IEEE Houston Section
Continuing Education On Demand

Seminar

Night 1 and Night 2

Presentation Code: 505
September 29-30, 2015

Basic Power System and Symmetrical Components Calculations Review

BASIC POWER SYSTEM AND SYMMETRICAL COMPONENTS CALCULATIONS - REVIEW

- **Synopsis:**

  Electrical Engineering basis of analysis, formulas, data, per unit calculations, "rule of thumb." Review of Fortescue theory and critical assumptions, unbalance system analysis (faults and voltage unbalance). Application, concerns and examples. Phase domain analysis of unbalance systems and comparison with Symmetrical Components methodology.

AGENDA

- What are the “power system calculations”
- Why basic calculations/shortcuts?
- Fundamentals
- The Per Unit Method
- Fortescue Theory
- Symmetrical Components
- Faults and Sequence Networks
- Component Modeling
- Phase Domain Modeling
- Where to Find Data for Calculations

WHAT ARE THE
"POWER SYSTEM CALCULATIONS"

WHAT ARE THE "POWER SYSTEM CALCULATIONS"

- A **calculation** is a deliberate process for transforming one or more inputs into one or more results, with variable change
- Electrical network (circuits) theory with simplifications or "easy" electrical engineering
- Complex math or degenerated form complex math

WHY BASIC CALCULATIONS/SHORTCUTS?

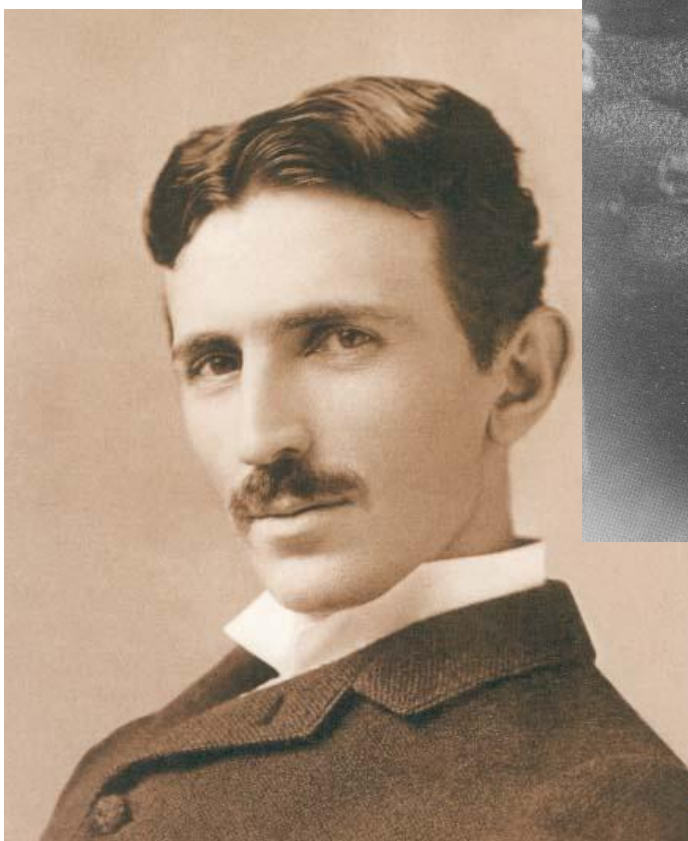
WHY BASIC CALCULATIONS/SHORTCUTS?

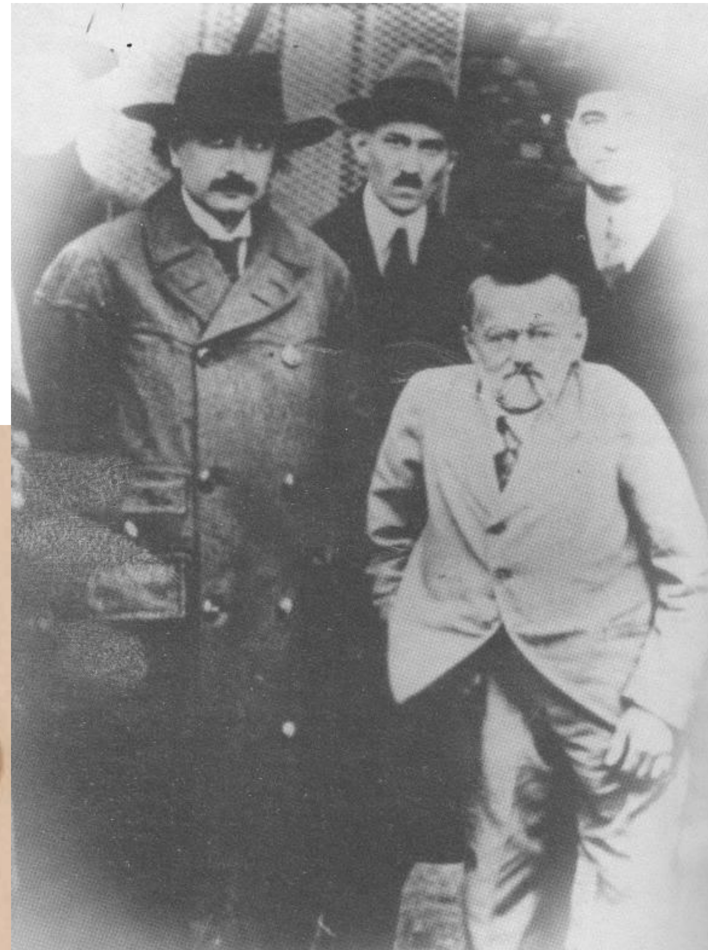
- History
  - Overwhelming
  - Lack of tools
  - Answer: speed and quality
- Approximate solution(s)
- Data interpretation
- Solution evaluation and interpretation

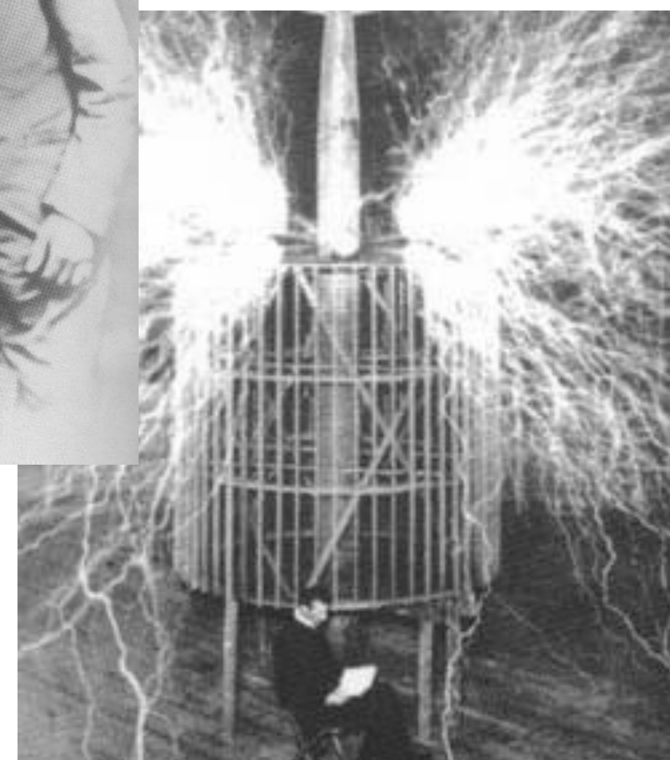
WHY BASIC CALCULATIONS/SHORTCUTS?

Most common causes of errors in circuit analysis:

- Failure to use a valid analytical procedures
- Misapplication of “cookbook” method(s)
- Improper use of a valid solution method
- Inaccurate simplifying assumption
- Improper model







FUNDAMENTALS

FUNDAMENTALS

- Definitions, acronyms, tagging and symbols
- Linearity and superposition
- Base elements and related equations, Ohm's Law
- Kirchhoff's laws
- The Thevenin and Norton equivalent circuit
- DC and AC
- The per unit method
- The symmetrical components analysis and related
- Some complex math
- The sinusoidal forcing function
- The phasors representation
- The single phase equivalent circuit

DEFINITION ACRONYMS, TAGGING AND SYMBOLS

$u(t), i(t)$	- instantaneous AC values of voltage and current at particular time t
$u_{DC}(t)$, $i_{DC}(t)$	- instantaneous DC value of voltage and current at particular time t
U_m, I_m	- root-mean-square (rms), one-cycle values of the sinusoidal voltage and current waveforms, or maximum values
U, I	- root-mean-square (rms) voltage or current; module value of vector
U_p, I_p	- peak or crest (ANSI) voltage
$\underline{U}$, $\underline{I}$, $\underline{S}$	- complex numbers or vectors
ω	- angular line frequency; $\omega = 2 \cdot \pi f$ with $f \equiv 60Hz$ system frequency for the signal
α	- "shift" angle in relation to assumed origin; in this case it is assumed inrelation to voltage; shift depends on type of load (resistive, capacitive, inductive) in the system; negative sign in front of the α reflects assumed convention for the circuit analysis;

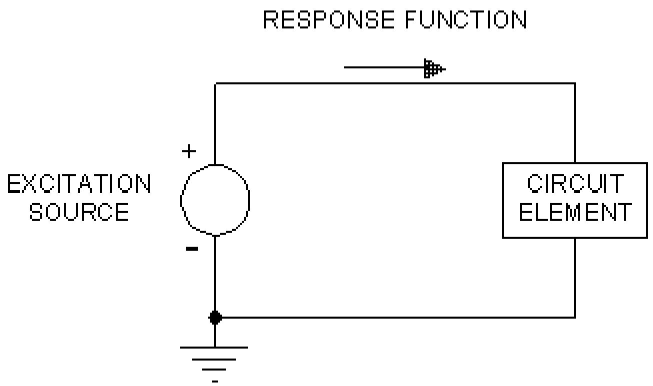
$$\alpha = \operatorname{atan}\left(\frac{\omega \cdot L}{R}\right) = \operatorname{atan}\left(\frac{X}{R}\right)$$

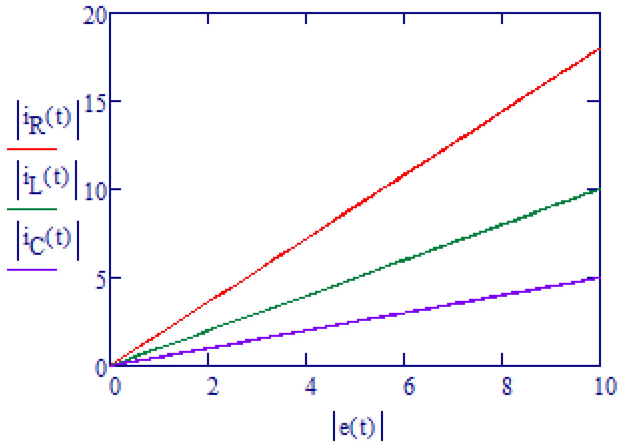
DEFINITION ACRONYMS, TAGGING AND SYMBOLS

R, L, C	- lumped resistance, inductance and capacitance
r, l, c	- distributed resistance, inductance and capacitance
j, i	- imaginary operator
η, eff	- efficiency
a	- 120 degree symmetrical component rotational operator
$\underline{\mathbf{Z}}$	- impedance; complex number (vector) i.e. $\underline{\mathbf{Z}} = R + j(X_l - X_c)$
$\lvert\underline{\mathbf{Z}}\rvert$, Z	- module value of impedance (scalar)
X_c	- capacitive reactance
X_l	- inductive reactance
U_n, I_n	- nominal system or apparatus parameters line-line voltage or line current (ANSI); rating of the system or apparatus are defined in standards - (IEC) nominal system line voltage
U_R, I_R	- (IEC) rated equipment line-line voltage or line current

LINEARITY

- DC Circuits:
  - The current doubles if the voltage is doubled.
- AC Circuits:
  - The frequency of the driving voltage is held constant, the current doubles if the voltage is doubled.

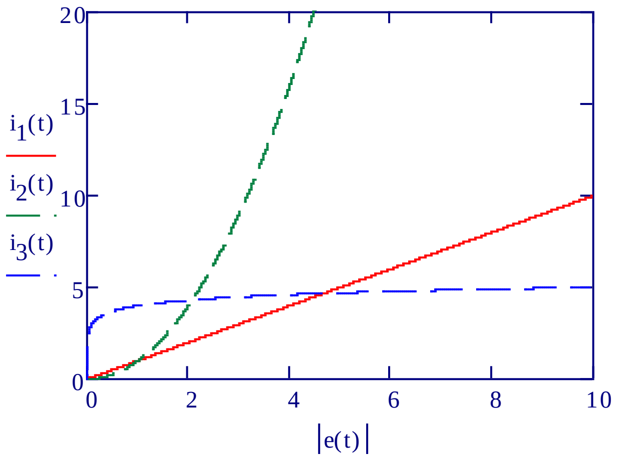




In this example, excitation is sinusoidal ideal voltage source and circuits elements are resistance, reactance or capacitance.

LINEARITY

- For the chosen excitation function of voltage and the chosen response function of current, both hidden-GREEN and hidden-BLUE are examples of the response characteristic of a nonlinear element.



LINEARITY

- With the circuit element represented by any of the response curves shown in Figures, the circuit will, in general, become nonlinear for a different response function (for example Power)
- An important limitation of linearity, therefore, is that it applies only to responses that are linear for the circuit conditions described (that is, a constant impedance circuit will yield a current that is linear with voltage).
- This restraint must be recognized in addition to the previously mentioned limitations of constant source excitation frequency for AC circuits and constant circuit element impedances for AC or DC circuits. Excitation sources, if not independent, must be linearly dependent. This restraint forces a source to behave just as would a linear response (which, by definition, is also linearly dependent).

SUPERPOSITION

- This very powerful principle is a direct consequence of linearity and can be stated as follows:
  - In any linear network containing several DC or fixed frequency AC excitation sources (voltages), the total response (current) can be calculated by algebraically adding all the individual responses caused by each independent source acting alone. All other sources inactivated (voltage sources shorted by their internal impedances, current sources opened). The equation written is for the sum of the currents from each individual source *V*1 and *V*2. Although Figure also illustrates a way this principle might actually be used, more often its main application is in support of other calculating methods. The only restraint associated with superposition is that the **network should be linear**. All limitations associated with linearity apply.

SUPERPOSITION

- Only applies to linear circuits and elements
- Best explained by example:
  - Example 1

SUPERPOSITION

- Example 1 – AC and AC Sources

$$I_{V1} + I_{V2} = I_L$$

$$I_{V1} = I_{c1}$$

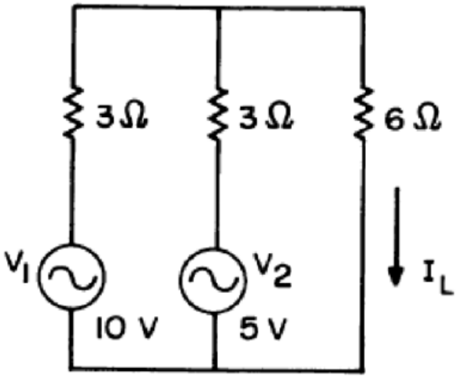
$$I_L = I_{c2}$$

$$\begin{pmatrix} Z_1 + Z_2 & -Z_2 \\ -Z_2 & Z_2 + Z_L \end{pmatrix} \cdot \begin{pmatrix} I_{c1} \\ I_{c2} \end{pmatrix} = \begin{pmatrix} V_1 - V_2 \\ V_2 \end{pmatrix}$$

$$I_{V1} := \frac{V_1}{\left(Z_1 + \|\left(Z_2, Z_L\right)\right)} \cdot \|\left(Z_2, Z_L\right) \cdot \frac{1}{Z_L} = 0.667\,A$$

$$I_{V2} := \frac{V_2}{\left(Z_2 + \|\left(Z_1, Z_L\right)\right)} \cdot \|\left(Z_1, Z_L\right) \cdot \frac{1}{Z_L} = 0.333\,A$$

$$I_L := I_{V1} + I_{V2} = 1\,A$$



SUPERPOSITION

- The non-applicability of superposition is why all but the very simplest nonlinear circuits are almost impossible to analyze using hand calculations.
- Although most real circuit elements are nonlinear to some extent, they can often be accurately represented by a linear approximation.
- Solutions to network problems involving such elements can be readily obtained. Problems involving complex networks having substantially nonlinear elements can practically be solved only through the use of certain simplification procedures, or **through the adjustment of calculated results to correct for nonlinearity. But both of these approaches can potentially lead to significant inaccuracy**. Tiresome iterative calculations performed in an instant by the digital computer make more accurate solutions possible when the nonlinear circuit elements can be described mathematically.

MORE ABOUT LINEARITY

- Linearization makes it possible to use tools for studying linear systems to analyze the behavior of a nonlinear function near a given point with certain restrictions.

- Based on the **Hartman-Grobman** or **Linearization Theorem,** it is an theorem about the local behavior of dynamical systems in the neighborhood of a hyperbolic fixed point.

- In simplicity, the theorem states that the behavior of a dynamical system near a hyperbolic fixed point is qualitatively the same as the behavior of its linearization near the origin. Therefore when dealing with such fixed points we can use the simpler linearization of the system to analyze its behavior.

MORE ABOUT LINEARITY

- Linearization makes it possible to use tools for studying linear systems to analyze the behavior of a nonlinear function near a given point with certain restrictions. The linearization of a function is the first order term of its Taylor expansion around the point of interest.
- For a system defined by the equation,

$$\frac{d\mathbf{x}}{dt} = \mathbf{F}(\mathbf{x}, t)$$

the linearised system can be written as for example

$$\frac{d\mathbf{x}}{dt} = D\mathbf{F}(\mathbf{x_0}, t) \cdot (\mathbf{x} - \mathbf{x_0})$$

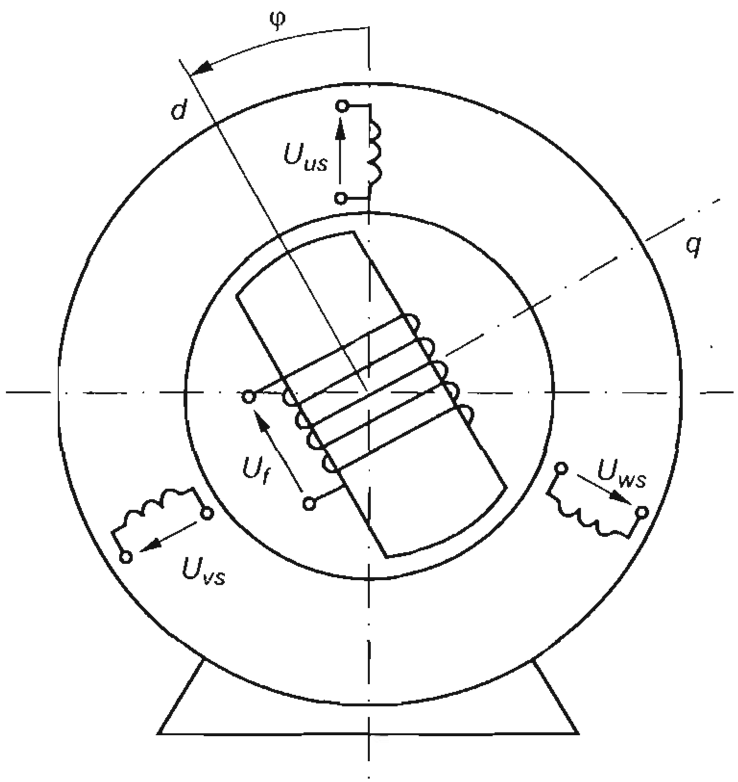
where: x_0 is the point of interest and D$F(x_0,t)$ *is* the Jacobian of $F(x)$ evaluated at point x_0.

MORE ABOUT LINEARITY

- Typical power system analysis tools steady-state / approximation of phasor methods i.e. use linearization models that are a simplified differential equation models around operating point of 50 or 60 Hz.
- Only electromagnetic transient software (EMTP) types use nonlinear differential equation for modeling with some approximation for some other attributes to find solution(s).
- How approximation is achieved is shown on example of Synchronous Machine

MORE ABOUT LINEARITY

- Synchronous Machine - General-Nonlinear Model



$$\mathbf{u}_s = \mathbf{R}_s \mathbf{i}_s + \frac{\mathrm{d}}{\mathrm{d}t} \mathbf{M}_{ss} \mathbf{i}_s + \frac{\mathrm{d}}{\mathrm{d}t} \mathbf{M}_{sf} \mathbf{i}_f$$

$$\mathbf{u}_f = \mathbf{R}_f \mathbf{i}_f + \frac{\mathrm{d}}{\mathrm{d}t} \mathbf{M}_{fs} \mathbf{i}_s + \frac{\mathrm{d}}{\mathrm{d}t} \mathbf{M}_{ff} \mathbf{i}_f$$

$$J \frac{\mathrm{d}^2 \varphi}{\mathrm{d}t^2} + k_d \frac{\mathrm{d}\varphi}{\mathrm{d}t} = T_m + T_e = T_m + \frac{1}{2} \frac{\partial}{\partial \varphi} [\mathbf{i}_{st} \ \mathbf{i}_f] \begin{bmatrix} \mathbf{M}_{ss} & \mathbf{M}_{sf} \\ \mathbf{M}_{fs} & \mathbf{M}_{ff} \end{bmatrix} \begin{bmatrix} \mathbf{i}_s \\ \mathbf{i}_f \end{bmatrix}$$

MORE ABOUT LINEARITY

- Synchronous Machine – Park's Model

$$\mathbf{T} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \cos\varphi & \cos(\varphi - 120^\circ) & \cos(\varphi - 240^\circ) \\ -\sin\varphi & -\sin(\varphi - 120^\circ) & -\sin(\varphi - 240^\circ) \end{bmatrix}$$

$$u_d = R_s i_d + \frac{\mathrm{d}}{\mathrm{d}t}\psi_d - \dot{\varphi}\psi_q$$

$$u_q = R_s i_q + \frac{\mathrm{d}}{\mathrm{d}t}\psi_q + \dot{\varphi}\psi_d$$

$$u_f = R_f i_f + \frac{\mathrm{d}}{\mathrm{d}t}\psi_f$$

$$J\frac{d^2\varphi}{dt^2} + k_d d\frac{\varphi}{dt} = T_e + T_m$$

$$T_e = \psi_d i_q - \psi_q i_d$$

MORE ABOUT LINEARITY

- Synchronous Machine – Steady State U_s=const

$$i_{us} = \sqrt{\frac{2}{3}}(I_d \cos\varphi - I_q \sin\varphi) = \frac{U_{sm}\cos\vartheta - E_{fm}}{X_d}\cos(\omega_0 t + \varphi_0) + \frac{U_{sm}\sin\vartheta}{X_q}\sin(\omega_0 t + \varphi_0) =$$

$$= \frac{U_{sm}\cos\vartheta - E_{fm}}{X_d}\sin(\omega_0 t + \vartheta) - \frac{U_{sm}\sin\vartheta}{X_q}\cos(\omega_0 t + \vartheta)$$

$$\begin{bmatrix} U_d \\ U_q \end{bmatrix} = \sqrt{\frac{3}{2}} U_{sm} \begin{bmatrix} \sin\vartheta \\ \cos\vartheta \end{bmatrix}$$

$$I_d = \sqrt{\frac{3}{2}} \frac{U_{sm}\cos\vartheta - E_{fm}}{X_d}$$

$$I_q = -\sqrt{\frac{3}{2}} \frac{U_{sm}}{X_q} \sin\vartheta$$

$$J\frac{d^2\varphi}{dt^2} + k_d d\frac{\varphi}{dt} = T_e + T_m$$

$$T_e = -\frac{1}{\omega_0}\left[\frac{3}{2}\frac{U_{sm}E_{fm}}{X_d}\sin\vartheta + \frac{3}{4}U_{sm}^2\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\vartheta\right]$$

MORE ABOUT LINEARITY

- Synchronous Machine – Steady State U_s=const

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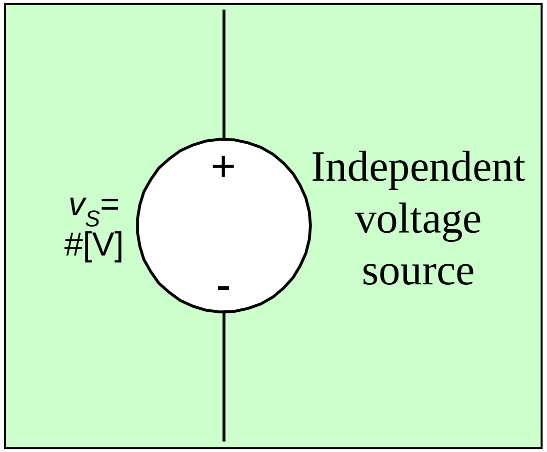
BASE ELEMENTS - INDEPENDENT SOURCES

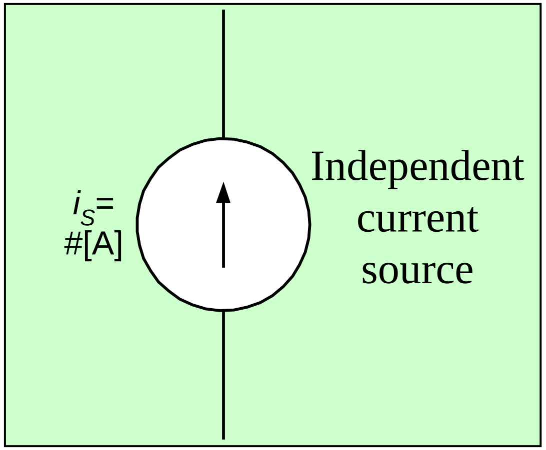
Voltage

- A voltage source maintains a voltage across its terminals no matter what you connect to those terminals

Current

- A current source is a two-terminal circuit element that maintains a current through its terminals



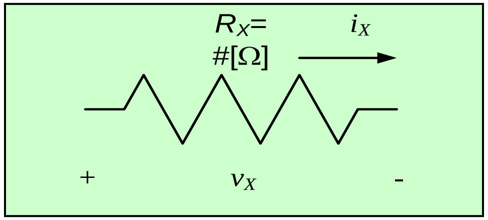


BASE ELEMENTS – R, L, C

Resistor

– Obeys the expression

$$R = \frac{v_R}{i_R}$$

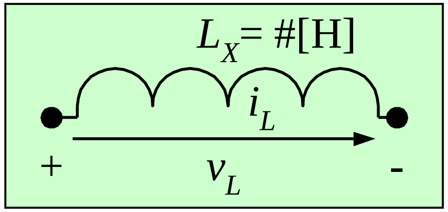


Inductor

– Obeys the expression

$$v_L = L_X \frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L_X}\int_{t_0}^{t} v_L(s)ds + i_L(t_0),$$



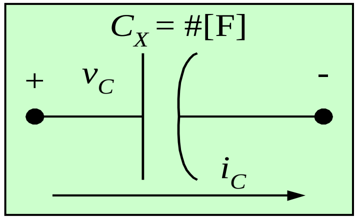
v_L is the voltage across the inductor, and i_L is the current through the inductor, L_X is called the inductance, and $i_L(t_0)$ is initial condition

Capacitor

– Obeys the expression

$$i_C = C_X \frac{dv_C}{dt}$$

$$v_C(t) = \frac{1}{C_X}\int_{t_0}^{t} i_C(s)ds + v_C(t_0),$$



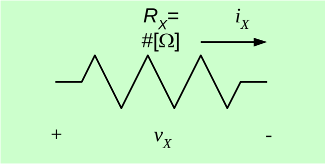
v_C is the voltage across the capacitor, and i_C is the current through the capacitor, C_X is called the capacitance, and $v_C(t_0)$ is initial condition

BASE ELEMENTS – R, L, C

Passive Sign Convention

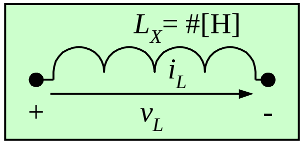
Resistor

$$R_X = \frac{v_X}{i_X}$$



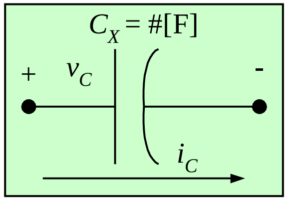
Inductor

$$v_L = L_X \frac{di_L}{dt}$$



Capacitor

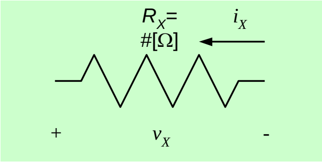
$$i_C = C_X \frac{dv_C}{dt}$$



Passive Sign Convention

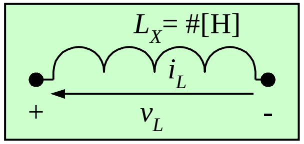
Resistor

$$R_X = -\frac{v_X}{i_X}$$



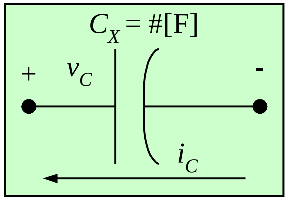
Inductor

$$v_L = -L_X \frac{di_L}{dt}$$



Capacitor

$$i_C = -C_X \frac{dv_C}{dt}$$



BASE ELEMENTS – R, L, C

	Resistor	Inductor	Capacitor
Energy Stored	N/A	$W_L = \frac{1}{2} \cdot L_x \cdot i_x^2$	$W_C = \frac{1}{2} \cdot C_x \cdot v_x^2$
Series Connection	$R_{EQ} = \sum_{i=1}^{N} R_i$	$L_{EQ} = \sum_{i=1}^{N} L_i$	$\frac{1}{C_{EQ}} = \sum_{i=1}^{N} \frac{1}{C_i}$
Parallel Connection	$\frac{1}{R_{EQ}} = \sum_{i=1}^{N} \frac{1}{R_i}$	$\frac{1}{L_{EQ}} = \sum_{i=1}^{N} \frac{1}{L_i}$	$C_{EQ} = \sum_{i=1}^{N} C_i$

KIRCHHOFF'S LAWS

- Kirchhoff's Current Law (KCL)
  - **The algebraic (or signed) summation of currents through any closed surface must equal zero.**

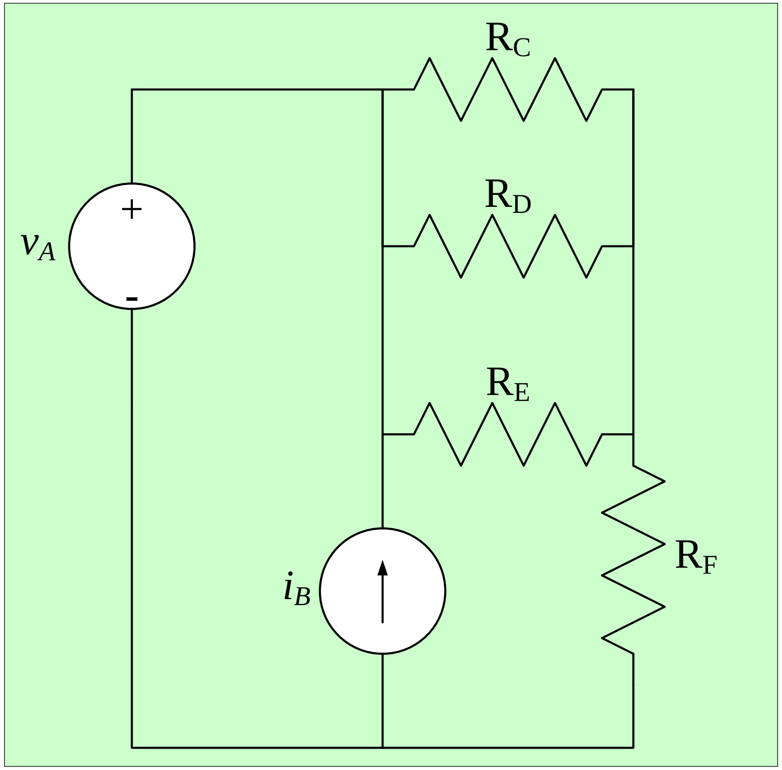
- Kirchhoff's Voltage Law (KVL)
  - **The algebraic (or signed) summation of voltages around any closed loop must equal zero.**

KIRCHHOFF'S LAWS

- Node

- Close Loop

  - Example 1
  - Example 2

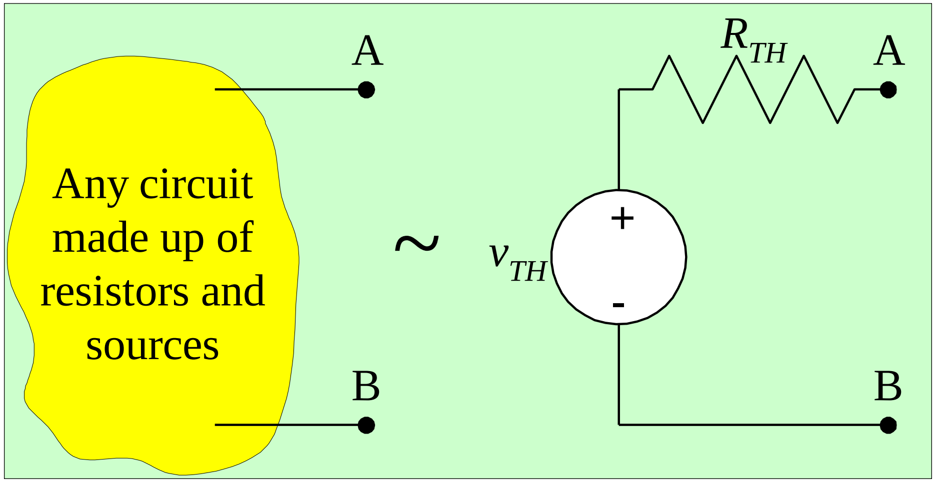


THE THEVENIN AND NORTON EQUIVALENT CIRCUITS

- Why?
  - Per unit calcs
  - Symmetrical components calcs
- Example of measurement, expectation, real value and explanation using Thevenin equivalent circuit

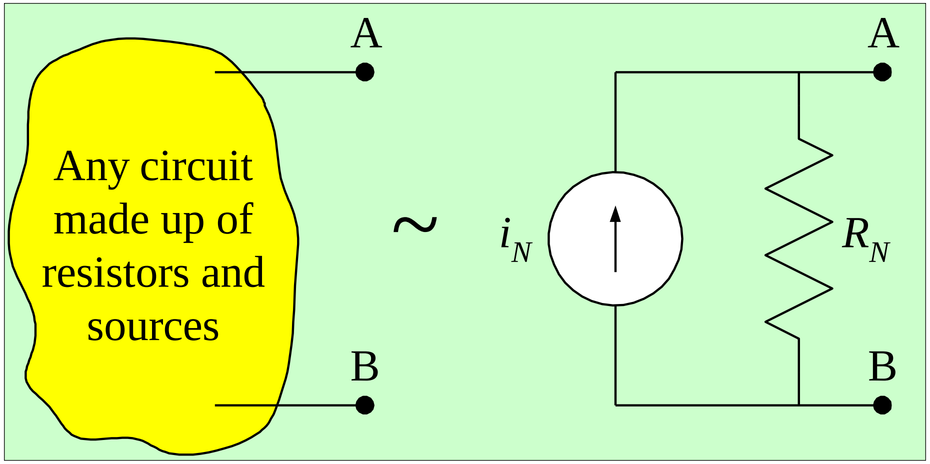
THE THEVENIN EQUIVALENT CIRCUITS

- Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a voltage source in series with a resistance.



THE NORTON EQUIVALENT CIRCUITS

- Any circuit made up of resistors and sources, viewed from two terminals of that circuit, is equivalent to a current source in parallel with a resistance.



SINUSOIDAL FORCING FUNCTION

- It is a most fortunate truth in nature that the excitation sources (driving voltage) for electrical networks, in general, have a sinusoidal character and can be represented by a sine wave type periodic functions
- A sinusoid is a *sine wave* or a *cosine wave*
- Sinusoids can represent many functions, but we will concentrate on voltages or currents, as a function of time
- The only restraint associated with the use of the sinusoidal forcing function concept is that the circuit must be comprised of linear elements, that is, *R* , *L* , and *C* are constant as current or voltage varies

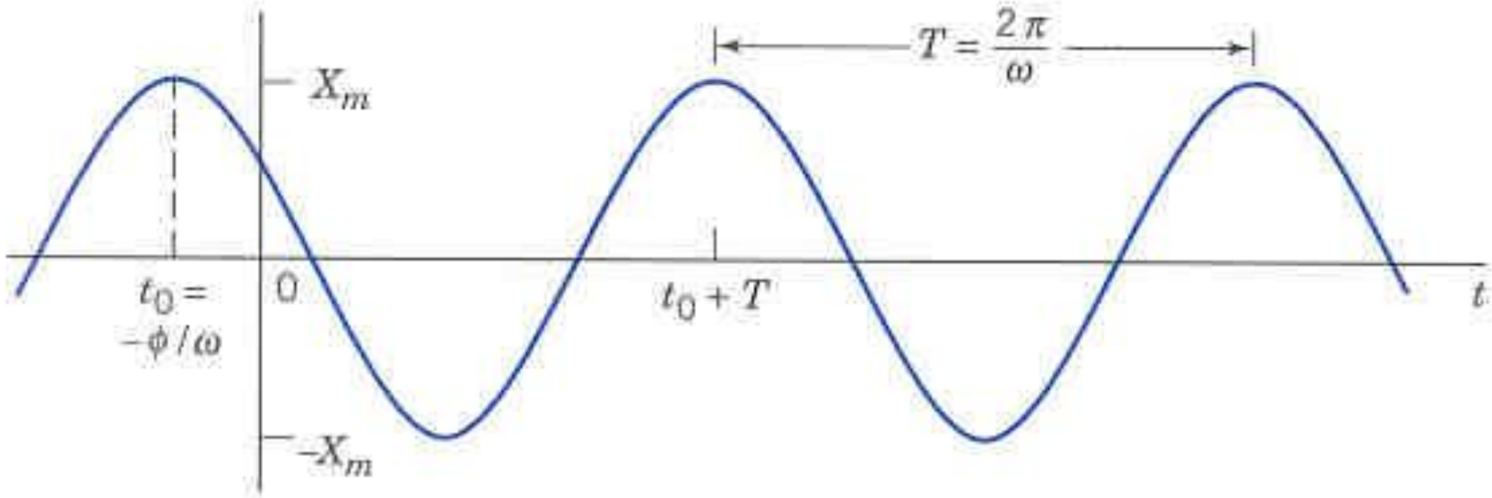
SINUSOIDAL FORCING FUNCTION

There are two important consequences of this circumstance:

– First, although the response (current) for a complex *R*, *L*, *C* network represents the solution to at least one second-order differential equation, the result will also be a sinusoid of the same frequency as the excitation and different only in magnitude and phase angle. The relative character of the current with respect to the voltage for simple *R*, *L*, and *C* circuits is also shown in previous figure.

– The second important concept is that when the sine wave shape of current is forced to flow in a general impedance network of *R*, *L*, and *C* elements, the voltage drop across each element will always exhibit a sinusoidal shape of the same frequency as the source. The sinusoidal character of all the circuit responses makes the application of the superposition technique to a network with multiple sources surprisingly manageable. The necessary manipulation of the sinusoidal terms is easily accomplished using the laws of vector algebra.

SINUSOIDAL FORCING FUNCTION

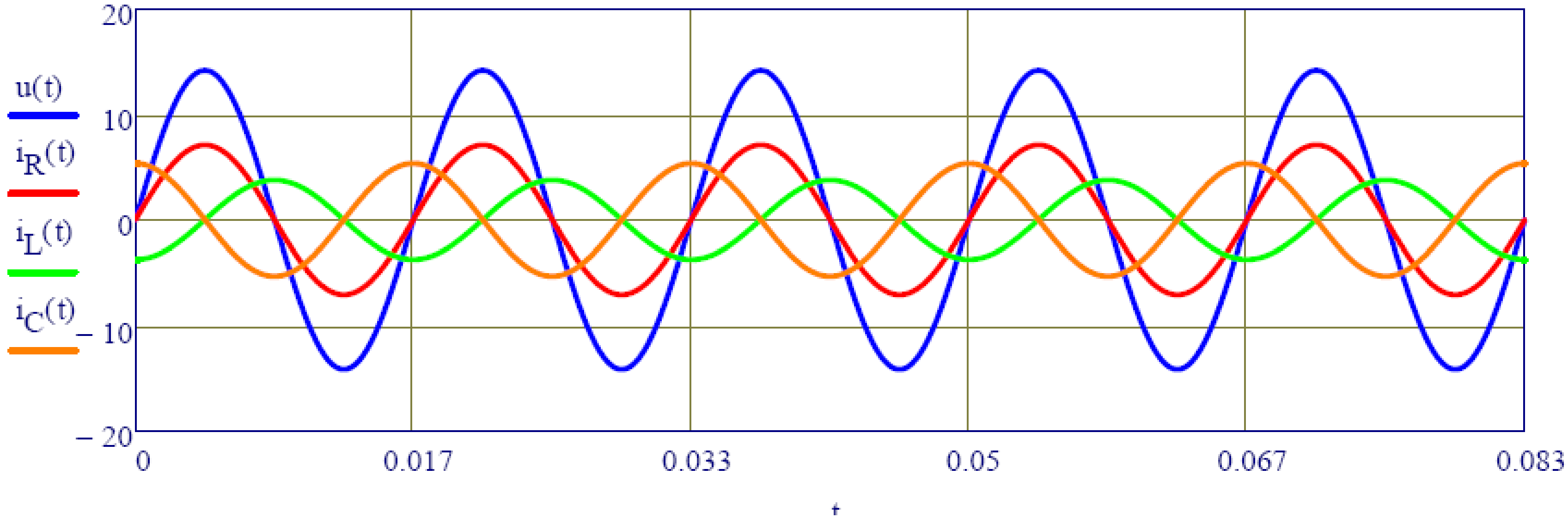


$$x(t) = X_m \cos(\omega t + \phi)$$

$$X_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} \left(x^2(t)\right) dt}.$$

$$X_{rms} = \frac{X_m}{\sqrt{2}}.$$

SINUSOIDAL FORCING FUNCTION



$$u(t) := \sqrt{2}\cdot U\cdot \sin(\omega\cdot t + \Theta)$$

$$i_R(t) := \sqrt{2}\cdot \frac{U}{R}\cdot \sin(\omega\cdot t + \Theta)$$

$$i_L(t) := \sqrt{2}\cdot \frac{U}{\omega\cdot L}\cdot \sin(\omega\cdot t + \Theta - 90deg)$$

$$i_C(t) := \sqrt{2}\cdot U\cdot C\cdot \omega\cdot \sin(\omega\cdot t + \Theta + 90deg)$$

SINUSOIDAL FORCING FUNCTION

- Limits
  - Liner elements
  - Independent on changes in voltage or current
- Results of the calculations contain sine wave type periodic functions
- Frequency of the calculations results sine wave type periodic functions is same as the source frequency

PHASORS

- A phasor is a transformation of a sinusoidal voltage or current to phase (or phasor; complex) domain
- Using phasors, and the techniques of phasor analysis, solving circuits with sinusoidal sources gets much easier, but…
- Only the steady state value of a solution is obtained with the phasor transform technique. Transient nature of sinusoidal voltage or current effect is lost in transformation.

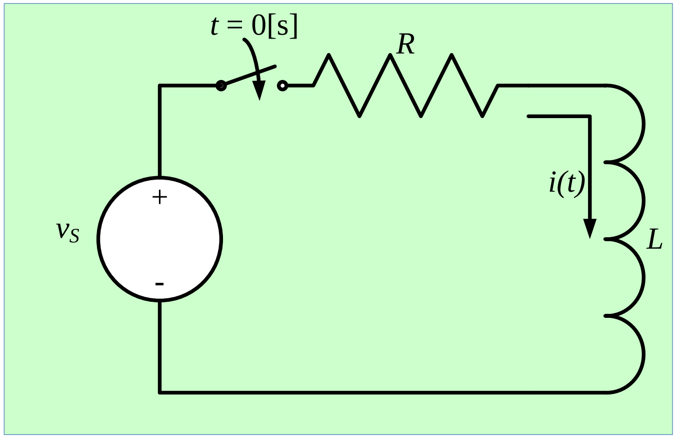
PHASORS

When forcing source is in form

$$v_S(t) = V_m \cos(\omega t + \phi).$$

solution to the circuit equation

$$V_m \cos(wt + \phi) = L\frac{di(t)}{dt} + i(t)R,$$



is in form

$$i(t) = \frac{-V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\phi - \tan^{-1}\frac{\omega L}{R}) e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}).$$

You can note that ***solution varies with time***….

PHASORS

You can note that solution varies with time…

$$i(t) = \frac{-V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\phi - \tan^{-1}\frac{\omega L}{R})e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t + \phi - \tan^{-1}\frac{\omega L}{R})$$

This part of the solution varies with time as a decaying exponential. In fact, you may recognize that it has a time constant $\tau = L/R$. After several time constants, it will die away and become relatively small. We call this part of the solution the **transient response**.

This part of the solution varies with time as a sinusoid. In fact, you may recognize that it is a sinusoid with the same frequency as the source, but with different amplitude and phase. This part of the solution does not die out with time. We call this part of the solution the **steady-state response**.

Only the steady state value of a solution is obtained with the phasor transform technique.

$$i_{SS}(t) = \frac{V_m}{\sqrt{R^2+\omega^2L^2}}\cos(\omega t + \phi - \tan^{-1}\frac{\omega L}{R})$$

PHASORS

Instantaneous value representation is not convenient and efficient for analysis of electrical circuits. This is why sinusoidal instantaneous voltages and currents as time functions are usually replaced with the phasor notation.

$$f(t) = K_m \cdot \sin(\omega \cdot t + \Phi) = \sqrt{2} \cdot K \cdot \sin(\omega \cdot t + \Phi) = \mathrm{Im}\left(\underline{K} \cdot e^{j \cdot \omega \cdot t}\right)$$

Where:

$\underline{K}$ - complex number; note vector notation (underlined K) with magnitude of $\sqrt{2} \cdot K$ or K_m and phase Φ and is defined as the phasor related to the sinusoidal function $f(t)$, thus

$$\underline{K} = \sqrt{2} \cdot K \cdot (\cos(\Phi) + j \cdot \sin(\Phi)) = \left(\sqrt{2} K\right) (\measuredangle) \; \Phi$$

In new format, current and voltage time function are represented by complex vectors in following way:

$$\underline{U} = U \; (\measuredangle) \; \Phi_u$$

$$\underline{I} = I \; (\measuredangle) \; \Phi_i$$

PHASORS

Consequently impedance is defined as:

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{\left[U\ (\measuredangle)\ \Phi_u\right]}{\left[I\ (\measuredangle)\ \Phi_i\right]}$$

and power is defined as:

$$\underline{S} = \underline{U} \cdot \bar{\underline{I}} = \left[U\ (\measuredangle)\ \Phi_u\right] \cdot \left[I\ (\measuredangle)\ -\Phi_i\right] = U \cdot I \cdot \cos\left(\Phi_u - \Phi_i\right) + j \cdot U \cdot I \cdot \sin\left(\Phi_u - \Phi_i\right)$$

with module value equal to:

$$\left|\underline{S}\right| = \sqrt{\left[\left(U \cdot I \cdot \cos\left(\Phi_u - \Phi_i\right)\right)^2 + \left(j \cdot U \cdot I \cdot \sin\left(\Phi_u - \Phi_i\right)\right)^2\right]} = S = U \cdot I$$

$$S^2 = P^2 + Q^2$$

and definition of the power factor:

$$PF = \cos\left(\Phi_u - \Phi_i\right) = \frac{P}{S}$$

PHASORS

For sinusoidal function

$$u(t) := \sqrt{2}\cdot 10\cdot \sin(\omega\cdot t + \theta)$$ with phase shift angle $\theta := -35deg$

and applying transformeation to complex domain (i.e. ignore rotating time space vector represented by $e^{j\cdot\omega\cdot t}$ from eqation:

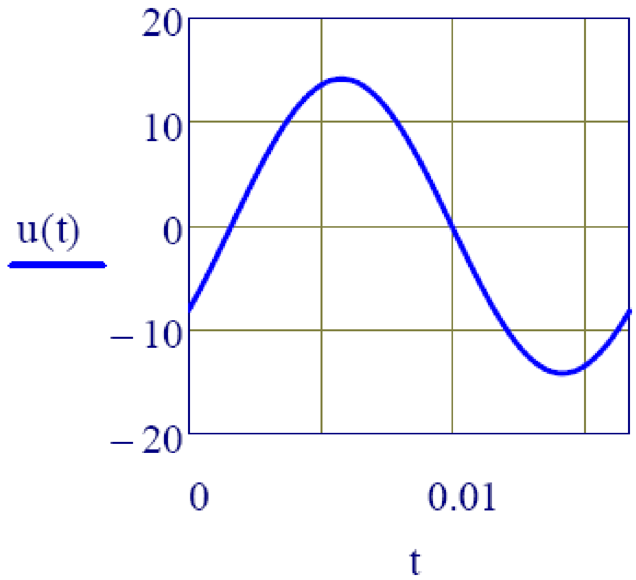
$$f(t) = \sqrt{2}\cdot U\cdot \sin(\omega\cdot t + \theta) = Im\left(\underline{U}\cdot e^{j\cdot\omega\cdot t}\right) = Im\left[\left[\left(\sqrt{2}U\right)\ (\measuredangle)\ \theta\right]\cdot e^{j\cdot\omega\cdot t}\right]$$

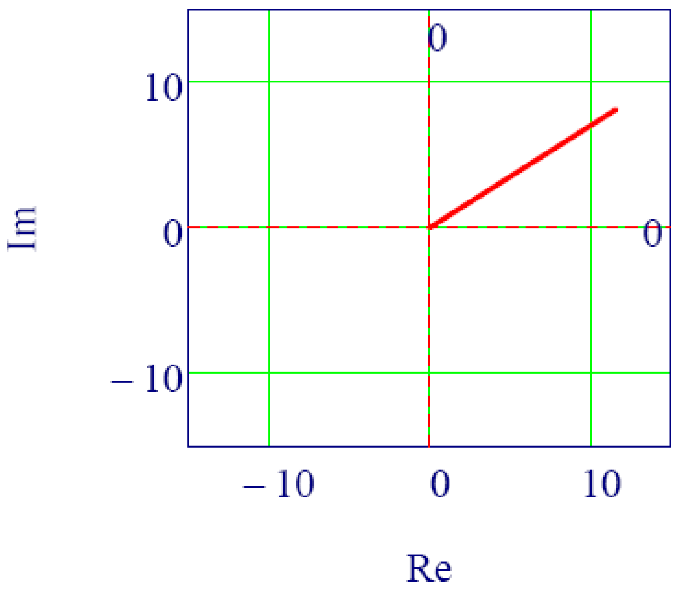
we obtain following:

$$\underline{U} := \left(z2r\theta\left(\sqrt{2}\cdot 10\cdot e^{-j-15deg}\right)\right) = \text{"(10.8847}\measuredangle\text{ -57.3°)"}$$

$$\underline{U} := \sqrt{2}\cdot 10\cdot e^{-j\cdot\theta} = 11.5846 + 8.1116j$$

PHASORS



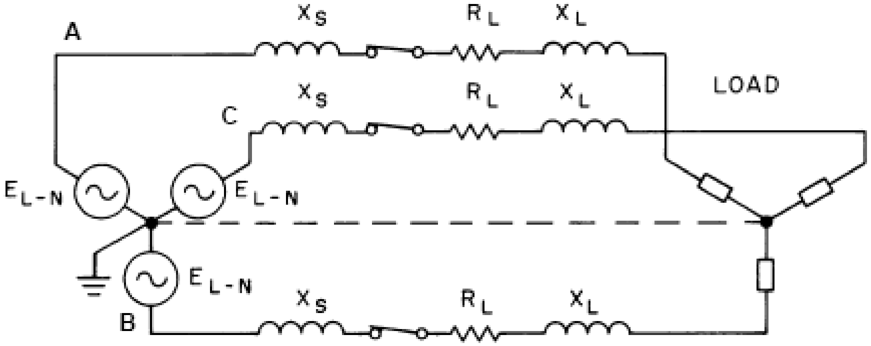


THE SINGLE-PHASE EQUIVALENT CIRCUIT

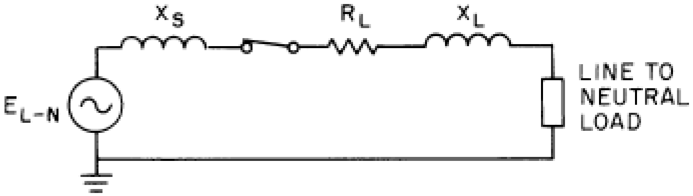
- Powerful tool for simplifying the analysis of balanced three-phase circuits, yet its restraints are probably most often disregarded. Its application is best understood by examining a three-phase diagram of a simple system and its single-phase equivalent.
- If a three-phase system has a perfectly balanced symmetrical source excitation (voltage) and load, as well as equal series and shunt system and line impedances connected to all three phases, imagine a conductor (shown as a dotted line) carrying no current connected between the effective neutrals of the load and the source. Under these conditions, the system can be accurately described by single phase equivalent

THE SINGLE-PHASE EQUIVALENT CIRCUIT

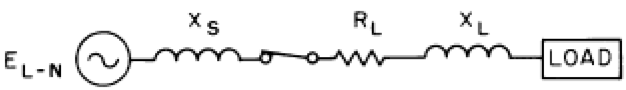
- Three Phase Diagram



- Simplification / diagrams



Single Phase Equivalent



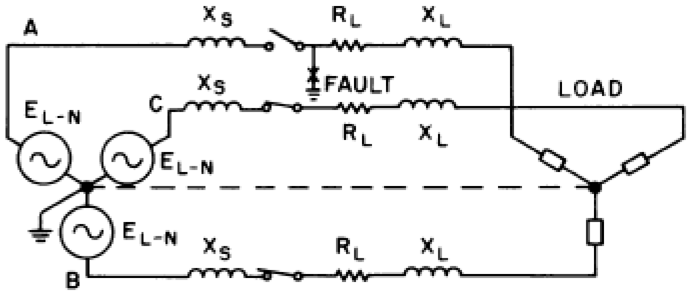
Single Line Impedance Diagram

THE SINGLE-PHASE EQUIVALENT CIRCUIT

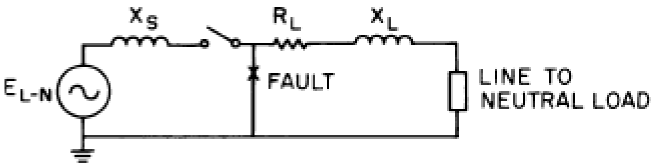
- The single-phase equivalent circuit is particularly useful since the solution to the classical loop equations is much easier to obtain than for the more complicated three-phase network. To determine the complete solution, it is only necessary to realize that the other two phases will have responses that are shifted by 120° and 240° but are otherwise identical to the reference phase.
- Anything that upsets the balance of the network renders the model **invalid**. A subtle way this might occur during asymmetrical faults.
- Neither the single-phase equivalent nor the single-line diagram representation is valid when unbalance or asymmetry occurs.
- The single-phase and the single-line diagram representations would imply that the load has been disconnected, it continues to be energized by single-phase power. This can cause serious damage to motors and result in unacceptable operation of certain load apparatus.

THE SINGLE-PHASE EQUIVALENT CIRCUIT

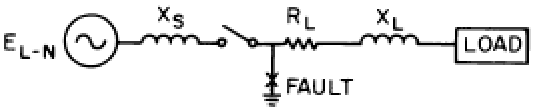
- Three Phase Diagram



- Single Phase Equivalent



- Single Line Impedance Diagram



THE SINGLE-PHASE EQUIVALENT CIRCUIT

- Restraints for this calculations:
  - Symmetry of the electrical system, including all switching devices and applied load.
  - Any of the other previously described restraints that apply to the analytical technique being used in combination with the single-phase equivalent.

THE PER UNIT METHOD

THE PER UNIT METHOD

- Definition:

  A **per-unit system** is the expression of system quantities as fractions of a defined base unit quantity of the same type.

$$system\ quanity\ [pu] = \frac{system\ quantity\ [in\ actual\ unit]}{system\ quantity_{base}\ [in\ actual\ unit]}$$

System quantities are power, voltage, current, frequency, impedance, admittance, torque, inertia etc.

THE PER UNIT METHOD

- Advantages Of Per Unit
  - *Equipment Parameters. For a given type of equipment, and* disregarding the size and voltage, the parameters in per unit are within a narrow, known range
  - *Eliminate Turn Ratio. For two adjacent networks of different* voltage levels, if the selected base power is the same throughout and the selected base voltages match the turn ratio of the transformer between the networks, then all quantities in per unit have the same value regardless of which voltage level they are defined. In essence, the transformer is eliminated.
  - *Eliminate Coefficients. For almost all equations with quantities* defined in per unit, the numerical coefficients are eliminated.
  - *Voltage. In per unit, the line-to-neutral voltage equals the phase-to-phase* voltage, and during normal operation both quantities are close to unity.

THE PER UNIT METHOD

- Per unit conversion requires us to select a base quantities
- How do we make the selection?
  - Answer: Select two quantities as the base from the following: voltage, current, power, impedance, admittance
- Which do we choose?
  - Answer: Generally choose voltage and power.

THE PER UNIT METHOD

- Why Voltage and Power?
  - *Voltage. For each voltage level in our system, we* know the rated voltage of equipment, and even if loading changes, the voltage does not deviate too much from the rated value.
  - *Power. The range of power flowing in a section of the* system is quadratically related with the voltage. As such, the range of expected power flow is known for an area. Note, for transmission level analysis, it is customary to select a base power of 100 MVA.

Note: The base power is usually selected to be the same for the entire network.

THE PER UNIT METHOD

- Select base quantities

$$V_{base} = V_{phase-phase}$$

$$S_{base} = S_{3phase}$$

  - Voltage:
    - Usually selected as the nominal phase-to-phase voltage at each voltage level
  - Power:
    - Usually selected in the range of 3-phase power flowing in the network (i.e. whatever network is being analyzed)
    - It is customary to select a base power of 10 or 100 MVA.
    - The base power is usually selected to be the same for the entire network.

THE PER UNIT METHOD

- Conversion
  - Each piece of equipment is different
  - Selection of base quantities could be different
  - Analysis requires common base
- Power Conversion

$$S_{new.base}[pu] = S_{old.base}[pu] * \frac{S_{old.base}[in\ actual\ units]}{S_{new.base}[in\ actual\ units]}$$

- Impedance Conversion

$$Z_{new.base}[pu] = Z_{old.base}[pu] * \frac{Z_{old.base}[\Omega]}{Z_{new.base}[\Omega]}$$

$$Z_{new.base}[pu] = Z_{old.base}[pu] * \left(\frac{V_{old.base}[V]}{V_{new.base}[V]}\right)^2 * \left(\frac{S_{new.base}[VA]}{S_{old.base}[VA]}\right)$$

THE PER UNIT METHOD

Single phase 1Φ power and voltage case

$U_{base} \equiv 69kV$ $S_{base} \equiv 100MVA$ $P_{base} := S_{base}$ $Q_{base} := P_{base}$

$$I_{base} := \frac{S_{base}}{U_{base}}$$

$$Z_{base} := \frac{U_{base}}{I_{base}}$$

$$Z_{base} := \frac{U_{base}^{2}}{S_{base}}$$

$$Y_{base} := Z_{base}^{-1}$$

$$Z_{pu}(Z_{\Omega}) := \frac{Z_{\Omega}}{Z_{base}} \quad Y_{pu}(Y_S) := \frac{Y_S}{Y_{base}} \quad I_{pu}(I_A) := \frac{I_A}{I_{base}} \quad U_{pu}(U_V) := \frac{U_V}{U_{base}}$$

$$P_{pu}(P_W) := \frac{P_W}{S_{base}} \quad Q_{pu}(Q_{VAr}) := \frac{Q_{VAr}}{S_{base}} \quad S_{pu}(S_{VA}) := \frac{S_{VA}}{S_{base}}$$

THE PER UNIT METHOD

EXAMPLE:

An impedance $\underline{\mathbf{Z}} := (2 + 2j)\Omega$ is connected to a single phase circuit with following paramters: base power $S_b := 5000\text{kVA}$ and base voltage $U_b := 10\text{kV}$. Calculate I_b, $Z_b$and $\underline{\mathbf{Z_{pu}}}$ of circuit and connected impedance.

Solution:

$$I_b := \frac{S_b}{U_b} = 500\,\text{A}$$

$$Z_b := \frac{U_b}{I_b} = 20\,\Omega$$

$$\underline{\mathbf{Z_{pu}}} := \frac{\underline{\mathbf{Z}}}{Z_b} = (0.1 + 0.1j)\cdot\text{pu}$$

THE PER UNIT METHOD

EXAMPLE:

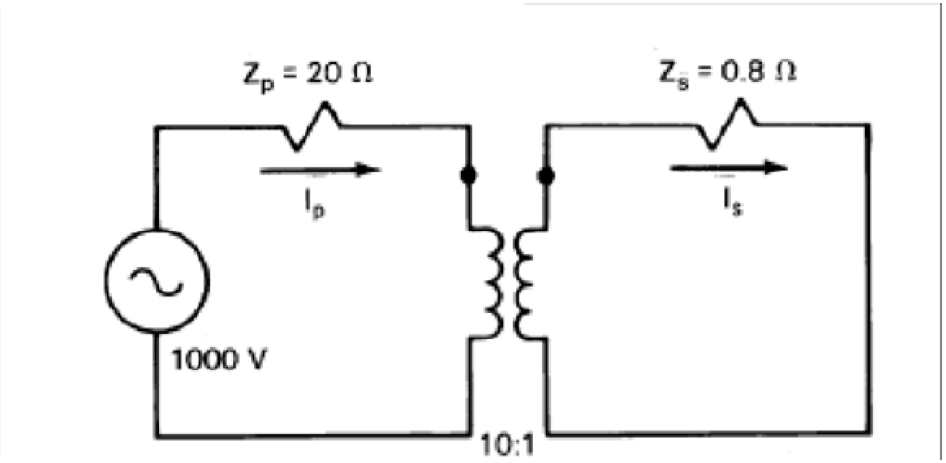
Data for following single phase circuit is:

$S_b := 5MVA$

$U_P := 1000V$ $U_S := 100V$

$Z_P \equiv 20\Omega$ $Z_S := 0.8\Omega$

Calcualte: I_{P_pu}, I_{S_pu}



THE PER UNIT METHOD

Solution:

$$S_b = 5 \cdot MVA \qquad S_b = 5 \cdot MVA$$

$$U_{b\_P} := U_P \qquad U_{b\_S} := U_S$$

$$Z_{b\_P} := \frac{{U_{b\_P}}^2}{S_b} = 0.2\,\Omega \qquad Z_{b\_S} := \frac{{U_{b\_S}}^2}{S_b} = 0.002\,\Omega$$

$$Z_{P\_pu} := \frac{Z_P}{Z_{b\_P}} = 100 \cdot pu \qquad Z_{S\_pu} := \frac{Z_S}{Z_{b\_S}} = 400 \cdot pu$$

$$U_{P\_pu} := \frac{U_P}{U_{b\_P}} = 1.00 \cdot pu$$

$$I_{b\_P} := \frac{S_b}{U_{b\_P}} = 5 \cdot kA \qquad I_{b\_S} := \frac{S_b}{U_{b\_S}} = 50 \cdot kA$$

$I_{P\_pu} = I_{S\_pu}$ and $I_{P\_pu} := \dfrac{U_{P\_pu}}{Z_{P\_pu} + Z_{S\_pu}} = 0.002$

$$I_P := I_{P\_pu} \cdot I_{b\_P} = 10\,A \qquad I_S := I_{P\_pu} \cdot I_{b\_S} = 100\,A$$

THE PER UNIT METHOD

Three phase 3Φ power and voltage case

Relating to equations for 1Φ

$$S_{base} = 3 \cdot U_{base\_phase} \cdot I_{base\_phase}$$

"...customarilly, rated kVA is given for the three phases and rated voltage is line-to-line voltage." *per E.Clarke*

$$U_{base} = U_{LL} \qquad I_{base} = I_L$$

$$S_{base} = \sqrt{3} \cdot U_{base} \cdot I_{base}$$

$$I_{base} := \frac{S_{base}}{\sqrt{3} \cdot U_{base}}$$

$$Z_{base} := \frac{\left(\frac{U_{base}}{\sqrt{3}}\right)^2}{\frac{S_{base}}{3}}$$

THE PER UNIT METHOD

$$Z_{base} := \frac{U_{base}{}^2}{S_{base}}$$

$$Y_{base} := Z_{base}{}^{-1}$$

$$Z_{pu}(Z_{\Omega}) := \frac{Z_{\Omega}}{Z_{base}} \qquad Y_{pu}(Y_S) := \frac{Y_S}{Y_{base}} \qquad I_{pu}(I_A) := \frac{I_A}{I_{base}} \qquad U_{pu}(U_V) := \frac{U_V}{U_{base}}$$

$$P_{pu}(P_W) := \frac{P_W}{S_{base}} \qquad Q_{pu}(Q_{VAr}) := \frac{Q_{VAr}}{S_{base}} \qquad S_{pu}(S_{VA}) := \frac{S_{VA}}{S_{base}}$$

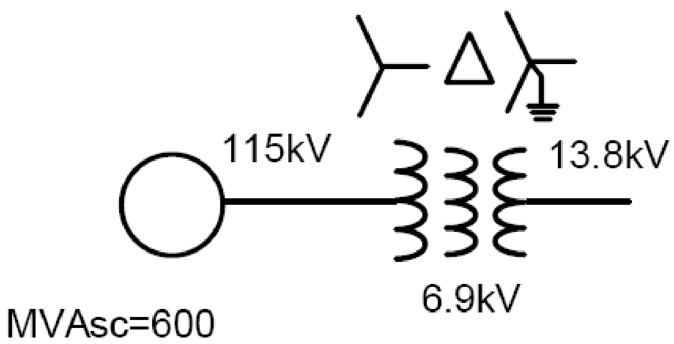
$$Z_{new} = Z_{old} \cdot \left(\frac{U_{old}}{U_{new}}\right)^2 \cdot \frac{S_{new}}{S_{old}}$$

THE PER UNIT METHOD

Example

This is a multiple stage example. For the system shown below:

a. Determine the source and equivalent star reactances of the transformer on a 30 MVA base.



X_{hx}=10% on 30MVA

X_{hy}=22.5% on 15MVA

X_{xy}=10% on 10MVA

h=115kV, x = 13.8kV, y = 6.9kV

Source Data (at 115kV):

$MVA_{sc} := 600MVA$

Transformer data:

$S_b := 30MVA$

$V_{b\_h} := 115kV$

$V_{b\_y} := 6.9kV$

$V_{b\_x} := 13.8kV$

THE PER UNIT METHOD

Compute Source Impedance:

Two options to compute per unit source impedance:

$$X_{source\_pu} := \frac{S_b}{MVA_{sc}} = 0.05\,pu$$

$$X_{source} := \frac{(115kV)^2}{MVA_{sc}} = 22.042\,\Omega$$

$$Z_{b\_h} := \frac{V_{b\_h}^{\ 2}}{S_b} = 440.833\,\Omega$$

$$X_{s\_pu} := \frac{X_{source}}{Z_{b\_h}} = 0.05\,pu$$

Note that:

$$X_{s\_pu} = \frac{\dfrac{(115kV)^2}{MVA_{sc}}}{\dfrac{Vb\_h^2}{S_b}} = \frac{S_b}{MVA_{sc}}$$

Compute Transformer Impedances for T-equivalent:

$X_{h\_x} := 0.1pu$ $S_{b\_hx} := 30MVA$

$X_{h\_y} := 0.225pu$ $S_{b\_hy} := 15MVA$

$X_{x\_y} := 0.11pu$ $S_{b\_xy} := 10MVA$

THE PER UNIT METHOD

Change of MVA base calculations.

$$X_{hx\_pu} := \frac{S_b}{S_{b\_hx}} \cdot X_{h\_x} \qquad X_{hx\_pu} = 0.1\,\text{pu}$$

$$X_{hy\_pu} := \frac{S_b}{S_{b\_hy}} \cdot X_{h\_y} \qquad X_{hy\_pu} = 0.45\,\text{pu}$$

$$X_{xy\_pu} := \frac{S_b}{S_{b\_xy}} \cdot X_{x\_y} \qquad X_{xy\_pu} = 0.33\,\text{pu}$$

Find Zh, Zx and Zy

$$X_h := (0.5)\cdot\left(X_{hy\_pu} + X_{hx\_pu} - X_{xy\_pu}\right) \qquad X_h = 0.11\,\text{pu}$$

$$X_x := (0.5)\cdot\left(X_{hx\_pu} + X_{xy\_pu} - X_{hy\_pu}\right) \qquad X_x = -0.01\,\text{pu}$$

$$X_y := (0.5)\cdot\left(X_{xy\_pu} + X_{hy\_pu} - X_{hx\_pu}\right) \qquad X_y = 0.34\,\text{pu}$$

Use the same values for the positive, negative and zero sequence networks.

THE PER UNIT METHOD

B. Sequence Networks: Ignore loads. Include impact of delta tertiary winding and ungrounded Y on 115kV.

$$X_1 := X_{s\_pu} + X_h + X_x \qquad X_1 = 0.15\,\text{pu}$$

$$X_2 := X_{s\_pu} + X_h + X_x \qquad X_2 = 0.15\,\text{pu}$$

$$X_0 := X_y + X_x \qquad X_0 = 0.33\,\text{pu}$$

C. Three phase fault currents at 13.8 kV terminals (ignore load)

$$V_f := j \cdot 1.0\text{pu}$$

$$I_{af} := \frac{V_f}{j \cdot X_1} \qquad I_{af} = 6.667\,\text{pu}$$

Positive Sequence Rotation:

$$I_{abc} := I_{af} \cdot \begin{pmatrix} 1 \\ a^2 \\ a \end{pmatrix} \qquad \overrightarrow{\left|I_{abc}\right|} = \begin{pmatrix} 6.667 \\ 6.667 \\ 6.667 \end{pmatrix} \qquad \overrightarrow{\arg\left(I_{abc}\right)} = \begin{pmatrix} 0 \\ -120 \\ 120 \end{pmatrix} \text{deg}$$

THE PER UNIT METHOD

D. SLG fault at 13.8kV terminals

$$I_0 := \frac{V_f}{j \cdot X_1 + j \cdot X_2 + j \cdot X_0} \qquad I_0 = 1.587\,\text{pu} \qquad I_1 := I_0 \qquad I_2 := I_0$$

$$I_{abc\_SLG} := A \cdot \begin{pmatrix} I_0 \\ I_1 \\ I_2 \end{pmatrix} \qquad \overrightarrow{\left| I_{abc\_SLG} \right|} = \begin{pmatrix} 4.762 \\ 0 \\ 0 \end{pmatrix} \qquad \overrightarrow{\arg\left(I_{abc\_SLG}\right)} = \begin{pmatrix} 0 \\ 116.565 \\ 116.565 \end{pmatrix} \text{deg}$$

$$I_{b\_x} := \frac{S_b}{\sqrt{3} \cdot V_{b\_x}} = 1255.109\,\text{A} \qquad I_{a\_x} := I_{abc\_SLG_0} \cdot I_{b\_x} \qquad \left| I_{a\_x} \right| = 5976.711\,\text{A}$$

E. Phase to neutral voltages at fault

$$V_1 := V_f - j \cdot X_1 \cdot I_1 \qquad V_1 = 0.762i\,\text{pu} \qquad V_0 := -j \cdot X_0 \cdot I_0 \qquad V_0 = -0.524i\,\text{pu}$$

$$V_2 := -j \cdot X_2 \cdot I_2 \qquad V_2 = -0.238i\,\text{pu}$$

$$V_{abcSLG} := A \cdot \begin{pmatrix} V_0 \\ V_1 \\ V_2 \end{pmatrix} \qquad \overrightarrow{\left| V_{abcSLG} \right|} = \begin{pmatrix} 0 \\ 1.169 \\ 1.169 \end{pmatrix} \qquad \overrightarrow{\arg\left(V_{abcSLG}\right)} = \begin{pmatrix} -90 \\ -42.216 \\ -137.784 \end{pmatrix} \text{deg}$$

THE PER UNIT METHOD

F. Find the phase currents and phase to neutral voltages on the 115kV side of the transformer.

Now $I_0 := 0pu$ since neutral of the Y is ungrounded. I1 and I2 unaffected.

$$I_{abcSLG} := A \cdot \begin{pmatrix} I_0 \\ I_1 \\ I_2 \end{pmatrix} \qquad \overrightarrow{|I_{abcSLG}|} = \begin{pmatrix} 3.175 \\ 1.587 \\ 1.587 \end{pmatrix} \qquad \overrightarrow{arg(I_{abcSLG})} = \begin{pmatrix} 0 \\ 180 \\ 180 \end{pmatrix} deg$$

$$I_{b\_h} := \frac{S_b}{\sqrt{3} \cdot V_{b\_h}} \qquad I_{a\_h} := I_{b\_h} \cdot I_{abcSLG_0} \qquad |I_{a\_h}| = 478.137\,A$$

$$V_1 := V_f - j \cdot X_{s\_pu} \cdot I_1 \qquad V_1 = 0.921i\,pu$$

$$V_2 := -j \cdot X_{s\_pu} \cdot I_2 \qquad V_2 = -0.079i\,pu$$

$$V_0 := -j \cdot X_0 \cdot I_0 \qquad V_0 = 0\,pu$$

$$V_{abcSLG} := A \cdot \begin{pmatrix} V_0 \\ V_1 \\ V_2 \end{pmatrix} \qquad \overrightarrow{|V_{abcSLG}|} = \begin{pmatrix} 0.841 \\ 0.963 \\ 0.963 \end{pmatrix} \qquad \overrightarrow{arg(V_{abcSLG})} = \begin{pmatrix} 90 \\ -25.906 \\ -154.094 \end{pmatrix} deg$$

THE PER UNIT METHOD

G. Current in the delta for the fault on 13.8kV bus

$I_0 := I_1$ $\quad I_1 := 0\text{pu}$ $\quad I_2 := 0$

$$I_{abcSLG} := A \cdot \begin{pmatrix} I_0 \\ I_1 \\ I_2 \end{pmatrix} \qquad \overrightarrow{\left|I_{abcSLG}\right|} = \begin{pmatrix} 1.587 \\ 1.587 \\ 1.587 \end{pmatrix} \qquad \overrightarrow{\arg\left(I_{abcSLG}\right)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \text{deg}$$

$$I_b := \frac{S_b}{3 \cdot V_{b\_y}} = 1449.275\,\text{A} \qquad I_b \cdot I_0 = 2300.437\,\text{A} \qquad I_{a\_y} := I_b \cdot I_0 = 2300.437\,\text{A}$$

THE PER UNIT METHOD

H. Make an ampere-turn check for the currents in the windings

$$N1 \cdot I1 + N2 \cdot I2 + N3 \cdot I3 = 0$$

$$N_{h\_h} := \frac{115}{115} \qquad N_{h\_h} = 1$$

$$N_{h\_y} := \frac{6.9}{\frac{115}{\sqrt{3}}} \qquad N_{h\_y} = 0.104$$

$$N_{h\_x} := \frac{\frac{13.8}{\sqrt{3}}}{\frac{115}{\sqrt{3}}} \qquad N_{h\_x} = 0.12$$

$$At_{115} := N_{h\_h} \cdot \left|I_{a\_h}\right| \qquad At_{115} = 478.137\,A$$

$$At_{13.8} := N_{h\_x} \cdot \left|I_{a\_x}\right| \qquad At_{13.8} = 717.205\,A$$

$$At_{6.9} := N_{h\_y} \cdot \left|I_{a\_y}\right| \qquad At_{6.9} = 239.068\,A$$

$$Sum := At_{115} - At_{13.8} + At_{6.9}$$

$$Sum = 0\,A$$

FORTESCUE THEORY

FORTESCUE THEORY

- Fortescue presented paper[(1)] demonstrating that an unbalanced set of N phasors in any polyphase system could be expressed as the sum of N-1 balanced N-phase systems of different phase sequence and one zero-phase sequence system. Set of phase-sequence system is known as symmetrical components set for a three phase system.
- The paper[(1)] was judged to be the most important power engineering paper in the twentieth century.
- Note: three phase system is a special case of a polyphase signal

(1) Fortescue, Charles. L. "Method of Symmetrical Co-Ordinates Applied to the Solution of Polyphase Networks“ , AIEE Transactions*, vol. 37, part II, pages 1027-1140 (1918)*. Annual convention of the AIEE (American Institute of Electrical Engineers).
URL: http://www.energyscienceforum.com/files/fortescue/methodofsymmetrical.pdf

SYMMETRICAL COMPONENTS

SYMMETRICAL COMPONENTS

- What are Symmetrical Components?
  - Any set of *N unbalanced phasors — that is, any such* “polyphase” signal — can be expressed as the sum of *N* symmetrical sets of balanced phasors.
  - Only a single frequency component is represented by the phasors. This is overcome by using techniques such as Fourier or Laplace transforms.
  - Absolutely general and rigorous and can be applied to both steady state and transient problems.
  - It is thoroughly established as preeminently the only effective method of analyzing general polyphase network problems

SYMMETRICAL COMPONENTS

- Three-Phase System Symmetrical Components
  - Three sets of symmetrical components, where each set is referred to as a *sequence.*
  - First set of phasors, called the *positive sequence, has* the same phase sequence as the system under study (say A-B-C)
  - The second set, the *negative sequence, has the reverse* phase sequence (A-C-B)
  - The third set, the *zero sequence, phasors A, B and C* are in phase with each other.
  - Method converts *any set of three phasors* (phase domain) into *three* sets of *symmetrical phasors*, which makes asymmetric analysis easily achievable.

SYMMETRICAL COMPONENTS

- For three phase system, set of three phasors X_a, X_b, and X_c can be represented as a sum of three sequence vector sets

$$\underline{X_a} = \underline{X_{a0}} + \underline{X_{a1}} + \underline{X_{a2}}$$
$$\underline{X_b} = \underline{X_{b0}} + \underline{X_{b1}} + \underline{X_{b2}}$$
$$\underline{X_c} = \underline{X_{c0}} + \underline{X_{c1}} + \underline{X_{c2}}$$

where

$\underline{X_{a0}}, \underline{X_{b0}}, \underline{X_{c0}}$ - the zero sequence set

$\underline{X_{a1}}, \underline{X_{b1}}, \underline{X_{c1}}$ - the positive sequence set

$\underline{X_{a2}}, \underline{X_{b2}}, \underline{X_{c2}}$ - the negative sequence set

SYMMETRICAL COMPONENTS

- Only one set of sequence values are unique, i.e. for one phase sequences 0, 1, 2 (usually phase A). Remaining phases can be determined from unique sequence.
- In symmetrical components analysis, a complex operator "λ" is defined by the Fortescue theory for N-phase system. When applied to three phase (i.e. n=3, i=2), we define popular notation and manipulation vector "a":

$$\lambda_i = e^{j\cdot\frac{2\cdot\pi}{n}\cdot(i-1)} = e^{j\cdot\frac{2\cdot\pi}{3}\cdot(2-1)} = e^{j\cdot\frac{2\cdot\pi}{3}} = e^{j\cdot 120\mathrm{deg}}$$

- For three phase system $\lambda_2 = \underline{a} = e^{j\cdot 120\mathrm{deg}}$

SYMMETRICAL COMPONENTS

- Some of useful properties of “a”…

rectangular notation	polar notation
$\underline{a} := -\frac{1}{2} + j \cdot \frac{\sqrt{3}}{2} = -0.5 + 0.87i$	$z2r\theta(\underline{a}) =$ "($1\measuredangle 120°$)"
$\underline{a}^2 = -0.5 - 0.87i$	$z2r\theta(\underline{a}^2) =$ "($1\measuredangle -120°$)"
$\underline{a}^3 = 1$	$z2r\theta(\underline{a}^3) =$ "($1\measuredangle 0°$)"
$\underline{a}^4 = -0.5 + 0.87i$	$z2r\theta(\underline{a}^4) =$ "($1\measuredangle 120°$)"
$\underline{a}^5 = -0.5 - 0.87i$	$z2r\theta(\underline{a}^5) =$ "($1\measuredangle -120°$)"
$\underline{a} + \underline{a}^2 + 1 = 0$	
$\underline{a} + \underline{a}^2 = -1$	$z2r\theta(\underline{a} + \underline{a}^2) =$ "($1\measuredangle 180°$)"
$\underline{a} - \underline{a}^2 = 1.73i$	$z2r\theta(\underline{a} - \underline{a}^2) =$ "($1.7321\measuredangle 90°$)"
$\underline{a}^2 - \underline{a} = -1.73i$	$z2r\theta(\underline{a}^2 - \underline{a}) =$ "($1.7321\measuredangle -90°$)"
$1 - \underline{a} = 1.5 - 0.87i$	$z2r\theta(1 - \underline{a}) =$ "($1.7321\measuredangle -30°$)"
$1 - \underline{a}^2 = 1.5 + 0.87i$	$z2r\theta(1 - \underline{a}^2) =$ "($1.7321\measuredangle 30°$)"
$a - 1 = -1.5 + 0.87i$	$z2r\theta(\underline{a} - 1) =$ "($1.7321\measuredangle 150°$)"
$a^2 - 1 = -1.5 - 0.87i$	$z2r\theta(\underline{a}^2 - 1) =$ "($1.7321\measuredangle -150°$)"
$1 + a = 0.5 + 0.87i$	$z2r\theta(1 + a) =$ "($1\measuredangle 60°$)"
$1 + \underline{a}^2 = 0.5 - 0.87i$	$z2r\theta(1 + \underline{a}^2) =$ "($1\measuredangle -60°$)"

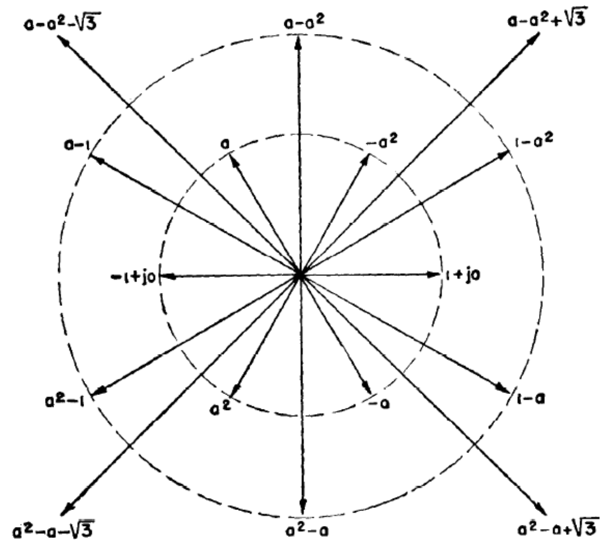
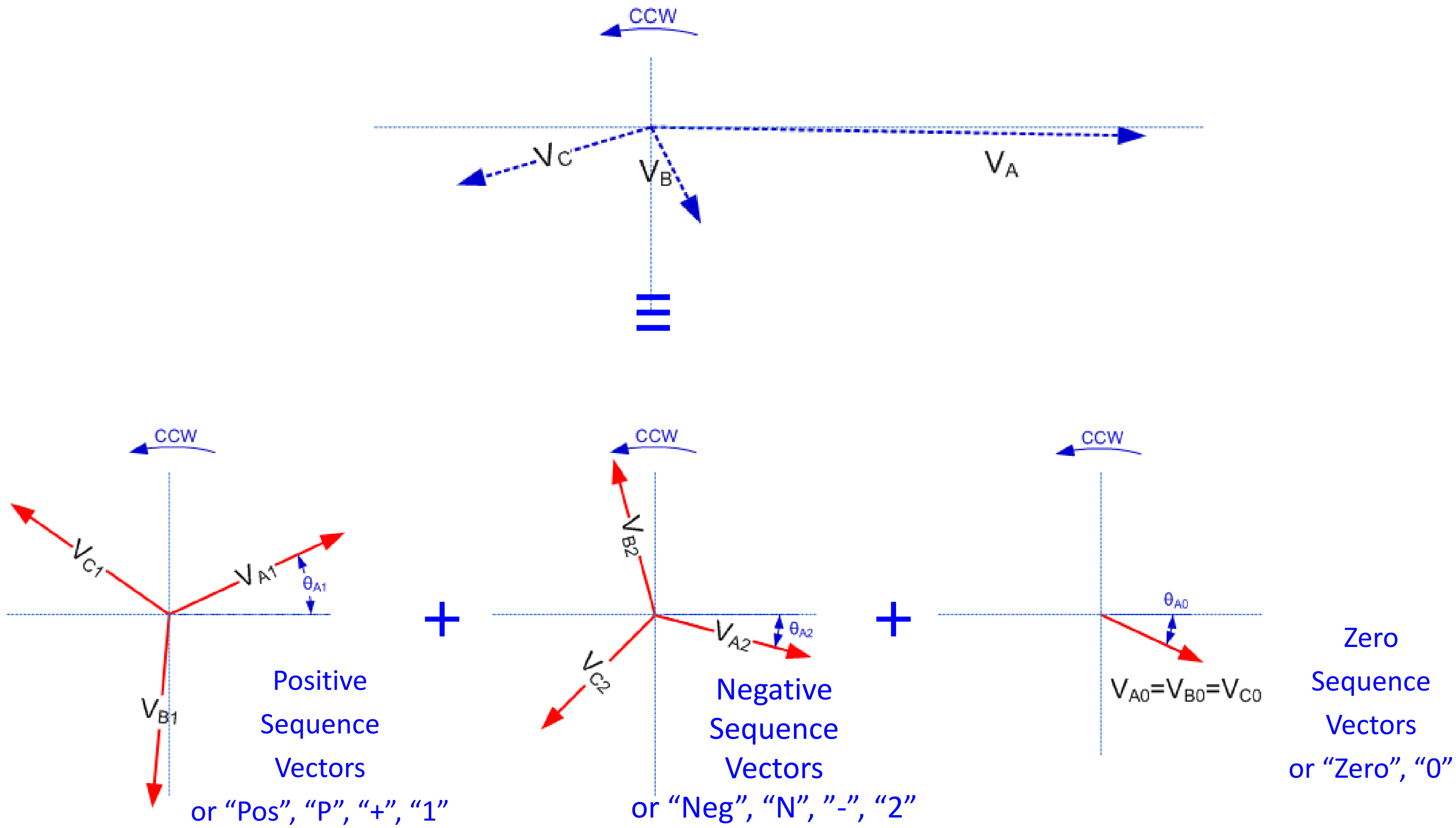


Fig. 2—Properties of the vector operator *a*.

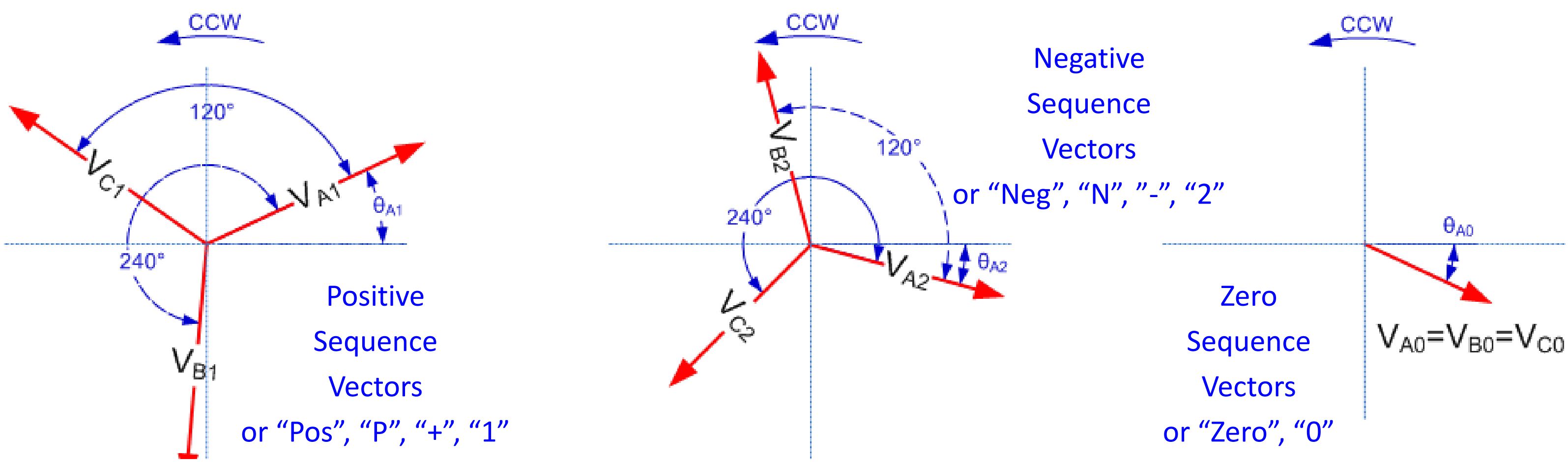
Reference [2]

SYMMETRICAL COMPONENTS

- Let's consider unbalanced voltage phasors of the three phase system (i.e. unbalanced set)



SYMMETRICAL COMPONENTS



$\underline{X}_{a1}$	$\underline{X}_{a2}$	$\underline{X}_{a0}$
$\underline{X}_{b1} = \underline{X}_{a1} \cdot \underline{a}^2$	$\underline{X}_{b2} = \underline{X}_{a2} \cdot \underline{a}$	$\underline{X}_{b0} = \underline{X}_{a0}$
$\underline{X}_{c1} = \underline{X}_{a1} \cdot \underline{a}$	$\underline{X}_{c2} = \underline{X}_{a2} \cdot \underline{a}^2$	$\underline{X}_{c0} = \underline{X}_{a0}$

Or

$$\underline{X}_{a} = \underline{X}_{a0} + \underline{X}_{a1} + \underline{X}_{a2}$$

$$\underline{X}_{b} = \underline{X}_{b0} + \underline{X}_{b1} + \underline{X}_{b2} = \underline{X}_{a0} + \underline{X}_{a1} \cdot \underline{a}^2 + \underline{X}_{a2} \cdot \underline{a}$$

$$\underline{X}_{c} = \underline{X}_{c0} + \underline{X}_{c1} + \underline{X}_{c2} = \underline{X}_{a0} + \underline{X}_{a1} \cdot \underline{a} + \underline{X}_{a2} \cdot \underline{a}^2$$

SYMMETRICAL COMPONENTS

$X_{abc} \rightarrow X_{012}$

$$\mathbf{X_{abc} = A \cdot X_{012}}$$

$$\begin{pmatrix} \underline{X_a} \\ \underline{X_b} \\ \underline{X_c} \end{pmatrix} = \frac{1}{h} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{pmatrix} \cdot \begin{pmatrix} \underline{X_{a0}} \\ \underline{X_{a1}} \\ \underline{X_{a2}} \end{pmatrix}$$

$$\mathbf{A} = \frac{1}{h} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{pmatrix}$$

$X_{012} \rightarrow X_{abc}$

$$\mathbf{X_{012} = A^{-1} \cdot X_{abc}}$$

$$\begin{pmatrix} \underline{X_{a0}} \\ \underline{X_{a1}} \\ \underline{X_{a2}} \end{pmatrix} = \frac{h}{3} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{pmatrix} \cdot \begin{pmatrix} \underline{X_a} \\ \underline{X_b} \\ \underline{X_c} \end{pmatrix}$$

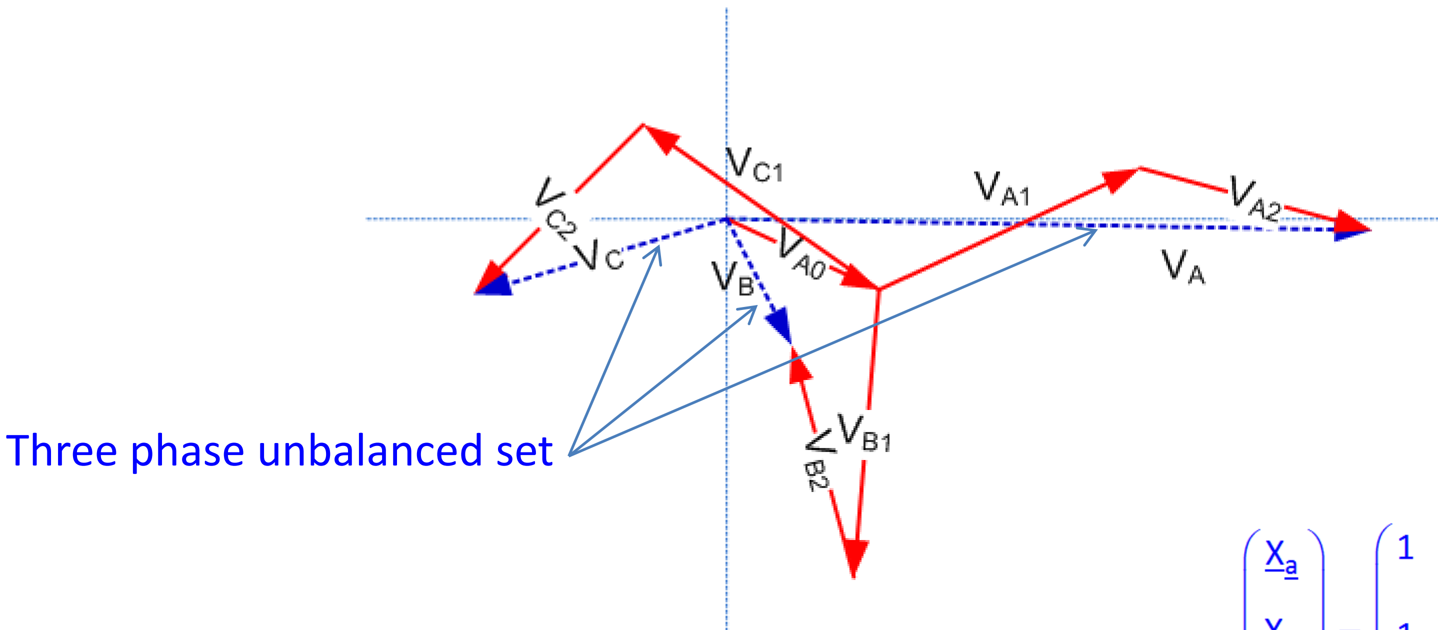
$$\mathbf{A}^{-1} = \frac{h}{3} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{pmatrix}$$

Note: $h = 1$ for the Fortescue transformation (this presentation)

$h = \sqrt{3}$ for the power invariant transformation

SYMMETRICAL COMPONENTS

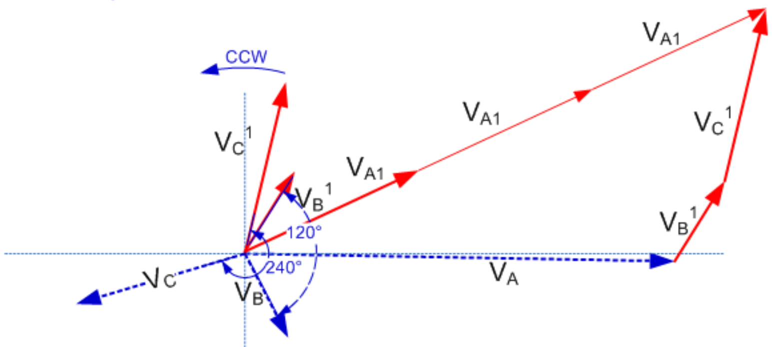
- Per phase voltage sets summation of symmetrical components to obtain an unbalanced set

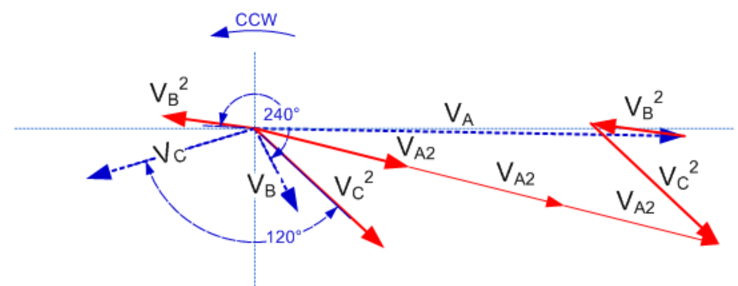


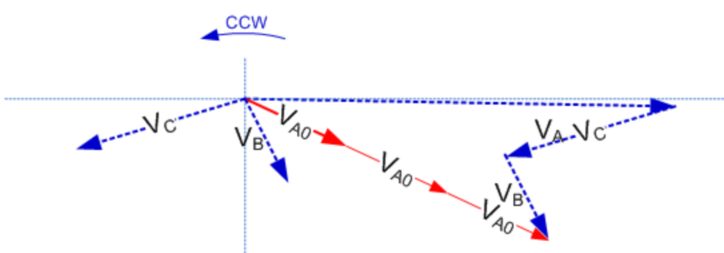
$$\begin{pmatrix} \underline{X_a} \\ \underline{X_b} \\ \underline{X_c} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{pmatrix} \cdot \begin{pmatrix} \underline{X_{a0}} \\ \underline{X_{a1}} \\ \underline{X_{a2}} \end{pmatrix}$$

SYMMETRICAL COMPONENTS

- Per phase voltage sets summation of unbalanced set with “a” transformation to obtain an symmetrical components







$$\begin{pmatrix} \underline{X}_{a0} \\ \underline{X}_{a1} \\ \underline{X}_{a2} \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{pmatrix} \cdot \begin{pmatrix} \underline{X}_a \\ \underline{X}_b \\ \underline{X}_c \end{pmatrix}$$

SYMMETRICAL COMPONENTS

- Example S1

$$\mathbf{I_{abc}} := \begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} = \begin{pmatrix} 10 \\ -5 - 8.66i \\ -5 + 8.66i \end{pmatrix} A \qquad z2r\theta M(\mathbf{I_{abc}}) = \begin{pmatrix} \text{"(10}\measuredangle \text{ 0°)"} \\ \text{"(10}\measuredangle \text{ -120°)"} \\ \text{"(10}\measuredangle \text{ 120°)"} \end{pmatrix}$$

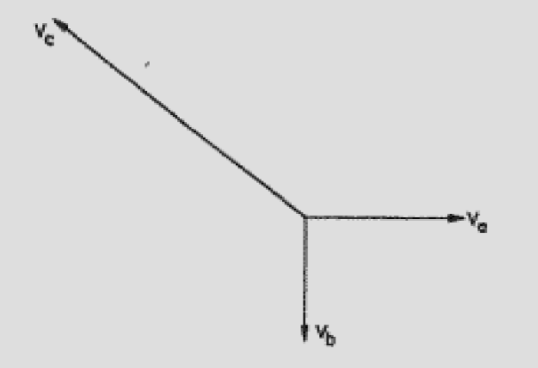
$$\mathbf{I_{012}} = \begin{pmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{pmatrix} \qquad \mathbf{I_{012}} := \mathbf{S}^{-1} \cdot \mathbf{I_{abc}} \qquad z2r\theta M(\mathbf{I_{012}}) = \begin{pmatrix} \text{"(0}\measuredangle \text{ -28.8634°)"} \\ \text{"(10}\measuredangle \text{ 0°)"} \\ \text{"(0}\measuredangle \text{ -90°)"} \end{pmatrix} A$$

$$\mathbf{V_{abc}} := \begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = \begin{pmatrix} 7 \\ -16i \\ -2.11 + 4.53i \end{pmatrix} V \qquad z2r\theta M(\mathbf{V_{abc}}) = \begin{pmatrix} \text{"(7}\measuredangle \text{ 0°)"} \\ \text{"(16}\measuredangle \text{ -90°)"} \\ \text{"(5}\measuredangle \text{ 115°)"} \end{pmatrix} V$$

$$\mathbf{V_{012}} = \begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{pmatrix} \qquad \mathbf{V_{012}} := \mathbf{S}^{-1} \cdot \mathbf{V_{abc}} \qquad z2r\theta M(\mathbf{V_{012}}) = \begin{pmatrix} \text{"(4.1554}\measuredangle \text{ -66.9203°)"} \\ \text{"(8.974}\measuredangle \text{ 16.3181°)"} \\ \text{"(3.4929}\measuredangle \text{ 158.1249°)"} \end{pmatrix} V$$

SYMMETRICAL COMPONENTS

- Example S2



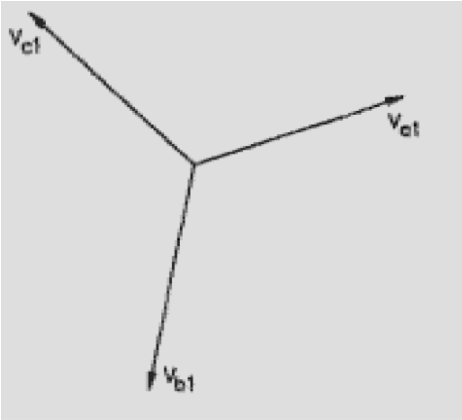
$V_a := 8V \cdot e^{j \cdot 0deg}$ $\quad V_b := 6V \cdot e^{j \cdot -90deg}$ $\quad V_c := 16V \cdot e^{j \cdot 143.1deg}$

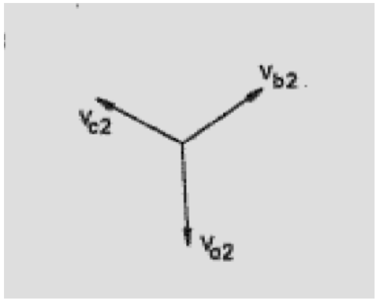
$$\begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{pmatrix} := \mathbf{S}^{-1} \cdot \begin{pmatrix} V_a \\ V_b \\ V_c \end{pmatrix} = \begin{pmatrix} -1.6 + 1.2i \\ 9.3 + 3.09i \\ 0.29 - 4.29i \end{pmatrix} V \qquad z2r\theta M\left(\begin{pmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{pmatrix}\right) = \begin{pmatrix} \text{"(2∡ 143.0498°)"} \\ \text{"(9.8049∡ 18.385°)"} \\ \text{"(4.3047∡ -86.0854°)"} \end{pmatrix} V$$

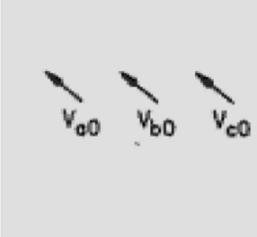
$$\begin{pmatrix} V_{b0} \\ V_{b1} \\ V_{b2} \end{pmatrix} := \begin{pmatrix} V_{a0} \\ a^2 \cdot V_{a1} \\ a \cdot V_{a2} \end{pmatrix} = \begin{pmatrix} -1.6 + 1.2i \\ -1.97 - 9.6i \\ 3.57 + 2.4i \end{pmatrix} V \qquad z2r\theta M\left(\begin{pmatrix} V_{b0} \\ V_{b1} \\ V_{b2} \end{pmatrix}\right) = \begin{pmatrix} \text{"(2∡ 143.0498°)"} \\ \text{"(9.8049∡ -101.615°)"} \\ \text{"(4.3047∡ 33.9146°)"} \end{pmatrix} V$$

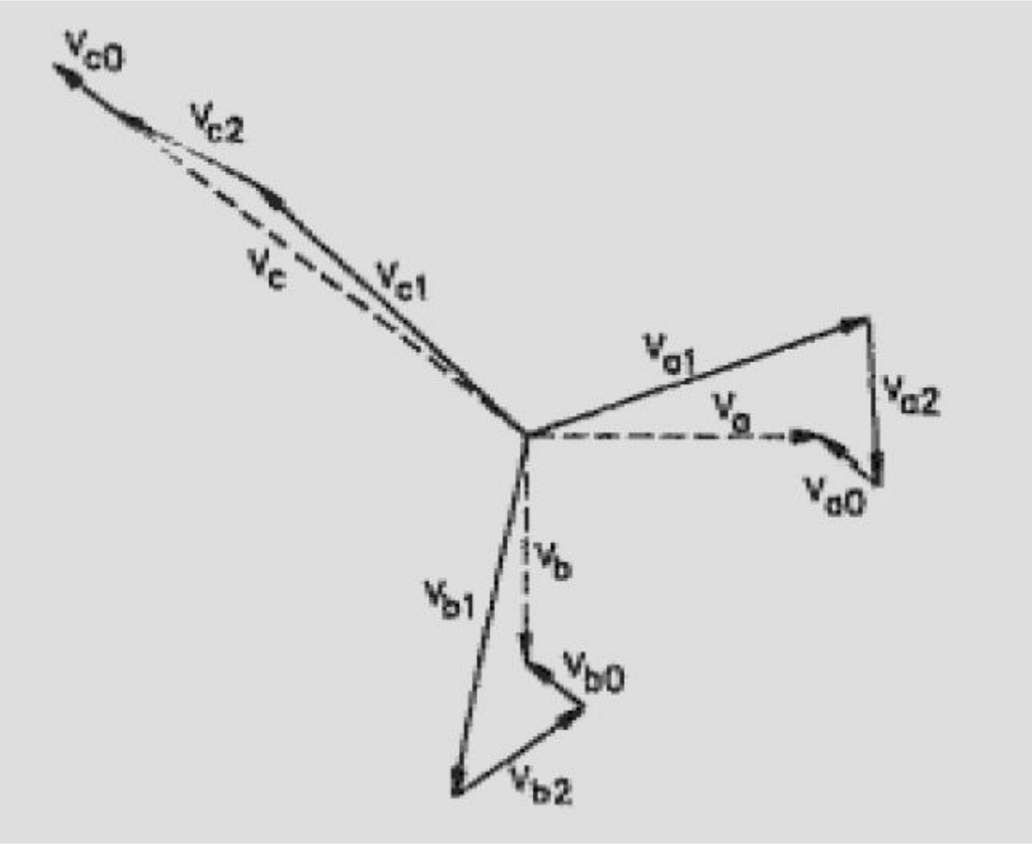
$$\begin{pmatrix} V_{c0} \\ V_{c1} \\ V_{c2} \end{pmatrix} := \begin{pmatrix} V_{a0} \\ a \cdot V_{a1} \\ a^2 \cdot V_{a2} \end{pmatrix} = \begin{pmatrix} -1.6 + 1.2i \\ -7.33 + 6.51i \\ -3.87 + 1.89i \end{pmatrix} V \qquad z2r\theta M\left(\begin{pmatrix} V_{c0} \\ V_{c1} \\ V_{c2} \end{pmatrix}\right) = \begin{pmatrix} \text{"(2∡ 143.0498°)"} \\ \text{"(9.8049∡ 138.385°)"} \\ \text{"(4.3047∡ 153.9146°)"} \end{pmatrix} V$$

SYMMETRICAL COMPONENTS









SYMMETRICAL COMPONENTS

- Comments & Notes:
  - Transformations apply only to balanced and symmetrical systems before unbalance occurs. Symmetrical networks can be solved using single phase techniques.
  - The transformations apply only for linear systems, that is, systems with constant parameters (impedance, admittance) independent of voltages and currents.
  - The quantities used for X_{abc} can be phase-to-neutral or phase-to-phase voltages, or line or line-to-line currents.
  - For some connections, the zero sequence component is always zero.
  - With symmetrical components we solve three interconnected symmetrical networks using single phase analysis.
  - Once solved, we use transformation equations to obtain phase quantities.

SYMMETRICAL COMPONENTS

- On equipment Modeling:
  - Symmetrical components has advantage that parameters in system components are easier to define as each sequence is a symmetrical three phase case, the parameters **can** be defined using typical three-phase tests.
  - In unbalanced and/or unsymmetrical systems modeling, the parameters **cannot** be defined using standard tests. As a result, symmetrical components principle cannot be used in systems that are unbalanced and/or unsymmetrical.

SYMMETRICAL COMPONENTS

- Complex Power with symmetrical components:

$$\mathbf{S_{abc}} = \begin{pmatrix} \underline{V}_{an} & \underline{V}_{bn} & \underline{V}_{cn} \end{pmatrix} \cdot \begin{pmatrix} \overline{\underline{I}_a} \\ \overline{\underline{I}_b} \\ \overline{\underline{I}_c} \end{pmatrix} = \mathbf{V_{abc.n}}^T \cdot \overline{\mathbf{I_{abc}}}$$

Knowing,

$$\mathbf{V_{abc.n}} = \mathbf{A} \cdot \mathbf{V_{012}} \qquad \mathbf{I_{abc}} = \mathbf{A} \cdot \mathbf{I_{012}}$$

Substituting

$$\mathbf{S_{abc}} = \left(\mathbf{A} \cdot \mathbf{V_{012}}\right)^T \cdot \overline{\left(\mathbf{A} \cdot \mathbf{I_{012}}\right)} = \mathbf{V_{012}}^T \cdot \mathbf{A}^T \cdot \overline{\mathbf{A}} \cdot \overline{\mathbf{V_{012}}}$$

Note that:

$$\mathbf{A}^T \cdot \overline{\mathbf{A}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot 3$$

$$\mathbf{S_{abc}} = 3 \cdot \mathbf{V_{012}}^T \cdot \overline{\mathbf{V_{012}}} = 3\left(\underline{V}_{a0} \cdot \overline{\underline{I}_{a0}} + \underline{V}_{a1} \cdot \overline{\underline{I}_{a1}} + \underline{V}_{a2} \cdot \overline{\underline{I}_{a2}}\right)$$

<u>Note:</u> For general case, with $h := \sqrt{3}$ i.e. power invariant transformation

$$\mathbf{S_{abc}} = \frac{3}{h^2}\left(\underline{V}_{a0} \cdot \overline{\underline{I}_{a0}} + \underline{V}_{a1} \cdot \overline{\underline{I}_{a1}} + \underline{V}_{a2} \cdot \overline{\underline{I}_{a2}}\right)$$

SYMMETRICAL COMPONENTS

- Consider the following D-Line (i.e series component):

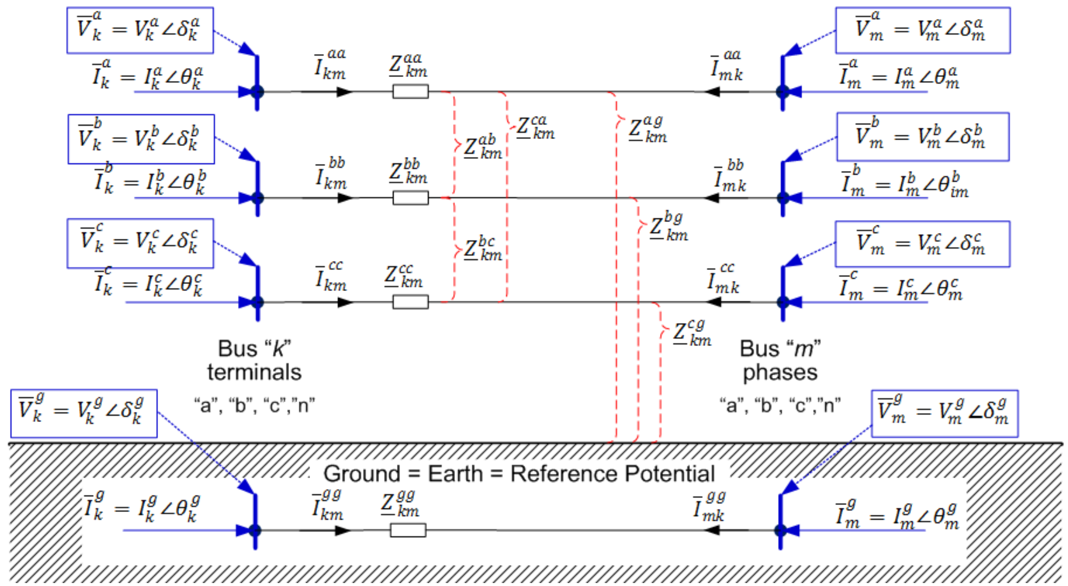
$Z_{aa} \neq Z_{bb} \neq Z_{cc}$

$Z_{ab} \neq Z_{bc} \neq Z_{ca}$

$Z_{ab} = Z_{ba}$

$Z_{bc} = Z_{cb}$

$Z_{ac} = Z_{ca}$



$$\begin{pmatrix} V_{km.a} \\ V_{km.b} \\ V_{km.c} \end{pmatrix} = \begin{pmatrix} Z_{km.aa} & Z_{km.ab} & Z_{km.ac} \\ Z_{km.ba} & Z_{km.bb} & Z_{km.bc} \\ Z_{km.ca} & Z_{km.cb} & Z_{km.cc} \end{pmatrix} \cdot \begin{pmatrix} I_{km.a} \\ I_{km.b} \\ I_{km.c} \end{pmatrix}$$

SYMMETRICAL COMPONENTS

- Transformation to Symmetrical Components….

$$\mathbf{V_{km.abc} = Z_{km} \cdot I_{km.abc}}$$

$$\mathbf{A \cdot V_{km.012} = Z_{km.abc} \cdot A \cdot I_{km.012}}$$

$$\mathbf{V_{km.012} = A^{-1} \cdot Z_{km.abc} \cdot A \cdot I_{km.012}}$$

$$\mathbf{V_{km.012} = Z_{km.012} \cdot I_{km.012}}$$

$$\mathbf{Z_{km.012}} = \begin{pmatrix} Z_{S0} + 2 \cdot Z_{M0} & Z_{S2} - Z_{M2} & Z_{S1} - Z_{M1} \\ Z_{S1} - Z_{M1} & Z_{S0} - Z_{M0} & Z_{S2} + 2 \cdot Z_{M2} \\ Z_{S2} - Z_{M2} & Z_{S1} + 2 \cdot Z_{M1} & Z_{S0} - Z_{M0} \end{pmatrix}$$

Where:

$$Z_{S0} = \frac{1}{3} \cdot \left(Z_{aa} + Z_{bb} + Z_{cc}\right)$$

$$Z_{S1} = \frac{1}{3} \cdot \left(Z_{aa} + a \cdot Z_{bb} + a^2 \cdot Z_{cc}\right)$$

$$Z_{S2} = \frac{1}{3} \cdot \left(Z_{aa} + a^2 \cdot Z_{bb} + a \cdot Z_{cc}\right)$$

$$Z_{M0} = \frac{1}{3} \cdot \left(Z_{bc} + Z_{ca} + Z_{ab}\right)$$

$$Z_{M1} = \frac{1}{3} \cdot \left(Z_{bc} + a \cdot Z_{ca} + a^2 \cdot Z_{ab}\right)$$

$$Z_{M2} = \frac{1}{3} \cdot \left(Z_{bc} + a^2 \cdot Z_{ca} + a \cdot Z_{ab}\right)$$

SYMMETRICAL COMPONENTS

- Note a problem… For example the positive sequence voltage drop

$$V_{km.1} = (Z_{S1} - Z_{M1}) \cdot I_{a0} + (Z_{S0} - Z_{M0}) \cdot I_{a1} + (Z_{S2} + 2 \cdot Z_{M2}) \cdot I_{a2}$$

  depends upon not only I_{a1} but I_{a0} and I_{a2} as well. This means there is mutual coupling between sequences. Also, $Z_{mn.012}$ is not symmetric, therefore mutual effect is not reciprocal.

- How to "decouple" sequences?
  - In general, there are three cases of decoupling, but only one is useful (Case 2)

SYMMETRICAL COMPONENTS

– Case 1:

Self and mutual impedances are symmetric with respect to phase "A"

$$Z_{bb} = Z_{cc} \qquad Z_{S0} = \frac{1}{3}\cdot\left(Z_{aa} + 2Z_{bb}\right)$$

$$Z_{ab} = Z_{ca} \qquad Z_{S1} = Z_{s2} = \frac{1}{3}\cdot\left(Z_{aa} - Z_{bb}\right)$$

$$Z_{M0} = \frac{1}{3}\cdot\left(Z_{bc} + 2\cdot Z_{ab}\right)$$

$$Z_{M1} = Z_{M2} = \frac{1}{3}\cdot\left(Z_{bc} - Z_{ab}\right)$$

$$\mathbf{Z_{km.012}} = \begin{pmatrix} Z_{S0} + 2\cdot Z_{M0} & Z_{S2} - Z_{M2} & Z_{S1} - Z_{M1} \\ Z_{S1} - Z_{M1} & Z_{S0} - Z_{M0} & Z_{S2} + 2\cdot Z_{M2} \\ Z_{S2} - Z_{M2} & Z_{S1} + 2\cdot Z_{M1} & Z_{S0} - Z_{M0} \end{pmatrix}$$

– Mutual coupling between sequences is not eliminated.

SYMMETRICAL COMPONENTS

– Case 2:

Self and mutual impedances are equal in all three phases:

$$Z_{S0} = Z_{aa} \qquad Z_{S1} = Z_{S2} = 0$$

$$Z_{M0} = Z_{bc} \qquad Z_{M1} = Z_{M2} = 0$$

$$\mathbf{Z_{km.012}} = \begin{pmatrix} Z_{S0} + 2 \cdot Z_{M0} & \xcancel{Z_{S2} - Z_{M2}} & \xcancel{Z_{S1} - Z_{M1}} \\ \xcancel{Z_{S1} - Z_{M1}} & Z_{S0} - Z_{M0} & \xcancel{Z_{S2} + 2 \cdot Z_{M2}} \\ \xcancel{Z_{S2} - Z_{M2}} & \xcancel{Z_{S1} + 2 \cdot Z_{M1}} & Z_{S0} - Z_{M0} \end{pmatrix} = \begin{pmatrix} Z_{S0} + 2 \cdot Z_{M0} & 0 & 0 \\ 0 & Z_{S0} - Z_{M0} & 0 \\ 0 & 0 & Z_{S0} - Z_{M0} \end{pmatrix}$$

- Off diagonal terms eliminated.
- Matrix is reciprocal and no coupling between sequences. This is the case that is used in all Symmetrical Components calculations.

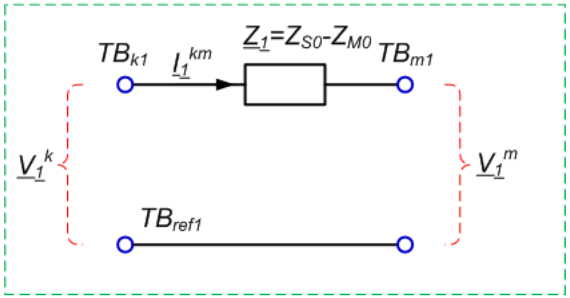
SYMMETRICAL COMPONENTS

Network equation and diagram for series component

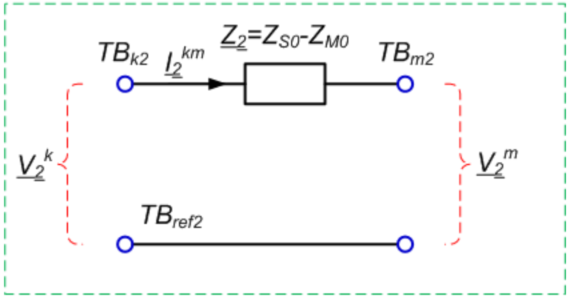
$$\mathbf{V_{km.012} = Z_{km.012} \cdot I_{km.012}}$$

$$\begin{pmatrix} V_{km.0} \\ V_{km.1} \\ V_{km.2} \end{pmatrix} = \begin{pmatrix} Z_{S0} + 2 \cdot Z_{M0} & 0 & 0 \\ 0 & Z_{S0} - Z_{M0} & 0 \\ 0 & 0 & Z_{S0} - Z_{M0} \end{pmatrix} \cdot \begin{pmatrix} I_{km.0} \\ I_{km.1} \\ I_{km.2} \end{pmatrix}$$

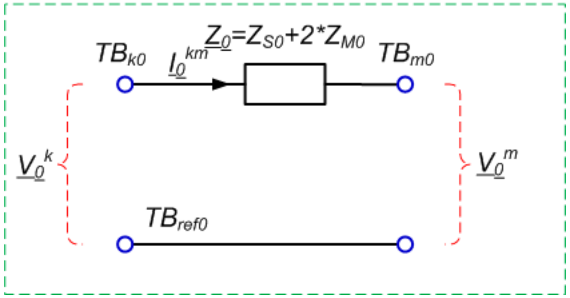
Positive Sequence Network



Negative Sequence Network



Zero Sequence Network



SYMMETRICAL COMPONENTS

- In the matrix modeling, the components in three-phase system exhibit characteristics in phase domain as follow:
  - Complete symmetry component

$$\begin{bmatrix} \underline{z_s} & \underline{z_m} & \underline{z_m} \\ \underline{z_m} & \underline{z_s} & \underline{z_m} \\ \underline{z_m} & \underline{z_m} & \underline{z_s} \end{bmatrix}$$

where $\underline{z_s}$ - self impedance

$\underline{z_m}$ - mutual impedance

  - Circulant symmetry component

$$\begin{bmatrix} \underline{z_s} & \underline{z_m} & \underline{z_n} \\ \underline{z_n} & \underline{z_s} & \underline{z_m} \\ \underline{z_m} & \underline{z_n} & \underline{z_s} \end{bmatrix}$$

where $\underline{z_s}$ - self impedance

$\underline{z_m}$, $\underline{z_n}$ - mutual impedances

  - Unbalanced component

$$\begin{bmatrix} \underline{z_{11}} & \underline{z_{12}} & \underline{z_{13}} \\ \underline{z_{21}} & \underline{z_{22}} & \underline{z_{23}} \\ \underline{z_{31}} & \underline{z_{32}} & \underline{z_{33}} \end{bmatrix}$$

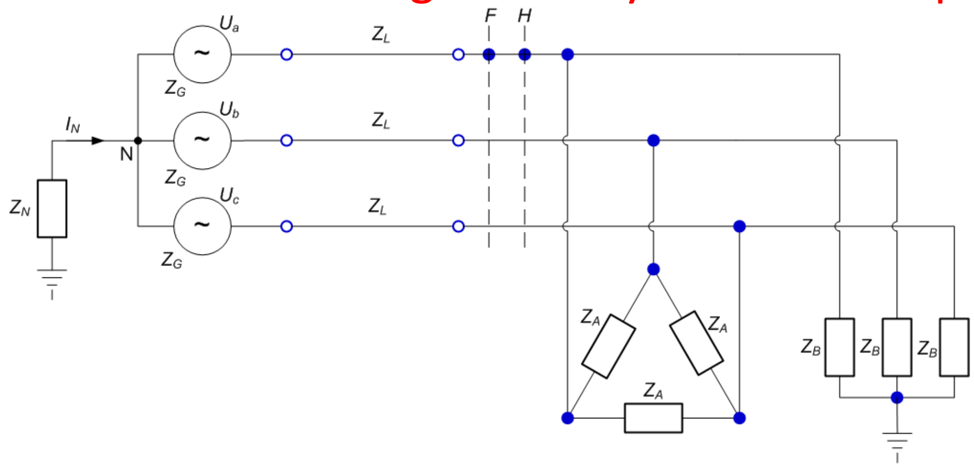
where $\underline{z_{11}}$, $\underline{z_{22}}$, $\underline{z_{33}}$ - self impedances in relation to each phase (in general not equal to each others),

$\underline{z_{12}}$, $\underline{z_{13}}$, $\underline{z_{21}}$, $\underline{z_{23}}$, $\underline{z_{31}}$, $\underline{z_{33}}$ - mutual impedance between each of phases primitive impedances (in general not equal to each other's and dependent on the mutual couplings).

UNBALANCE NETWORKS MODELING PRINCIPLES

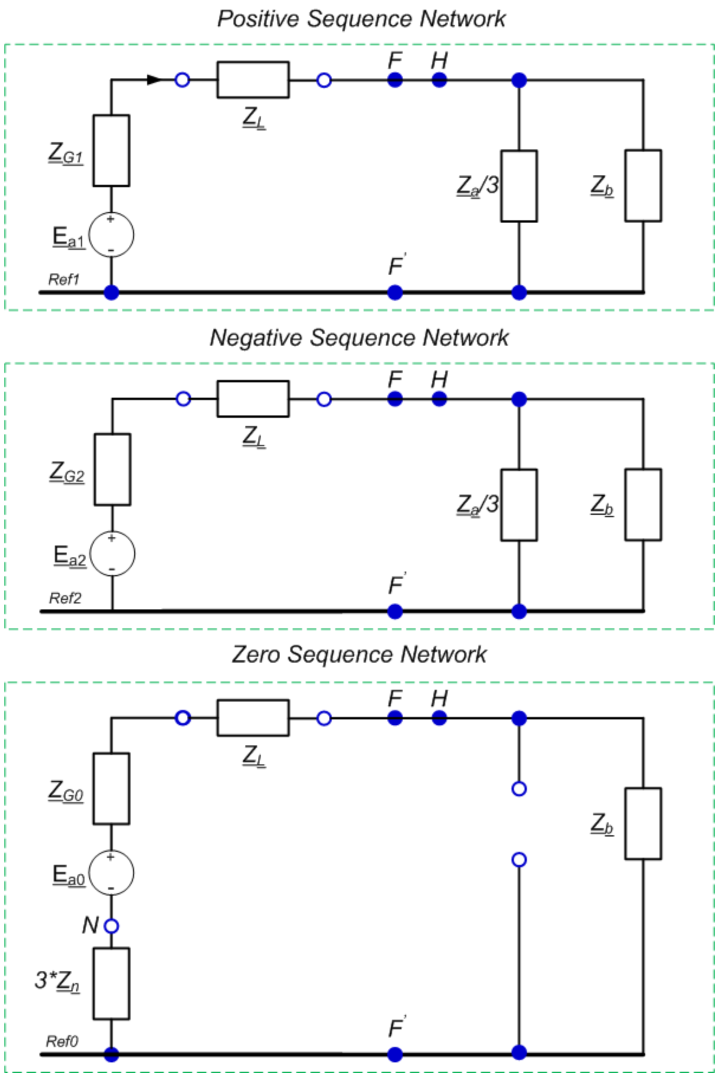
NETWORKS MODELING – UNBALANCED SOURCE WITH BALANCED CIRCUIT

- Symmetrical 3-phase circuit powered up with symmetrical (balanced) generator:
  - » Circuit currents are symmetrical; i.e. there are symmetrical components of current and voltages with symmetrical impedances but only positive sequence (for CCW rotation)
- Symmetrical 3-phase circuit powered up with unsymmetrical (unbalanced) generator
  - » Circuit currents are unsymmetrical; i.e. there are 0, 1, and 2 symmetrical components of current and voltages with symmetrical impedances in circuit



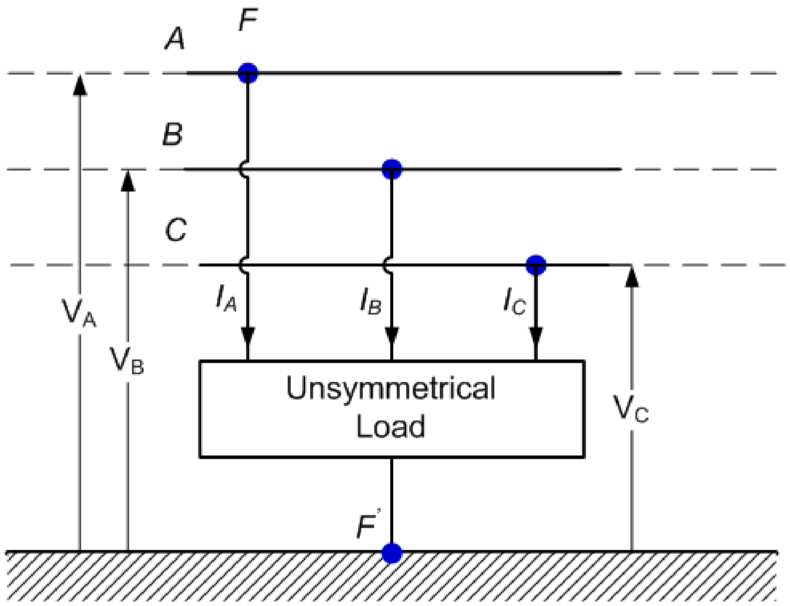
NETWORKS MODELING – UNBALANCED SOURCE WITH BALANCED CIRCUIT

- In symmetrical circuits, we can analyze each symmetrical component of current and voltage independent of other components networks (i.e. networks are "decoupled"). This results in three independent networks.
- Currents & voltages can be calculated for symmetrical network with unbalanced generator.



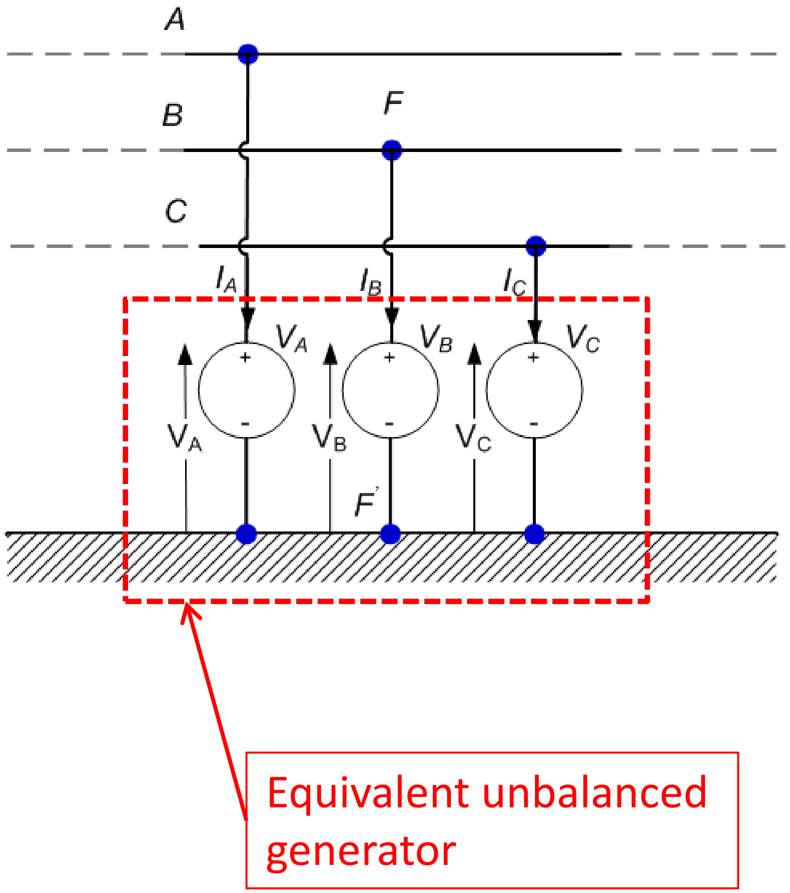
NETWORKS MODELING – SHUNT UNBALANCE

- Usually, 3-phase circuits are powered up with symmetrical (balanced) generators, but in certain operating conditions, symmetry in network is lost due to unsymmetrical load (i.e. fault)



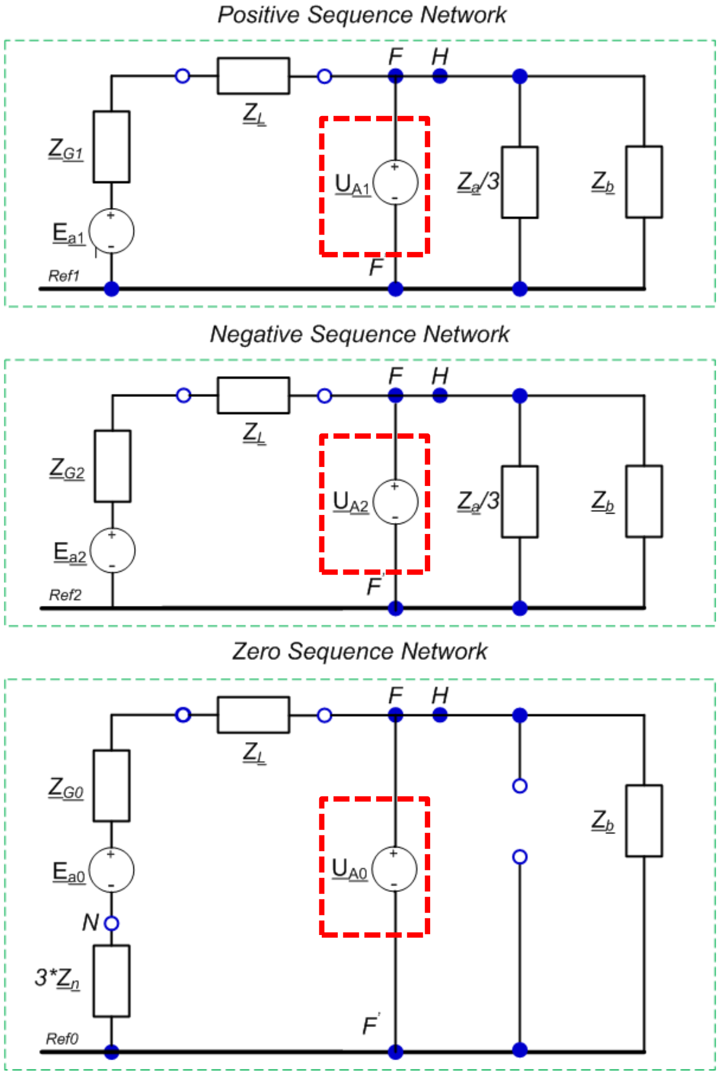
NETWORKS MODELING – SHUNT UNBALANCE

- In order to utilize symmetrical components methodology, different approach from previous method has to be developed.
  - » In place where unbalanced load was connected (point F), power sources with voltages U_A, U_B, and U_C equal to voltages at point F are connected; i.e. unbalanced generator replaced unbalanced load.
  - » No current flow and voltage values in the circuit are altered.
  - » <u>As a result, we are analyzing balanced circuit with unbalanced generator.</u>



NETWORKS MODELING – SHUNT UNBALANCE

- After converting U_A, U_B, and U_C to symmetrical components voltages U_0, U_1, and U_2 ,they are inserted in the network.



NETWORKS MODELING – SHUNT UNBALANCE

- Utilizing Thevenin Theorem

$$\frac{1}{Z_1} = \frac{1}{Z_{G1} + Z_L} + \frac{1}{\frac{1}{3}\cdot Z_a} + \frac{1}{Z_b}$$

$$\frac{1}{Z_2} = \frac{1}{Z_{G2} + Z_L} + \frac{1}{\frac{1}{3}\cdot Z_a} + \frac{1}{Z_b}$$

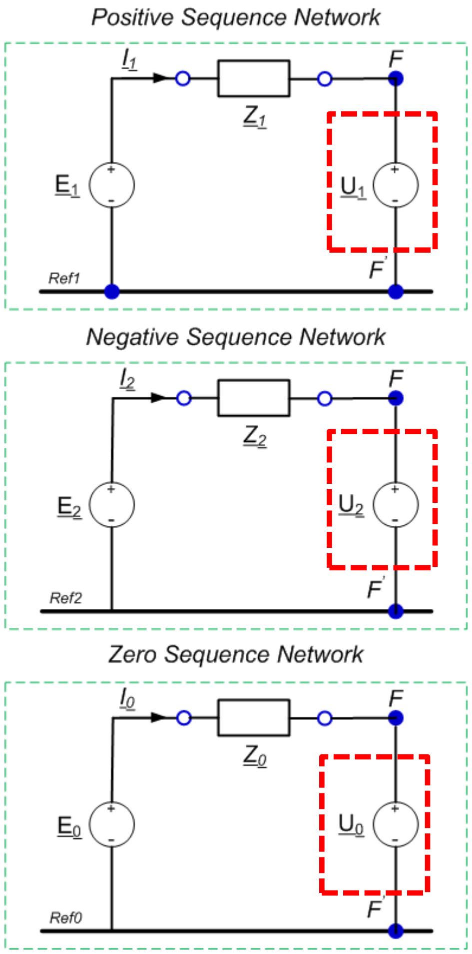
$$\frac{1}{Z_0} = \frac{1}{Z_{G0} + Z_L + 3\cdot Z_N} + \frac{1}{Z_b}$$

- Circuit solution is described by

$$\underline{U}_1 = \underline{E}_1 - Z_1\cdot \underline{I}_1$$
$$\underline{U}_2 = \underline{E}_2 - Z_2\cdot \underline{I}_2$$
$$\underline{U}_0 = \underline{E}_0 - Z_0\cdot \underline{I}_0$$

$$\underline{U}_1 = \underline{E}_1 - Z_1\cdot \underline{I}_1$$
$$\underline{U}_2 = -Z_2\cdot \underline{I}_2$$
$$\underline{U}_0 = -Z_0\cdot \underline{I}_0$$

For balanced and symmetrical generator



NETWORKS MODELING – BALANCED SOURCE WITH BALANCED CIRCUIT & UNBALANCED LOAD

- Example

$\underline{I}_B = 0$

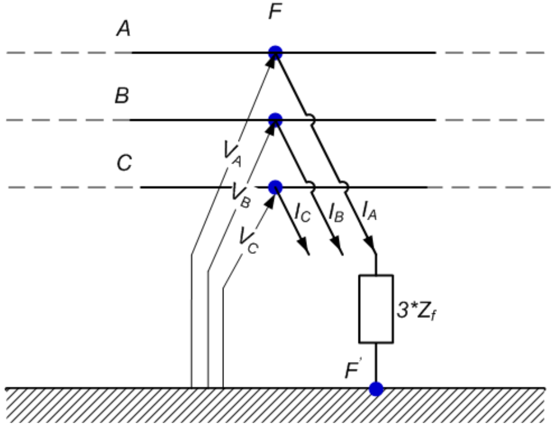
$\underline{I}_C = 0$

$\underline{U}_A = \underline{I}_A \cdot \underline{Z}_f$

$$\begin{pmatrix} \underline{I}_0 \\ \underline{I}_1 \\ \underline{I}_2 \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{pmatrix} \cdot \begin{pmatrix} \underline{I}_A \\ 0 \\ 0 \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} \underline{I}_A \\ \underline{I}_A \\ \underline{I}_A \end{pmatrix}$$

$\underline{I}_0 = \underline{I}_1 = \underline{I}_2$

$\underline{U}_A = \underline{I}_A \cdot \underline{Z}_f$



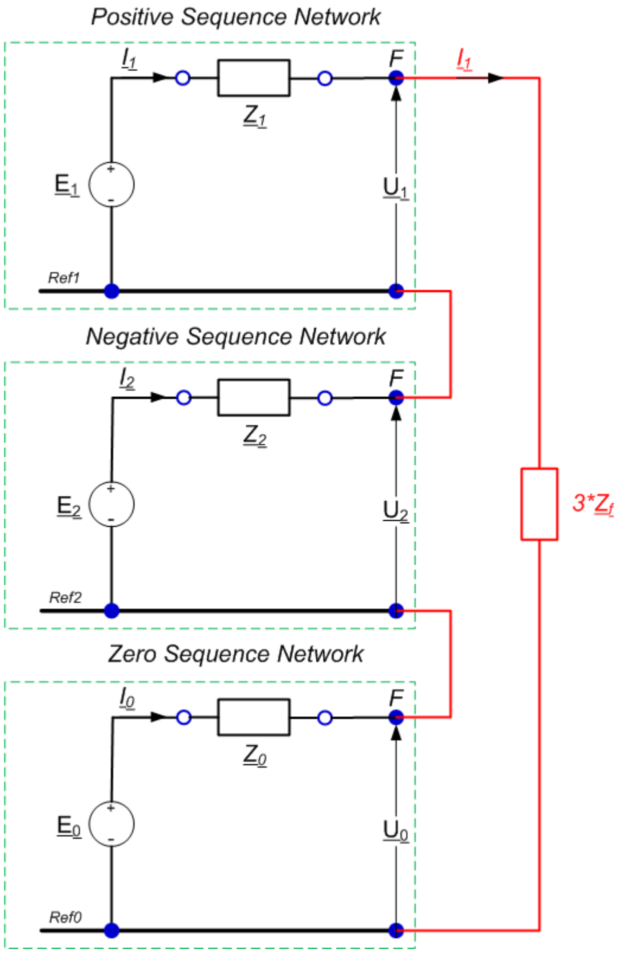
NETWORKS MODELING – BALANCED SOURCE WITH BALANCED CIRCUIT & UNBALANCED LOAD

- Example

$$\underline{U}_0 + \underline{U}_1 + \underline{U}_2 = 3 \cdot \underline{I}_1 \cdot \underline{Z}_f$$

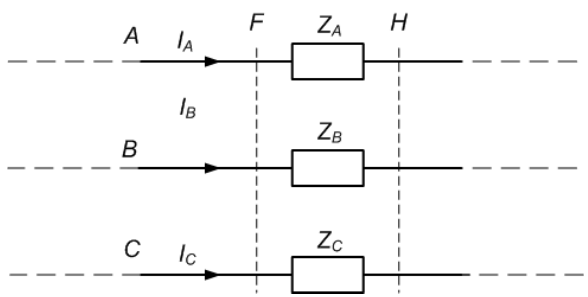
$$-\underline{Z}_0 \cdot \underline{I}_0 + \underline{E}_1 - \underline{Z}_1 \cdot \underline{I}_1 - \underline{Z}_2 \cdot \underline{I}_2 = 3 \cdot \underline{I}_1 \cdot \underline{Z}_f$$

$$\underline{I}_1 = \frac{\underline{E}_1}{\underline{Z}_1 + \underline{Z}_2 + \underline{Z}_0 + 3 \cdot \underline{Z}_f}$$



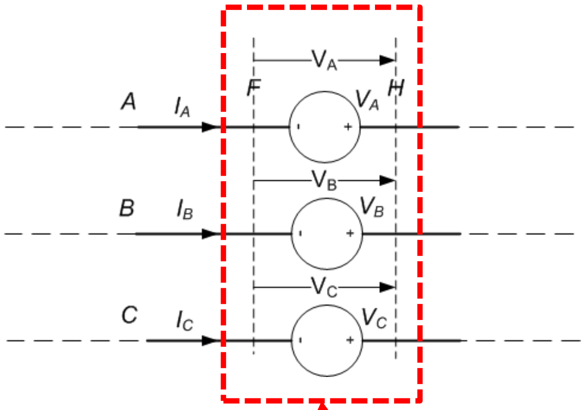
NETWORKS MODELING – SERIAL UNBALANCE

- Usually, 3-phase circuits are powered up with symmetrical (balanced) generators, but in certain operating conditions, symmetry in network is lost due to unsymmetrical load (i.e. fault)



NETWORKS MODELING – SERIAL UNBALANCE

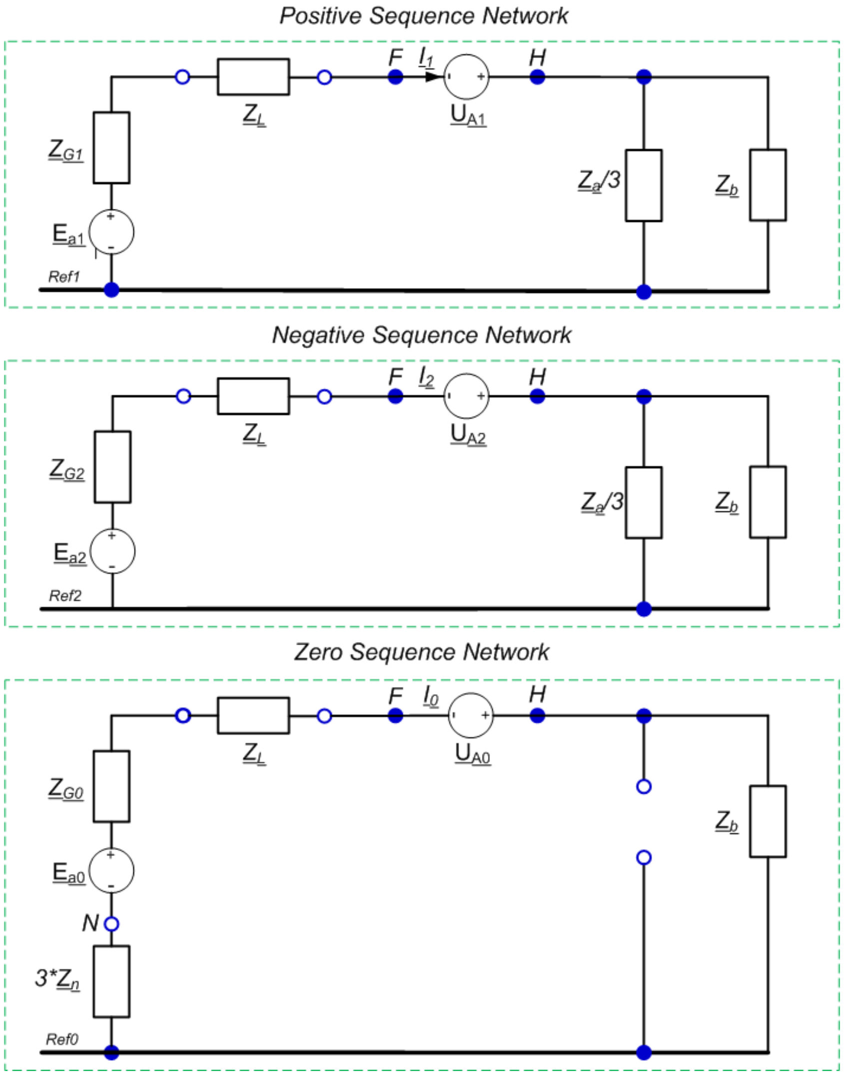
- In order to utilize symmetrical components methodology, different approach from previous method has to be developed.
  - » In place where unbalanced load was connected (point F), power sources with voltages U_A, U_B, and U_C equal to voltages at point F are connected; i.e. unbalanced generator replaced unbalanced load.
  - » No current flow and voltage values in the circuit are altered.
  - » <u>As a result, we are analyzing balanced circuit with unbalanced generator.</u>



Equivalent unbalanced generator

NETWORKS MODELING – SERIAL UNBALANCE

- After converting U_A, U_B, and U_C to symmetrical components voltages U_0, U_1, and U_2 ,they are inserted in the network.



NETWORKS MODELING – SERIAL UNBALANCE

- Utilizing Thevenin Theorem

$$Z_1 = Z_{G1} + Z_L + \frac{\frac{1}{3} \cdot Z_a \cdot Z_b}{\frac{1}{3} \cdot Z_a + Z_b}$$

$$Z_2 = Z_{G2} + Z_L + \frac{\frac{1}{3} \cdot Z_a \cdot Z_b}{\frac{1}{3} \cdot Z_a + Z_b}$$

$$Z_0 = Z_{G0} + Z_L + 3 \cdot Z_N + Z_b$$

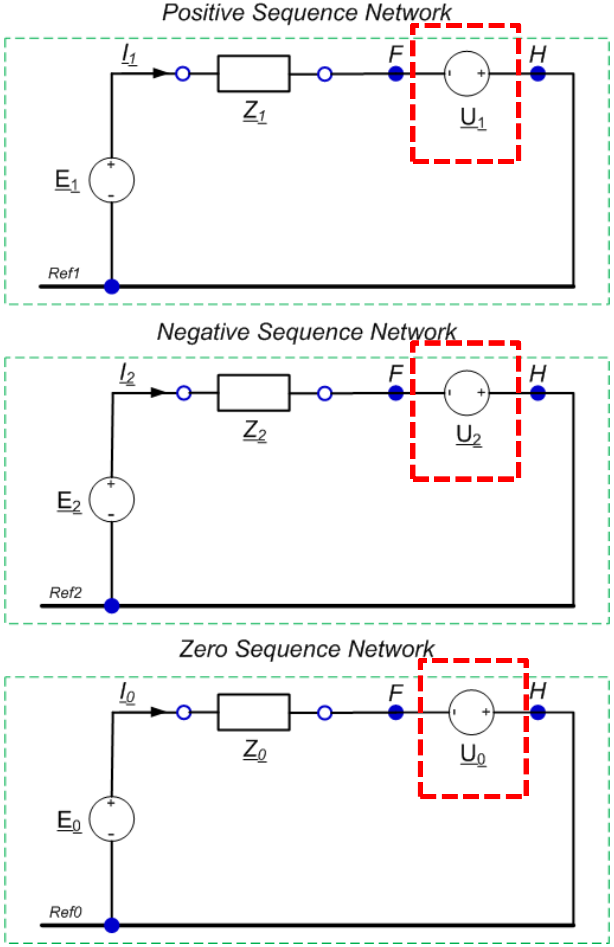
- Circuit solution is described by

$$\underline{U}_1 = \underline{E}_1 - Z_1 \cdot \underline{I}_1 \qquad \underline{U}_1 = \underline{E}_1 - Z_1 \cdot \underline{I}_1$$

$$\underline{U}_2 = \underline{E}_2 - Z_2 \cdot \underline{I}_2 \qquad \underline{U}_2 = -Z_2 \cdot \underline{I}_2$$

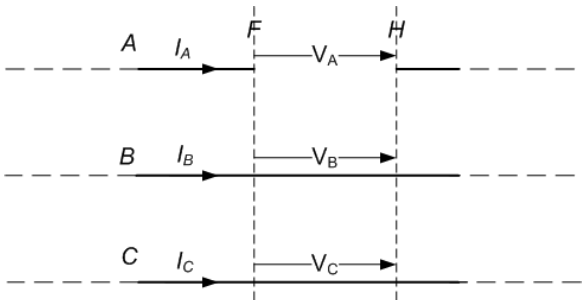
$$\underline{U}_0 = \underline{E}_0 - Z_0 \cdot \underline{I}_0 \qquad \underline{U}_0 = -Z_0 \cdot \underline{I}_0$$

For balanced and symmetrical generator



NETWORKS MODELING – SERIAL UNBALANCE

- Example



$\underline{I}_A = 0$

$U_B = U_C = 0$

$$\begin{pmatrix} \underline{U}_0 \\ \underline{U}_1 \\ \underline{U}_2 \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{pmatrix} \cdot \begin{pmatrix} \underline{U}_A \\ 0 \\ 0 \end{pmatrix} = \frac{1}{3} \cdot \begin{pmatrix} \underline{U}_A \\ \underline{U}_A \\ \underline{U}_A \end{pmatrix}$$

$$\boxed{\underline{U}_0 = \underline{U}_1 = \underline{U}_2}$$

$\underline{I}_A = 0$

$$\begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{pmatrix} \cdot \begin{pmatrix} \underline{I}_{a0} \\ \underline{I}_{a1} \\ \underline{I}_{a2} \end{pmatrix}$$

$$\boxed{\underline{I}_A = \underline{I}_{a0} + \underline{I}_{a1} + \underline{I}_{a2} = 0}$$

NETWORKS MODELING – SERIAL UNBALANCE

- Example

$$\underline{I}_A = \underline{I}_{a0} + \underline{I}_{a1} + \underline{I}_{a2} = 0$$

$$\underline{U}_1 = \underline{E}_1 - \underline{Z}_1 \cdot \underline{I}_1$$

$$\underline{U}_2 = -\underline{Z}_2 \cdot \underline{I}_2$$

$$\underline{U}_0 = -\underline{Z}_0 \cdot \underline{I}_0$$

$$-\underline{Z}_0 \cdot \underline{I}_0 = -\underline{Z}_2 \cdot \underline{I}_2$$

$$\underline{I}_0 = \frac{\underline{Z}_2 \cdot \underline{I}_2}{\underline{Z}_0}$$

$$\frac{\underline{Z}_2 \cdot \underline{I}_2}{\underline{Z}_0} + \underline{I}_1 + \underline{I}_2 = 0$$

$$\underline{I}_2 = -\underline{I}_1 \cdot \frac{\underline{Z}_0}{\underline{Z}_0 + \underline{Z}_2}$$

$$\underline{I}_0 = \frac{\underline{Z}_2}{\underline{Z}_0} \cdot \left(-\underline{I}_1 \cdot \frac{\underline{Z}_0}{\underline{Z}_0 + \underline{Z}_2} \right) = -\underline{I}_1 \cdot \frac{\underline{Z}_2}{\underline{Z}_0 + \underline{Z}_2}$$

NETWORKS MODELING – SERIAL UNBALANCE

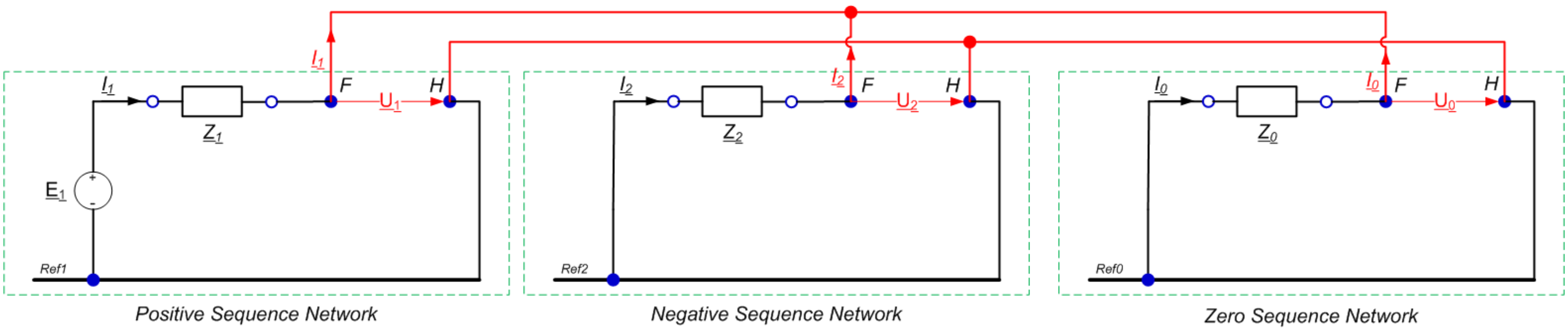
- Example

$$\underline{U}_0 = \underline{U}_1 = \underline{U}_2 = -\underline{Z}_2 \cdot \underline{I}_2 = -\underline{Z}_2 \cdot \left(-\underline{I}_1 \cdot \frac{\underline{Z}_0}{\underline{Z}_0 + \underline{Z}_2} \right) = \underline{I}_1 \cdot \frac{\underline{Z}_2 \cdot \underline{Z}_0}{\underline{Z}_0 + \underline{Z}_2}$$

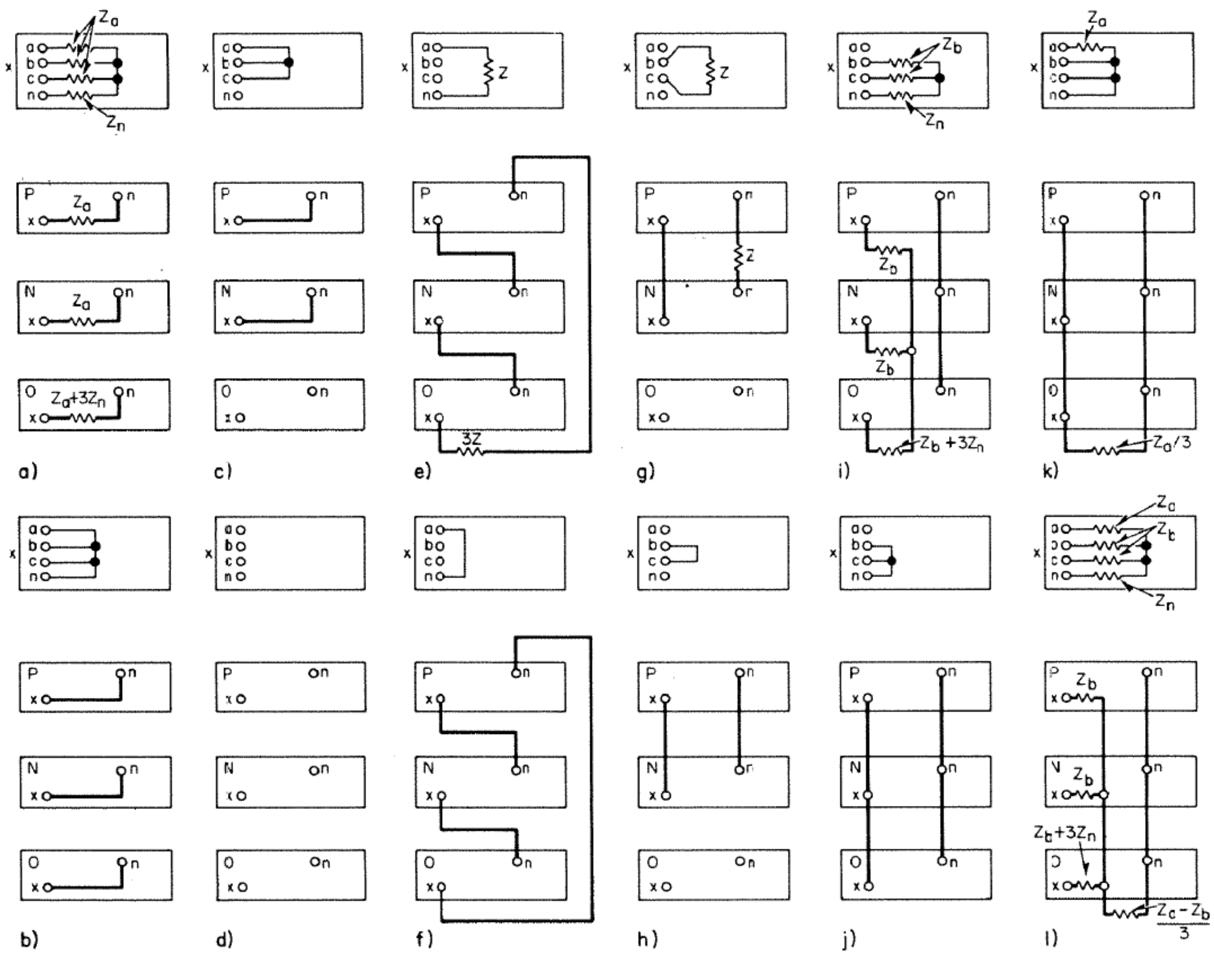
$$\underline{U}_1 = \underline{E}_1 - \underline{Z}_1 \cdot \underline{I}_1$$

$$\underline{I}_1 \cdot \frac{\underline{Z}_2 \cdot \underline{Z}_0}{\underline{Z}_0 + \underline{Z}_2} = \underline{E}_1 - \underline{Z}_1 \cdot \underline{I}_1$$

$$\underline{I}_1 = \frac{\underline{E}_1}{\underline{Z}_1 + \frac{\underline{Z}_2 \cdot \underline{Z}_0}{\underline{Z}_0 + \underline{Z}_2}}$$



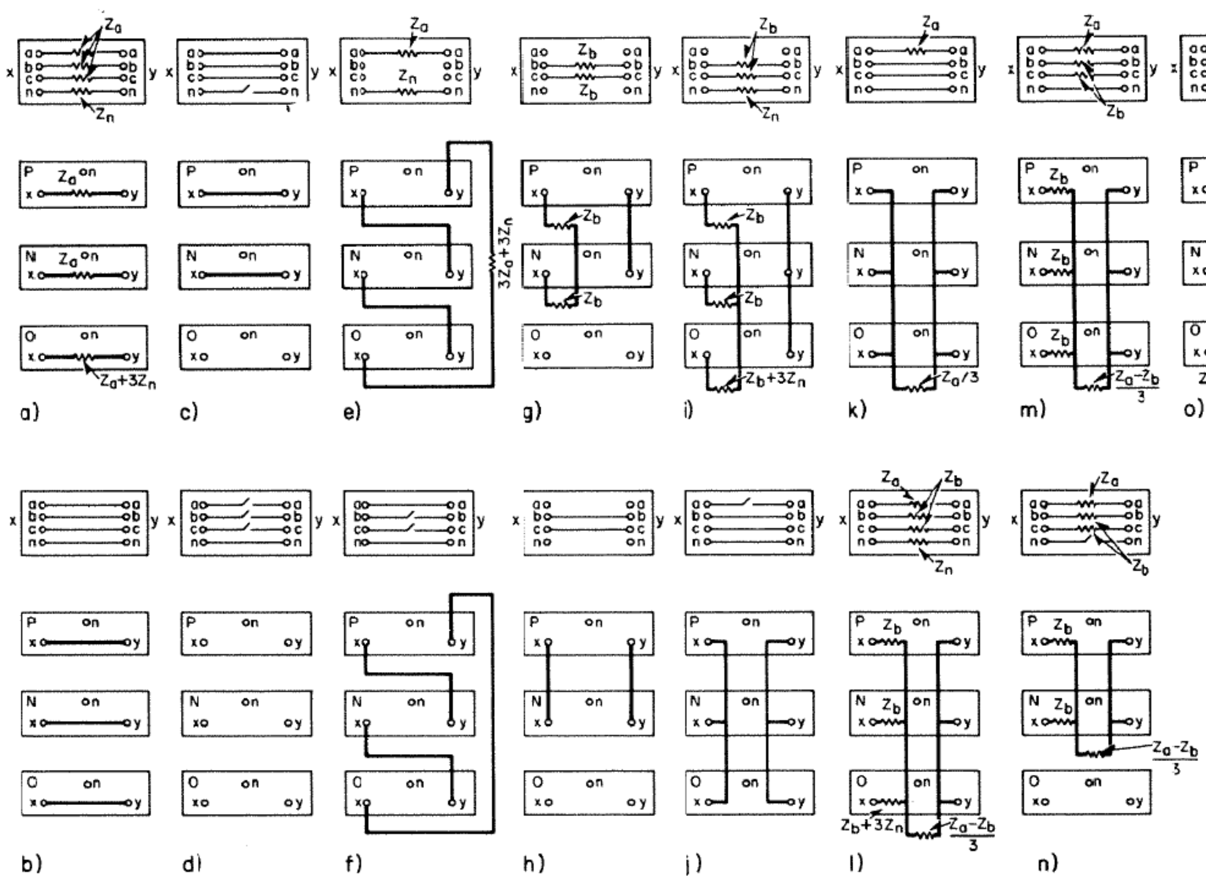
NETWORKS UNBALANCE MODELING – SHUNT FAULTS



Note:

a) Balanced load or three-line-to-ground fault with impedances.
b) A three-line-to-ground fault.
c) A three-phase fault.
d) A shunt circuit open.
e) A line-to-ground fault through an impedance.
f) A line-to-ground fault.
g) A line-to-line fault through impedance.
h) A line-to-line fault.
i) A two-line-to-ground fault with impedance.
j) A two-line-to-ground fault.
k) A three-line-to-ground fault with impedance in phase a.
l) Unbalanced load or three-line-to-ground fault with impedance.

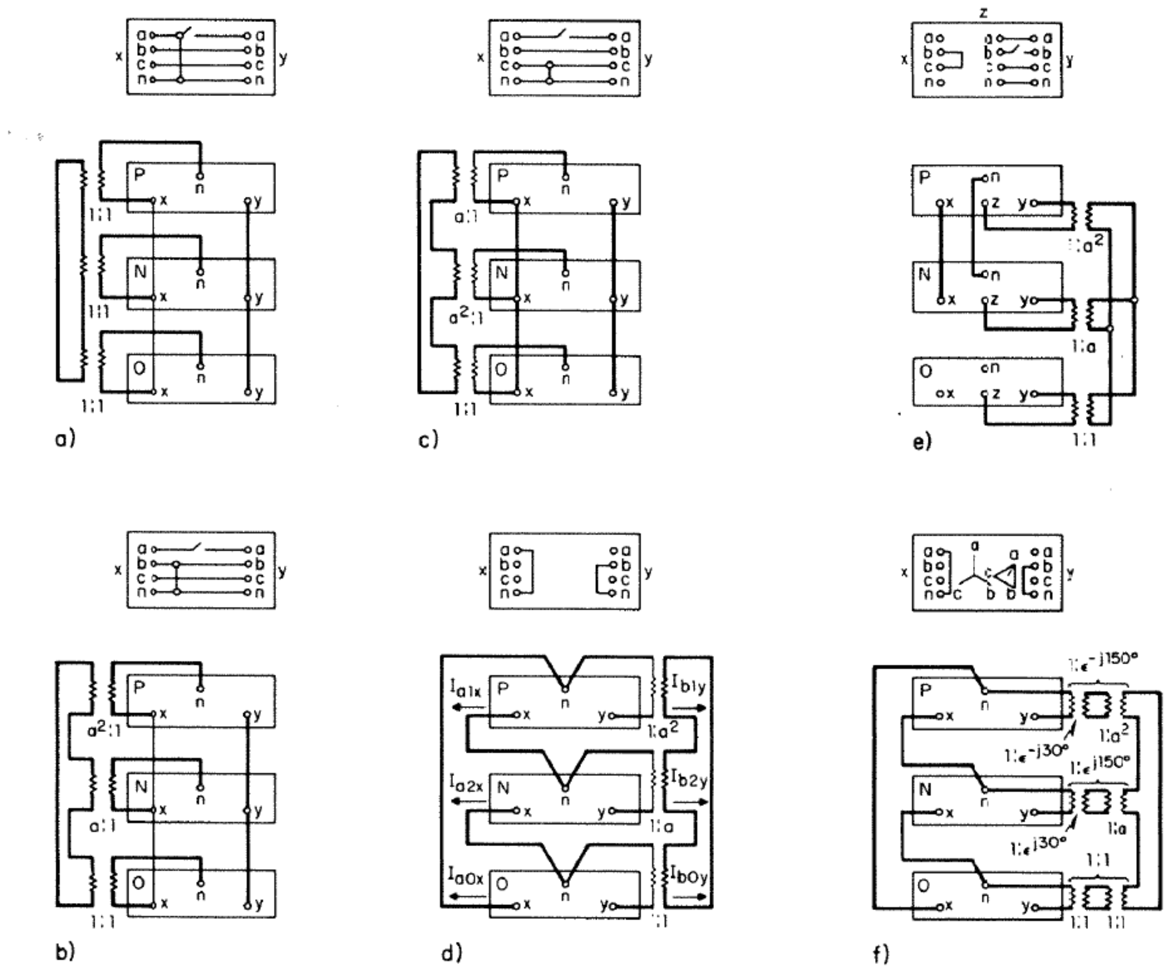
NETWORKS UNBALANCE MODELING – SERIES FAULTS



Note:

a) Equal impedances in three phases.
b) Normal conditions.
c) Neutral open.
d) Any three or four phases open.
e) Phases b and c open, impedances in phases a and neutral.
f) Phases b and c open.
g) Phases a and neutral open, impedance in b and c.
h) Phases a and neutral open.
i) Phase a open, impedances in b, c, and neutral.
j) Phase a open.
k) Impedance in phase a.
l) Equal impedances in b and c phases, and neutral.
m) Equal impedances in b and c phases.
n) Equal impedances in b and c phases, neutral open.
o) Impedances in phase a and neutral.

NETWORKS UNBALANCE MODELING – SIMULTANEOUS FAULTS / CROSS COUNTRY



Note:

a) One phase open and a fault to ground.
b) Phase a open and a b-phase-to-ground fault.
c) Phase a open and c-phase-to-neutral fault.
d) Phase a-to-ground fault at x and a b-phase-to-ground fault at y.
e) A b-to-c fault at x, and b phase open z to y.
f) Phase a-to-neutral fault at x, phase b-to-neutral fault on other side of star-delta transformer bank at y. x is taken as the reference point.

COMPONENTS MODELING

COMPONENTS MODELING

- Utility Sources and AC Generators
- Induction Machines
- Transformers
- Transmission Lines
- Notes on modeling

COMPONENTS MODELING – UTILITY SOURCES

- Request data from utility
  - Define inter-tie point location (GIS or other)
  - Define what is required:
    - I_{3ph} with X/R
    - I_{1ph} with X/R
    - Ops Conditions: max, min, normal
    - Special:

      Future growth

      Other prediction for

      PCC (or POI)

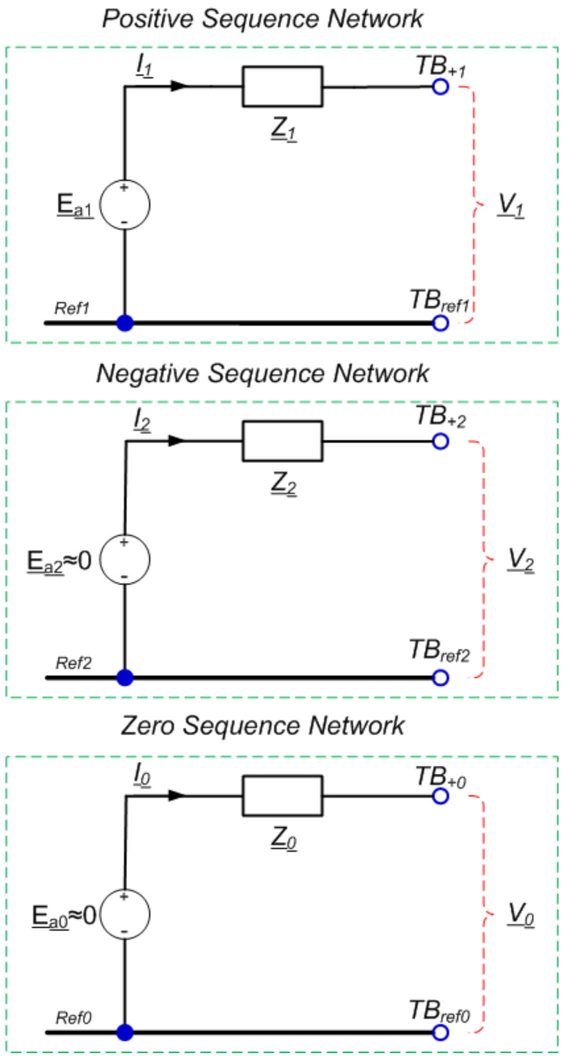
The following information was provided in per unit on a 100MVA base @ 138kV:

System Configuration/ Scenario	Three Phase (I_A)	Single Phase ($3I_0$)	Thevenin's Equivalent Voltage Source	Thevenin's Equivalent System Impedance (p.u.)					
				Positive		Negative		Zero	
				Z	X/R	Z	X/R	Z	X/R
Both 138kV Lines In-Service	6620 MVA	4314 MVA	1.031 p.u.	0.0161	9.8	0.1595	9.7	0.0417	5.7
Ckt #84 to Channelview Out-of-Service	2513 MVA	1618 MVA	1.032 p.u.	0.0424	9.7	0.0424	9.6	0.1110	5.8
Ckt #84 to Jacinto Out-of-Service	4284 MVA	2729 MVA	1.032 p.u.	0.0249	9.5	0.0249	9.4	0.0674	5.5

COMPONENTS MODELING – UTILITY SOURCES

- Utility is modeled as a fixed source with fixed impedances
- ANSI methodology allows for close by generation but does not adjust impedance. It is expected that system engineer does this calcs on his own per application (i.e. decrement curve calcs).

$$R = \frac{1}{\left(1 + XR^2\right)^{\frac{1}{2}}} \cdot Z \qquad X = \frac{XR}{\left(1 + XR^2\right)^{\frac{1}{2}}} \cdot Z$$



COMPONENTS MODELING – AC GENERATOR

- Fault Behavior
  - Sudden change in voltage and current, such as those in faults, produces transients
  - Armature current divided into two components:
    - Symmetrical AC component – whose associated component in the field is a DC current
    - DC component – whose associated component in the field is an AC current

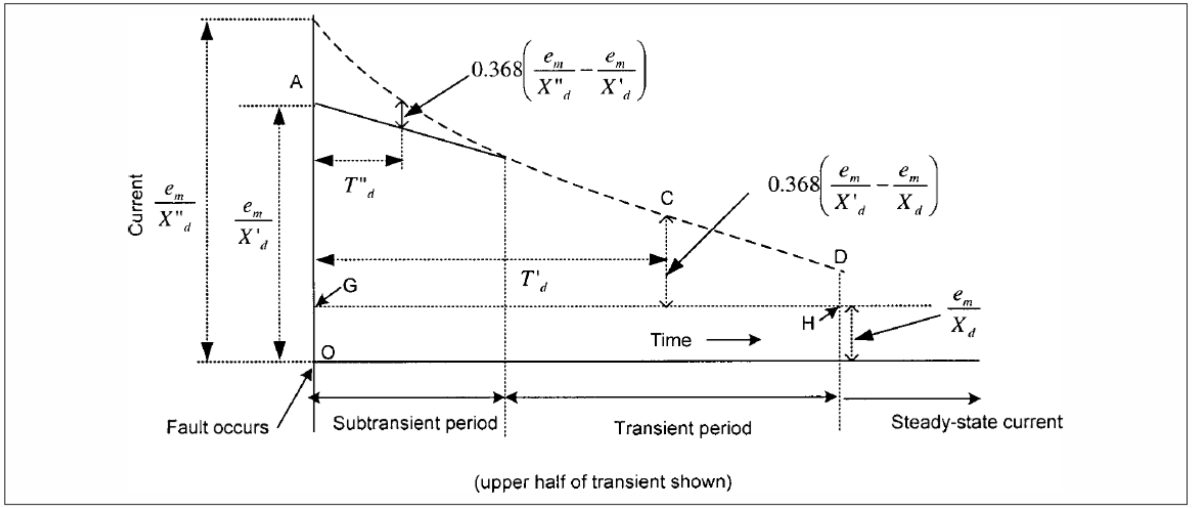


FIGURE 10-5 Decaying ac component of the short-circuit current from the instance of occurrence of fault to steady state. The subtransient, transient, and steady-state periods and decay characteristics of short-circuit current are shown.

COMPONENTS MODELING – AC GENERATOR

- Reactances change with time, i.e. model changes with time

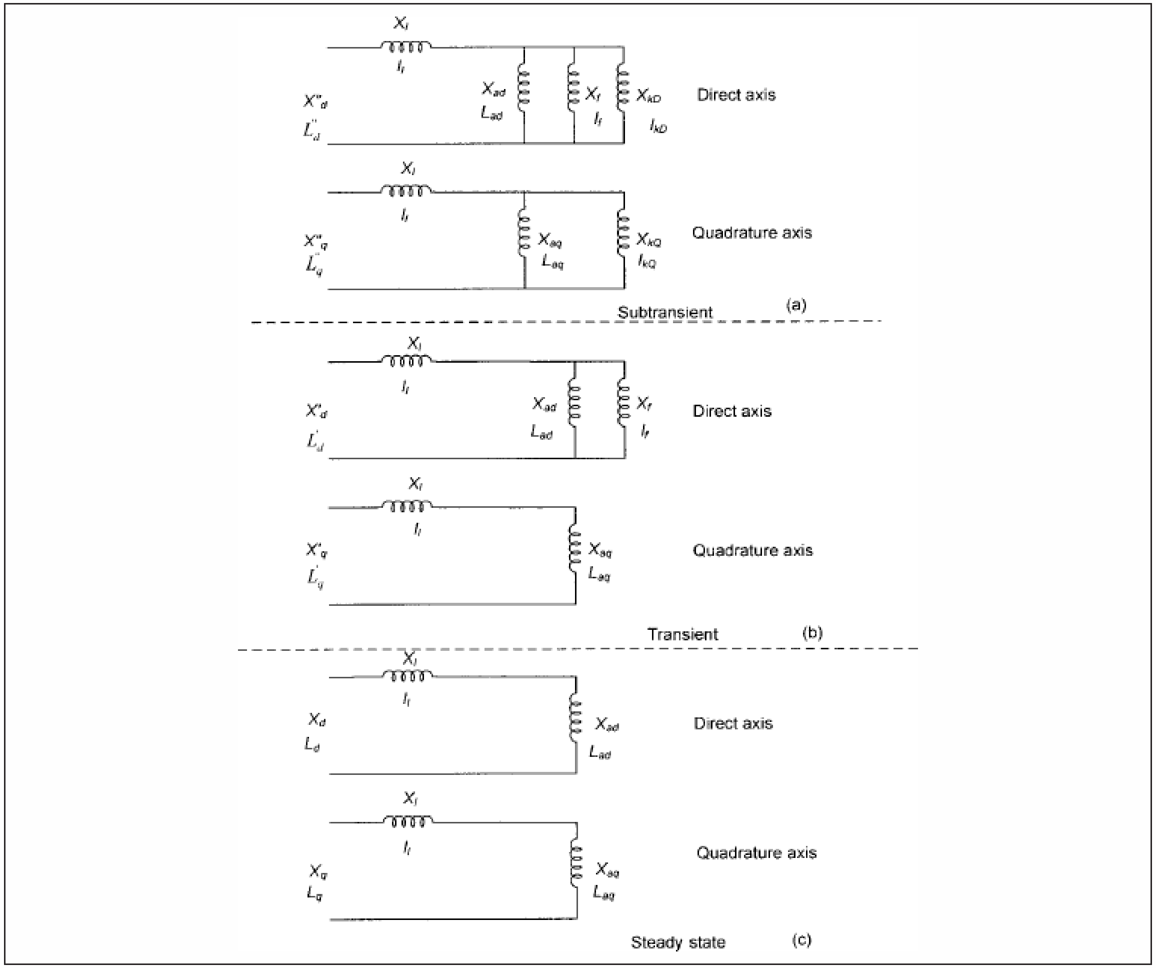


FIGURE 10-6 Equivalent circuits of a synchronous generator during subtransient, transient, and steady-state periods after a terminal fault.

COMPONENTS MODELING – AC GENERATOR

- Symmetrical Component Modeling
  - Principle concern is with symmetrical component and its associated constants
  - DC component often eliminated from studies
    - Usually not necessary to apply or set protective relays
    - If necessary (e.g. circuit breaker applications), various factors are available from standards, manufacturers, or other sources
  - For synchronous machines, symmetrical AC component can be resolved into three distinct components
    - Subtransient component – the double prime (“) values
    - Transient component – the single prime (‘) values
    - The steady-state component

COMPONENTS MODELING – AC GENERATOR

- Subtranscient Component
  - Occurs during commencement of fault
  - Subtransient reactance (*Xd") approaches armature* leakage reactance but is higher as a result of damper windings, and so on.
  - Subtransient time constant (*Td") is very low* (because damper windings have relatively high resistance), typically around 0.01–0.05 seconds

COMPONENTS MODELING – AC GENERATOR

- Transient Component
  - Armature current demagnetizes the field and decrease flux linkages with the field winding
  - Transient reactance (*Xd') includes effect of both* armature and field leakages and is higher than armature leakage reactance, and thus higher than the subtransient reactance
  - Transient time constant (*Td') varies typically from 0.35* to 3.3 seconds
- Steady-State Component
  - Transient eventually decays
  - For faults, eventually becomes unsaturated direct axis reactance (*Xd)*

COMPONENTS MODELING – AC GENERATOR

- Negative Sequence
  - Subtransient reactance can be measured by blocking the rotor with the field winding shorted and applying single phase voltage across any two terminals
  - As position of rotor is changed, measured reactance varies considerably if machine has salient poles without dampers (and very little damper winding exists) or if the machine has a round rotor
  - For negative sequence, similar phenomenon exists except rotor is at 2*f with relation to field set up by* applied voltage
  - Good approximation: $X_2 = \frac{1}{2}\left(X_d'' + X_q''\right)$

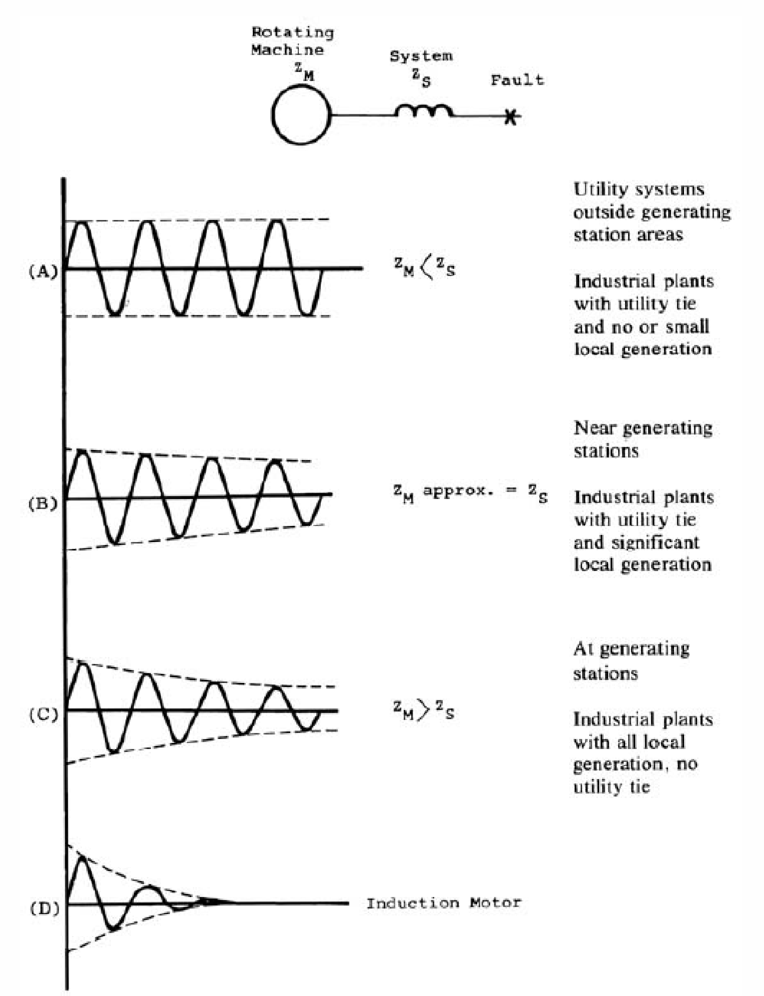
COMPONENTS MODELING – AC GENERATOR

- Zero Sequence
  - Varies quite a lot
  - Depends largely on pitch and breadth factors of armature winding
  - Generally, *X_0 is much smaller than X_1 and X_2* values

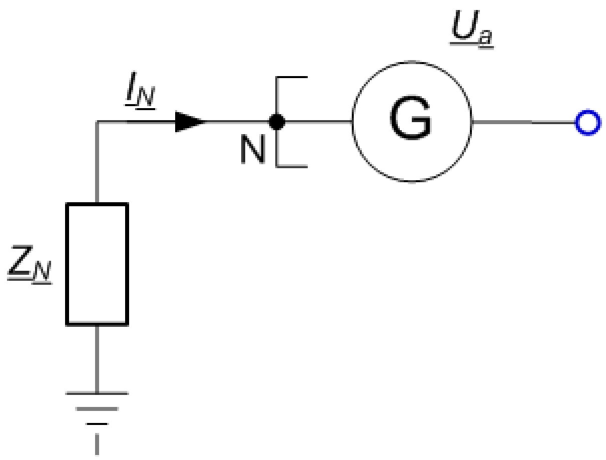
COMPONENTS MODELING – AC GENERATOR

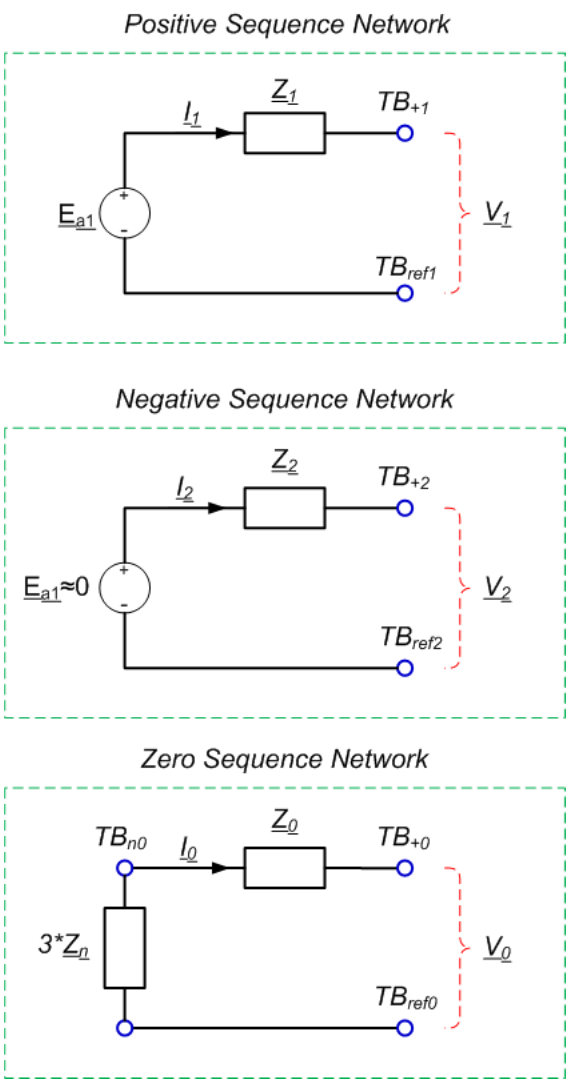
Close and Remote Generation

- Assume $X_1 = X_2$
- Calculate X_1 *from 3PH* fault duty
- Calculate X_0 *from SLG* fault duty



COMPONENTS MODELING – AC GENERATOR





COMPONENTS MODELING – AC GENERATOR

Turbine generators	Two-pole		Four-pole	
	Conventional cooled	Conductor cooled	Conventional cooled	Conductor cooled
X_d (unsat)	1.65/1.0–1.75	1.85/1.5–2.25	1.65/1.0–1.75	1.85/1.5–2.25
X_q rated current	1.61/0.96–1.71	1.81/1.46–2.21	1.61/.96–1.71	1.81/1.46–2.21
X'_d rated voltage	0.17/0.12–0.25	0.28/0.20–0.35	0.25/0.2–0.3	0.35/0.25–0.45
X''_d rated voltage	0.12/0.08–0.18	0.22/0.15–0.28	0.16/0.12–0.20	0.28/0.20–0.32
X_2 rated current	$= X''_d$	$= X''_d$	$= X''_d$	$= X''_d$
X_o rated current$^{(1)}$				
x_p Potier reactance	0.07–0.17	0.2–0.45	0.12–0.24	0.25–0.45
$r_2^{(2)}$	0.025–0.04	0.025–0.04	0.03–0.045	0.03–0.045
$r_1^{(3)}$	0.004–0.011	0.001–0.008	0.003–0.008	0.001–0.008
$r_a^{(3)}$	0.001–0.007	0.001–0.005	0.001–0.005	0.001–0.005
T'_{do}	5	5	8	6
T'_d	0.6	0.75	1.0	1.2
T''_d	0.035	0.035	0.035	0.035
T_a	0.13–0.45	0.2–0.55	0.2–0.4	0.25–0.55
H	2.5–3.5	2.5–3.5	3–4	3–4

Salient Pole Generators and Motors:

With dampers—$X'_d = 0.37/0.25–0.5$, $X''_d = 0.24/0.13–0.32$, $X_2 = X''_2$ ← X''_d

Without dampers—$X'_d = 0.35/0.25–0.5$, $X''_d = 0.32/0.20–0.5$, $X_2 = 0.4/0.3–0.45$.

Synchronous condensers:

$X'_d = 0.40$, $X''_d = 0.25$, $X_2 = 0.24$.

Notes: (1) X_o varies so critically with armature winding pitch that an average value can hardly be given. Variation is from 0.1 to 0.7 of X''_d; (2) r_2 varies with damper resistance; (3) r_1 and r_a vary with machine rating.

* Based on W. D. Stevenson Jr. “Elements of Power System Analysis”, 1982

COMPONENTS MODELING – AC GENERATOR

- Example of vendor provided data

DATA SHEET SYNCHRONOUS MACHINES

BASIC DATA

Date:		
Order no.:		
Ordered by:		
Destination:		

Type:		
Rated voltage:	13800	V, 60 Hz
Rated load:	22210	kW, at p.f. 0,85
Speed:	1800	rpm
Rated current:	1093	A
Rated temperature:	95	°F

Method of cooling:	IC616
Degree of protection:	IP55
Mounting arrangement:	IM1005

Insulation class Stator:	F, temp rise to B
Insulation class Rotor:	F, temp rise to B
Insulation class Exciter:	F, temp rise to B

Type of excitation: brushless

Exciter type:	DGBP60/15		
P.M.G.:	540/40 (6000 VA; 225 V; 1 phase)		
Exciter response:	2,39	1/sec	
	Stator	Rotor	
No load volt.	13	20	V
No load current	2,2	264	A
Exc. rated volt.	41	56	V
Rated current	6,3	744	A
Ceiling volt.	73,5	110	V
Ceiling current	12,1	1449	A

Doc. nr: SPE20624:WGN Version 1 ITEM

ELECTRICAL DATA

Standards: NEMA MG1

Gen. stator resistance:	0,0135	Ohm at 20 °C
Gen. rotor resistance	0,0586	Ohm at 20 °C
Exc. stator resistance	4,7	Ohm at 20 °C

REACTANCES	Unsaturated		saturated	
d-axis synchronous:	1,97	p.u.	1,48	p.u.
transient:	0,31	p.u.	0,24	p.u.
sub transient:	0,20	p.u.	0,17	p.u.
q-axis synchronous:	1,02	p.u.	0,76	p.u.
sub transient:	0,40	p.u.	0,28	p.u.
Negative sequence:	0,30	p.u.	0,23	p.u.
Zero sequence:	0,11	p.u.		
Potier reactance	0,28	p.u.		
Short Circuit Ratio:	0,55			

TIME CONSTANTS		
d-axis transient short circuit:	1,29	sec.
d-axis transient open circuit:	8,2	sec.
d-axis subtransient short circuit:	0,04	sec.
d-axis subtransient open circuit:	0,06	sec.
q-axis subtransient short circuit:	0,04	sec.
q-axis subtransient open circuit:	0,10	sec.
Armature:	0,24	sec.

Short Circuit Conditions				
3 phase Peak	17337	A	15,9	p.u.
3 phase RMS	6052	A	5,5	p.u.
2 phase Peak	11042	A	10,1	p.u.
Steady state	1853	A	1,7	p.u.

LANG

MECHANICAL PROPERTIES

Acceleration time Tj	3,1	sec
Inertia constant H	1,33	kWsec/kVA
Damping factor kd	16,1	MW/Hz
Direction of rotation:	CW/CCW	
Rotortype:	salient poles	
Poles are:	massive without damperwinding	
Shaft extension:	Flange	

LOSSES AT NOMINAL RATING		
Friction and windage losses	160	kW
Core losses	146	kW
Stray load losses	57	kW
Armature I^2R losses @ 95°C	63	kW
Field I^2R losses:	42	kW

EFFICIENCIES, tol. on losses acc. to IEC 60034

	4/4	3/4	1/2	1/4
p.f. 1.0	98,17	97,83	97,05	94,58
p.f. 0,85	97,92	97,64	96,91	94,49

COMPONENTS MODELING – INDUCTION MACHINES

- Positive Sequence
  - Changes from stalled to running
  - ~0.15 pu stalled (*Xd”)*
  - 0.9–1.0 pu running
- Negative Sequence
  - Remains effectively constant
  - ~0.15 pu (*Xd”)*
- Zero Sequence
  - 0.0 if WYE ungrounded or DELTA connected

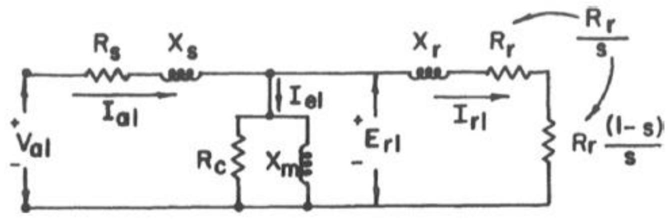


Fig. 6.18. Positive sequence induction motor equivalent circuit.

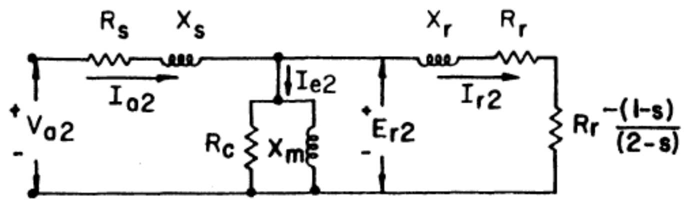


Fig. 6.20. Negative sequence induction motor equivalent circuit.

R_s = 0.01 pu = Stator Resistance

jX_s = Stator Leakage Reactance at Rated Frequency

R_r = 0.01 pu = Rotor Resistance

jX_r = Rotor Leakage Reactance at Rated Frequency

JX_m = j3.0 pu = Shunt Exciting Reactance

$jX = jX_s + jX_r = jX_d'' = j0.15$ pu

Values shown are typical for an induction motor and are per unit on the motor kVA and kV base

$$S = \frac{\text{Synchronous RPM - Rotor RPM}}{\text{Synchronous RPM}} = \begin{array}{l}\text{1.0 for stalled condition}\\ \text{0+ for running conditions}\end{array}$$

COMPONENTS MODELING – INDUCTION MACHINES

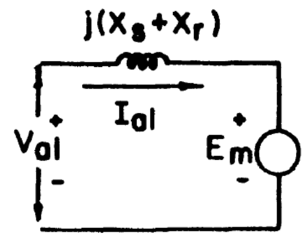
Table 6.3. Approximate Constants for Three-Phase Induction Motors

Rating	Full Load Efficiency	Full Load Power Factor	Full Load Slip	R and X in per Unit*			
				$X_s + X_r^{\dagger}$	X_m	R_s	R_r
(HP)	(%)	(%)	(%)	(pu)	(pu)	(pu)	(pu)
Up to 5	75–80	75–85	3.0–5.0	0.10–0.14	1.6–2.2	0.040–0.06	0.040–0.06
5–25	80–88	82–90	2.5–4.0	0.12–0.16	2.0–2.8	0.035–0.05	0.035–0.05
25–200	86–92	84–91	2.0–3.0	0.15–0.17	2.2–3.2	0.030–0.04	0.030–0.04
200–1000	91–93	85–92	1.5–2.5	0.15–0.17	2.4–3.6	0.025–0.03	0.020–0.03
Over 1000	93–94	88–93	~1.0	0.15–0.17	2.6–4.0	0.015–0.02	0.015–0.025

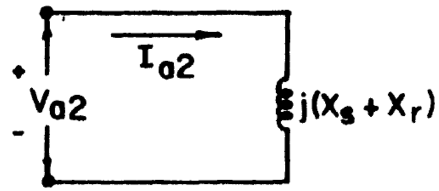
Source: Clarke [11, vol. 2]. Used with permission of the publisher.
*Based on full load kVA rating and rated voltage.
†Assume that $X_s = X_r$ for constructing the equivalent circuit.

$$E_m = V_{a1} - jI_{a1}(X_s + X_r)$$

$$T_r = (X_s + X_r)/\omega_1 R_r \quad \text{sec}$$



POSITIVE SEQUENCE

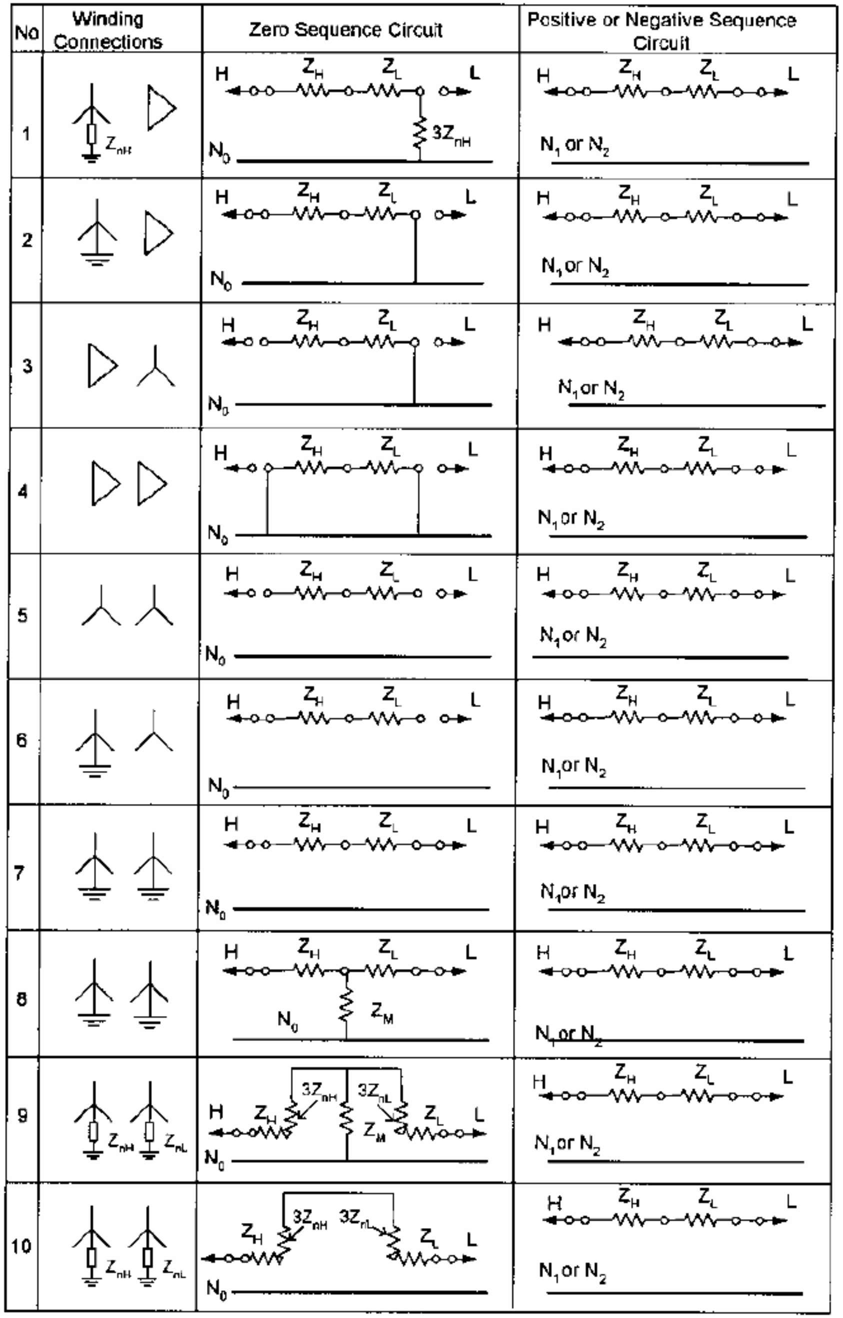


NEGATIVE SEQUENCE

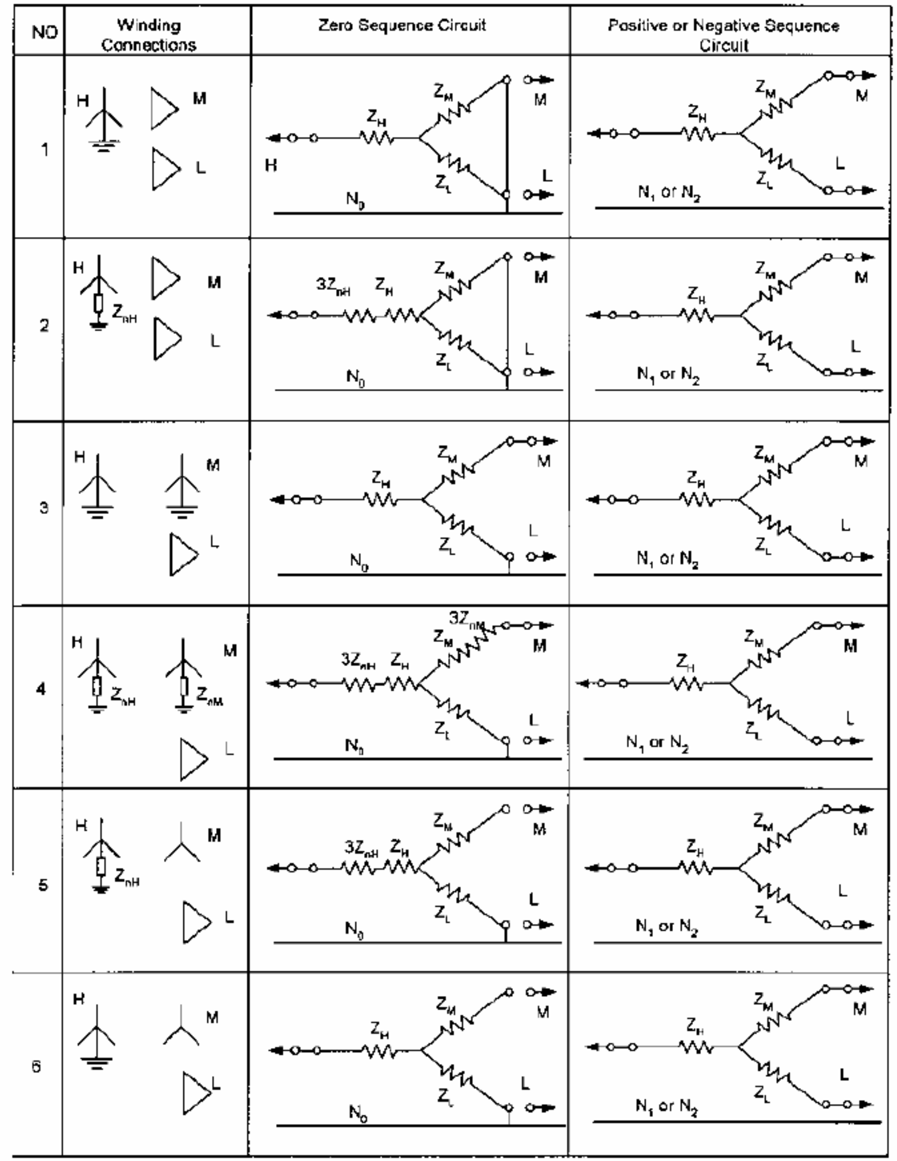
COMPONENTS MODELING – TRANSFORMERS

- Modeling
  - Usually modeled as a series impedance
  - Shunt parameters can be calculated by review of transformer tests
  - Shunt parameters don't generally impact analysis
  - Transformer winding configuration determines sequence networks
  - Three winding transformers have interesting sequence networks, but close inspections shows them to be intuitive

COMPONENTS MODELING – TRANSFORMERS



COMPONENTS MODELING – TRANSFORMERS



COMPONENTS MODELING – TRANSFORMERS

- 3 Winding Transformer Impedance
  - Usually given as a winding-to-winding (delta) impedances in %
  - Convert to equivalent WYE impedances for sequence network analysis
  - Often times, the base power is different for various impedances
  - Ex: 100 MVA autotransformer with 35 MVA tertiary
    May show Z_{HM} *on 100 MVA base*
    May show Z_{HL} *and* Z_{ML} *on 35 MVA base*

Must convert delta impedance to common base before converting to equivalent WYE network

COMPONENTS MODELING – TRANSFORMERS

- 3 Winding Transformer Impedances
  - Delta-WYE Conversion Formula

$$Z_H = \frac{1}{2}(Z_{HM} + Z_{HL} - Z_{ML})$$

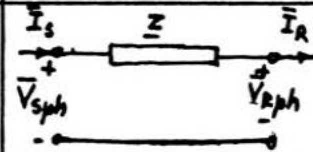
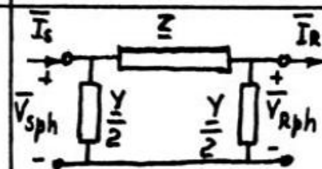
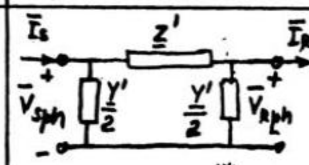
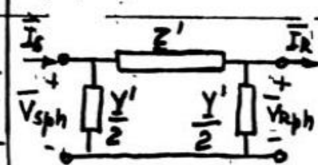
$$Z_M = \frac{1}{2}(Z_{ML} + Z_{HM} - Z_{HL})$$

$$Z_L = \frac{1}{2}(Z_{HL} + Z_{ML} - Z_{HM})$$

Z_{HM} = leakage impedance between the H and M windings, as measured on the H winding with M winding short-circuited and L winding open circuited; Z_{HL} = leakage impedance between the H and L windings, as measured on the H winding with L winding short-circuited and M winding open circuited; Z_{ML} = leakage impedance between the M and L windings, as measured on the M winding with L winding short-circuited and H winding open circuited.

COMPONENTS MODELING – TRANSMISSION LINES

Equivalent circuit and equations for overhead lines

Line type \ Characteristics	Length range [Km/mile]	K_z	K_y	Equivalent circuit (π)	Equations
short-lines	$\ell \le 80/50$	1	1		$\bar{V}_{sph} = \bar{V}_{Rph} + \underline{Z}\,\bar{I}_R$; $\bar{I}_s = \bar{I}_R$
medium-length lines	$80/50 \le \ell \le 240/150$	1	1		$\bar{V}_{sph} = \left(1+\frac{\underline{Z}\,\underline{Y}}{2}\right)\bar{V}_{Rph} + \underline{Z}\,\bar{I}_R$; $\bar{I}_s = \underline{Y}\left(1+\frac{\underline{Z}\,\underline{Y}}{4}\right)\bar{V}_{Rph} + \left(1+\frac{\underline{Z}\,\underline{Y}}{2}\right)\bar{I}_R$
long lines	$240/150 \le \ell$	$\frac{\sinh\sqrt{\underline{Z}\,\underline{Y}}}{\sqrt{\underline{Z}\,\underline{Y}}}$	$\frac{2(\cosh\sqrt{\underline{Z}\,\underline{Y}}-1)}{\sqrt{\underline{Z}\,\underline{Y}}\,\sinh\sqrt{\underline{Z}\,\underline{Y}}}$		$\bar{V}_{sph} = \left(1+\frac{\underline{Z}'\underline{Y}'}{2}\right)\bar{V}_{Rph} + \underline{Z}'\bar{I}_R$; $\bar{I}_s = \underline{Y}'\left(1+\frac{\underline{Z}'\underline{Y}'}{4}\right)\bar{V}_{Rph} + \left(1+\frac{\underline{Z}'\underline{Y}'}{2}\right)\bar{I}_R$
	$\left(240/150 \le \ell \le 320/200\right)^{*)}$	$1+\frac{\underline{Z}\,\underline{Y}}{6}+\frac{(\underline{Z}\,\underline{Y})^2}{120}+\cdots$	$\frac{1+\frac{\underline{Z}\,\underline{Y}}{12}+\frac{(\underline{Z}\,\underline{Y})^2}{360}+\cdots}{1+\frac{\underline{Z}\,\underline{Y}}{6}+\frac{(\underline{Z}\,\underline{Y})^2}{120}+\cdots}$		$\bar{V}_{sph} = \left(1+\frac{\underline{Z}'\underline{Y}'}{2}\right)\bar{V}_{Rph} + \underline{Z}'\bar{I}_R$; $\bar{I}_s = \underline{Y}'\left(1+\frac{\underline{Z}'\underline{Y}'}{4}\right)\bar{V}_{Rph} + \left(1+\frac{\underline{Z}'\underline{Y}'}{2}\right)\bar{I}_R$

Comment: $\underline{Z}' = K_z \cdot \underline{Z}$; $\underline{Y}' = K_y \cdot \underline{Y}$; *) Particular length range for long lines.

From - O. Crisan, “ECE-6127 Power Transmission and Distribution Lectures”, University of Houston, 1993

COMPONENTS MODELING – TRANSMISSION LINES

Positive and Negative Sequence Impedance

- Passive component
- Assume line symmetry and transpositions

- Calculations with simplified formula types:

$$L_1 = L_2 = \frac{\mu_0}{2\pi} \ln \frac{D_m}{D_s} \ [\mathrm{H/m}]$$

$$X = \omega L = 2\pi f L$$

$$X_1 = X_2 = 4\pi \times 10^{-7} \ln \frac{D_m}{D_s} \ [\Omega/\mathrm{m}]$$

$$R_1 = R_2 = \text{conductor AC resistance (tables)}$$

- From tables:

$$\mathbf{Z} = r_a + j(X_a + X_d)$$

COMPONENTS MODELING – TRANSMISSION LINES

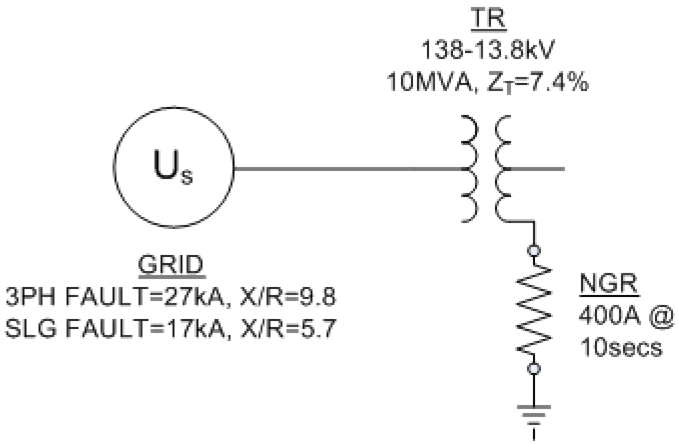
- Zero Sequence Impedance
  - More involved
  - Assumptions
    - Zero sequence current divides equally between conductors
    - Conductors are parallel to ground
    - Earth is a solid with a plane surface, infinite in extent, and of uniform conductivity
  - None of the assumptions are true
  - We get acceptable error with these assumptions
  - Line design affects calculation techniques

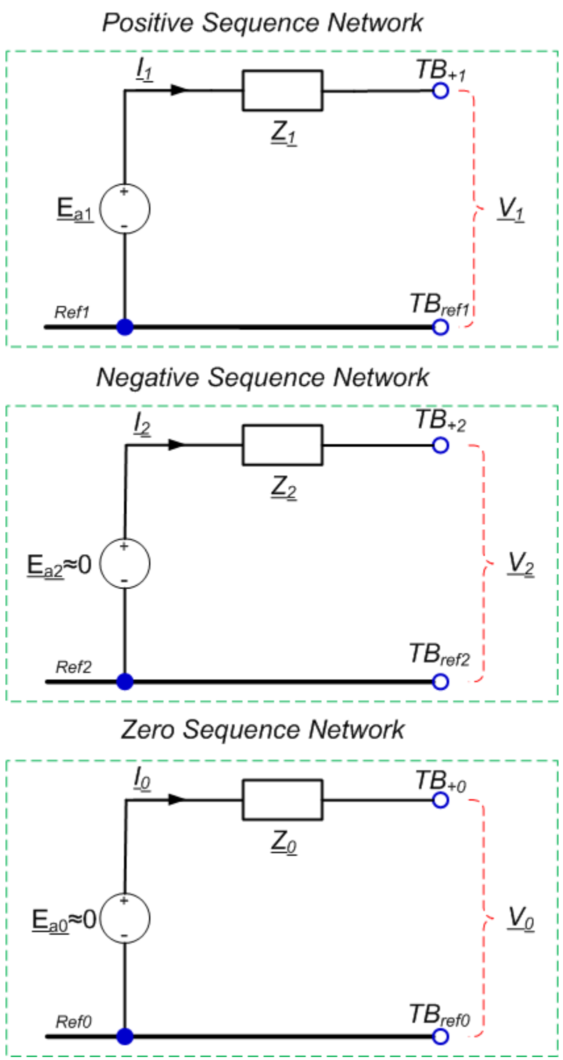
COMPONENTS MODELING – TRANSMISSION LINES

Positive and Negative Sequence Impedance

- Cannot assume line symmetry or transpositions
  - General principle has to be applied and all self and mutual impedances are calculated from formulas
  - No tabulation of impedances
- T-line series impedance (or admittance) matrix is not symmetrical. Similarly, shunt susceptances and conductance is not symmetrical as well.

SYMMETRICAL COMPONENTS - EXAMPLES





Assumptions:

$\underline{E_{a1}} := (1 + j\cdot 0)\text{pu} \qquad \underline{E_{a2}} = \underline{E_{a0}} = 0$

Base Quantites:

$S_{base} := 10\text{MVA}$

$V_{base1} := 138\text{kV}$

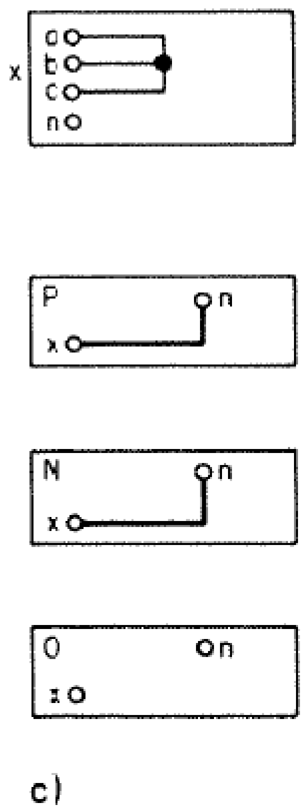
$$I_{base1} := \frac{S_{base}}{\sqrt{3}\cdot V_{base1}} = 41.84\ \text{A}$$

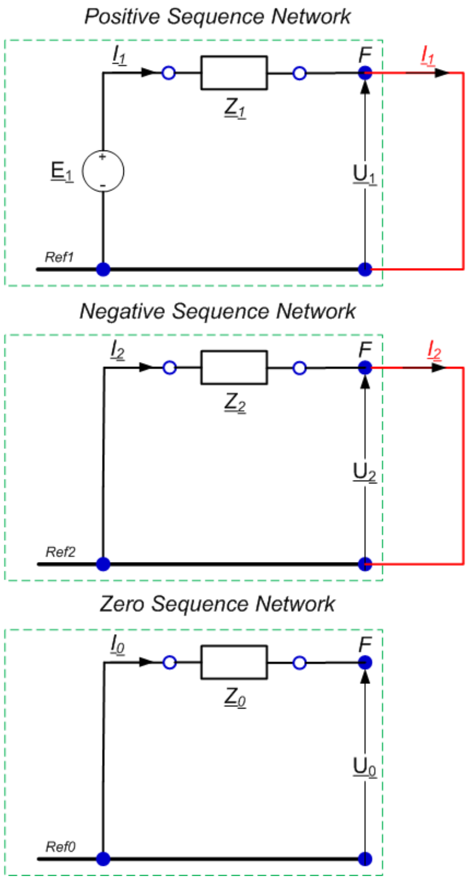
$V_{base2} := 13.8\text{kV}$

$$I_{base2} := \frac{S_{base}}{\sqrt{3}\cdot V_{base2}} = 418.37\ \text{A}$$

SYMMETRICAL COMPONENTS - EXAMPLES

- 3PH Fault @ HS of TR, w/o Z_f , w/o ground





$$I_{sc\_3PH.a.pu} := \frac{27kA}{I_{base1}} \cdot e^{-j\cdot(atan(9.8))} = (65.51 - 642.03i)\cdot pu$$

$$z2r\theta\left(I_{sc\_3PH.a.pu}\right) = \text{"(645.3621∡ -84.17°)"}\cdot pu$$

$$I_{sc\_3PH.b.pu} := a^2 \cdot I_{sc\_3PH.a.pu} = (-588.77 + 264.28i)\cdot pu$$

$$z2r\theta\left(I_{sc\_3PH.b.pu}\right) = \text{"(645.3621∡ 155.83°)"}\cdot pu$$

$$I_{sc\_3PH.c.pu} := a \cdot I_{sc\_3PH.a.pu} = (523.26 + 377.75i)\cdot pu$$

$$z2r\theta\left(I_{sc\_3PH.c.pu}\right) = \text{"(645.3621∡ 35.83°)"}\cdot pu$$

$$\begin{pmatrix} \underline{I_0} \\ \underline{I_1} \\ \underline{I_2} \end{pmatrix} := \frac{1}{3}\cdot\begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a} & \underline{a}^2 \\ 1 & \underline{a}^2 & \underline{a} \end{pmatrix}\cdot\begin{pmatrix} I_{sc\_3PH.a.pu} \\ I_{sc\_3PH.b.pu} \\ I_{sc\_3PH.c.pu} \end{pmatrix} = \begin{pmatrix} 0 \\ 65.51 - 642.03i \\ 0 \end{pmatrix} pu$$

$$z2r\theta M\left(\begin{pmatrix} \underline{I_0} \\ \underline{I_1} \\ \underline{I_2} \end{pmatrix}\right) = \begin{pmatrix} \text{"(0∡ 14.0362°)"} \\ \text{"(645.3621∡ -84.1737°)"} \\ \text{"(0∡ -45°)"} \end{pmatrix} pu$$

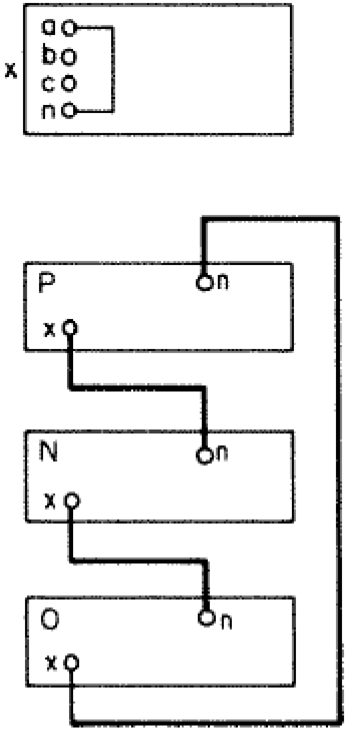
$$\underline{Z_1} := \frac{\underline{E_{a1}}}{\underline{I_1}} = (0.0001573 + 0.00154151i)\ pu$$

$$z2r\theta\left(\underline{Z_1}\right) = \text{"(0.0015∡ 84.17°)"}\ pu$$

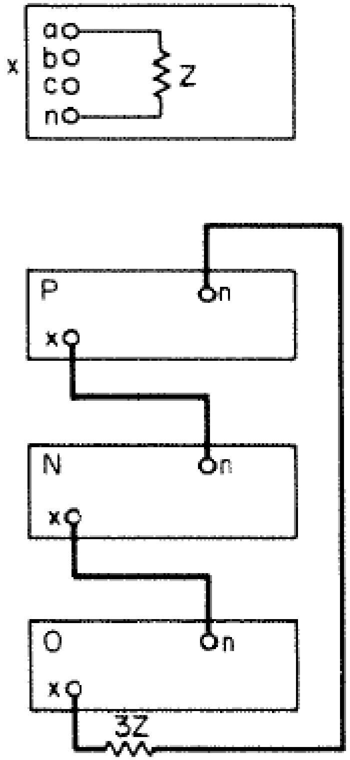
$$\underline{Z_2} = \underline{Z_1}$$

SYMMETRICAL COMPONENTS - EXAMPLES

- SLG Fault @ HS of TR, w/o Z_f



f)



e)

Assumptions:

$$\underline{I_0} = \frac{\underline{E_{a1}}}{\underline{Z_1} + \underline{Z_2} + \underline{Z_0} + 3 \cdot \underline{Z_f}} \qquad \underline{Z_f} = 0 \qquad \underline{Z_1} = \underline{Z_2}$$

$$\underline{I_{a0.pu}} := \frac{17kA}{I_{base1}} \cdot e^{-j \cdot (\mathrm{atan}(5.7))} = (70.22 - 400.23i) \cdot pu$$

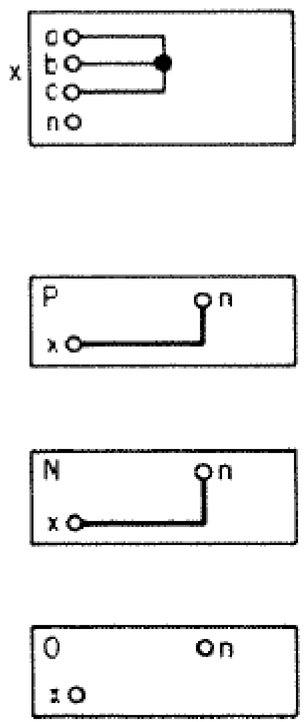
$$z2r\theta\left(\underline{I_{a0.pu}}\right) = \text{"(406.3391} \measuredangle \text{ -80.05°)"} \cdot pu$$

$$\underline{Z_0} := \frac{\underline{E_{a1}}}{\underline{I_{a0.pu}}} - 2 \cdot \underline{Z_1} = 0.00011066 - 0.00065905i$$

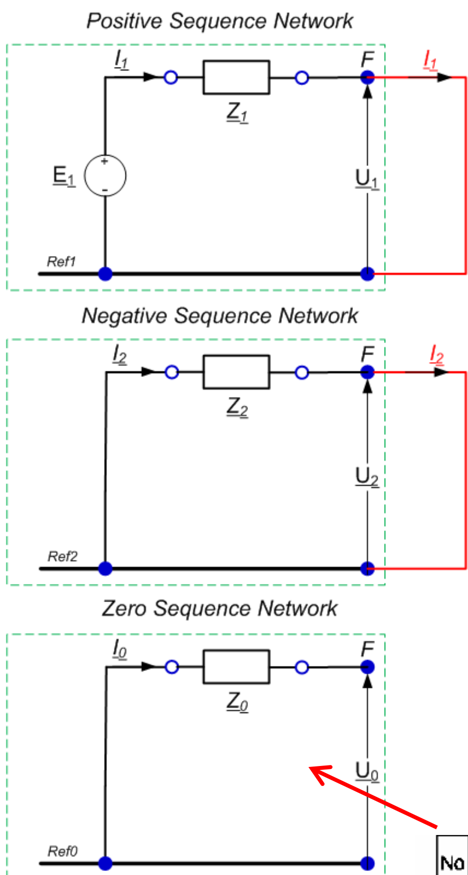
$$z2r\theta\left(\underline{Z_0}\right) = \text{"(0.0007} \measuredangle \text{ -80.47°)" pu}$$

SYMMETRICAL COMPONENTS - EXAMPLES

- 3PH Fault @ LS of TR, w/o Z_f , w/o ground



c)



Assumptions:

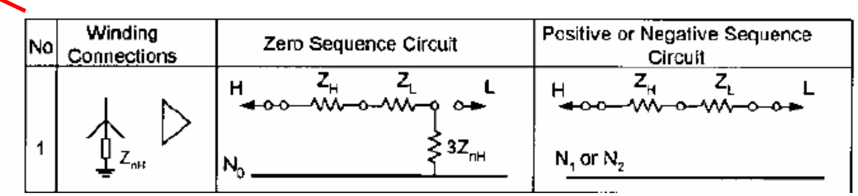
$\underline{Z}_{TR0} = \underline{Z}_{TR1} = \underline{Z}_{TR2} = j \cdot 0.074pu \qquad \underline{Z}_1 = (0.000157 + 0.001542i)\ pu$

$$\underline{I}_1 := \frac{\underline{E}_{a1}}{\underline{Z}_1 + \underline{Z}_{TR1}} = (0.02756436 - 13.23769705i)\ pu$$

$$z2r\theta\left(\underline{I}_1\right) = \text{"(13.2377∡ -89.88°)"}\ pu$$

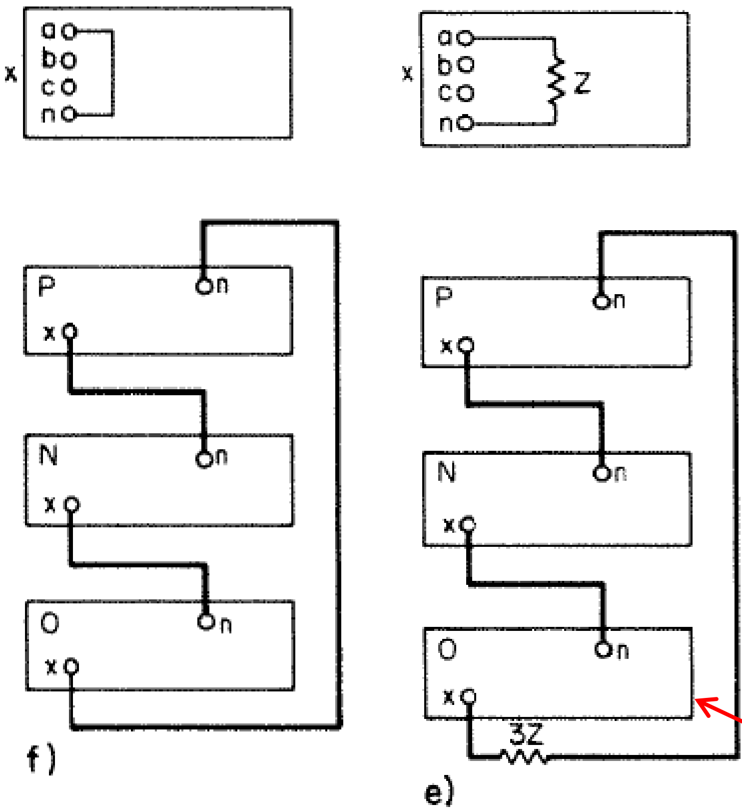
$$\begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & \underline{a}^2 & \underline{a} \\ 1 & \underline{a} & \underline{a}^2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ \underline{I}_1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.03 - 13.24i \\ -11.48 + 6.59i \\ 11.45 + 6.64i \end{pmatrix} pu$$

$$z2r\theta M\left(\begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix}\right) = \begin{pmatrix} \text{"(13.2377∡ -89.8807°)"} \\ \text{"(13.2377∡ 150.1193°)"} \\ \text{"(13.2377∡ 30.1193°)"} \end{pmatrix} pu$$



SYMMETRICAL COMPONENTS - EXAMPLES

- SLG Fault @ LS of TR, w/o Z_f



Assumptions:

$I_1 = I_2 = I_0$ $\quad Z_f = 0$

$Z_1 = Z_1 + Z_{TR}$ $\quad Z_2 = Z_1$ $\quad Z_0 = Z_{TR} + 3 \cdot Z_f = Z_{TR}$

$$E_{a1} = I_0 \cdot (Z_1 + Z_2 + Z_0 + 3 \cdot Z_f)$$

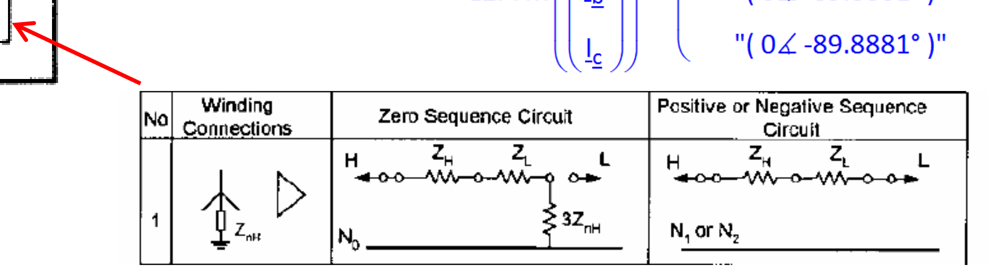
$$E_{a1} = I_0 \cdot (Z_1 + Z_{TR} + Z_1 + Z_{TR} + Z_{TR})$$

$$I_0 := \frac{E_{a1}}{(Z_1 + Z_{TR} + Z_1 + Z_{TR} + Z_{TR})} = 0.00620962 - 4.44279635i$$

$$z2r\theta(I_0) = \text{"(4.4428∡ -89.92°)"}\ \text{pu}$$

$$\begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} := \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \cdot \begin{pmatrix} I_0 \\ I_0 \\ I_0 \end{pmatrix} = \begin{pmatrix} 0.02 - 13.33i \\ 0 \\ 0 \end{pmatrix} \text{pu}$$

$$z2r\theta M \left(\begin{pmatrix} I_a \\ I_b \\ I_c \end{pmatrix} \right) = \begin{pmatrix} \text{"(13.3284∡ -89.9199°)"} \\ \text{"(0∡ -89.8881°)"} \\ \text{"(0∡ -89.8881°)"} \end{pmatrix} \text{pu}$$



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- In the Past…
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  - Papers
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  - http search engines
  - WWW
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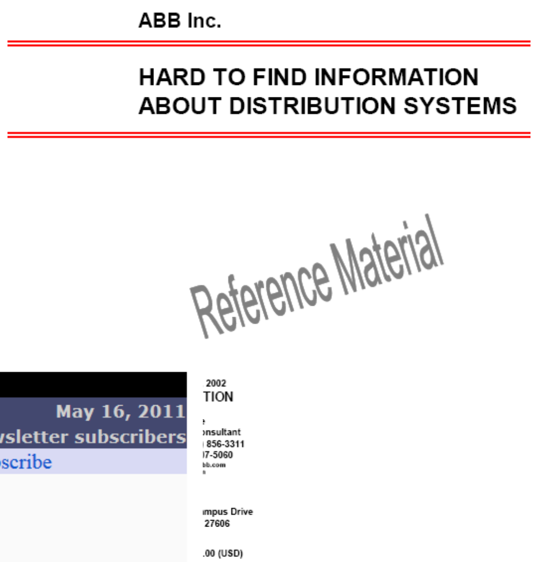

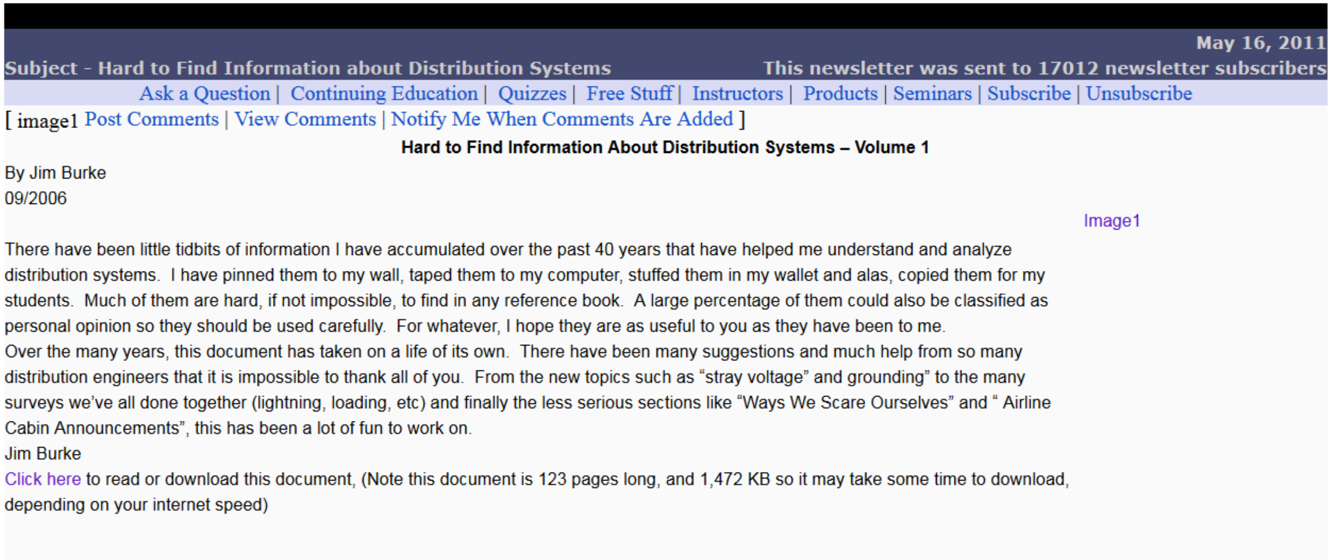

May 16, 2011

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Hard to Find Information About Distribution Systems – Volume 1

By Jim Burke
09/2006

Image1

There have been little tidbits of information I have accumulated over the past 40 years that have helped me understand and analyze distribution systems. I have pinned them to my wall, taped them to my computer, stuffed them in my wallet and alas, copied them for my students. Much of them are hard, if not impossible, to find in any reference book. A large percentage of them could also be classified as personal opinion so they should be used carefully. For whatever, I hope they are as useful to you as they have been to me.
Over the many years, this document has taken on a life of its own. There have been many suggestions and much help from so many distribution engineers that it is impossible to thank all of you. From the new topics such as "stray voltage" and grounding" to the many surveys we've all done together (lightning, loading, etc) and finally the less serious sections like "Ways We Scare Ourselves" and " Airline Cabin Announcements", this has been a lot of fun to work on.
Jim Burke
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THE END

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