CONTINUOUS –TIME ANALOG FILTERS
A GROUP DELAY PERSPECTIVE

- INTRODUCTION
- CONCEPTS
- PROPERTIES
- APPLICATIONS
- CONCLUSION

I. Introduction

Continuous-Time (CT) Analog Filters are used for :

1. Interface Applications –

   Anti-Aliasing or Preconditioning, and, Smoothing or Reconstruction functions. The "real" signal processing is performed in the Digital domain .

2. Direct Filtering –

   At high frequencies, CT filters perform all of the signal processing in the analog domain to realize Magnitude and/or Phase (Delay) Shaping.

Introduction contd.

- High-Frequency (HF) relates to the Technology available for implementation and Applications under consideration (Active and Passive) ; currently HF starts from Hundreds of MHz , and extends up to tens of GHz.
- With 0.13um Technology and lumped elements (as opposed to distributed), limited functionality is achievable up to ~ 10 GHz.
- At these frequencies, we (are forced to) appreciate the interdependence between Magnitude and Phase/Delay.
- This interaction is very important in direct HF filtering unlike in Interface filtering where, in general, only magnitude shaping is of prime concern.

II. CONCEPTS - Magnitude and Phase/Delay

For a transfer function H(s), at real frequencies, with s=jω,

$$H(j\omega) = \left|H(j\omega)\right|.e^{j\theta(\omega)} = G(\omega).e^{j\theta(\omega)}$$

where G(ω) and θ(ω) are the gain-magnitude, or simply the gain, and the phase components respectively.

Phase Delay Pd(ω) is defined as ,

$$Pd(\omega) = -\frac{\theta(\omega)}{\omega}$$

Group Delay τ(ω) is defined as,

$$\tau(\omega) = -\frac{\partial\theta(\omega)}{\partial\omega}$$

CONCEPTS – contd.

- In general, both Pd(ω) and τ(ω) are functions of frequency.
- The Phase delay Pd(ω) represents the absolute delay, and is of little significance.
- The Group Delay τ(ω) is used as the criterion to evaluate phase nonlinearity.
- A linear phase variation with frequency (over a band of frequencies) implies a constant Group Delay and no phase distortion in that frequency band.
- In order to preserve the integrity of the pulse through a system, it is mandatory that the group delay of the system be constant up to the maximum frequency component of the pulse.

III. PROPERTIES

For the First Order Low Pass Transfer Function (LPF1) :

$$H(s) = \frac{\omega_0}{s + \omega_0} = \frac{1}{1 + \frac{s}{\omega_0}}$$

$$G(j\omega) = \left| H(j\omega) \right| = \frac{1}{\sqrt{1 + \left(\omega / \omega_0 \right)^2}}$$

$$\theta(\omega) = -\arctan(\frac{\omega}{\omega_0})$$

$$\tau(\omega) = \frac{1}{\omega_0}\left[\frac{1}{1+\frac{\omega^2}{\omega_0^2}}\right]$$

$$\frac{\tau(\omega)}{\tau_0} = \left[\frac{1}{1+\frac{\omega^2}{\omega_0^2}}\right] = [G(\omega)]^2$$, where, $$\tau_0 = \left[\frac{1}{\omega_0}\right]$$

- Note τ_0 is the value at dc. At $\omega=\omega_0$, $\tau(\omega)$ is –6 dB relative to its dc value.
- Only for the first order function, the group delay response is the square of the magnitude response.

PROPERTIES – contd.

For the Second Order Low Pass Transfer Function (LPF2) :

$$H(s) = \frac{\omega_0^2}{\left(s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2\right)} = \frac{1}{\left(1 + \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0^2}\right)}$$

$$\tau(\omega) = \frac{1}{\omega_0 Q}\left[\frac{1 + \left(\frac{\omega^2}{\omega_0^2}\right)}{1 - \left(1 - \frac{1}{2Q^2}\right)\frac{2\omega^2}{\omega_0^2} + \frac{\omega^4}{\omega_0^4}}\right]$$

- $\tau_0 = \left[\frac{1}{\omega_0 Q}\right]$, and is inversely proportional to ω_o and Q.

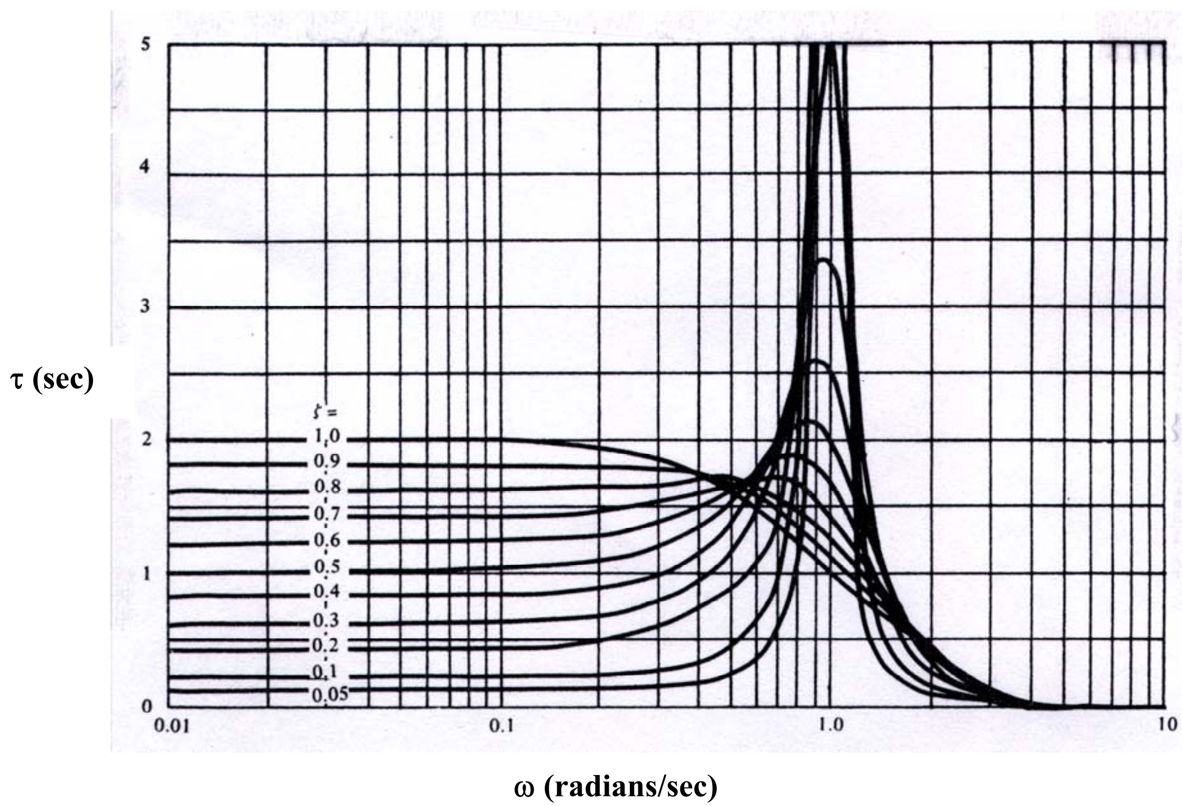
- $\tau(\omega)$ peaks for Q > 0.577, and , $$\frac{\tau_{peak}}{\tau_0} = Q^2\left[1+\frac{1}{\sqrt{1-\frac{1}{4Q^2}}}\right]$$

- From τ_0 and ($\tau_{peak}/\ \tau_0$), ω_o and Q can be calculated in real applications.

- $\tau(\omega)$ is a real and even function of ω .

Fig. 1 Normalized Group Delay vs. radian frequency for LPF2 [1]

$$Q = \left[\frac{1}{2\zeta}\right]$$



PROPERTIES - Magnitude and Phase (Delay) Relationships.

- Theoretically speaking, Magnitude and Phase responses of Transfer Functions are independent.
- However, in practice, the Magnitude and Phase functions are closely related.

I. Fixed Magnitude – Variable Delay filters:

- For a given magnitude function, the transfer function H(s) can be made unique by requiring that H(s) be a Minimum Phase Function (MPF) .
- A MPF is defined as a function for which the phase, at zero radian frequency minus the phase, at infinite radian frequency, is the minimum that can be associated with that Magnitude function.
- A MPF has no RHS poles (for stability reasons) and no RHS zeroes.

- A non-MPF, $H_{n\text{-}mpf}(s)$ can be realized with the same Magnitude function as the MPF, $H_{mpf}(s)$ in the following manner : $H_{n-mpf}(s) = H_{mpf}(s).A(s)$

where, A(s) is an All-Pass Function (APF) defined as : $A(s) = \frac{Q(-s)}{Q(s)}$, where Q(s) is a Hurwitz polynomial.

Examples of A(s) are : $A(s) = \frac{\left(1 - \frac{s}{\omega_0}\right)}{\left(1 + \frac{s}{\omega_0}\right)}$, $A(s) = \frac{\left(1 - \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0^2}\right)}{\left(1 + \frac{s}{\omega_0 Q} + \frac{s^2}{\omega_0^2}\right)}$, and so on.

- For s=jω , the magnitude of APF is unity. Therefore A(s) can be used to vary the delay of $H_{n\text{-}mpf}(s)$ without affecting the magnitude as specified by $H_{mpf}(s)$.
- This is the general principle of realizing **Fixed Magnitude – Variable Delay filters**. Examples are delay shaping circuits for cable equalizers.

II. Fixed Delay – Variable Magnitude filters:

- For a given phase function, the system function is definitely not unique.
- However, for a specified phase function, the system transfer function H(s) can be made unique, within a constant scalar (frequency-independent) multiplier, by requiring the system function be a minimum phase function $H_{mpf}(s)$ for that specified magnitude function.
- In order to provide flexibility in magnitude shaping without affecting the phase characteristics, the $H_{mpf}(s)$ function is multiplied with a strictly magnitude function to yield the required function $H_{actual}(s)$ as follows –

$$H_{actual}(s) = H_{mpf}(s).M(s)$$

where $M(s) = Q(s).Q(-s)$, and Q(s) is a Hurwitz polynomial.

- Note for $s=j\omega$, $M(s)$ is $Q(\omega^2)$, which is strictly a magnitude function, and the phase/Group delay of $H_{actual}(s)$ is same as that of $H_{mpf}(s)$.

- Examples of M(s) are $$M(s)=\left(1-\frac{s^2}{\omega_0^2}\right)$$

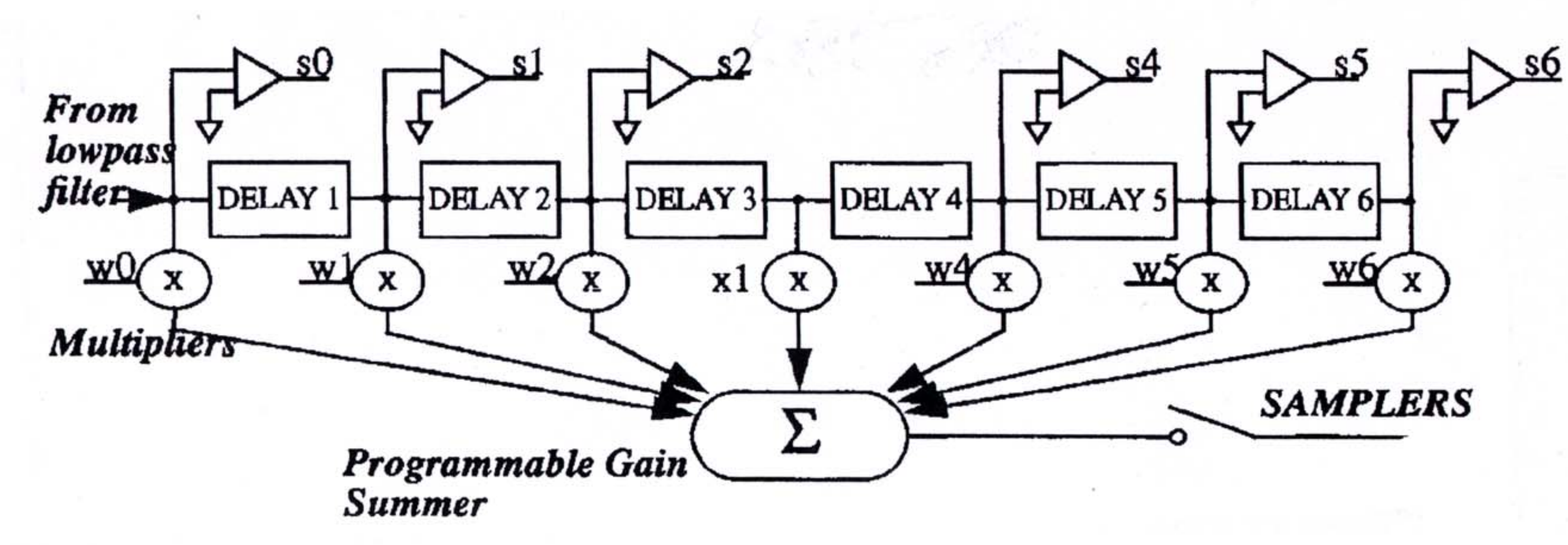
- This has been used for read-channel IC filters (for HDD applications) for almost 20 years to provide a magnitude boost around a specified frequency ω_0.

IV. Application

7 tap Analog Transversal FIR Equalizer for Read Channels [2]

Objective :

- Perform high speed signal equalization (in Analog domain).



Features :

- Six identical delay cells, where the nominal delay is one read clock period.
- 8-bit multipliers for each tap (except for center tap which has a fixed gain of unity)
- Adaptation uses decision directed sign-sign LMS algorithm

Delay Cell :

- Each Delay element is a Fourth order transfer function with a linear-phase, flat magnitude response given by the equation :

$$H(s) = \left[\frac{(1.5867\omega_0)^2}{\left(s^2 + s.\frac{1.5867\omega_0}{1.065} + (1.5867\omega_0)^2 \right)} \right] . \left[\frac{-0.5s^2 + (1.0752\omega_0)^2}{s^2 + \frac{s.1.0752\omega_0}{0.557} + (1.0752\omega_0)^2} \right]$$

Note that this transfer function is an example of a Fixed Delay – Variable Magnitude transfer function, and is realized as the product of a MPF phase function modified by a magnitude function of the form M(s) = Q(s).Q(-s).

- The resulting Group Delay is $\tau(\omega) = \dfrac{2.2624}{\omega_0}$, and is set to T, the clock period corresponding to the data rate.
- The delay is maintained constant over temperature and process and supply variations via a standard master-slave clock/RC tuning circuit.
- In order to accommodate variable data rate, the clock period is programmed within a +/- 25% range.
- The delay cell is realized as a cascade of two biquads using Gm-C integrators, shown below.

Fig. 3. Gm-C Based Biquad Design

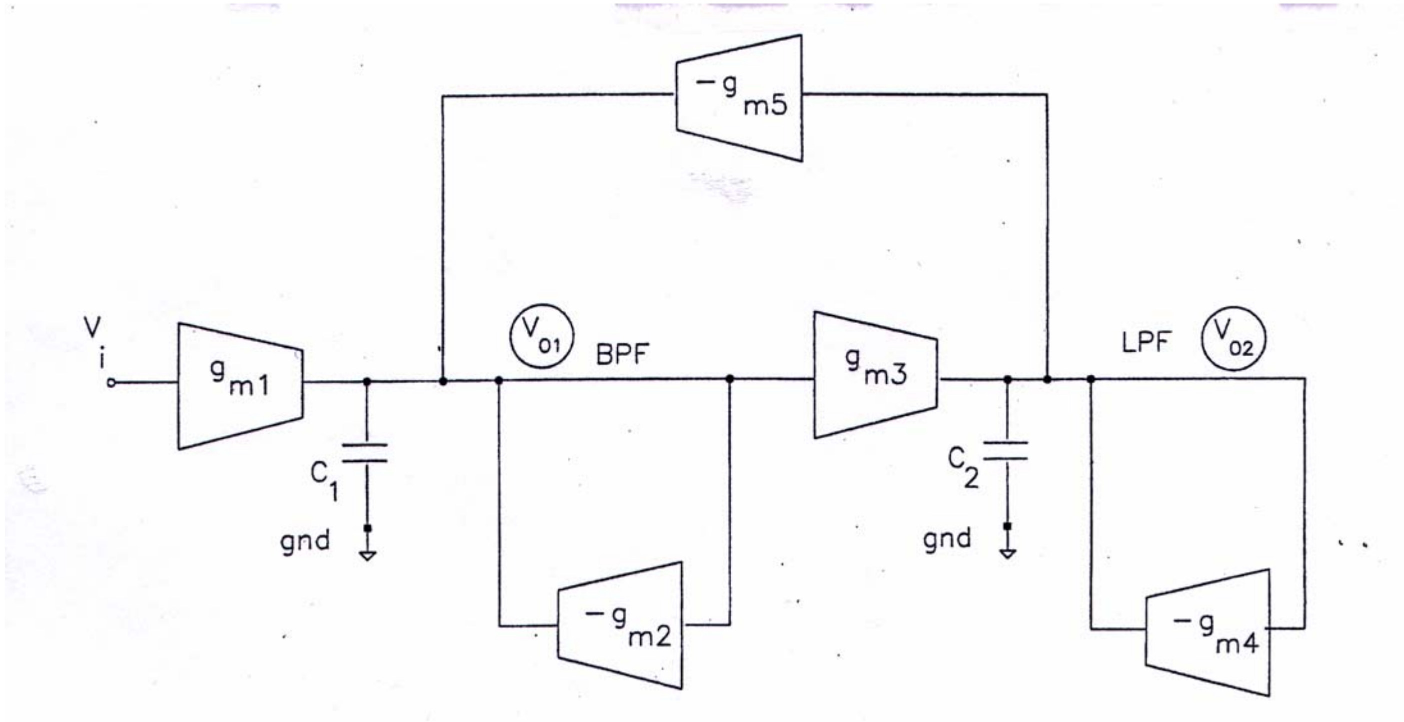


Fig.4. Schematic of a Gm-C cell

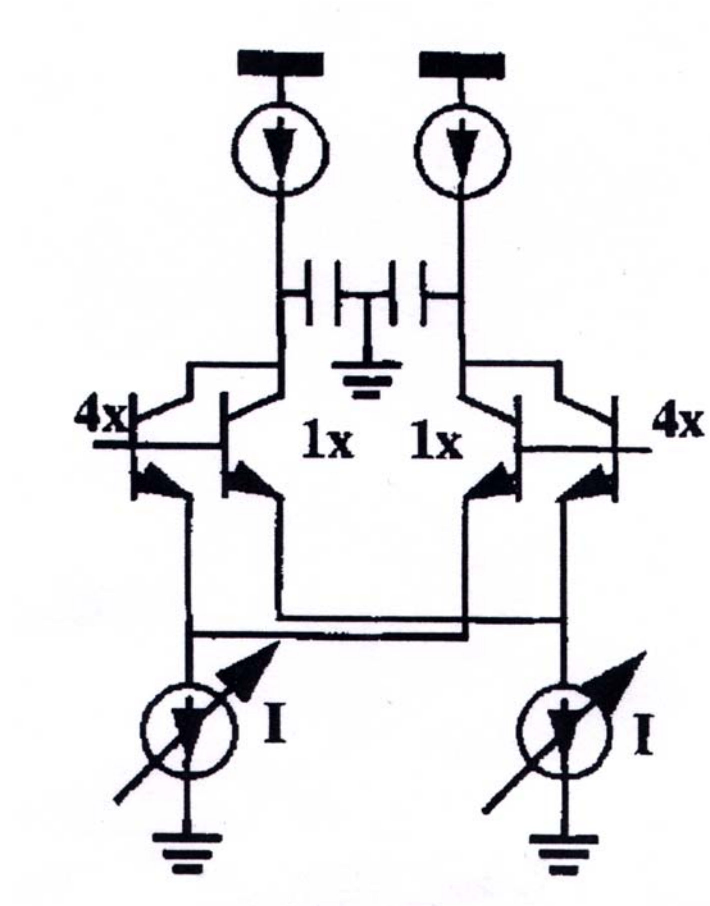
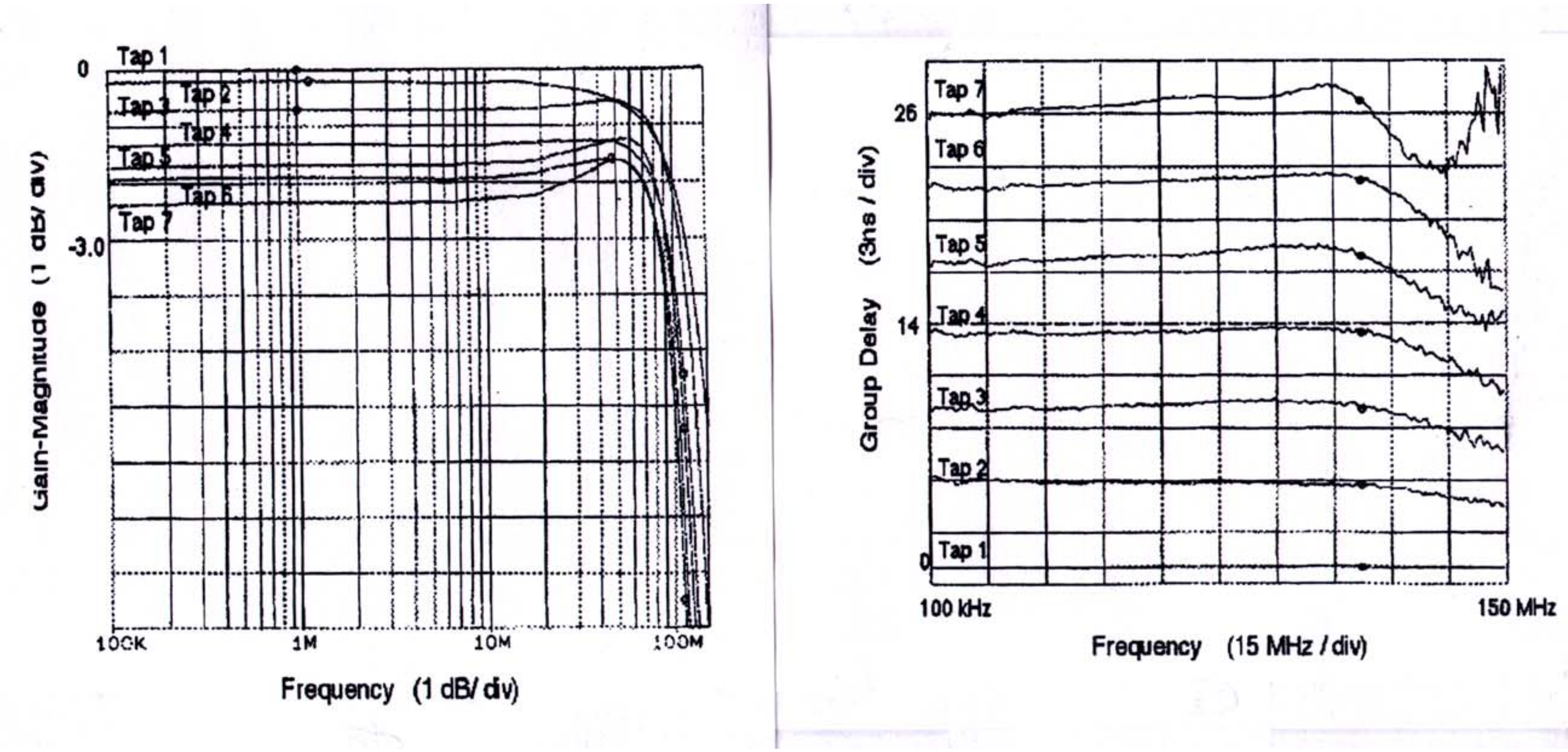


Fig.5. Magnitude and Group-Delay Response of Analog Equalizer



Advantages :

- Analog FIR filter achieves highest speed of operation for the given technology.
- Equalization is achieved without requiring a sampling clock and with reduced latency.
- The simple analog architecture results in lower power consumption and smaller die size.

V.CONCLUSIONS

- Analog equalizers are becoming increasingly popular as both the field of application and the frequency of operation increase.
- What is required is a basic analog delay cell which is realized using a Gm-C structure. To a first approximation where the pole at the output dominates, the delay is $\tau = \frac{1}{2.\pi f_0}$, where f_o is the BW in Hz.
- For a 10 pS delay, the BW should be 16 GHz ➔ Small delays imply large BW .
- For a Gm-C cell driving a resistive load R, τ is same as RC, assuming that the BW of the Gm circuit is much larger than f_o !
- A variable delay cell is realized using a variable Gm or a variable C (varactor).
- A major challenge is to maintain the delay constant over process, supply and temperature fluctuations.

References

1. C.S.Lindquist, "Active Network Design with Signal Filtering Applications", Steward & Sons, 1977, pp.134.
2. "A PRML Read/Write Channel IC Using Analog Signal Processing for 200 Mb/s HDD", K. Parsi, et al, IEEE Journal of Solid-State Circuits, Vol.31, No.11, pp.1817-1830, Nov. 1996.
3. "A 150 MHz Continuous-Time Seventh-Order 0.05^{o} Equiripple Linear-Phase Filter", N.Rao et al, ISCAS Proceedings, vol.II, pp.664-667, June 1999.
4. L.Weinberg, "Network Analysis and Synthesis", McGraw Hill, 1962, pp.231.

Properties – Appendix [4]

A very general property of the Group Delay is :

$$\tau(\omega) = -\mathrm{Re}al\left[\frac{\partial H(s)}{\partial s}.\frac{1}{H(s)}\right], \qquad \text{for } s = j(\omega)$$

$$\tau(\omega) = -\mathrm{Re}al\left[\frac{H'(s)}{H(s)}\right], \qquad \text{for } s = j(\omega)$$

$$\tau(\omega) = -\mathrm{Re}al\left[\frac{S_s^{H(s)}}{s}\right], \qquad \text{for } s = j(\omega)$$

where $S_s^{H(s)}$ is the Sensitivity of H(s) with respect to s.