have knowledge of a working transmit and receive pair of Q-omni directions
- The working two pairs can be transformed into best pairs by enabling Q-omni training (see next slide)
- The best Q-omni pairs are used for any further data and command exchange in CAP

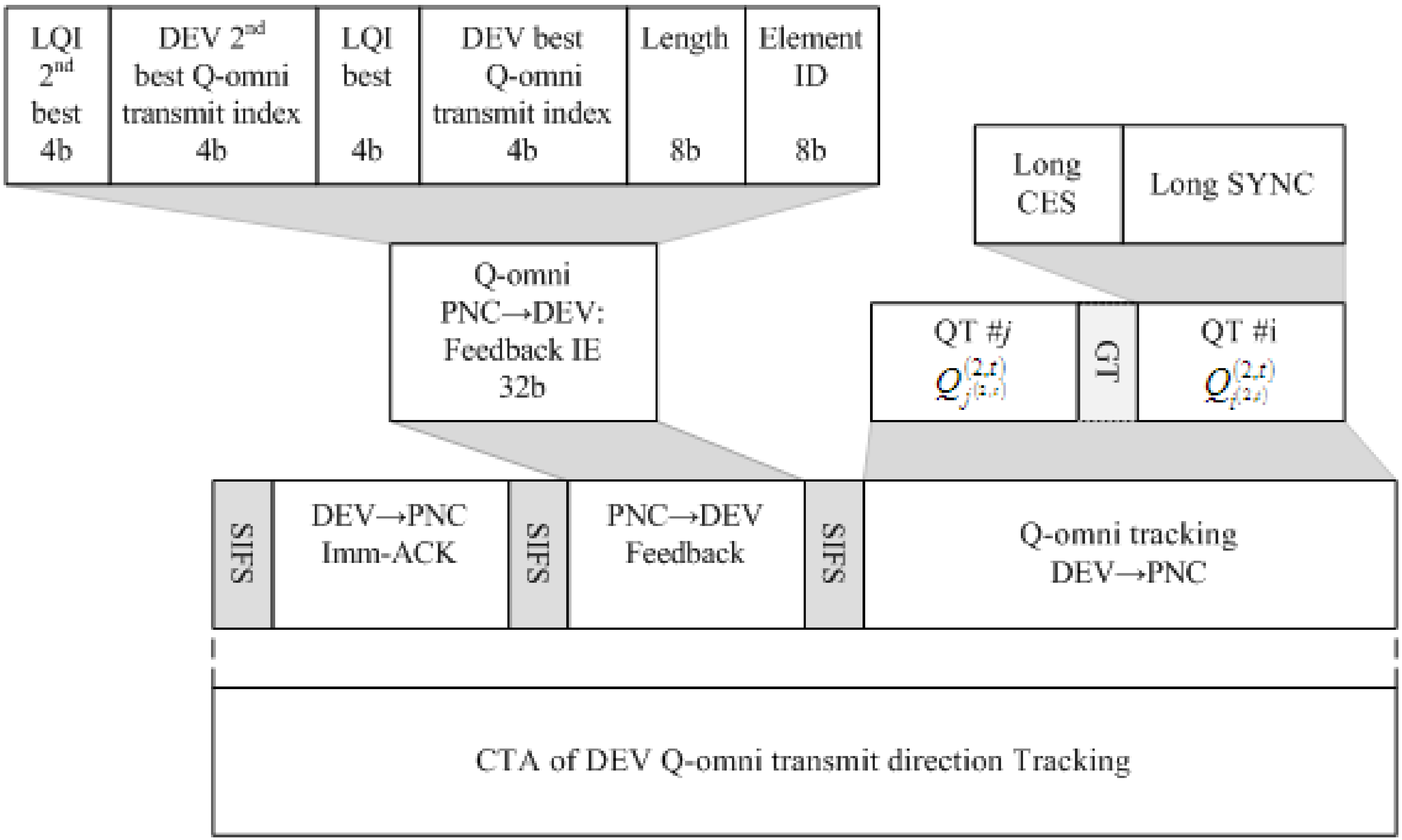
DEV Q-omni training

- PNC finds a DEV's best and 2nd best Q-omni directions



DEV Q-omni Tracking

- PNC tracks a DEV’s best and 2nd best Q-omni directions



Appendix I
Beamforming System Model

Beamforming System Model

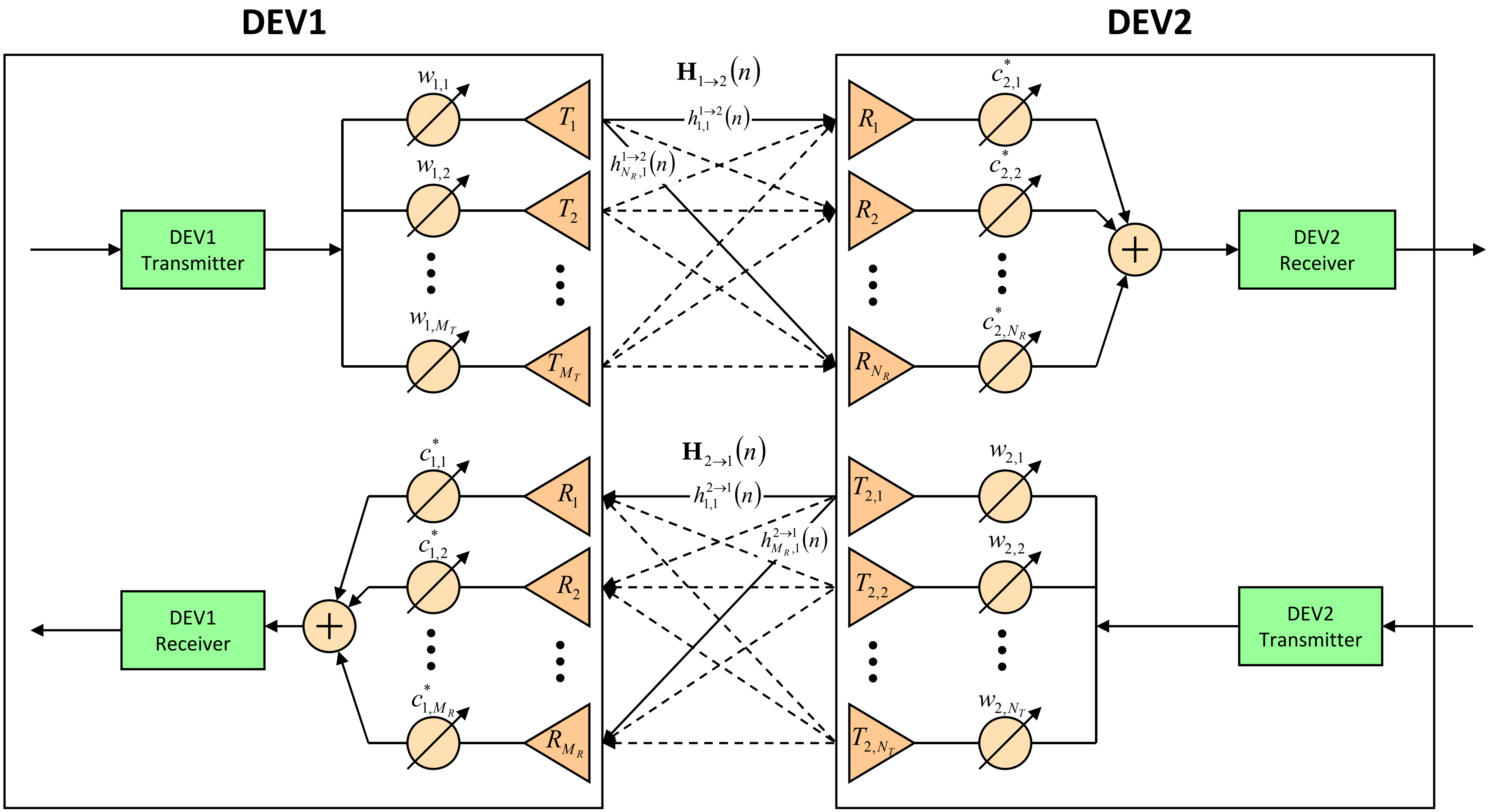
- Antenna Symmetric System (ASS): DEV1→DEV2 channel is the same as DEV2 → DEV1 channel, otherwise we have an Antenna Asymmetric System (AAS). ASS is a special case of AAS;

- Consider a link between two AAS devices, DEV1 and DEV2, as shown in next slide:
  - DEV1 has M_{T} Transmit antennas and M_{R} Receive antennas;
  - DEV2 has N_T transmit antennas and N_R receive antennas;

- The developed model makes no assumptions regarding the antenna arrangements whether these are 1-D or 2-D antenna arrays, or any other arrangement;

- A cyclic prefixed OFDM system with an FFT length of N subcarriers and a cyclic prefixed Single Carrier (SC) system with a burst length of N have the same system model. It is assumed that the cyclic prefix is longer than any multipath delay spread between any pair of antenna elements;

Beamforming System Model



Beamforming System Model

- Consider an OFDM symbol (SC burst) transmitted from DEV1 to DEV2:

$$x(t)=\sum_{k=0}^{N-1} s_k \delta(t-kT_c)$$

where T_c is the sample (chip) duration and s_k are the complex data $s_k \in C$

- At DEV1's transmitter, the bit stream is modulated by the ***beamformer*** *vector*:

$$\mathbf{w}_1=\begin{bmatrix} w_{1,1} & w_{1,2} & \cdots & w_{1,M_T} \end{bmatrix}^T$$

and then transmitted into a MIMO channel with a frequency domain Channel State Information (CSI) $\mathbf{H}_{1\to 2}(n) \in C^{M_T \times N_R}$ at frequency bin number n:

$$\mathbf{H}_{1\to 2}(n)=\begin{bmatrix} h_{1,1}^{1\to 2}(n) & h_{1,2}^{1\to 2}(n) & \cdots & h_{1,N_R}^{1\to 2}(n) \\ h_{2,1}^{1\to 2}(n) & h_{2,2}^{1\to 2}(n) & \cdots & h_{2,N_R}^{1\to 2}(n) \\ \vdots & \vdots & \ddots & \vdots \\ h_{M_T,1}^{1\to 2}(n) & h_{M_T,2}^{1\to 2}(n) & \cdots & h_{M_T,N_R}^{1\to 2}(n) \end{bmatrix}$$

where $h_{i,j}(n)$ denotes the cascade of the transmit/receive filtering, and the channel response between DEV1 *j*-th transmit antenna and DEV2 *i*-th receive antennas;

Beamforming System Model

- At DEV2's receiver, the received signals are processed through the ***combiner*** *vector*:

$$\mathbf{c}_2 = \begin{bmatrix} c_{2,1} & c_{2,2} & \cdots & c_{2,N_R} \end{bmatrix}^T$$

- The equivalent channel between DEV1's transmitter and DEV2's receiver is a Single Input Single Output (SISO) channel, with frequency response at frequency bins 0 to *N*-1 given by:

$$P_n = \mathbf{c}_2^H \mathbf{H}_{1\to 2}(n)\mathbf{w}_1 \quad \text{for} \quad n = 0,1,\ldots,N-1$$

- The discrete-frequency received signal model becomes:

$$Y_n = P_n S_n + B_n \quad \text{for} \quad n = 0,1,\ldots,N-1$$

where $\begin{bmatrix} S_0 & S_1 & \cdots & S_N \end{bmatrix}$ is the OFDM data symbol (FFT of the SC data burst), and $\begin{bmatrix} B_0 & B_1 & \cdots & B_N \end{bmatrix}$ is the additive white Gaussian noise vector;

- The equivalent model from DEV2's transmitter to DEV1's receiver have the same formulation but with a different SISO channel (see previous figure):

$$Q_n = \mathbf{c}_1^H \mathbf{H}_{2\to 1}(n)\mathbf{w}_2 \quad \text{for} \quad n = 0,1,\ldots,N-1$$

Beamforming Cost Function

- For both OFDM and SC, the Signal to Noise Ratio (SNR) on the *n*th subcarrier is given by:

$$SNR_n^{1\to 2} = \frac{E_s|P_n|^2}{N_0} = \frac{E_s\left|\mathbf{c}_2^H\mathbf{H}_{1\to 2}(n)\mathbf{w}_1\right|^2}{N_0}, \quad SNR_n^{2\to 1} = \frac{E_s|Q_n|^2}{N_0} = \frac{E_s\left|\mathbf{c}_1^H\mathbf{H}_{2\to 1}(n)\mathbf{w}_2\right|^2}{N_0}$$

- Define the Effective SNR (ESNR) as a mapping from the instantaneous subcarriers SNRs to an equivalent SNR that takes into account the FEC (Forward Error Correction). There are many methods that can be used to compute the ESNR:
  - Mean of SNRs over the different subcarriers
  - QSM (Quasi-Static Method: 3GPP2 1xEV-DV/DO method 1)
  - CESM (Capacity effective SINR mapping: 3GGP2 1xEV-DV/DO method 2)
  - CECM (CESM based on Convex Metric: 3GGP2 1xEV-DV/DO method 3)
  - EESM (Exponential Effective SIR Mapping: 3GGP)
  - Etc,...

- *The ultimate objective of a beamforming algorithm is to select the optimal beamformer vectors* $\mathbf{w}_1$(DEV1) and $\mathbf{w}_2$(DEV2) *and optimal combiner vectors* $\mathbf{c}_2$(DEV2) and $\mathbf{c}_1$(DEV1) *that maximize the effective SNR (or any other optimality criterion);*

Beamforming Cost function

- Different ESNR methods can be used for SC and OFDM. A MMSE SC equalizer for example tends to have an ESNR which can be approximated by the average of the SNRs over the different subcarriers, whereas OFDM tends to have an ESNR which can be approximated by the geometric mean of the SNRs over the different subcarriers. More accurate methods that we listed before take into account the FEC, imperfections in the receiver, BER, …

- For an ASS we have the following:

$$\mathbf{c}_1 = \mathbf{w}_1 \quad \text{and} \quad \mathbf{c}_2 = \mathbf{w}_2$$

- And consequently, it is enough to consider one direction of the link, i.e. DEV1-Tx to DEV2-Rx or DEV2-Tx to DEV1-Rx but not both;

- For an AAS, both directions of the link should be considered to estimate the four vectors $\mathbf{w}_1$(DEV1), $\mathbf{w}_2$(DEV2), $\mathbf{c}_1$(DEV1), and $\mathbf{c}_2$(DEV2);

- So for the general case of AAS, the following two steps are required:
  - 1. DEV1-Tx → DEV2-Rx, in order to estimate $\mathbf{w}_1$(DEV1) and $\mathbf{c}_2$(DEV2), DEV2 has to acquire the CSI matrices $\mathbf{H}_{1\to 2}(n)$ for $n = 0,1,\ldots,N-1$;
  - 2. DEV2-Tx → DEV1-Rx, in order to estimate $\mathbf{w}_2$(DEV2) and $\mathbf{c}_1$(DEV1), DEV1 has to acquire the CSI matrices $\mathbf{H}_{2\to 1}(n)$ for $n = 0,1,\ldots,N-1$

Appendix II
Beam Codebooks

Beam & Sector Codebooks

- Beam codebooks for *N* antennas and *M* beams (*M* ≥ *N*) can be generated as follows:

$$W(n,m) = j^{\mathrm{fix}\left[\frac{n \times \mathrm{mod}(m+(M/2),M)}{(M/4)}\right]} \quad \text{for } n = 0 : N-1 \text{ and } m = 0 : M-1$$

- Sector (multi-beam) codebooks for *N* antennas and *M* = *N*/2 beams can be generated as follows:

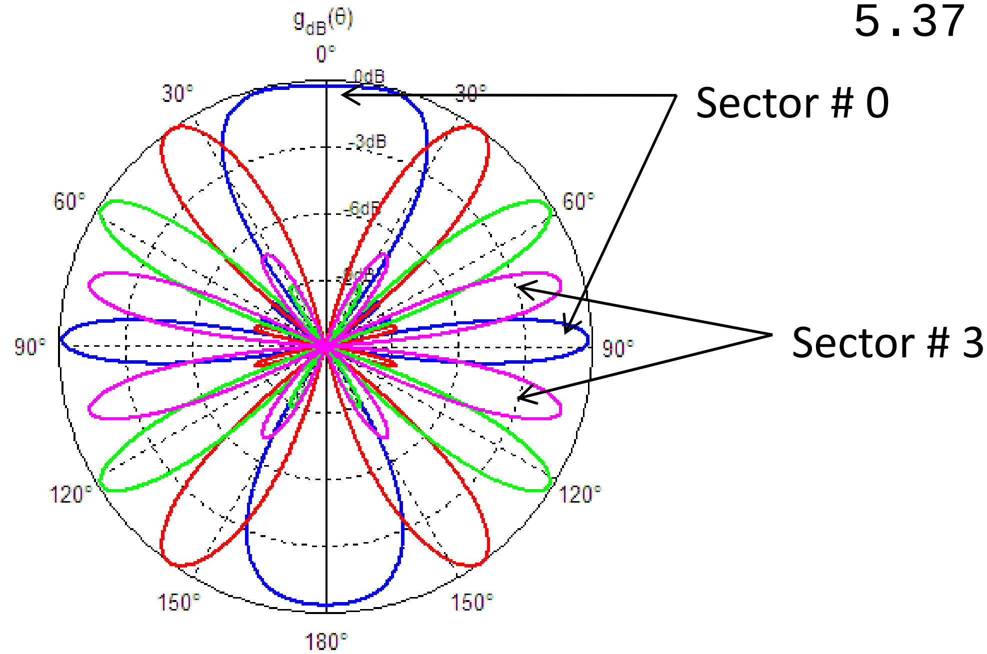
$$W(n,m) = \begin{cases} (-j)^{\mathrm{mod}(n,2)} & m = 0 \\ (-1)^{\mathrm{fix}\left[\frac{n \times \mathrm{mod}(m+(M/2),M)}{(M/2)}\right]} & n = 0 : N-1 \text{ and } m = 1 : M-1 \end{cases}$$

- Fix can be substituted with round().

Sectors

- Ex: Linear array with 8 elements:
- Sector Level: Four sectors (each sector = 2 beams):

- r = 1, Dthe_deg= 53.54, TheMax_deg = 13.78, GdB = 6.22
- r = 2, Dthe_deg= 20.35, TheMax_deg = 35.69, GdB = 6.36
- r = 3, Dthe_deg= 14.75, TheMax_deg = 58.07, GdB = 6.22
- r = 4, Dthe_deg= : 5.37

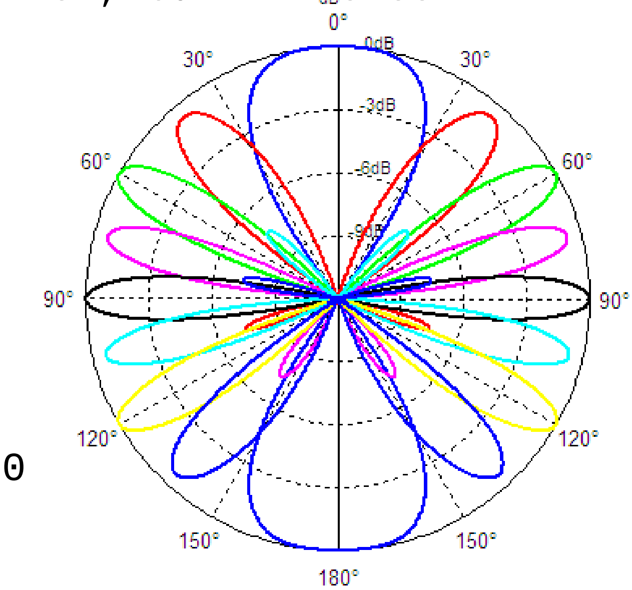


- +1 +1 +1 +1
- -j -1 -1 -1
- +1 +1 -1 -1
- -j -1 +1 -1
- +1 -1 +1 -1
- -j +1 -1 +1
- +1 -1 -1 +1
- -j +1 +1 +1

Beams

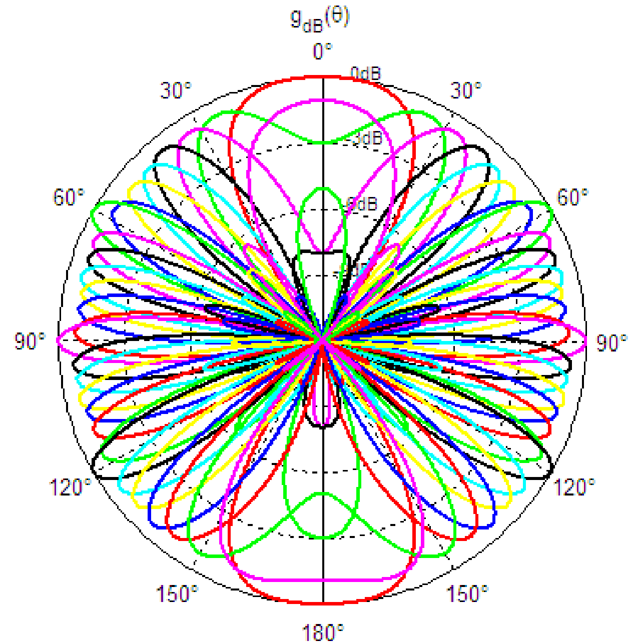
- Beam level: 8 beams
- r = 1, Dthe_deg= 54.63, TheMax_deg = 0.00, GdB = 9.03
- r = 2, Dthe_deg= 20.13, TheMax_deg = 40.32, GdB = 8.38
- r = 3, Dthe_deg= 14.85, TheMax_deg = 60.01, GdB = 9.03
- r = 4, Dthe_deg= 13.19, TheMax_deg = 74.79, GdB = 8.38
- r = 5, Dthe_deg= 12.81, TheMax_deg = 89.98, GdB = 9.03
- r = 6, Dthe_deg= 13.13, TheMax_deg = 103.74, GdB = 8.38
- r = 7, Dthe_deg= 14.85, TheMax_deg = 119.99, GdB = 9.03
- r = 8, Dthe_deg= 19.32, TheMax_deg = 137.52, GdB = 8.38

- +1 +1 +1 +1 +1 +1 +1 +1
- -1 -1 -j -j +1 +1 +j +j
- +1 +j -1 -j +1 +j -1 -j
- -1 -j +j -1 +1 +j -j +1
- +1 -1 +1 -1 +1 -1 +1 -1
- -1 +1 -j +j +1 -1 +j -j
- +1 -j -1 +j +1 -j -1 +j
- -1 +j +j +1 +1 -j -j -1
- condition number = 1.000, GSUM = 69.63710



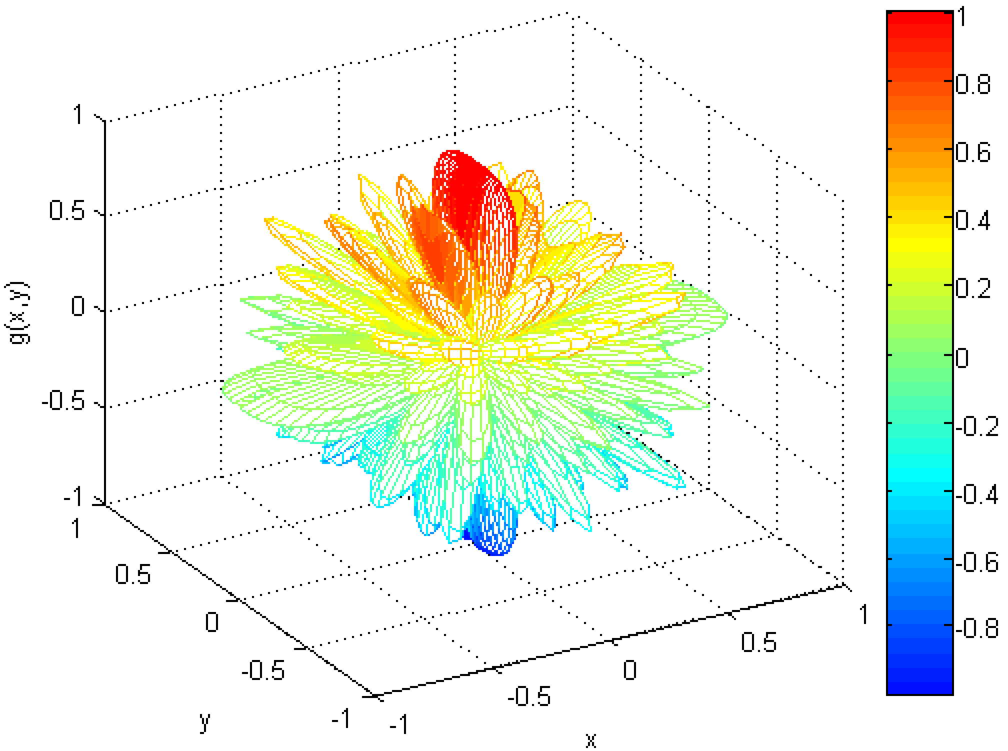
High Resolution Beams

- HR Beams: 32 beams
  - r = 1, Dthe_deg= 54.63, TheMax_deg =   0.00, GdB =   9.03
  - r = 2, Dthe_deg= 54.63, TheMax_deg =   0.00, GdB =   9.03
  - r = 3, Dthe_deg= 46.04, TheMax_deg = 25.16, GdB =   8.39
  - r = 4, Dthe_deg= 24.85, TheMax_deg = 32.91, GdB =   8.30
  - r = 5, Dthe_deg= 20.13, TheMax_deg = 40.32, GdB =   8.38
  - r = 6, Dthe_deg= 18.13, TheMax_deg = 45.38, GdB =   8.18
  - r = 7, Dthe_deg= 16.60, TheMax_deg = 50.88, GdB =   8.18
  - r = 8, Dthe_deg= 16.19, TheMax_deg = 57.42, GdB =   8.25
  - r = 9, Dthe_deg= 14.85, TheMax_deg = 60.01, GdB =   9.03
  - r = 10, Dthe_deg= 14.85, TheMax_deg = 60.01, GdB =   9.03
  - r = 11, Dthe_deg= 13.69, TheMax_deg = 66.11, GdB =   8.39
  - r = 12, Dthe_deg= 13.50, TheMax_deg = 70.17, GdB =   8.30
  - r = 13, Dthe_deg= 13.19, TheMax_deg = 74.79, GdB =   8.38
  - r = 14, Dthe_deg= 13.03, TheMax_deg = 78.33, GdB =   8.18
  - r = 15, Dthe_deg= 12.91, TheMax_deg = 82.48, GdB =   8.18
  - r = 16, Dthe_deg= 13.63, TheMax_deg = 87.80, GdB =   8.25
  - r = 17, Dthe_deg= 12.81, TheMax_deg = 89.98, GdB =   9.03
  - r = 18, Dthe_deg= 12.81, TheMax_deg = 89.98, GdB =   9.03
  - r = 19, Dthe_deg= 12.56, TheMax_deg = 95.45, GdB =   8.39
  - r = 20, Dthe_deg= 12.85, TheMax_deg = 99.24, GdB =   8.30
  - r = 21, Dthe_deg= 13.13, TheMax_deg = 103.74, GdB =   8.38
  - r = 22, Dthe_deg= 13.38, TheMax_deg = 107.30, GdB =   8.18
  - r = 23, Dthe_deg= 13.81, TheMax_deg = 111.64, GdB =   8.18
  - r = 24, Dthe_deg= 15.47, TheMax_deg = 117.49, GdB =   8.25
  - r = 25, Dthe_deg= 14.85, TheMax_deg = 119.99, GdB =   9.03
  - r = 26, Dthe_deg= 14.85, TheMax_deg = 119.99, GdB =   9.03
  - r = 27, Dthe_deg= 15.69, TheMax_deg = 126.52, GdB =   8.39
  - r = 28, Dthe_deg= 17.10, TheMax_deg = 131.34, GdB =   8.30
  - r = 29, Dthe_deg= 19.32, TheMax_deg = 137.52, GdB =   8.38
  - r = 30, Dthe_deg= 22.10, TheMax_deg = 142.90, GdB =   8.18
  - r = 31, Dthe_deg= 29.60, TheMax_deg = 150.34, GdB =   8.18
  - r = 32, Dthe_deg= 55.85, TheMax_deg = 164.09, GdB =   8.25
  - +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1 +1
  - -1 -1 -1 -1 -1 -1 -1 -1 -j -j -j -j -j -j -j -j +1 +1 +1 +1 +1 +1 +1 +1 +j +j +j +j +j +j +j +j
  - +1 +1 +1 +1 +j +j +j +j -1 -1 -1 -1 -j -j -j -j +1 +1 +1 +1 +j +j +j +j -1 -1 -1 -1 -j -j -j -j
  - -1 -1 -1 -j -j -j +1 +1 +j +j +j -1 -1 -1 -j -j +1 +1 +1 +j +j +j -1 -1 -j -j -j +1 +1 +1 +j +j
  - +1 +1 +j +j -1 -1 -j -j +1 +1 +j +j -1 -1 -j -j +1 +1 +j +j -1 -1 -j -j +1 +1 +j +j -1 -1 -j -j
  - -1 -1 -j -j +1 +j +j -1 -j -j +1 +1 +j -1 -1 -j +1 +1 +j +j -1 -j -j +1 +j +j -1 -1 -j +1 +1 +j
  - +1 +1 +j -1 -j -j +1 +j -1 -1 -j +1 +j +j -1 -j +1 +1 +j -1 -j -j +1 +j -1 -1 -j +1 +j +j -1 -j
  - -1 -1 -j +1 +j -1 -j +1 +j +j -1 -j +1 +j -1 -j +1 +1 +j -1 -j +1 +j -1 -j -j +1 +j -1 -j +1 +j



Planar Array

- Planar arrays can be obtained from 1-D linear arrays by specifying the codebooks along the z-axis and x-axis .
- Ex: Planar array 8x8 showing all 64 beams



Planar Array

- Contour of planar array 8x8 showing all 64 beams