Electromagnetics Made Compatible

To Study, To Learn, To Work With

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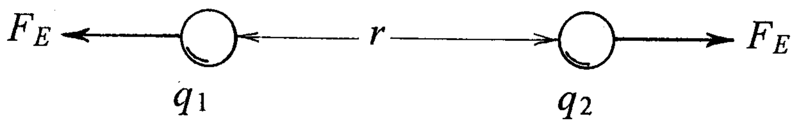
Electromagnetics Made Compatible

I. Forces Due to Stationary Charges

If two stationary charges, q_1 and q_2, both have the same sign, then $q_1 q_2 > 0$, and



where $$F_E = \frac{Z_m v_p}{4\pi r^2} q_1 q_2 \quad \text{(newtons)}$$

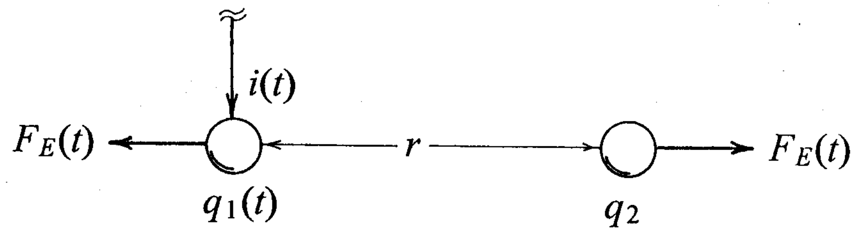
A Medium's Characteristics:

$$Z_m = \textit{characteristic impedance} = \sqrt{\frac{\mu_m}{\varepsilon_m}}$$

$$v_p = \textit{force propagation velocity} = \frac{1}{\sqrt{\mu_m \varepsilon_m}}$$

$$\left(\varepsilon_m = \text{"permittivity"} = \frac{1}{Z_m v_p}\right)$$

$$\left(\mu_m = \text{"permeability"} = \frac{Z_m}{v_p}\right)$$



$$F_E(t) = \frac{Z_m v_p}{4\pi r^2}\left[q_1(t-t_p) + \frac{r}{v_p}\frac{dq_1(t-t_p)}{dt}\right] q_2$$

(newtons)

$$t_p = \frac{r}{v_p} = \textit{force propagation time}$$

Assume q_2 is a test charge, then the E-fields of q_1 and $q_1(t)$ are

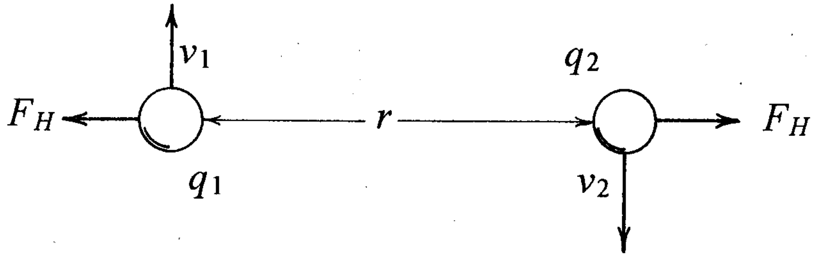
$$E_r = \frac{F_E}{q_2}\left(\frac{\text{newtons}}{\text{coulomb}}\right) = \frac{Z_m v_p}{4\pi r^2} q_1 \left(\frac{\text{volts}}{\text{meter}}\right)$$

and

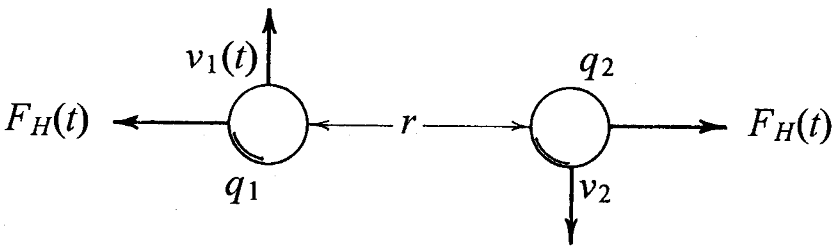
$$E_r(t) = \frac{F_E(t)}{q_2}\left(\frac{\text{newtons}}{\text{coulomb}}\right)$$

$$= \frac{Z_m v_p}{4\pi r^2}\left[q_1(t-t_p) + \frac{r}{v_p}\frac{dq_1(t-t_p)}{dt}\right]\left(\frac{\text{volts}}{\text{meter}}\right)$$

II. Forces Due to Charge Movement



$$F_H = \frac{Z_m}{v_p} \frac{q_1 v_1 q_2 v_2}{4\pi r^2} \quad \text{(newtons)}$$



$$F_H(t) = \frac{Z_m}{v_p} \frac{q_1 q_2}{4\pi r^2} \left[v_1(t) + t_p \frac{dv_1(t)}{dt} \right] v_2 \quad \text{(newtons)}$$

This is a snapshot taken at time t, with $r \to 0$, so the effects of propagation can be temporarily neglected. Also, $q_1 q_2 > 0$, in both cases.

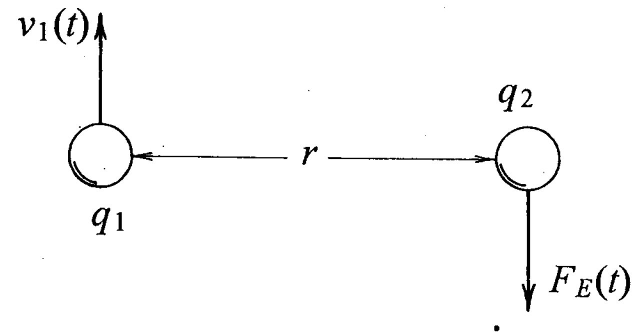
Using q_2 as a test charge, the H-fields that leave $q_1 v_1$ and $q_1 v_1(t)$ at snapshot time are seen to be

$$H_\phi = \frac{v_p}{Z_m} \frac{F_H}{q_2 v_2} = \frac{q_1 v_1}{4\pi r^2} \left(\frac{\text{amperes}}{\text{meter}} \right), \quad \text{and}$$

$$H_\phi(t) = \frac{v_p}{Z_m} \frac{F_H(t)}{q_2 v_2}$$

$$= \frac{q_1}{4\pi r^2} \left[v_1(t) + \frac{r}{v_p} \frac{dv_1(t)}{dt} \right] \left(\frac{\text{amperes}}{\text{meter}} \right)$$

In addition, because $dv_1(t)/dt \neq 0$,



where

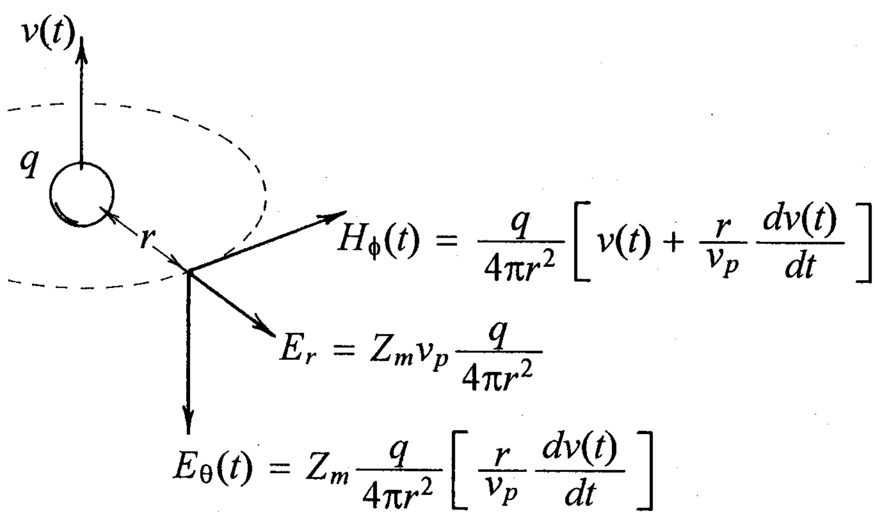
$$F_E(t) = Z_m \frac{q_1 q_2}{4\pi r^2} \left[\frac{r}{v_p} \frac{dv_1(t)}{dt} \right] \quad \text{(newtons)}$$

So, the additional E-field leaving $q_1 v_1(t)$ is

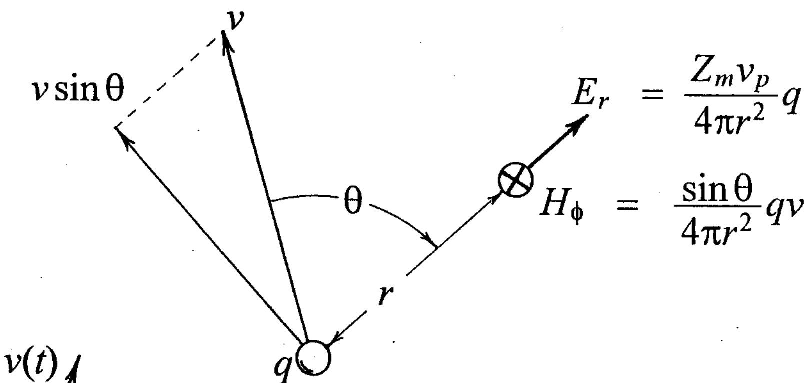
$$E_\theta(t) = \frac{F_E(t)}{q_2} \quad \left(\frac{\text{newtons}}{\text{coulomb}}\right)$$

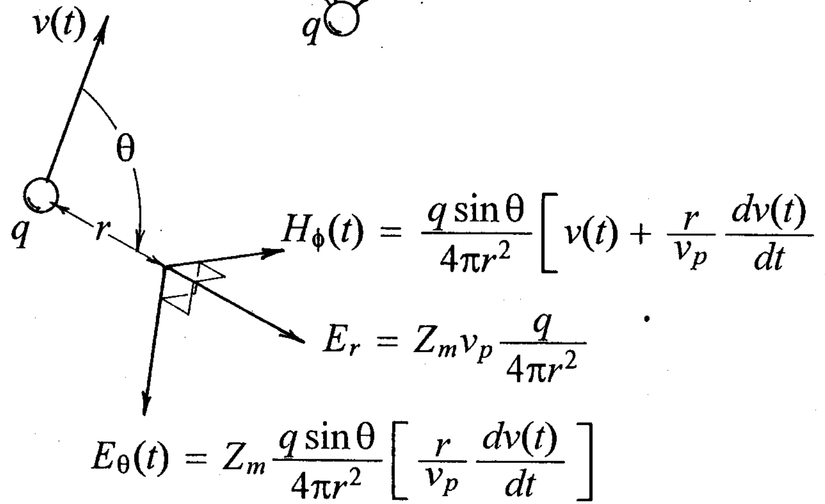
$$= \frac{Z_m q_1}{4\pi r^2}\left[\frac{r}{v_p}\frac{dv_1(t)}{dt}\right] \quad \left(\frac{\text{volts}}{\text{meter}}\right)$$

Thus, as a charge, q, passes through any plane normal to its velocity $v(t)$, for $r \to 0$, the fields of q in that plane are E_r, $H_\phi(t)$, and $E_\theta(t)$.



But, in any r-direction, the fields H_ϕ, $H_\phi(t)$, and $E_\theta(t)$, also have $\sin\theta$ as a factor, where θ is the angle between the r-direction and v, or $v(t)$. For then, either $v\sin\theta$, or $v(t)\sin\theta$, will be the velocity component of q that is normal to the r-direction.





III. The Fields of Electric Currents

An electric current is a field-forced movement of electric charges. In metals, the movable charges are electrons. So, the k-th differential length of a current-carrying metal conductor, is a stationary point with moving charge equal to the product of

q_e = electric charge per electron;
$\frac{n_m}{V}$ = movable electron density per unit volume;
dA_k = conductor cross-sectional area at point k;
v_k, or $v_k(t)$ = electron drift velocity at point k.

$$i_k = \left(q_e \frac{n_m}{V} dA_k\right) v_k = \frac{Q_k}{dl} v_k \ \left(\frac{\text{coulombs}}{\text{second}}\right)$$

and

$$i_k(t) = \left(q_e \frac{n_m}{V} dA_k\right) v_k(t) = \frac{Q_k}{dl} v_k(t) \ \left(\frac{\text{coulombs}}{\text{second}}\right)$$

So, a differential length of current $i_k dl$, or $i_k(t)dl$, is a fixed point past which the charge Q_k moves with drift velocity v_k, or $v_k(t)$. That is,

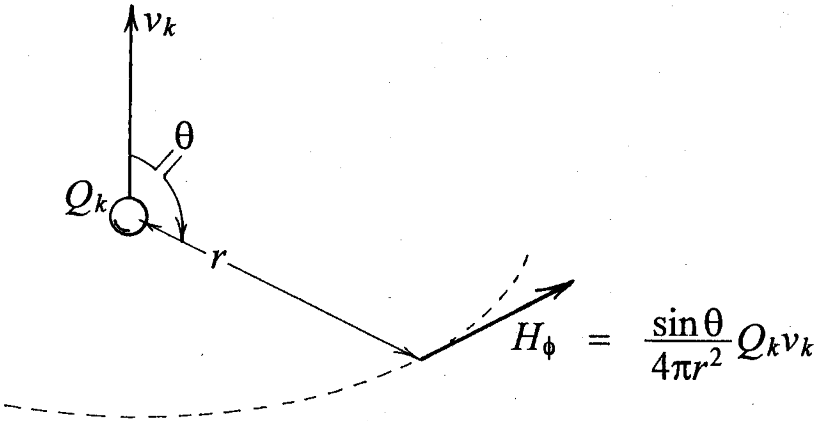
$$\begin{aligned} i_k dl &= \left(q_e \frac{n_m}{V} dA_k\right) v_k dl \\ &= \left(q_e \frac{n_m}{V} dA_k dl\right) v_k = Q_k v_k \end{aligned}$$

And, the H-field that leaves a charge Q_k moving with constant velocity v_k is

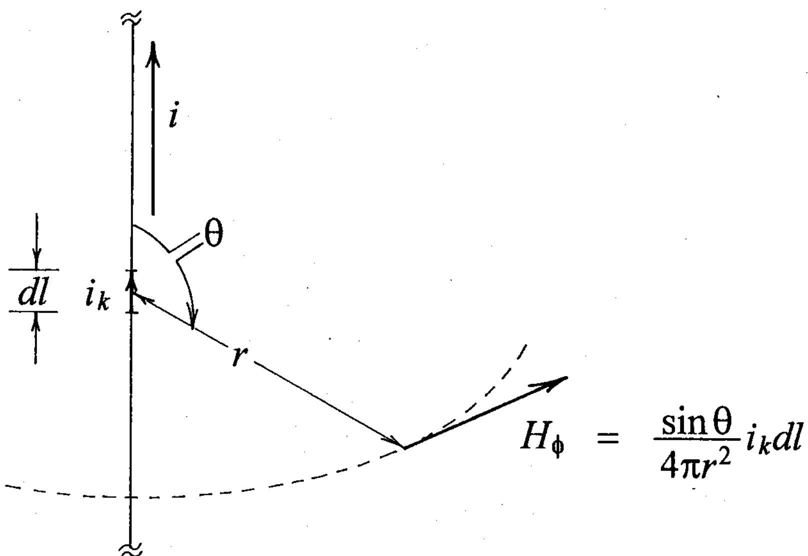
$$H_\phi = \frac{\sin\theta}{4\pi r^2} Q_k v_k$$

So, if $i_k dl = Q_k v_k$, it is obvious that the H-field leaving $i_k dl$ is

$$H_\phi = \frac{\sin\theta}{4\pi r^2} i_k dl$$



The H-field leaving $Q_k v_k$



The H-field of $i_k dl$

Similarly, the H-field leaving a charge Q_k that is moving with the time-varying velocity $v_k(t)$ is

$$H_\phi(t) = \frac{\sin\theta}{4\pi r^2}\left[v_k(t) + \frac{r}{v_p}\frac{dv_k(t)}{dt} \right] Q_k$$

So, if $i_k(t)dl = Q_k v_k(t)$, then the H-field leaving $i_k(t)dl$ is

$$H_\phi(t) = \frac{\sin\theta}{4\pi r^2}\left[i_k(t) + \frac{r}{v_p}\frac{di_k(t)}{dt} \right] dl$$

Also, the θ-directed E-field leaving $i_k(t)dl$ is the same as that leaving $Q_k v_k(t)$. That is,

$$E_\theta(t) = Z_m \frac{\sin\theta}{4\pi r^2}\left[\frac{r}{v_p}\frac{dv_k(t)}{dt} \right] Q_k$$

$$= Z_m \frac{\sin\theta}{4\pi r^2}\left[\frac{r}{v_p}\frac{di_k(t)}{dt} \right] dl$$

However, dl is not moving. So, for *all* r

$$H_\phi(t) = \frac{\sin\theta}{4\pi r^2}\left[i_k(t-t_p) + \frac{r}{v_p}\frac{di_k(t-t_p)}{dt} \right]dl$$

and

$$E_\theta(t) = Z_m \frac{\sin\theta}{4\pi r^2}\left[\frac{r}{v_p}\frac{di_k(t-t_p)}{dt} \right]dl$$

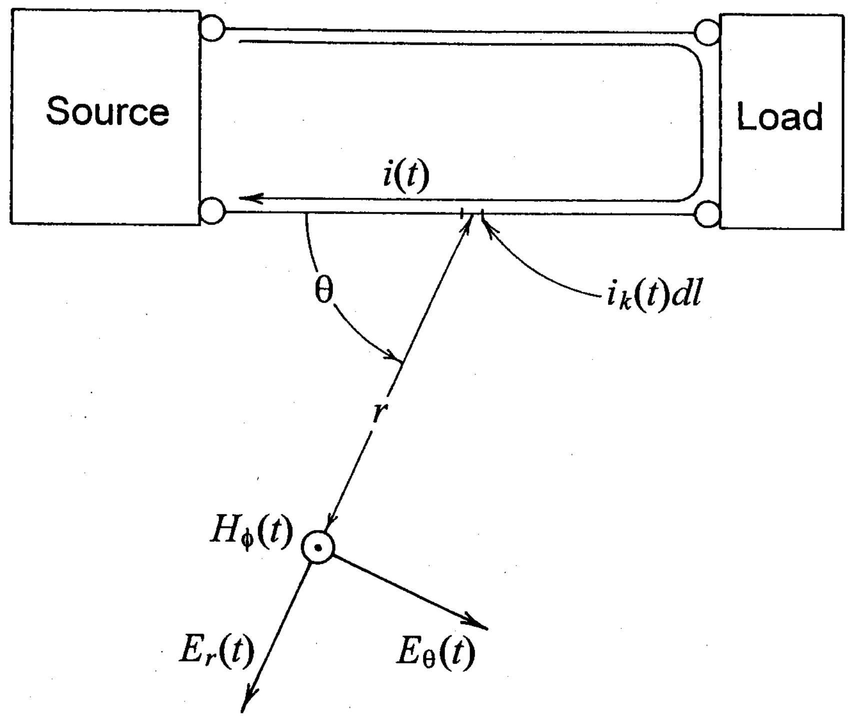
where $t_p = r/v_p$ is field propagation time, or the time it takes the fields to travel the distance r.

Also, propagation of $i(t)$ makes $i_{k-1}(t) \neq i_{k+1}(t)$, and

$$q_k(t) = \int_0^t [i_{k-1}(t) - i_{k+1}(t)]dt \neq 0$$

So, point k also has an r-directed E-field

$$E_r(t) = \frac{Z_m v_p}{4\pi r^2}\left[q_k(t-r/v_p) + \frac{r}{v_p}\frac{dq_k(t-r/v_p)}{dt} \right]$$

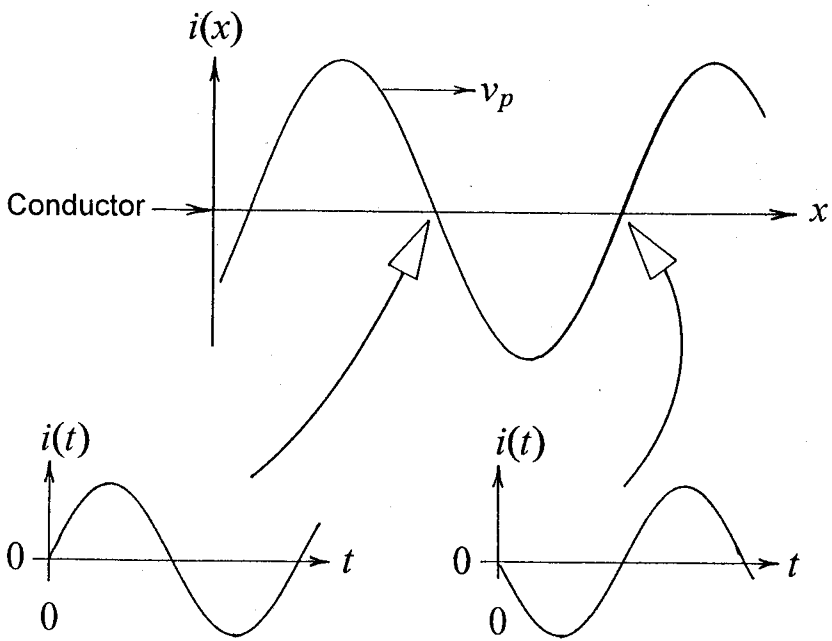


$$H_\phi(t) = \frac{\sin\theta}{4\pi r^2}\left[i_k(t-t_p) + \frac{r}{v_p}\frac{di_k(t-t_p)}{dt} \right]dl$$

$$E_\theta(t) = Z_m \frac{\sin\theta}{4\pi r^2}\left[\frac{r}{v_p}\frac{di_k(t-t_p)}{dt}. \right]dl$$

$$E_r(t) = \frac{Z_m v_p}{4\pi r^2}\left[q_k(t-t_p) + \frac{r}{v_p}\frac{dq_k(t-t_p)}{dt} \right]$$

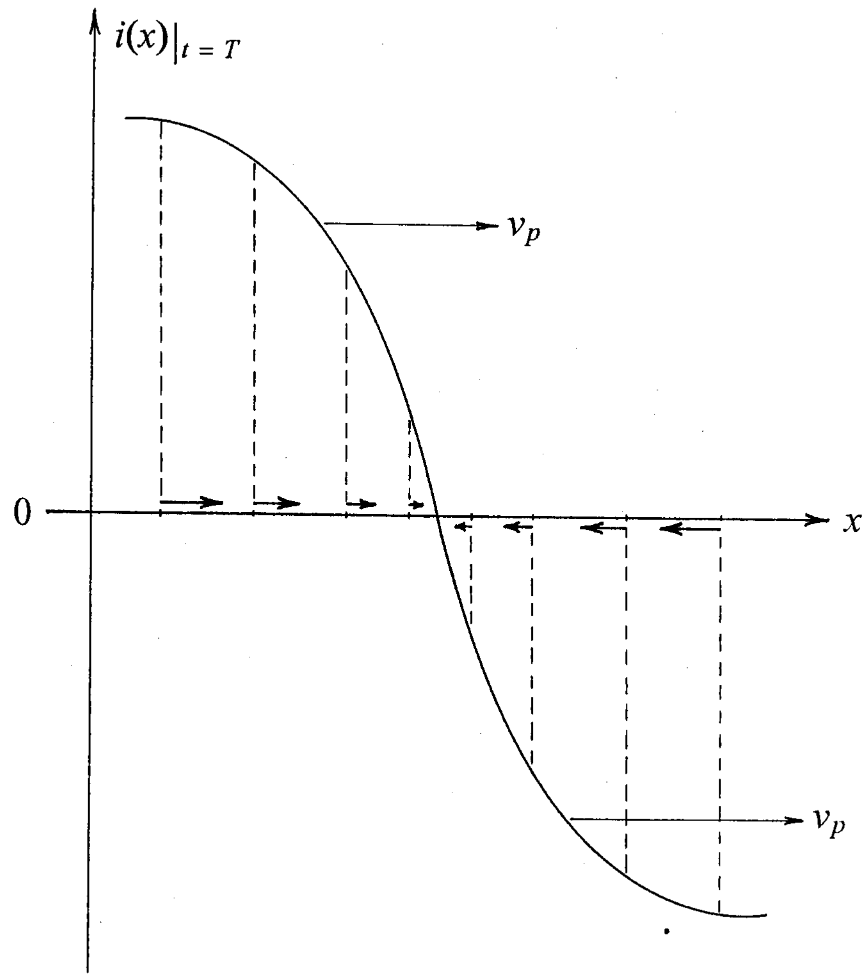
IV. Circuit Current Behavior



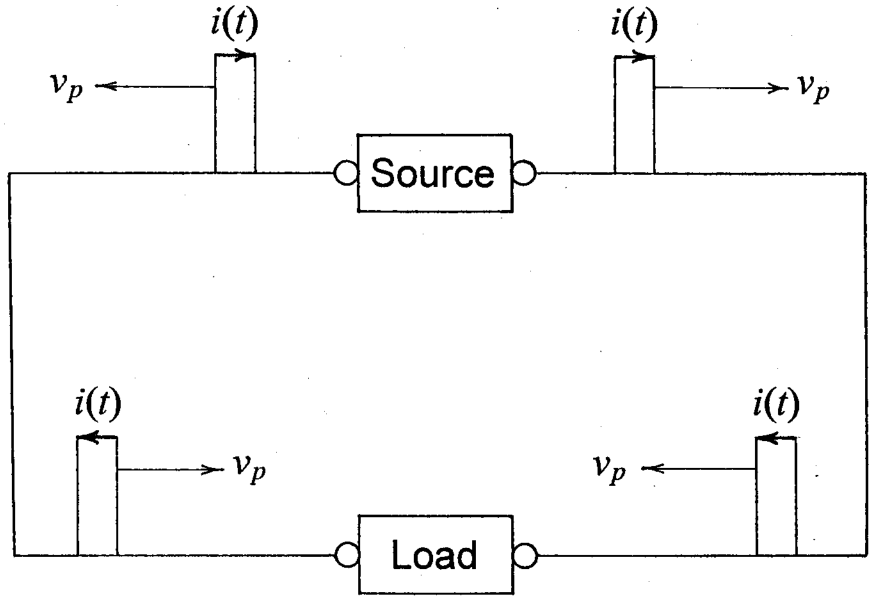
$$\lambda = \frac{v_p}{f} \quad \text{and} \quad v_p \approx 2 \times 10^8 \ \frac{\text{meters}}{\text{second}}$$

$$f = 10\ MHz \quad \Rightarrow \quad \lambda = \frac{2 \times 10^8}{10 \times 10^6} = 20 \text{ meters}$$

$$f = 10\ GHz \quad \Rightarrow \quad \lambda = \frac{2 \times 10^8}{10 \times 10^9} = 0.02 \text{ meter}$$

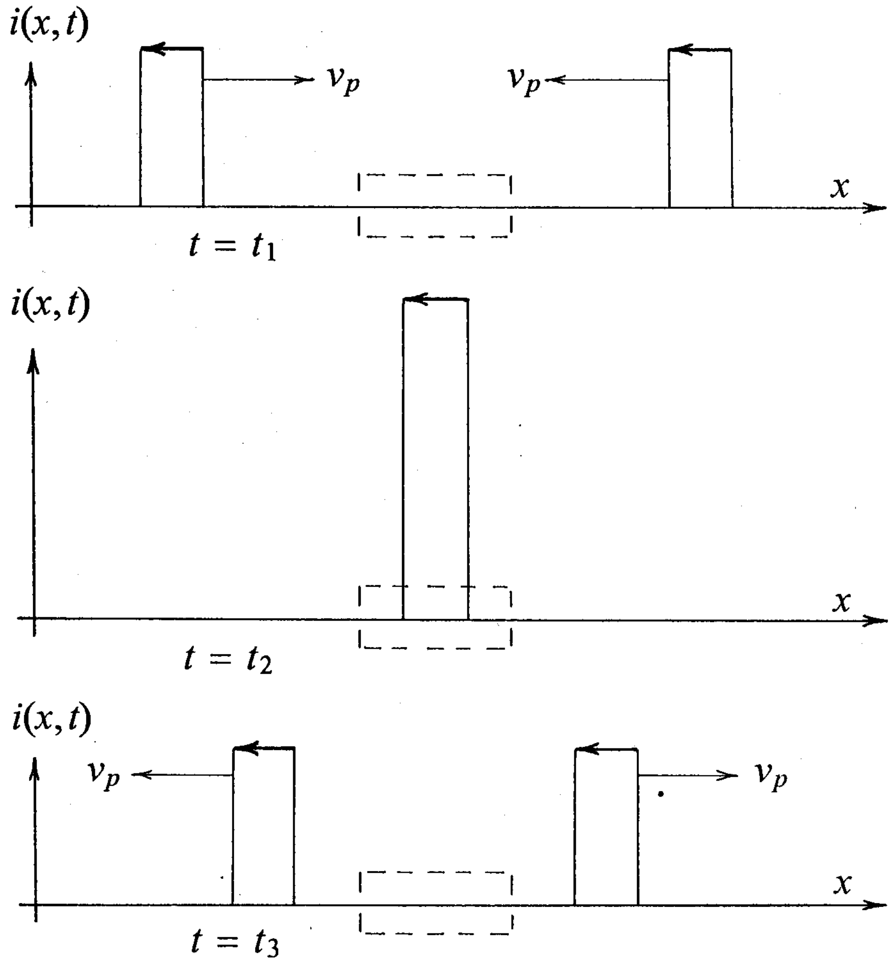


Current Amplitude and Charge Flow

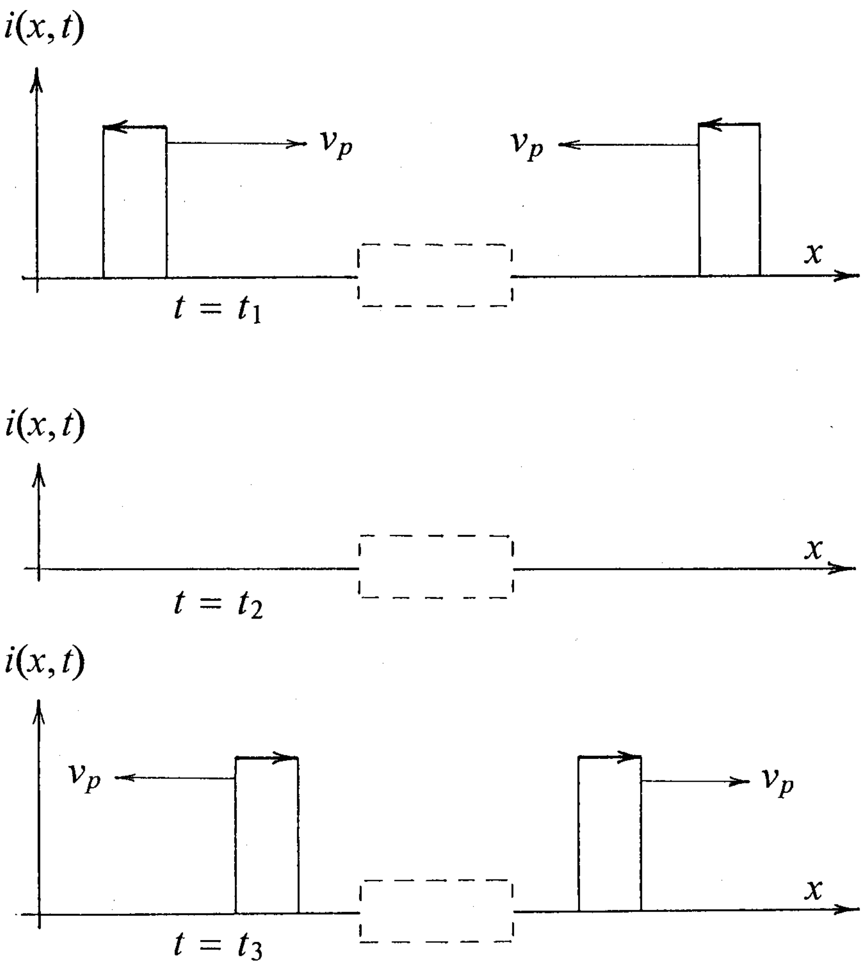


Both the "forward" and the "return" current of a circuit consist of electrons moving with the drift velocities of their conductor. And, both currents propagate from source to load with propagation velocities equal to that of the electric field in the conductor that causes them to move.

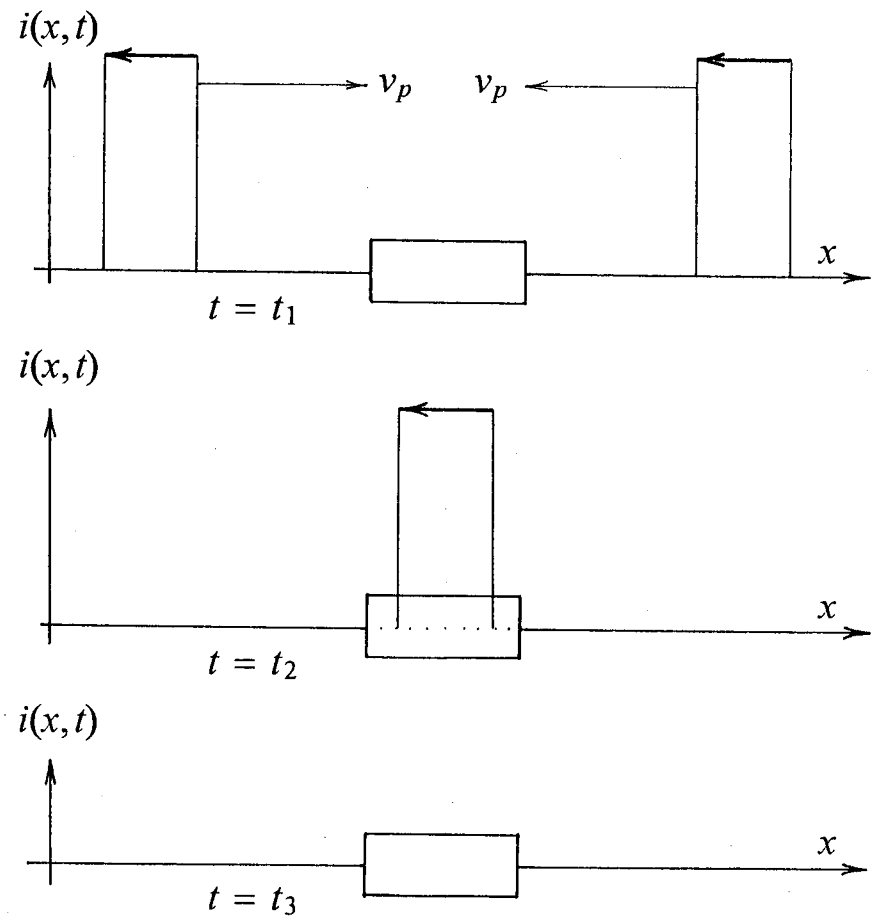
Both current components continue to propagate back to the source, if the load is zero.



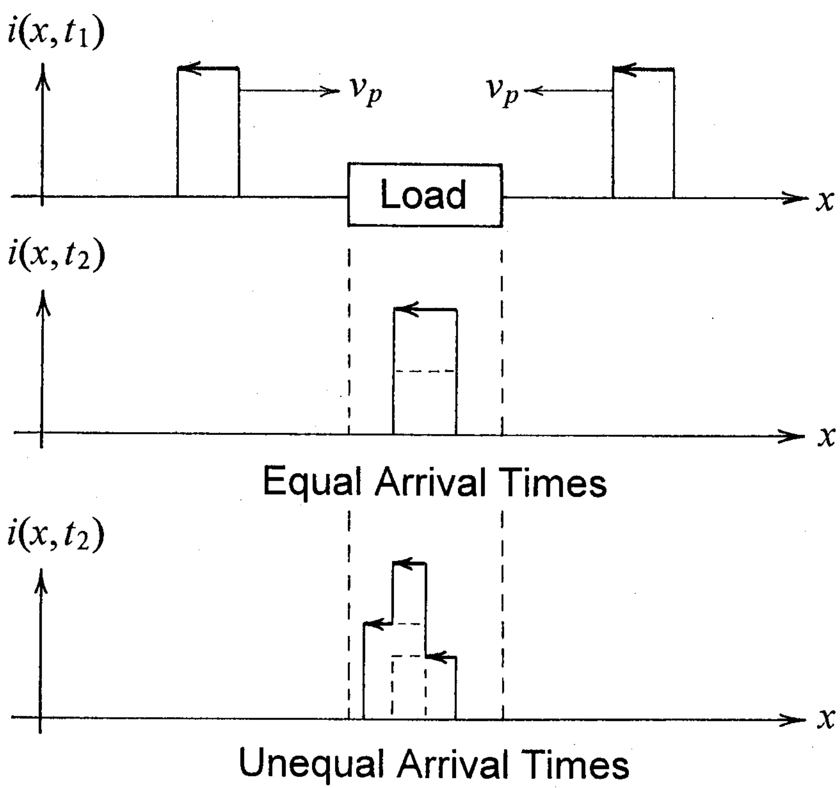
Both current components are reflected back to the source, if the load is infinite.



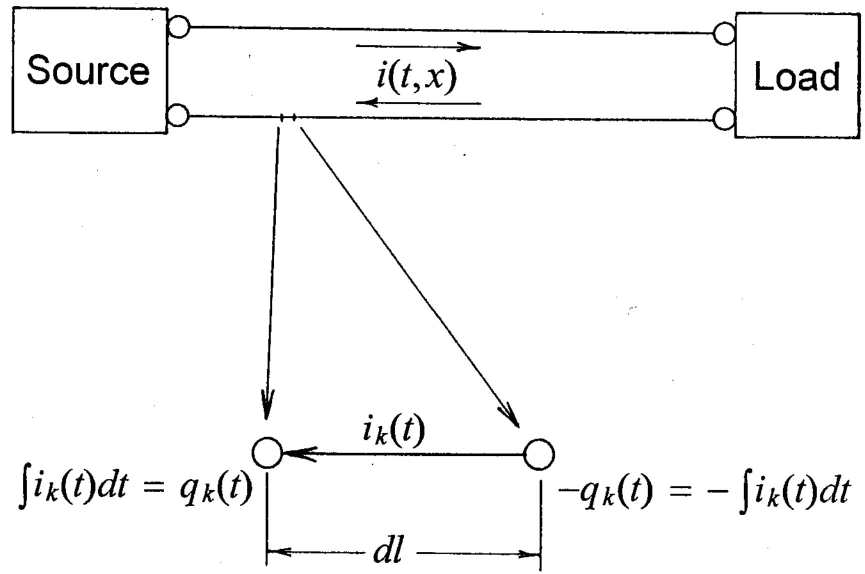
Currents do not continue to propagate, nor are they reflected, when a circuit's source and load are matched.



The propagation times of a current from source to load must be the same for both paths. If they are not equal, the load current will be corrupted. For example, if the load and source impedances are matching resistances,



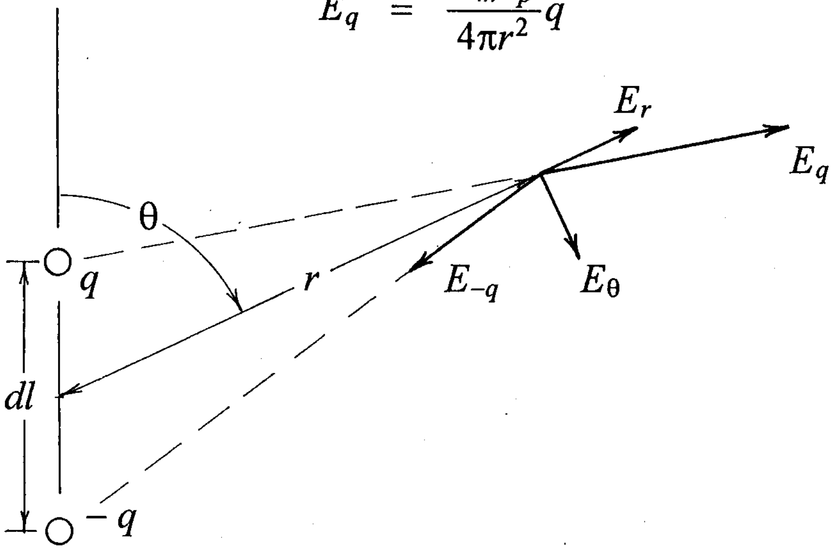
V. Circuit Current Models



Hertz's model of a circuit current is made up of a large number of differential dipoles, or current elements, in superposition. And, each of those elements has the structure zoomed in on above.

A single stationary charge, q, has the electric field

$$E_q = \frac{Z_m v_p}{4\pi r^2} q$$



So, two charges, q and $-q$, separated by $dl << r$, with the geometry shown, have the fields

$$E_\theta = \frac{\sin\theta dl}{r} E_q = Z_m v_p \frac{\sin\theta dl}{4\pi r^3} q$$

and

$$E_r = \frac{2\cos\theta dl}{r} E_q = Z_m v_p \frac{\cos\theta dl}{2\pi r^3} q$$

A single stationary point of time-varying electric charge, $q(t)$, has the electric field

$$E_r(t) = \frac{Z_m v_p}{4\pi r^2}\left[q(t-t_p) + \frac{r}{v_p}\frac{dq(t-t_p)}{dt} \right]$$

So, with the geometry assumed, the charges on the end points of Hertz's dipole, together, cause the fields

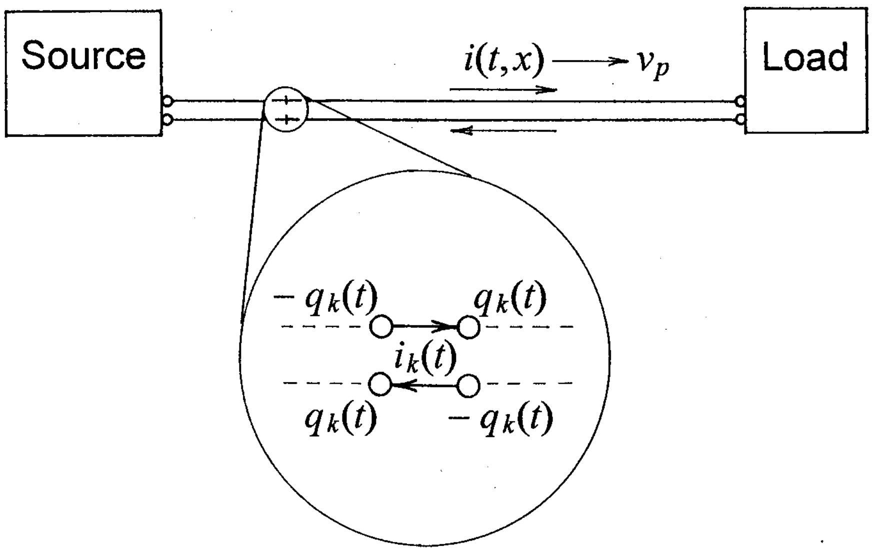
$$E_\theta(t) = Z_m v_p \frac{\sin\theta dl}{4\pi r^3}\left[q_k(t-t_p) + \frac{r}{v_p}\frac{dq_k(t-t_p)}{dt} \right]$$

and

$$E_r(t) = Z_m v_p \frac{\cos\theta dl}{2\pi r^3}\left[q_k(t-t_p) + \frac{r}{v_p}\frac{dq_k(t-t_p)}{dt} \right]$$

These, together with the fields of $i_k(t)dl$, are the electromagnetic fields of any differential dipole, or current element, of any electric current.

Now note, if the paths leaving a circuit current's source have the same structure and geometry, their point pairs equidistant from the source will have equal and opposite currents and charges.



So, to minimize the electromagnetic fields of any circuit current, the source-to-load paths must be equal in length. And, all pairs of points – one per path, at equal distances from the source – must be placed as close together as possible.

The fields of $i_k(t)dl$ with end charges $\pm q_k(t)$ are

$$E_r(t) = Z_m v_p \frac{\cos\theta dl}{2\pi r^3}\left[q_k(t-t_p) + \frac{r}{v_p}\frac{dq_k(t-t_p)}{dt} \right]$$

$$E_\theta(t) = Z_m v_p \frac{\sin\theta dl}{4\pi r^3}\left[q_k(t-t_p) + \frac{r}{v_p}\frac{dq_k(t-t_p)}{dt} \right] + Z_m v_p \frac{\sin\theta dl}{4\pi r^2}\left[\frac{r}{v_p}\frac{di_k(t-t_p)}{dt} \right]$$

and

$$H_\phi(t) = \frac{\sin\theta dl}{4\pi r^2}\left[i_k(t-t_p) + \frac{r}{v_p}\frac{di_k(t-t_p)}{dt} \right]$$

Typical textbook versions of these fields are

$$E_r = \frac{\cos\theta dl}{2\pi\varepsilon_m}\left(\frac{1}{j\omega r^3} + \frac{1}{v_p r^2} \right) I_0 e^{j(\omega t-\beta r)}$$

$$E_\theta = \frac{\sin\theta dl}{4\pi\varepsilon_m}\left(\frac{1}{j\omega r^3} + \frac{1}{v_p r^2} + \frac{j\omega}{v_p^2 r} \right) I_0 e^{j(\omega t-\beta r)}$$

and

$$H_\phi = \frac{\sin\theta dl}{4\pi}\left(\frac{1}{r^2} + \frac{j\omega}{v_p r} \right) I_0 e^{j(\omega t-\beta r)}$$

To compare the equation sets, note that in texts on electromagnetics it is generally assumed that the current is sinusoidal. So,

$$i_k(t - t_p) = I_0 e^{j(\omega t - \beta r)}$$

$$\frac{di_k(t - t_p)}{dt} = j\omega I_0 e^{j(\omega t - \beta r)}$$

$$q_k(t - t_p) = \int i_k(t - t_p)dt = \frac{1}{j\omega} I_0 e^{j(\omega t - \beta r)}$$

and

$$\frac{dq_k(t - t_p)}{dt} = i_k(t - t_p) = I_0 e^{j(\omega t - \beta r)}$$

Also,

$$\frac{1}{\varepsilon_m} = Z_m v_p \quad \text{and} \quad \beta = \frac{\omega}{v_p}$$

VI. Summary and Conclusions

A. Electric fields are caused by electric charges that are not moving.

B. Magnetic fields are caused by unaccelerated movement of electric charges.

C. Electric and Magnetic fields are caused by accelerated movement of electric charges.

D. "Current flow" is the bidirectional propagation of bidirectional charge flows.

E. The current paths of a circuit from source to load should be equal in length, and all point pairs equally distant from the source should be parallel with minimal separation. That will minimize the electric and magnetic fields, as well as distortion of the current waveshape.