The Hebbian-LMS Algorithm

By

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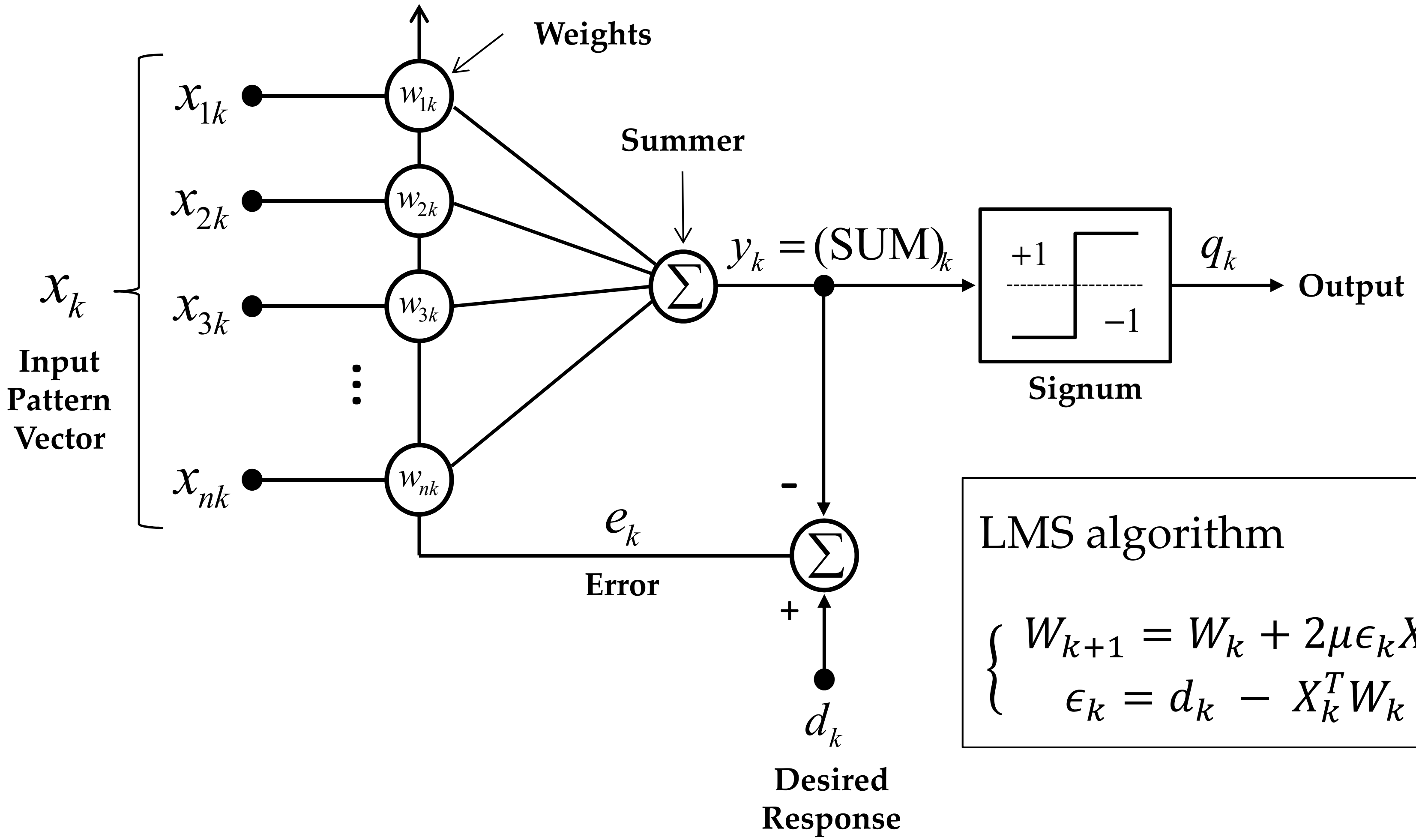


LMS algorithm

$$\begin{cases} W_{k+1} = W_k + 2\mu\epsilon_k X_k \\ \epsilon_k = d_k - X_k^T W_k \end{cases}$$

Figure 1: Adaline. An adaptive linear neuron

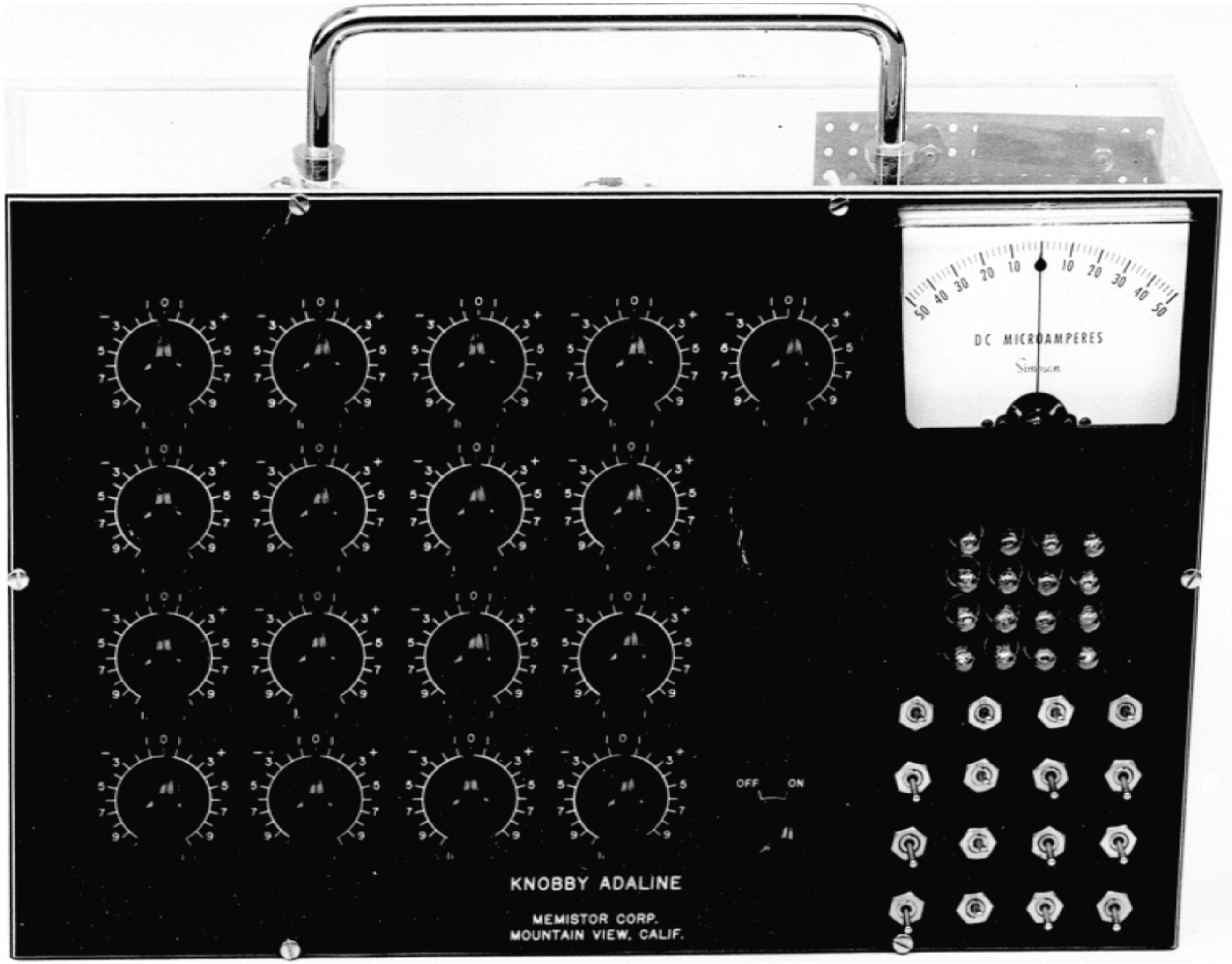


Figure 2: Knobby adaline

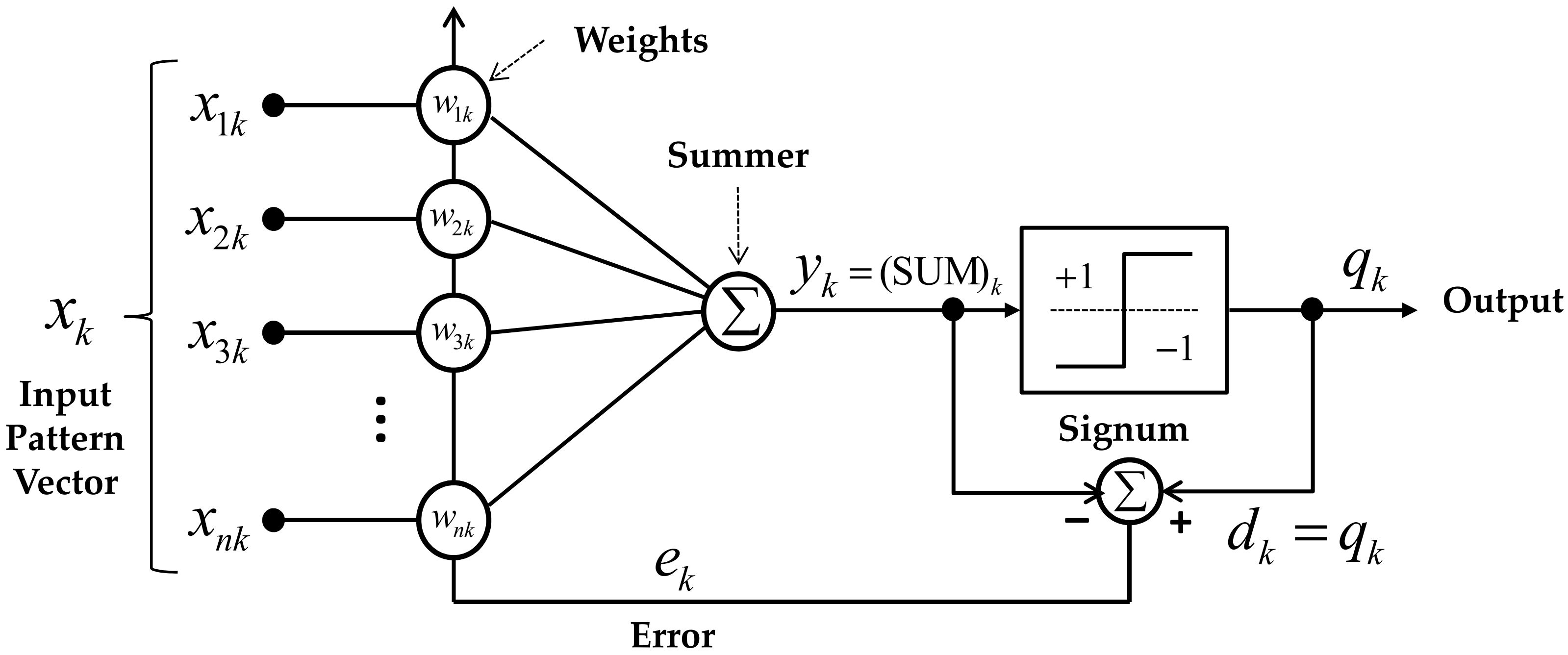


Figure 3: Adaline with bootstrap learning.

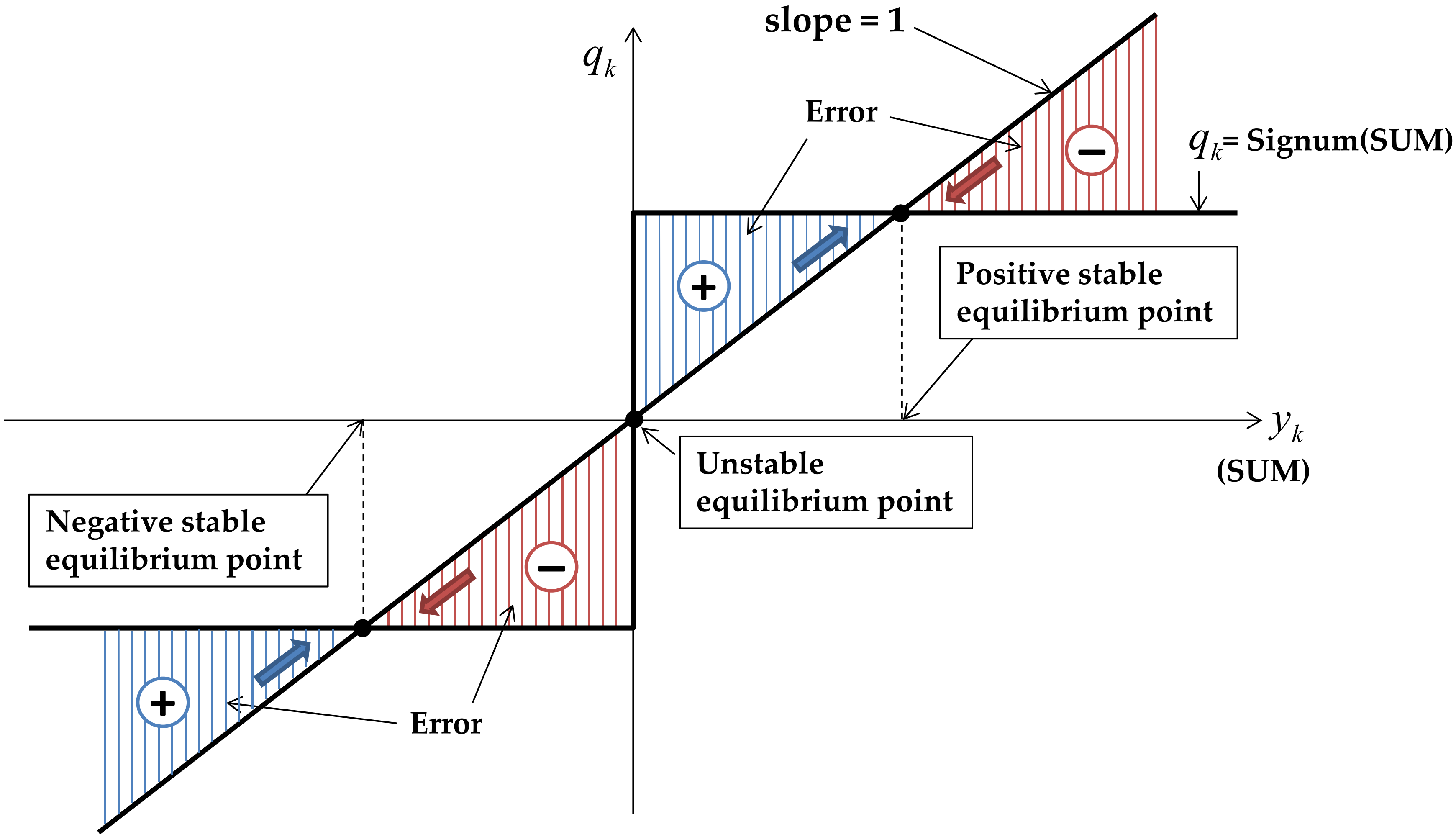


Figure 4-(a): Bootstrap learning. The quantized output, the sum, and the error vs (SUM).

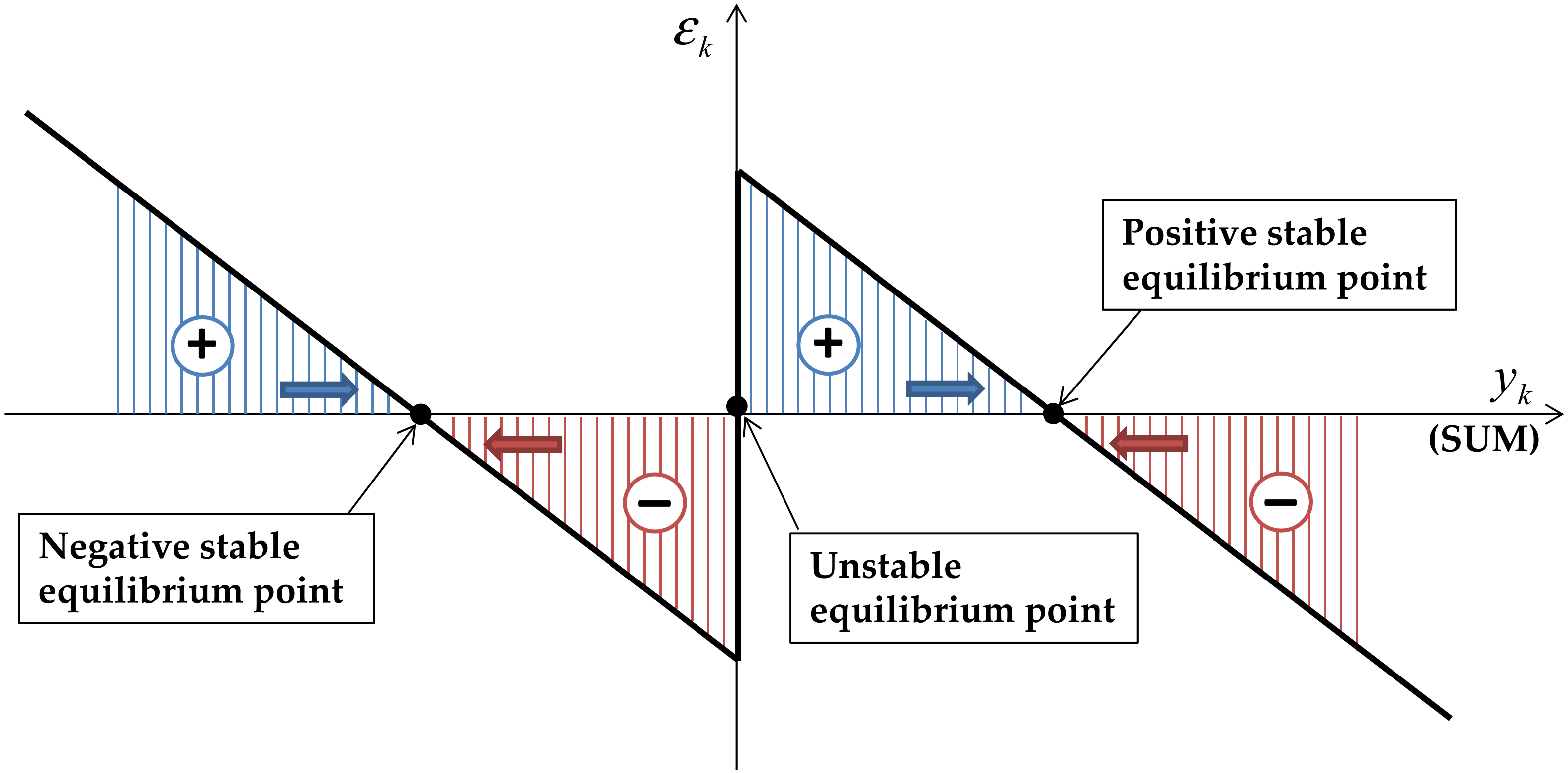


Figure 4-(b): Bootstrap learning. The error vs (SUM).

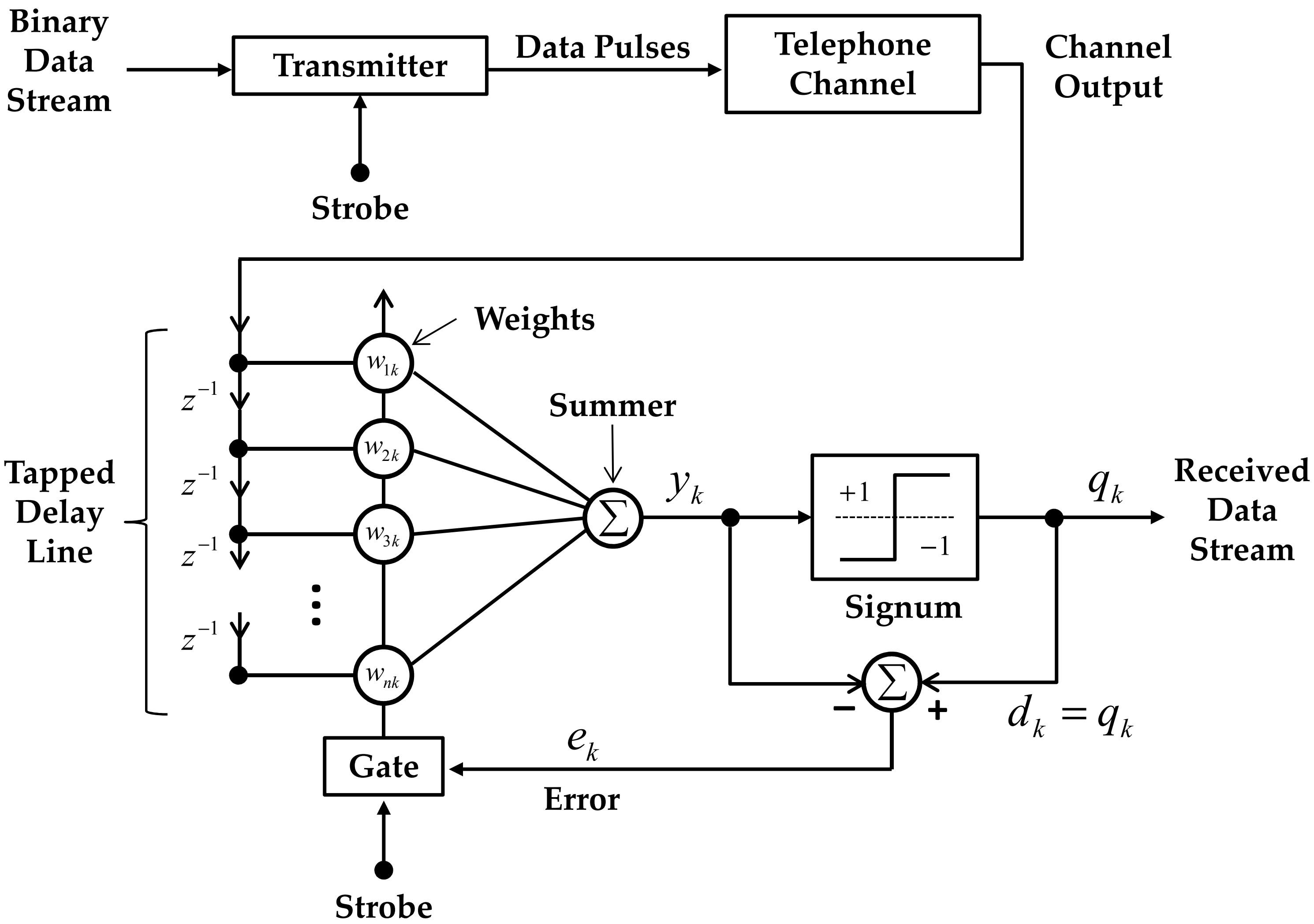
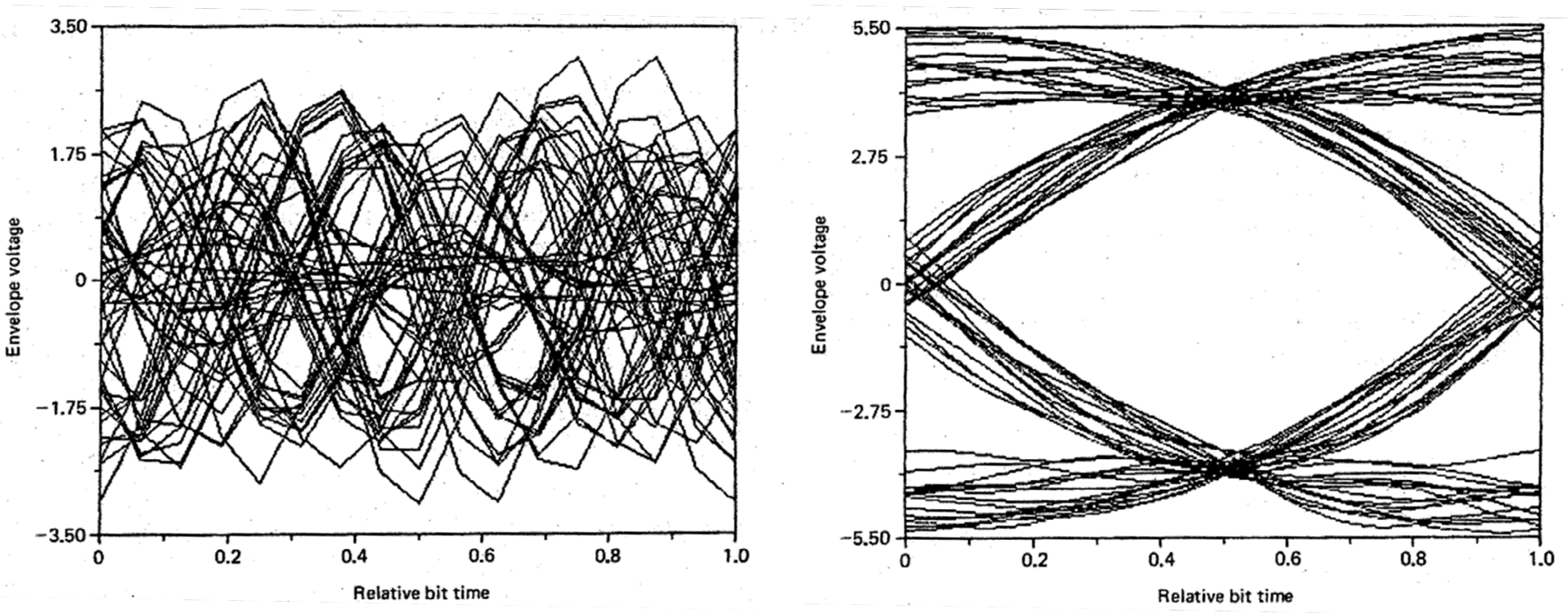


Figure 5: Decision-directed learning for channel equalization.



(a) Before equalization. (b) After equalization.

Figure 6: Eye patterns produced by overlaying cycles of the received waveform.

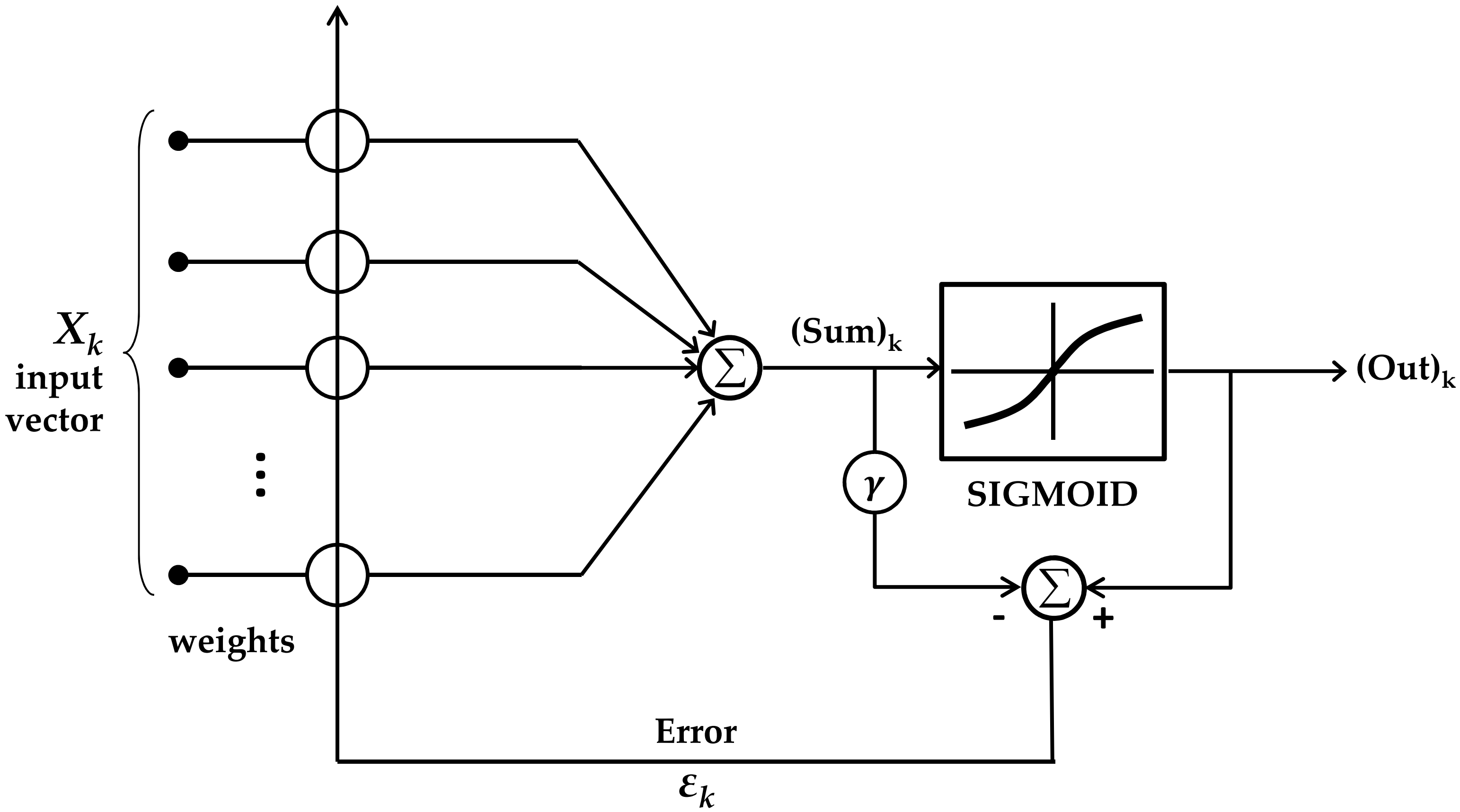


Figure 7: A sigmoidal neuron trained with bootstrap learning.

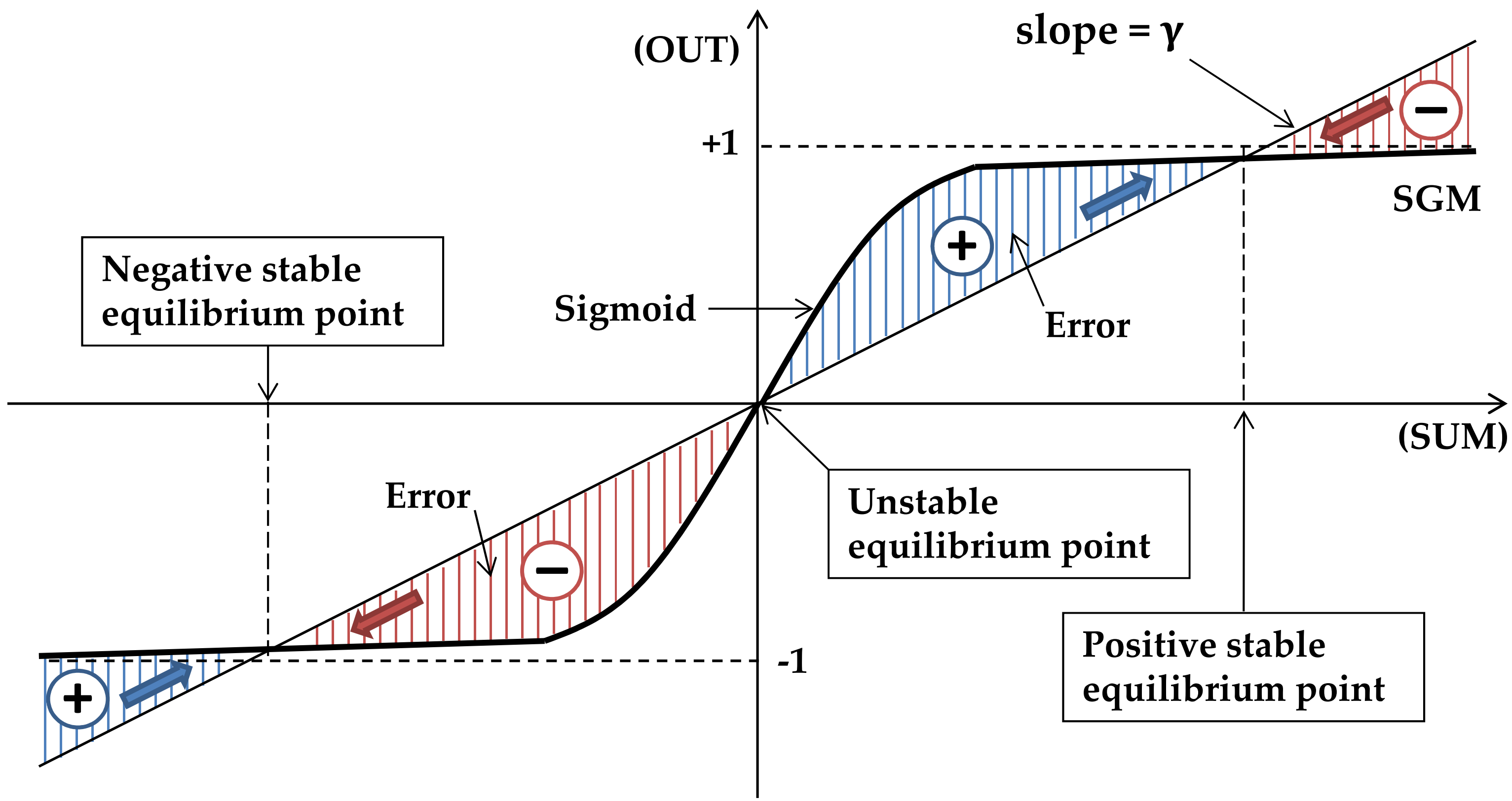


Figure 8-(a): The error of a sigmoidal neuron with bootstrap learning.

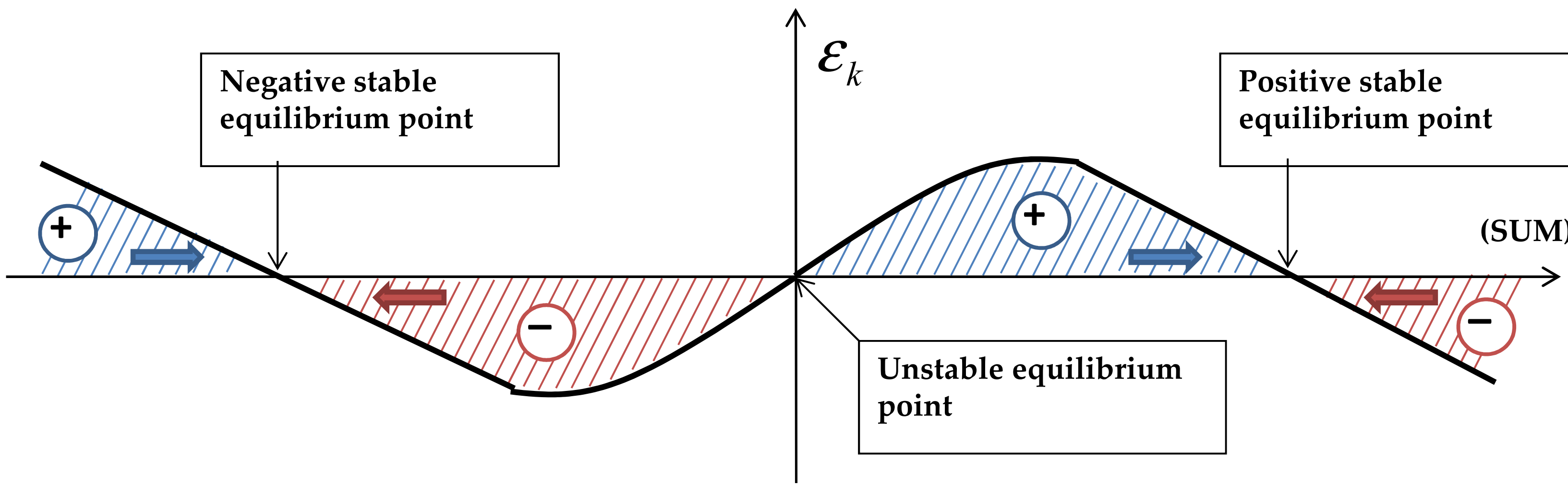


Figure 8-(b): The error function.

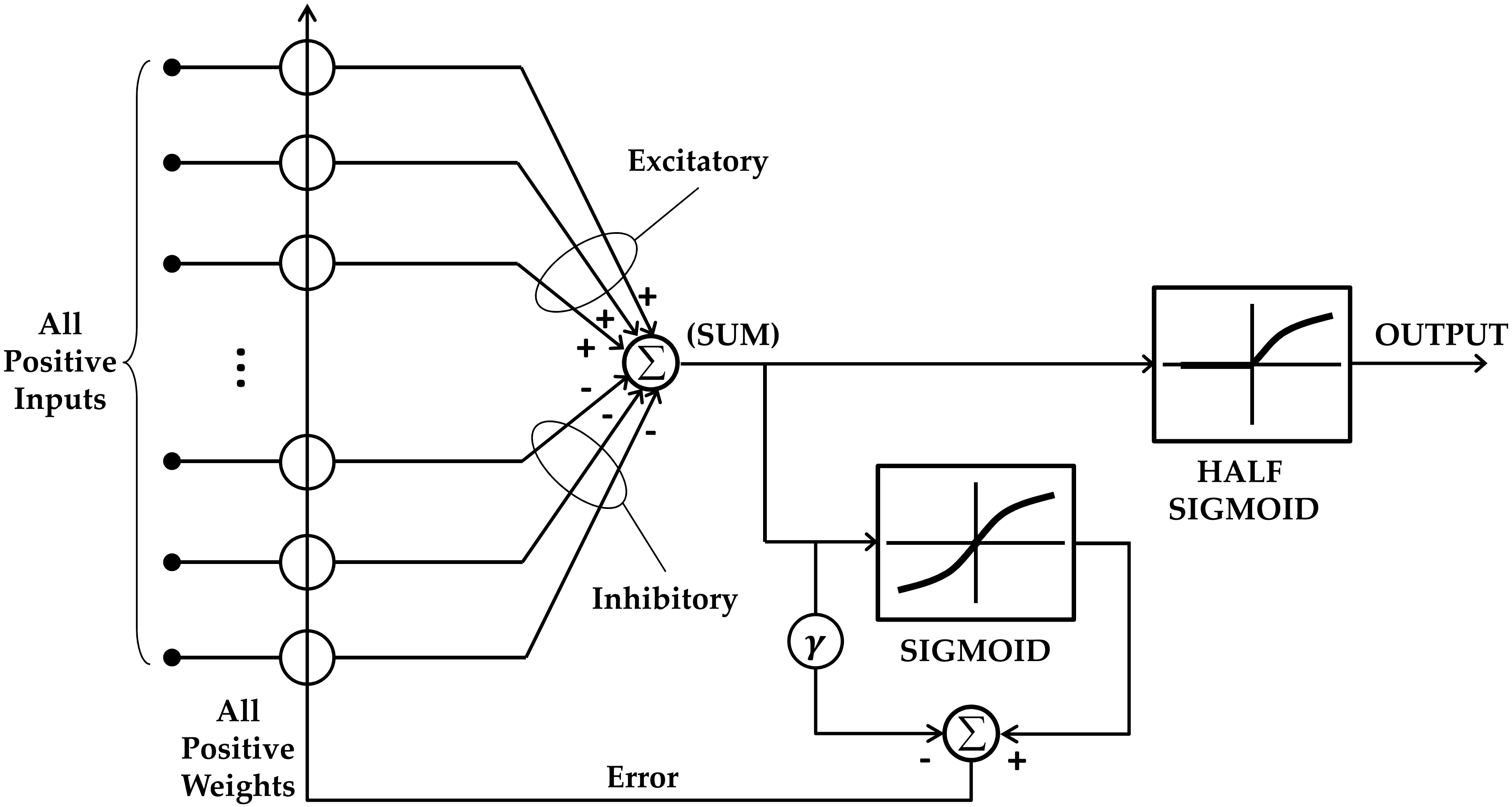


Figure 9: A post-synaptic neuron with excitatory and inhibitory inputs and all positive weights trained with LMS bootstrap learning. All outputs are positive after rectification

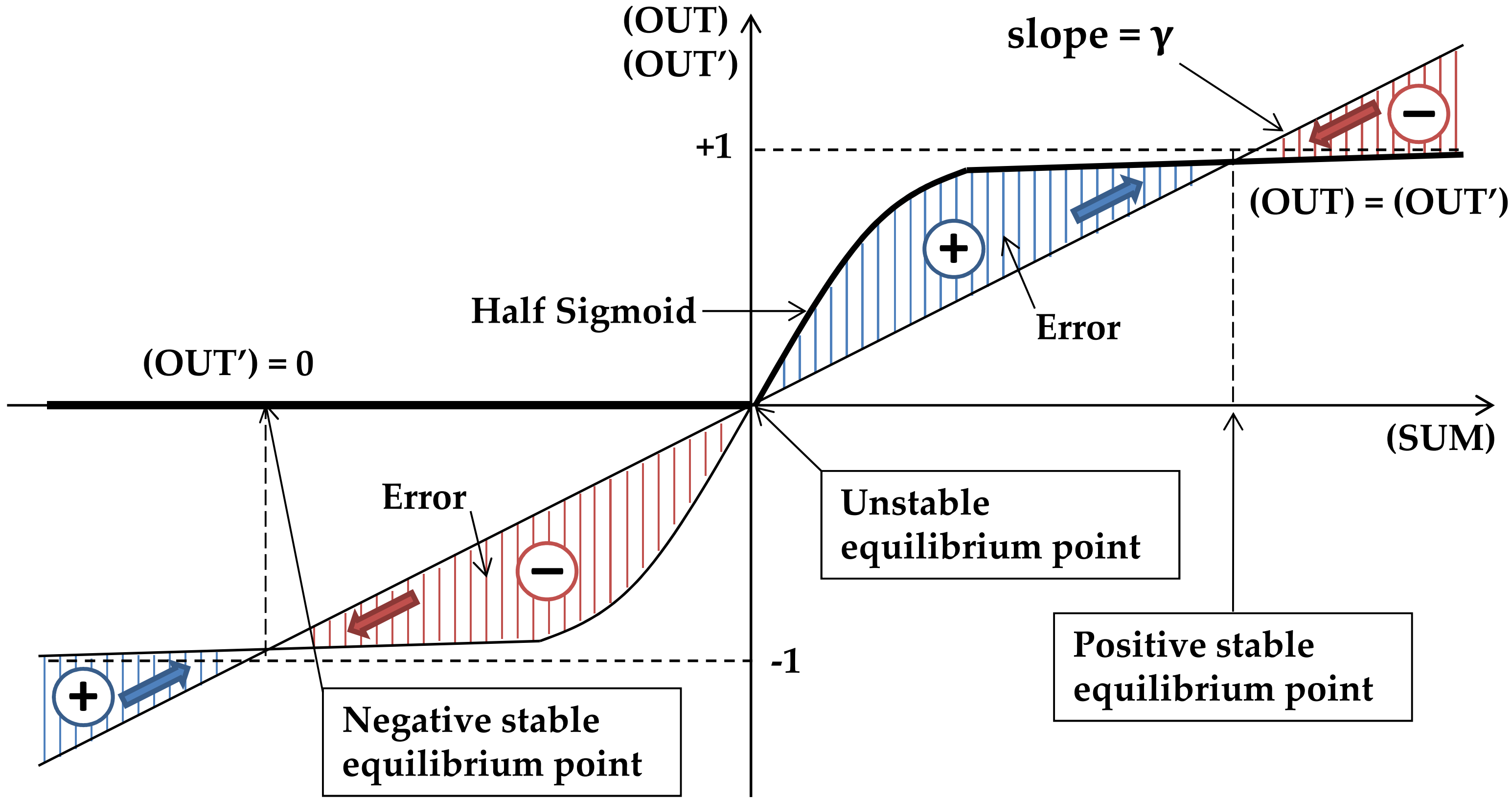


Figure 10-(a): The error of the sigmoidal neuron with rectified output, trained with bootstrap learning.

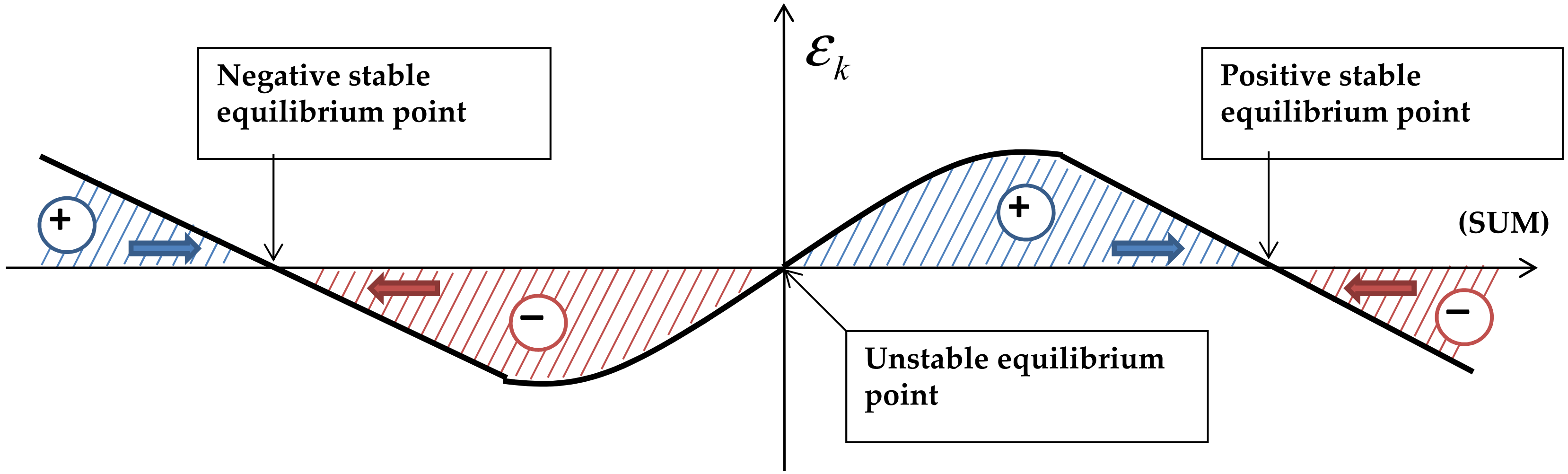


Figure 10-(b): The error function.

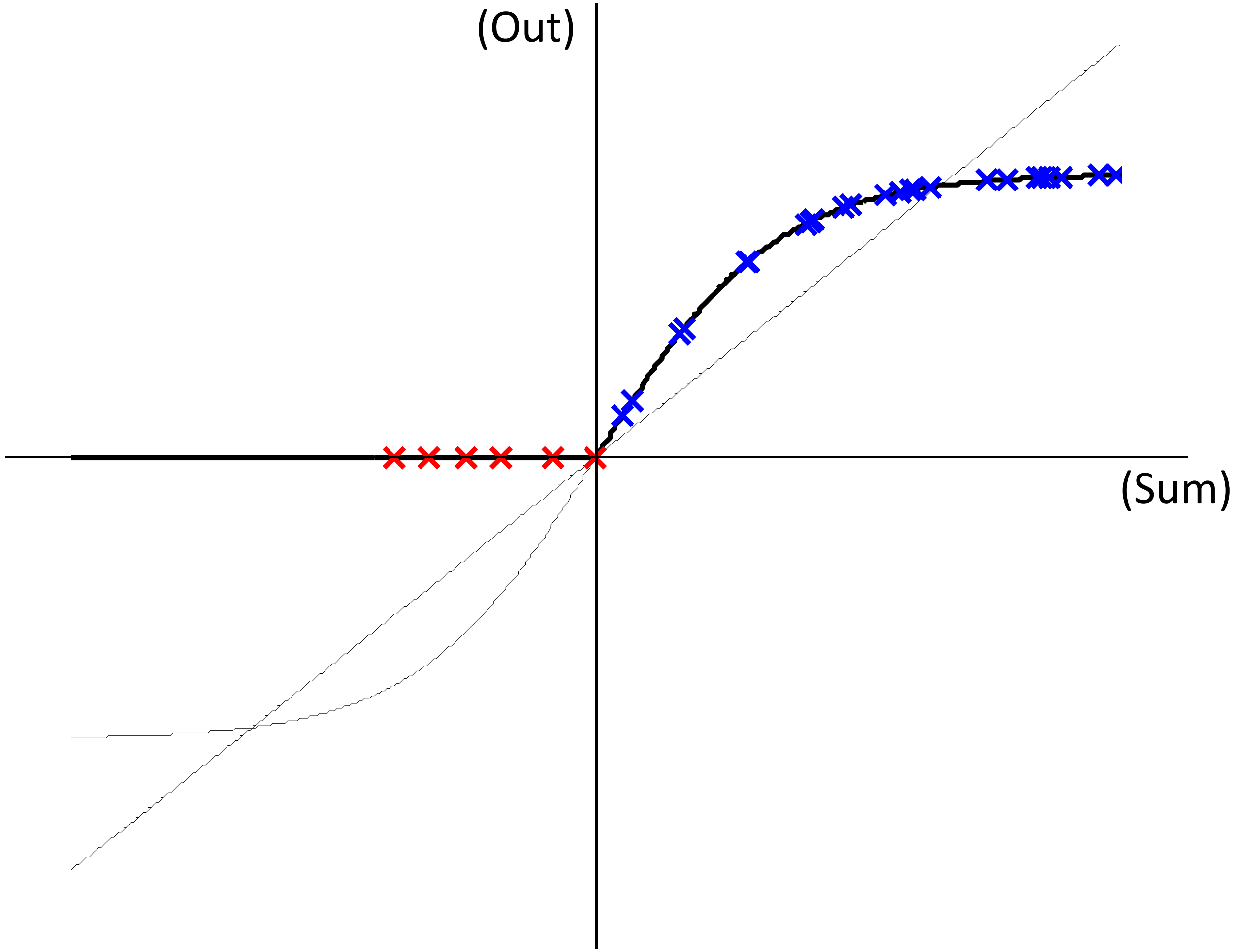


Figure 11-(a): Hebbian-LMS learning process. Initial condition.

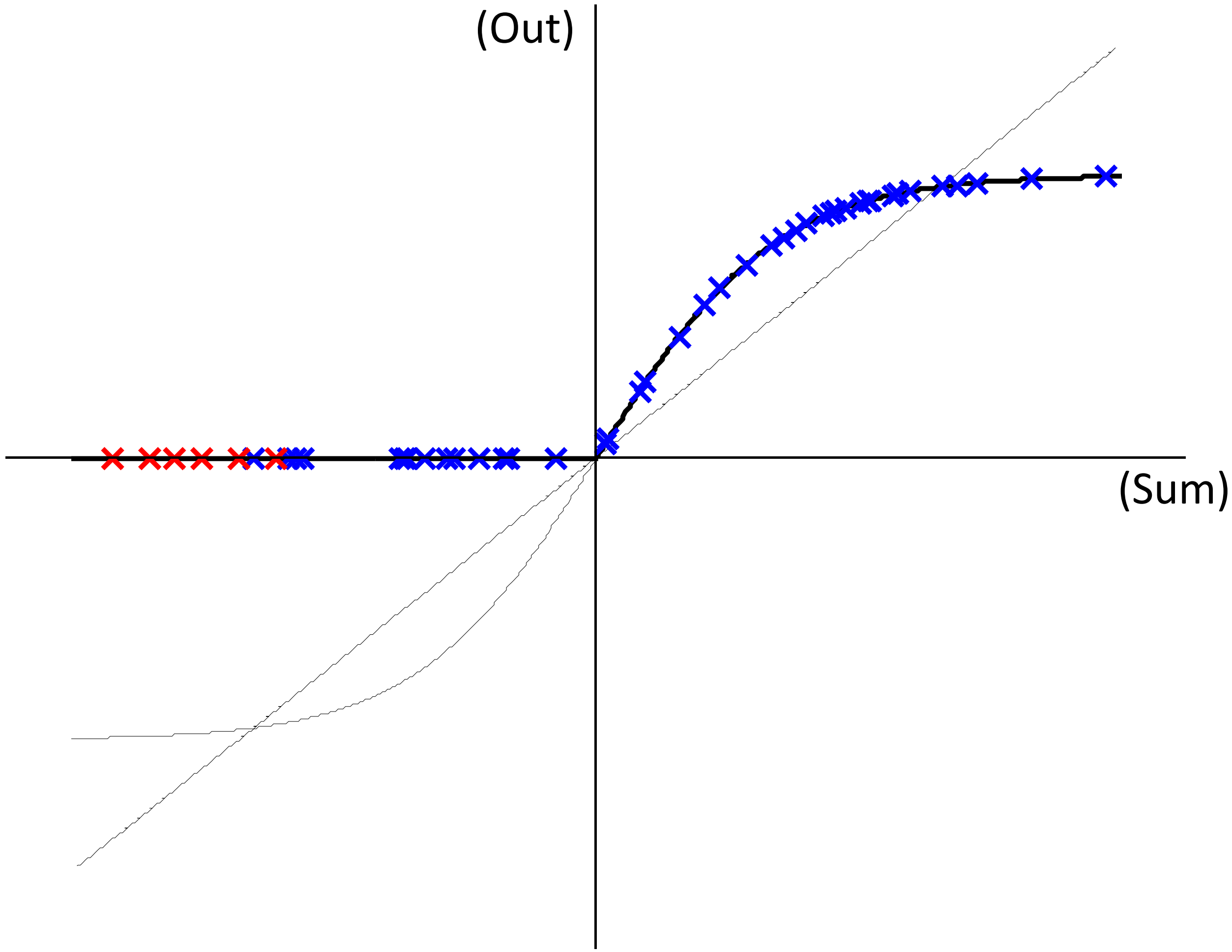


Figure 11-(b): Hebbian-LMS learning process. After 100 iterations

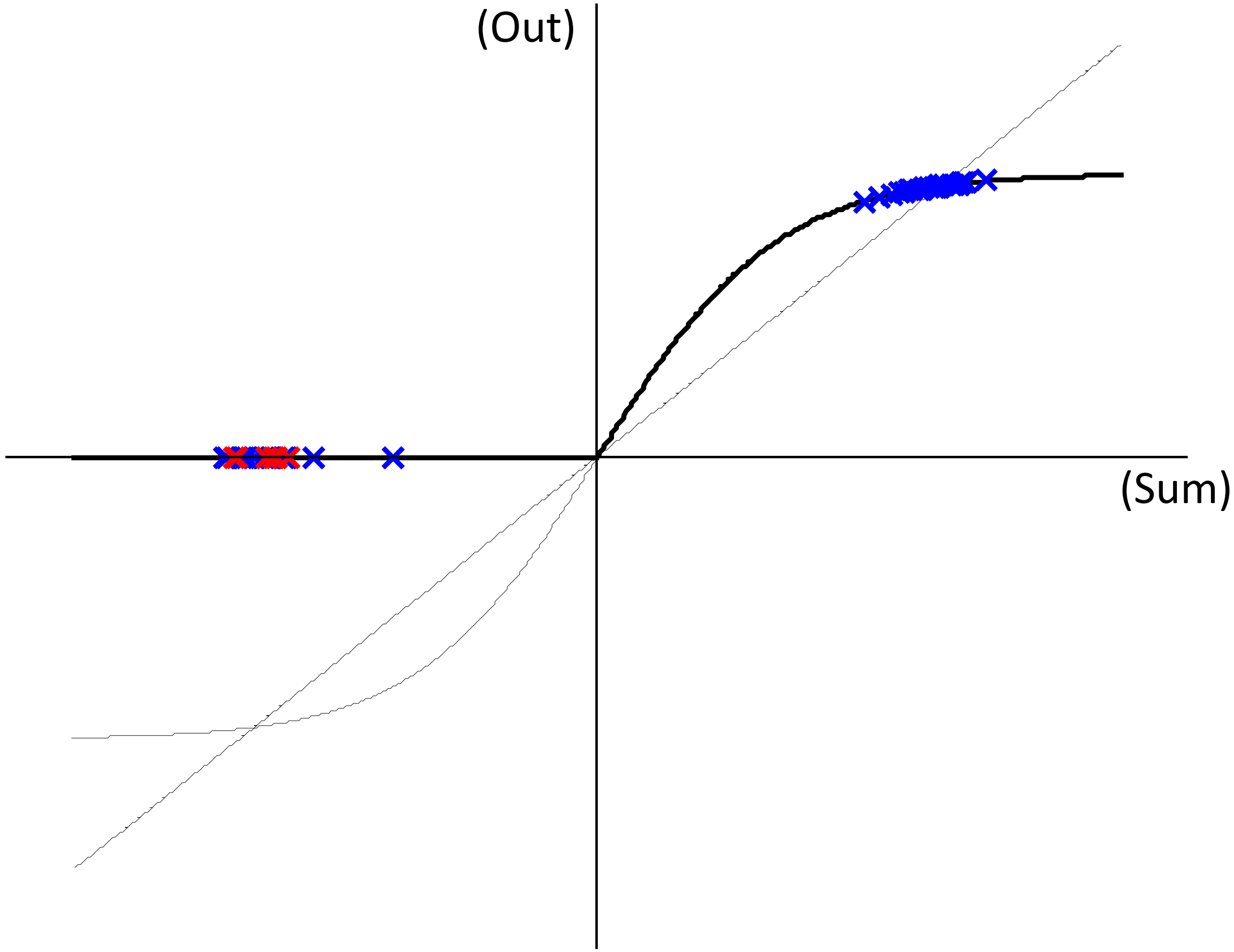


Figure 11-(c): Hebbian-LMS learning process. After 2000 iterations

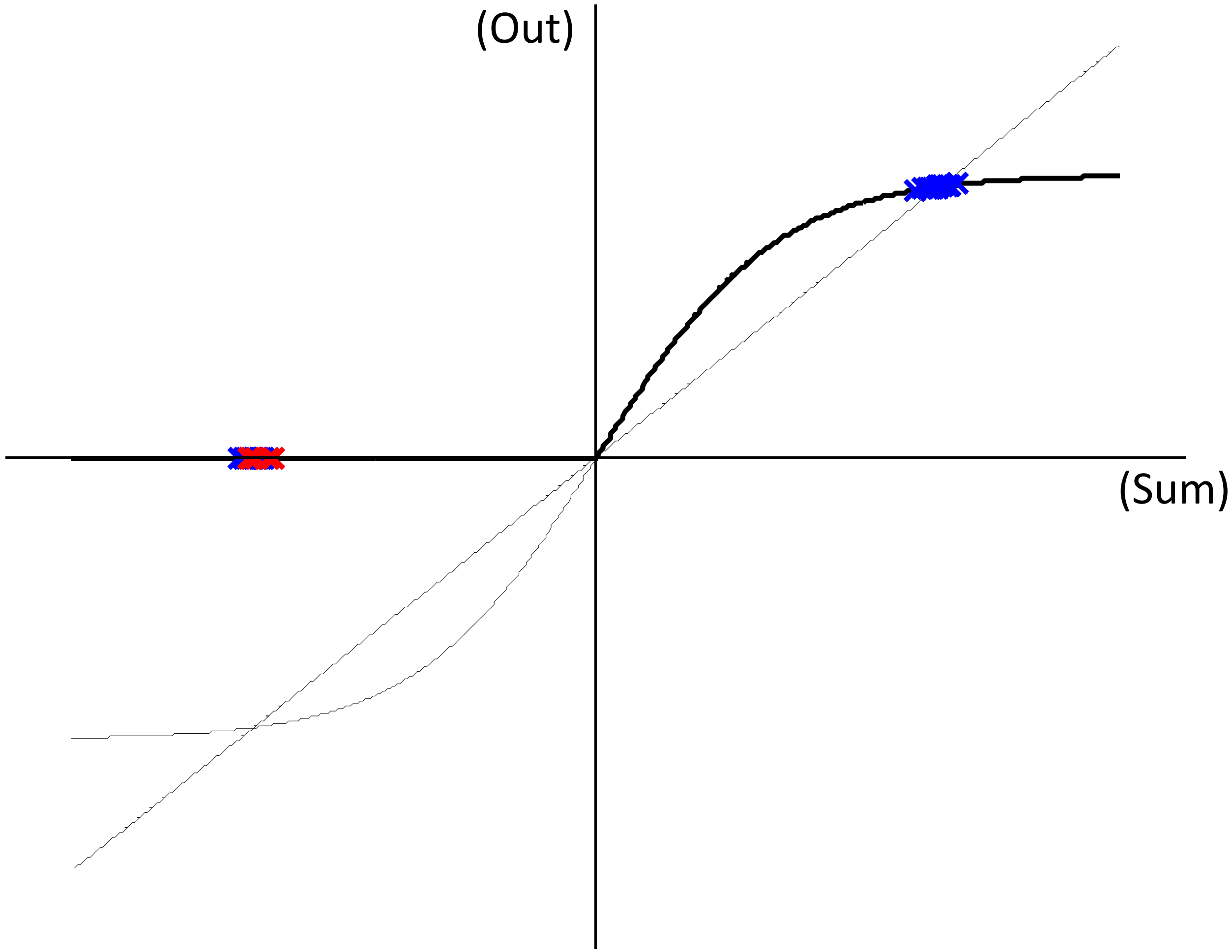


Figure 11-(d): Hebbian-LMS learning process. After 5000 iterations

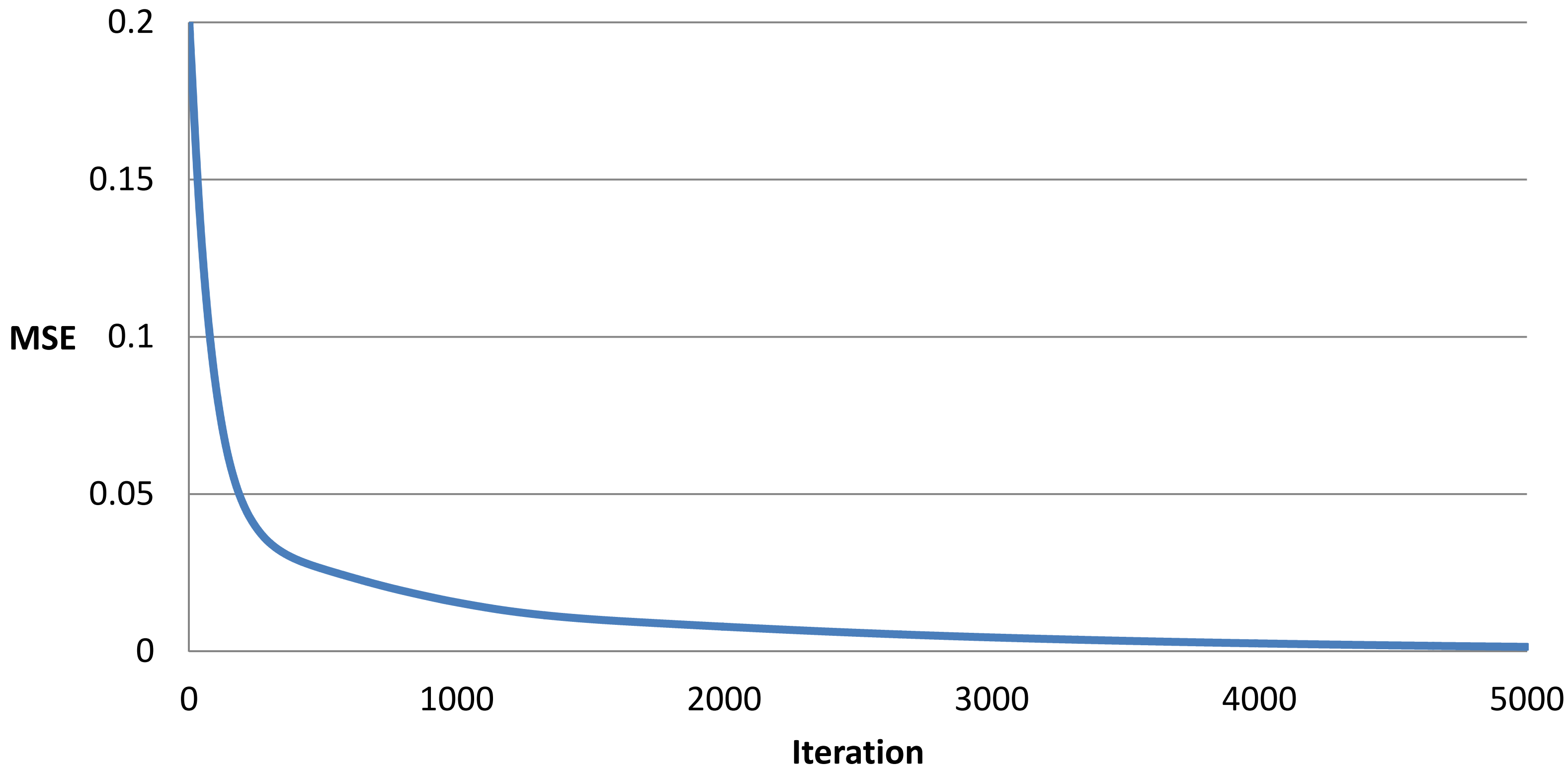


Figure 11-(e): Hebbian-LMS learning curve.

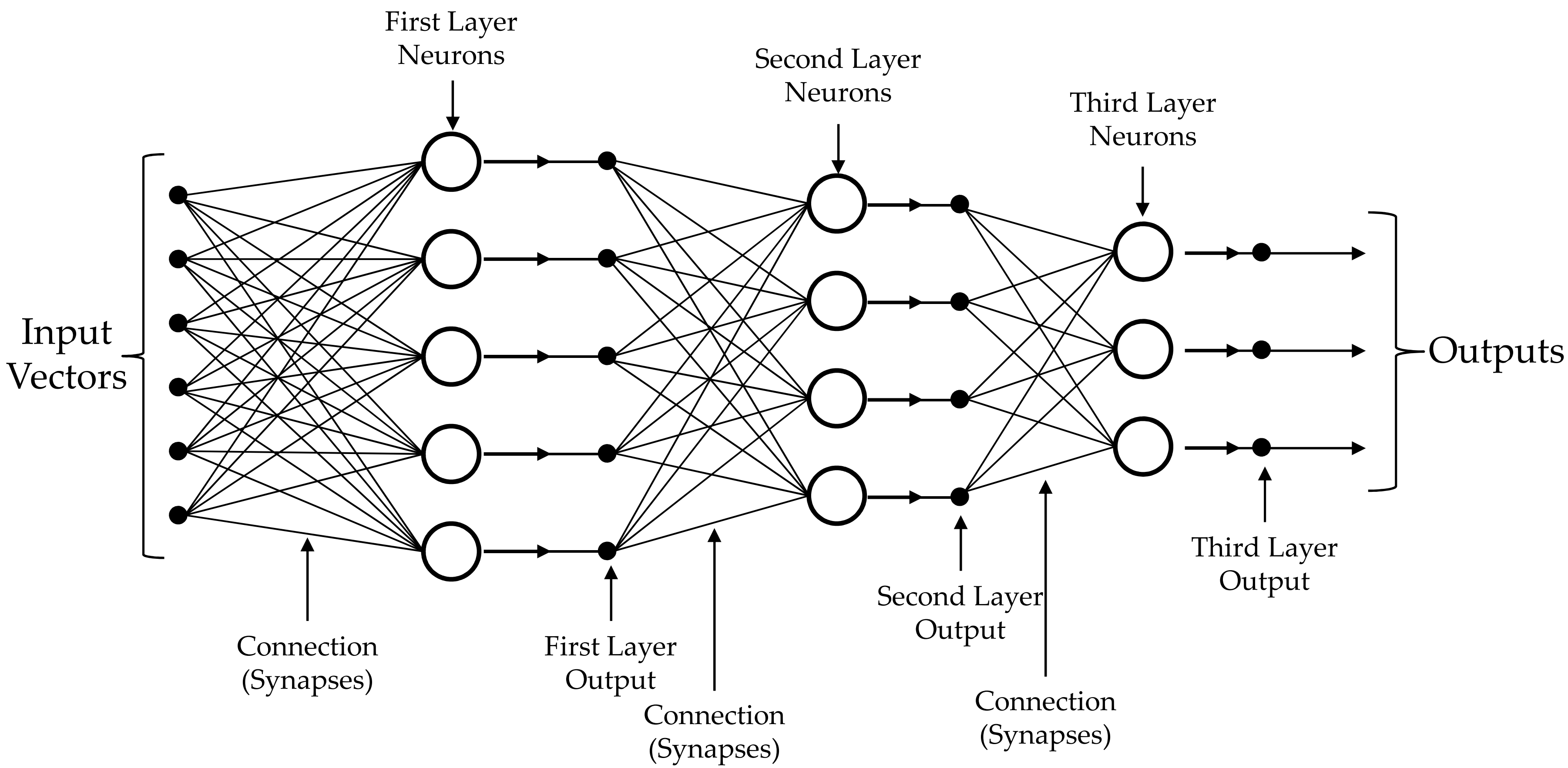


Figure 12: An example of a layered neural network.

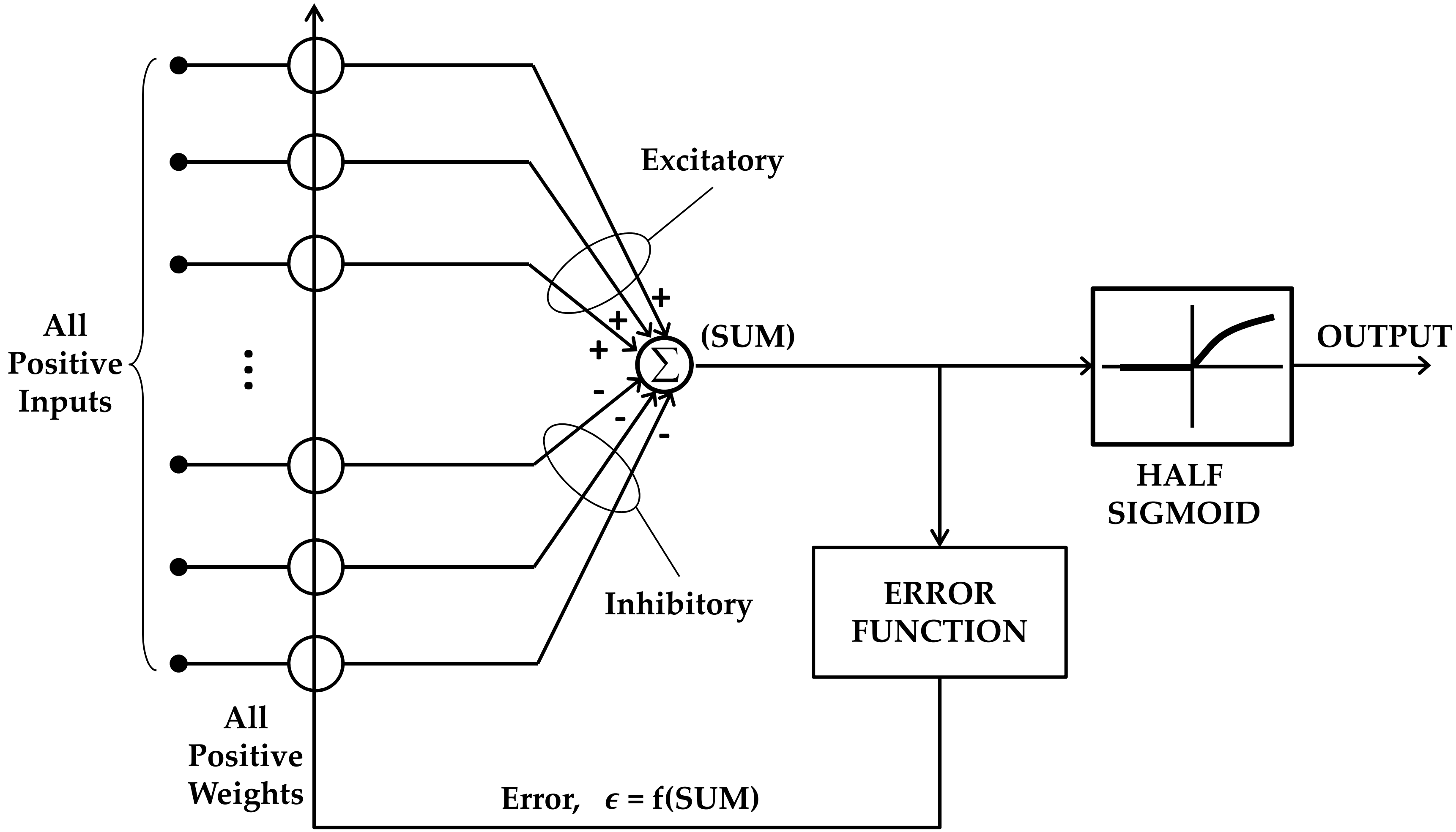
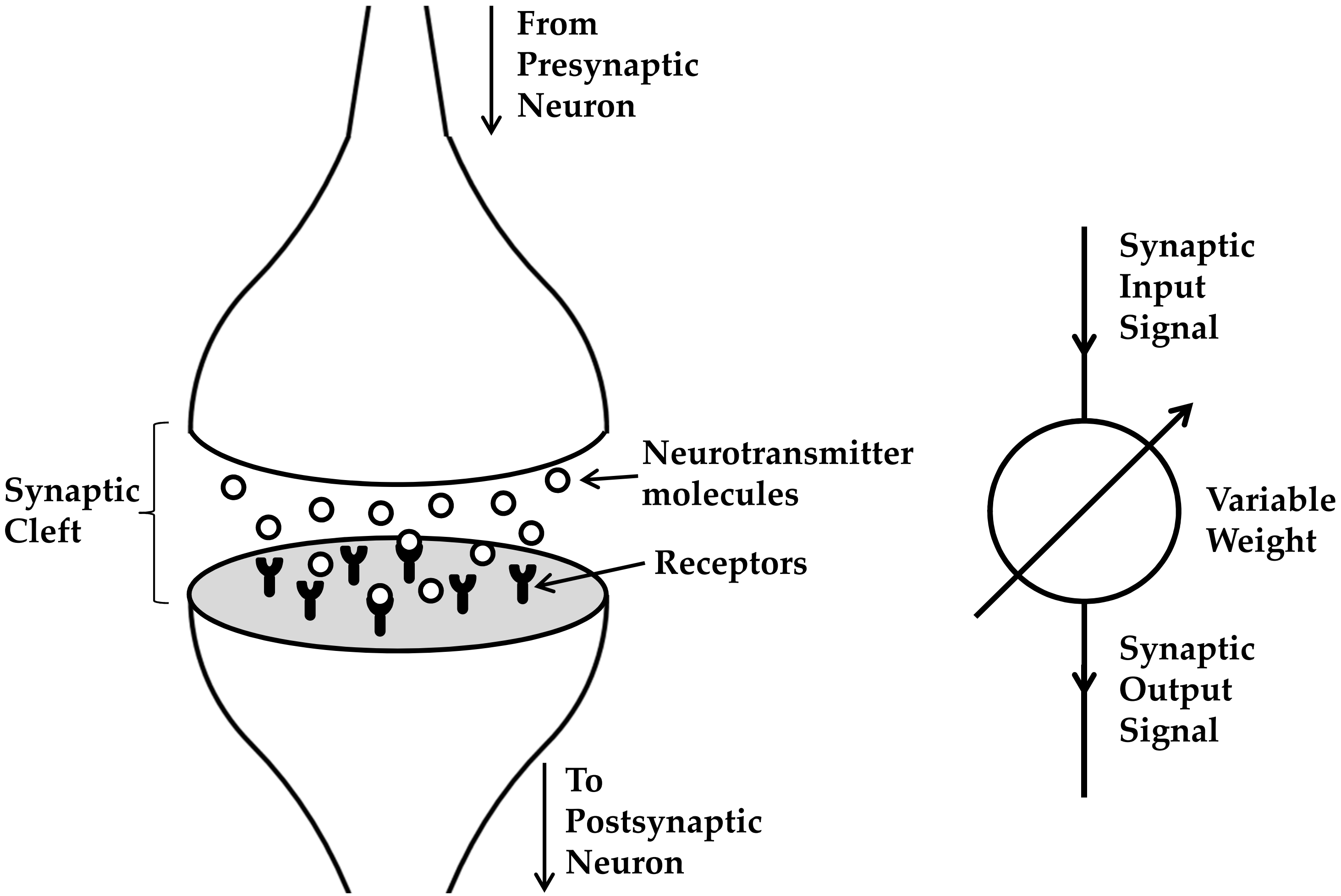


Figure 13: A general form of Hebbian-LMS.



(a) Synapse (b) A Variable Weight

Figure 14: A synapse corresponding to a variable weight.

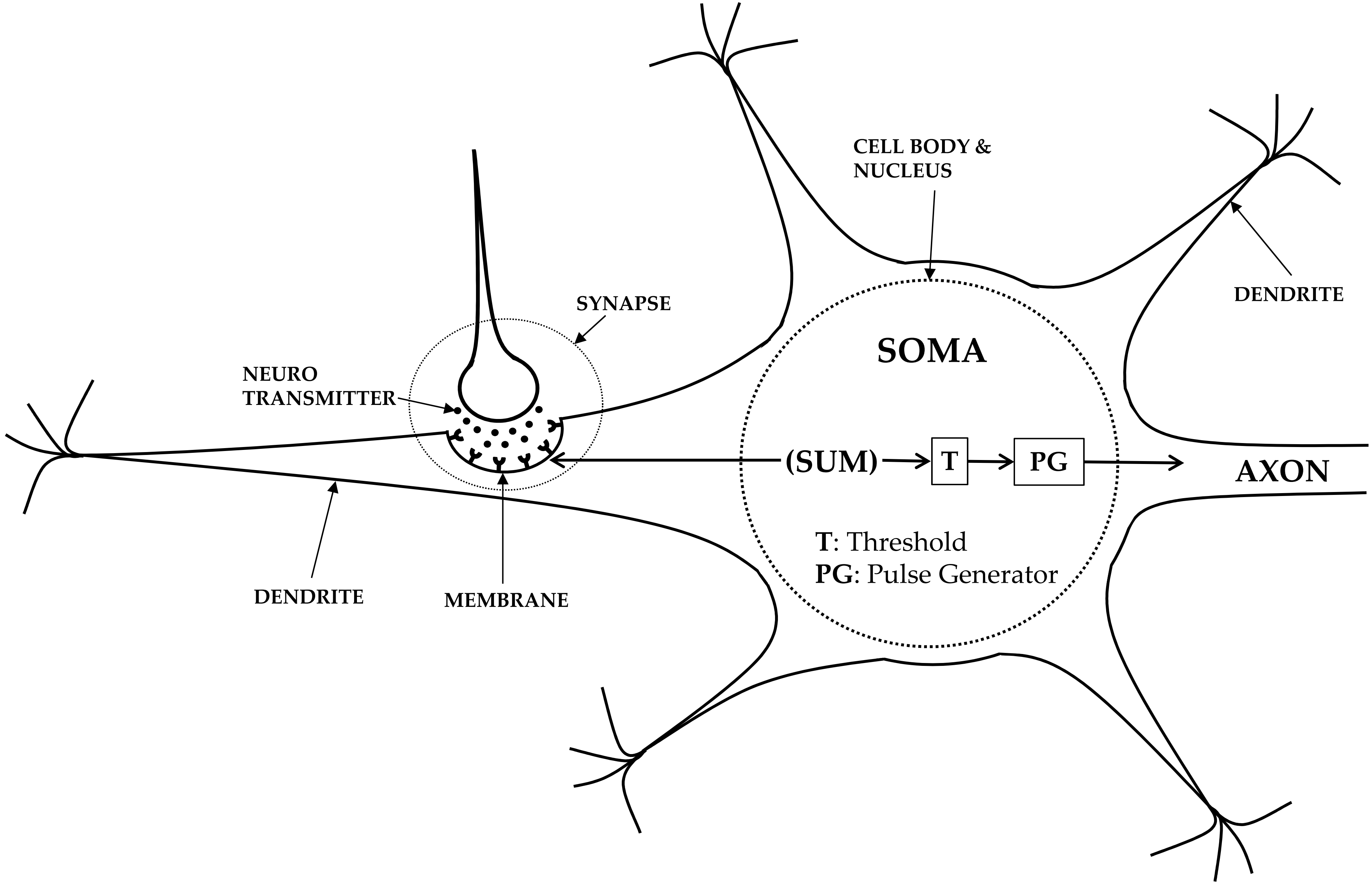


Figure 15: A neuron, dendrite, and synapse.

- When the pre-synaptic neuron is not firing, there will be no neurotransmitter in the gap and there will be no weight change. This applies to both excitatory and inhibitory synapses.

- When the pre-synaptic neuron is firing, and the post-synaptic neuron is also firing, there will be neurotransmitter in the gap and the post-synaptic membrane voltage will be positive since the (SUM) is positive, and the number of neuroreceptors will gradually increase, thus increasing the weight. This applies to excitatory synapses.

- When the pre-synaptic neuron is firing, and the post-synaptic neuron is not firing, there will be neurotransmitter in the gap and the post-synaptic membrane voltage will be negative since the (SUM) is negative and its number of neuroreceptors will gradually decrease, thus decreasing the weight. This applies to excitatory synapses.

- The opposite of these rules apply to inhibitory synapses.

Figure 16: Postulates of synaptic plasticity

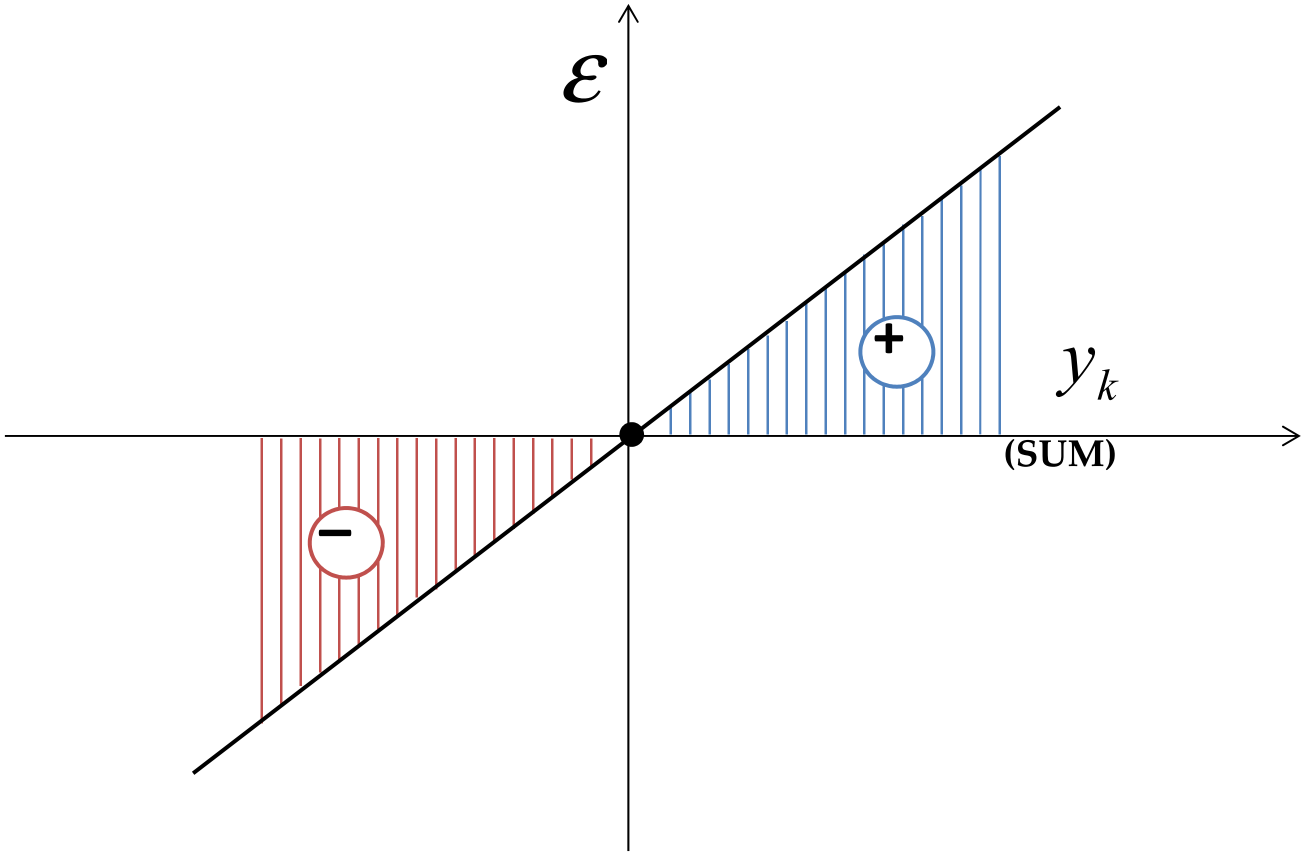


Figure 17: A linear error function.

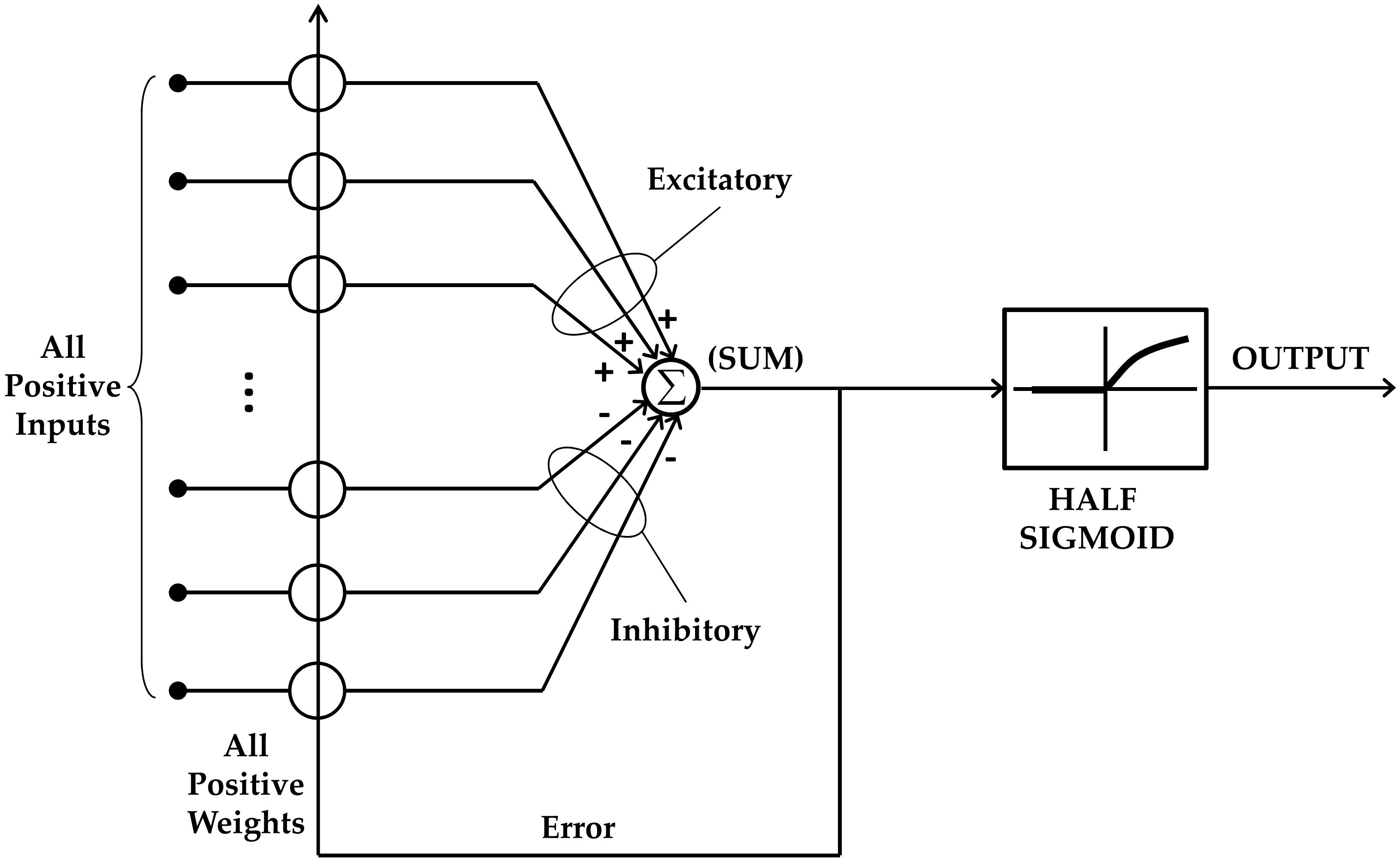


Figure 18: Hebbian-LMS with a linear error function.