An LMI Approach to the Control of a Compact Disc Player

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Overview of my Ph.D. research project

Joint project between:

- Systems and Control Group, Delft University of Technology, The Netherlands.
- Philips Research Laboratories.

- Theoretical side:

  Development of LMI algorithms for analysis and synthesis of control systems.

- Practical side:

  Application to a Compact Disc Player system:

  - Multi-objective design.
  - Gain-scheduling design.

Outline

Theoretical part

- LMIs in control theory.
- Gain-scheduling for LPV systems.

Application

- Gain-scheduling design for CD player.
- Experimental set-up.
- Implementation results.

Conclusions and discussion.

What are LMIs?

A generic Linear Matrix Inequality is:

$$F(x) < 0$$

$x \in \boldsymbol{R}^m$, $F(x)$ is a real symmetric matrix

$F(.)$ is an affine mapping

'$<$' means negative definite.

Form with matrix variables:

$$X := \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$$

$$A'XB + B'X'A + C < 0$$

Solving an LMI is a convex optimization problem:

$$\min t$$

$$\lambda_{max}(A'XB + B'X'A + C) < t$$

Lyapunov stability criterion

$\dot{x} = Ax$ is asymptotically stable iff

$$X > 0, \quad A'X + XA < 0$$

Bounded Real Lemma

The system

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

has $\mathcal{H}_\infty$ norm smaller than 1 iff

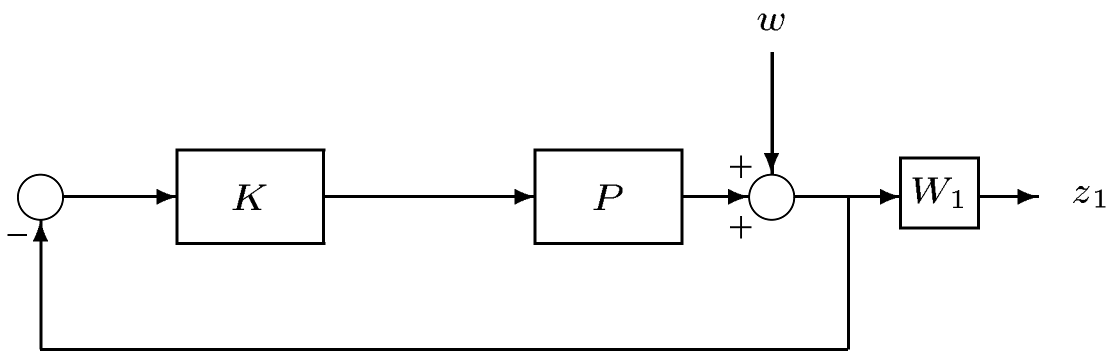
$$X > 0, \quad A'X + XA + XBB'X + C'C < 0$$

iff

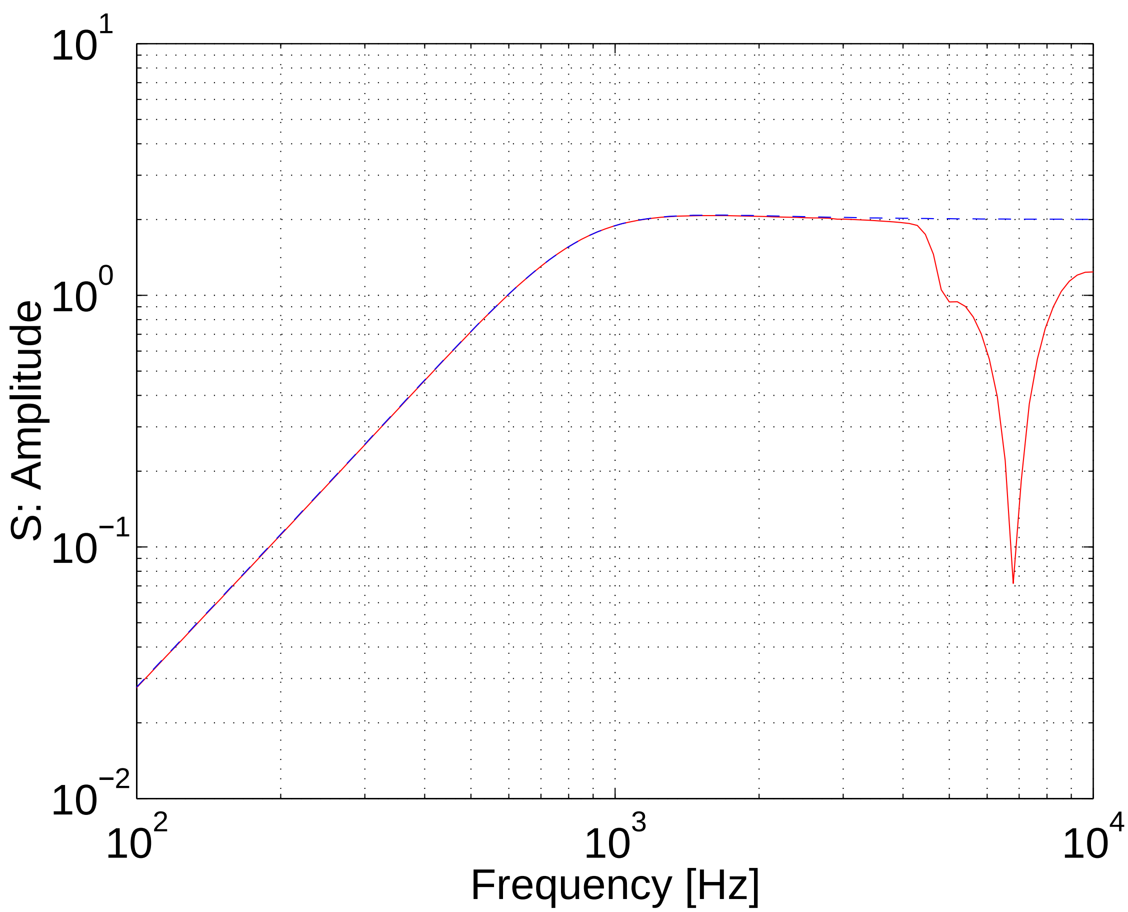
$$X > 0, \quad \begin{bmatrix} A'X + XA + C'C & XB \\ B'X & -I \end{bmatrix} < 0$$

H_∞ loopshaping: S scheme

Disturbance attenuation

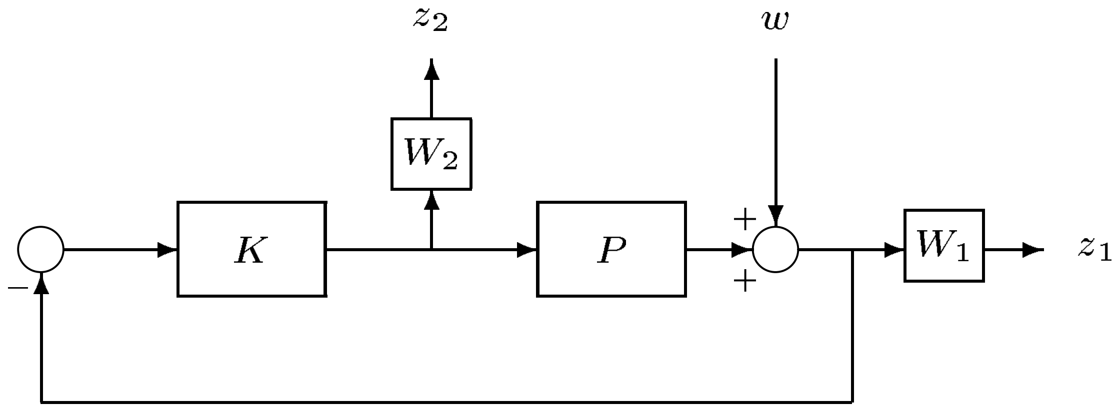


$$\|W_1 S\|_\infty \leq \gamma \iff |S(j\omega)| \leq \frac{\gamma}{|W_1(j\omega)|}, \text{ for all } \omega \in \mathbb{R}.$$



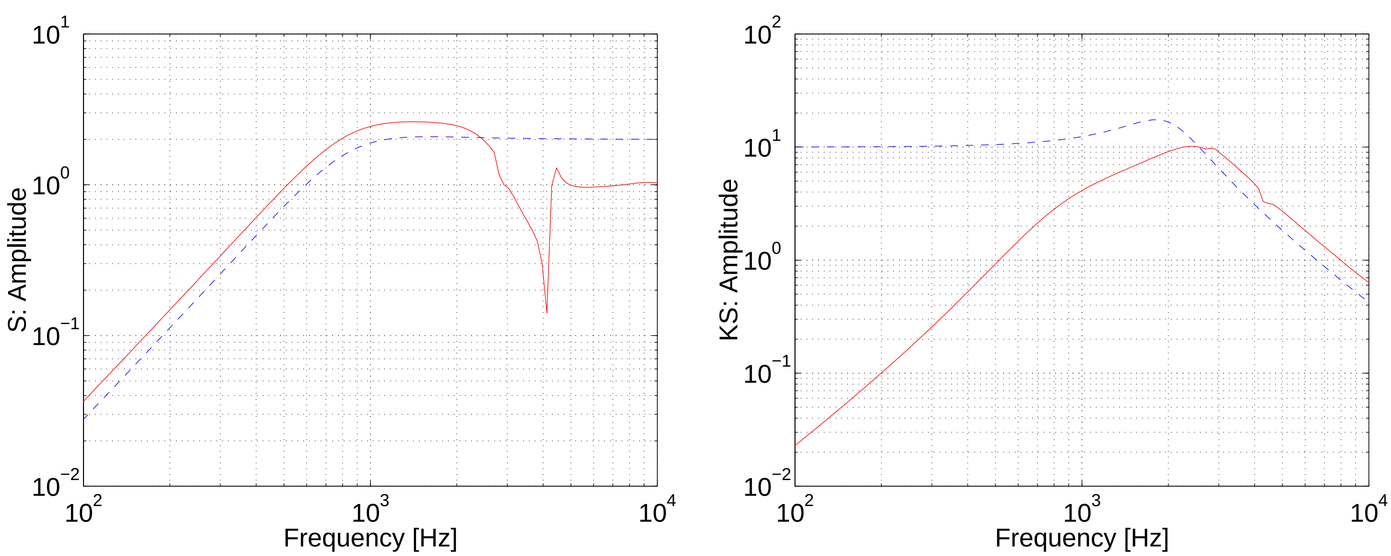
H_∞ loopshaping: S/KS scheme

Disturbance attenuation with bounded control



$$\left\| \begin{pmatrix} W_1 S \\ W_2 K S \end{pmatrix} \right\|_\infty \leq \gamma \iff$$

$$\sqrt{|W_1(j\omega)S(j\omega)|^2 + |W_2(j\omega)K(j\omega)S(j\omega)|^2} \leq \gamma, \; \forall\, \omega \in \mathbb{R}.$$



Benefits of LMI formulation

- Recent development of powerful interior point methods for convex optimization (1994).

  Available software packages:
  LMI Toolbox for Matlab: Gahinet, Nemirovskij, Laub, Chilali
  SP: Boyd, Vandenberghe
  LMITOOL: El Ghaoui, Delebecque, Nikoukhah
  SDPpack: Overton, Alizadeh et al.

- Through LMIs it is possible to numerically solve problems otherwise unsolvable.

  Multi-objective Control
  Linear Parametrically Varying Control

Motivation

Gain scheduling: design controller for nonlinear systems using linear design tools.

- Classical approach:
  - linearize the system at various operating points
  - design linear controller at each point
  - interpolate to get a 'global' controller

Drawback: no systematic way to perform interpolation.

- LPV approach:
  - systematic design method
  - no need for interpolation step
  - based on LMI techniques

From nonlinear to LPV systems

Given the nonlinear system

$$\dot{x} = a(x, q_1) + b_1(x, q_1)w + b_2(x, q_1)u$$

$$z = c_1(x, q_1) + d_1(x, q_1)w + d_2(x, q_1)u$$

$$y = c(x, q_1) + d(x, q_1)w$$

Suppose $x = 0$ is an equilibrium for all q_1.

Rewrite

$$\dot{x} = A(x, q_1)x + B_1(x, q_1)w + B_2(x, q_1)u$$

$$z = C_1(x, q_1)x + D_1(x, q_1)w + D_2(x, q_1)u$$

$$y = C(x, q_1)x + D(x, q_1)w$$

Arrive at LPV system:

- Replace x by q_2
- Define $p = \begin{pmatrix} q_1 \\ q_2 \end{pmatrix} \in \Pi$

From nonlinear to LPV systems

LPV system

$$\begin{aligned} llpv\ \dot{x} &= A(p(t))x + B_1(p(t))w + B_2(p(t))u \\ z &= C_1(p(t))x + D_1(p(t))w + D_2(p(t))u \\ y &= C(p(t))x + D(p(t))w. \end{aligned}$$

where $p(t) \in \Pi$ for all t.

Example:

$$\dot{x} = x \,\mathsf{sin}(x)$$

can be transformed into

$$\dot{x} = p\, x, \quad p \in [-1, 1]$$

Analysis result for LPV systems

The LPV system

$$\dot{x}(t) = A(p(t))x(t) + B(p(t))w_1(t)$$
$$z_1(t) = C(p(t))x(t) + D(p(t))w_1(t)$$

is exponentially stable and has L_2 gain $w_1 \to z_1$ smaller than γ if

$\exists$ Lyapunov matrix $X > 0$ s.t. $\forall p \in \Pi$

$$\begin{bmatrix} A(p)'X + XA(p) & XB_1(p) & C_1(p)' \\ B_1(p)'X & -\gamma I & D_1(p)' \\ C_1(p) & D_1(p) & -\gamma I \end{bmatrix} < 0$$

Infinitely many LMI's in X. Two ways to proceed:

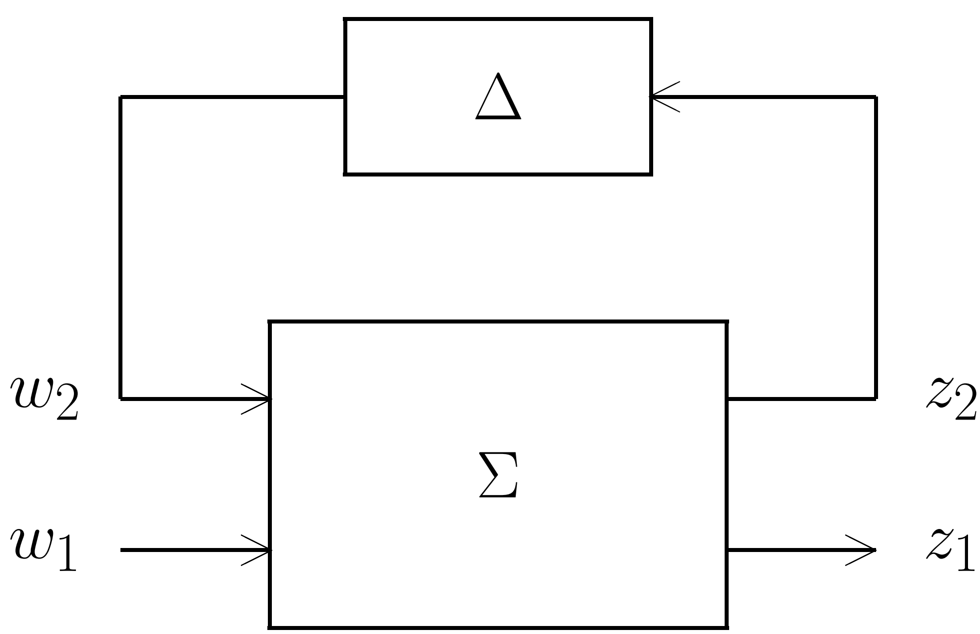
- Gridding techniques
- Introduction of scalings

LFT representation of LPV systems

"Pulling out" the parameter p:

$$\begin{bmatrix} \dot{x} \\ z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_1 & D_{12} \\ C_2 & D_{21} & D_2 \end{bmatrix} \begin{bmatrix} x \\ w_1 \\ w_2 \end{bmatrix}, \; w_2 = \Delta(p) z_2$$

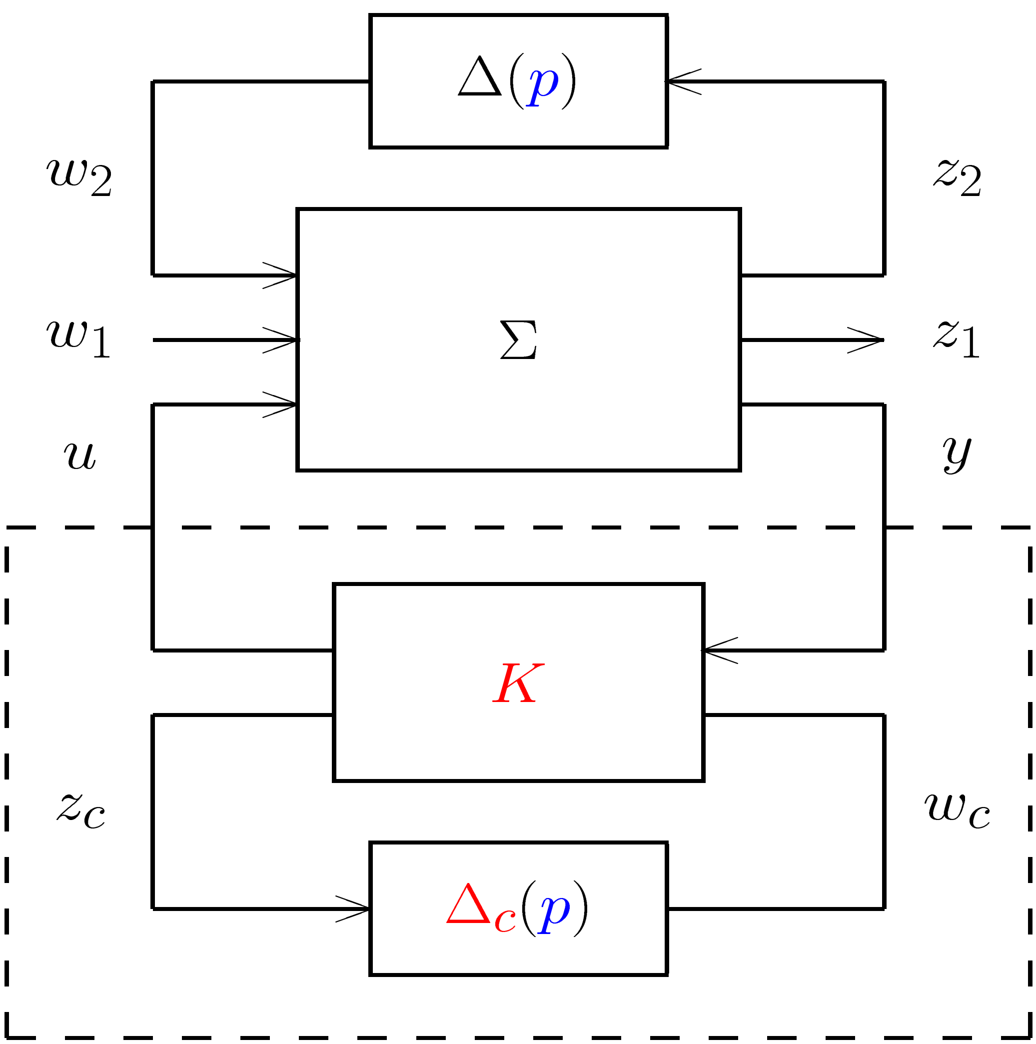
where $\Delta(.)$ is continuous



Controller synthesis

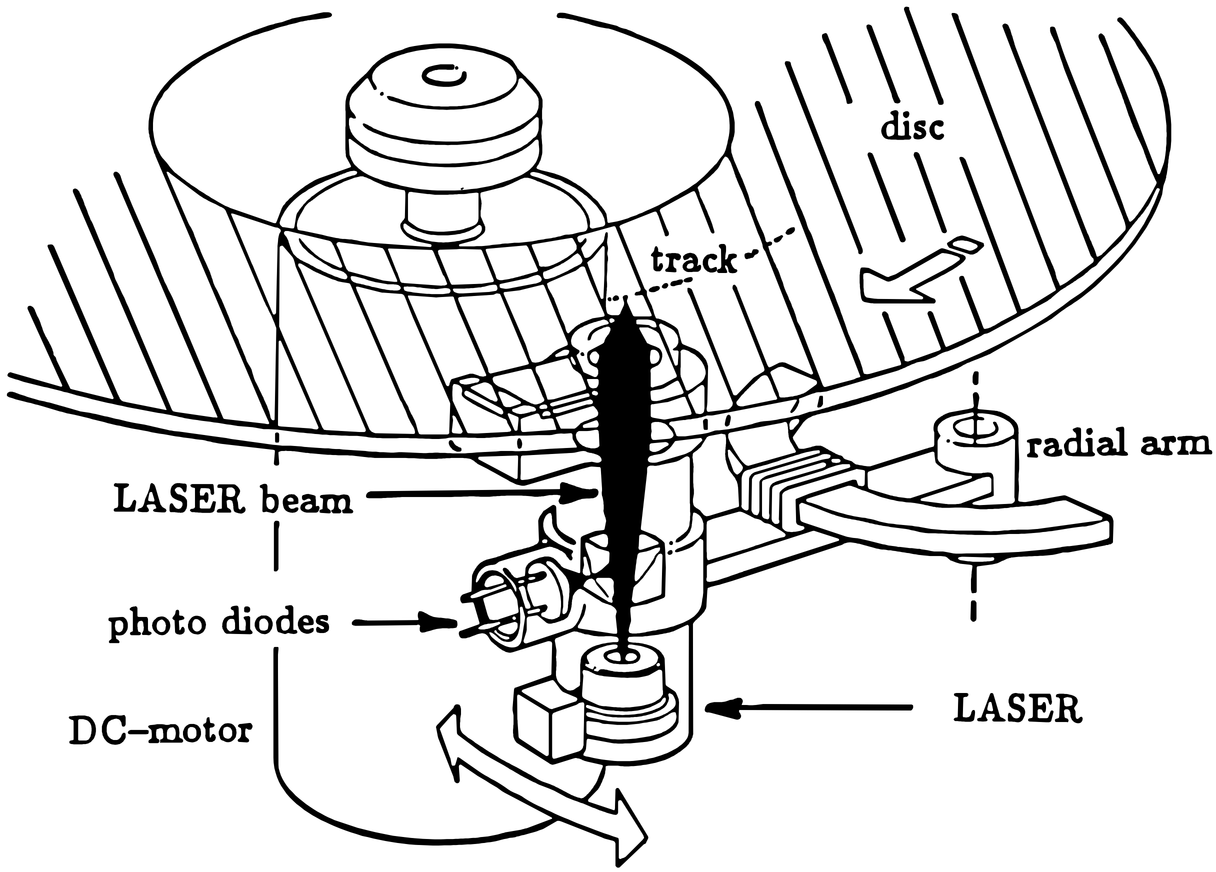
Controller with same structure of the plant

$$\begin{bmatrix} \dot{x} \\ u \\ z_c \end{bmatrix} = \begin{bmatrix} A_c & B_{c1} & B_{c2} \\ C_{c1} & D_{c1} & D_{c12} \\ C_{c2} & D_{c21} & D_{c2} \end{bmatrix} \begin{bmatrix} x \\ y \\ w_c \end{bmatrix}, \; w_c = \Delta_c(p) z_c$$

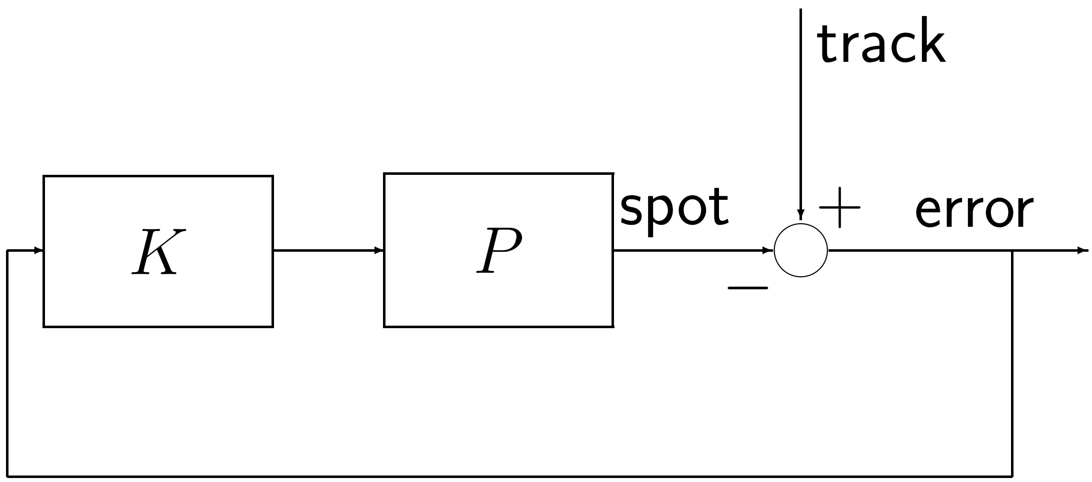


Synthesis algorithms based om LMI techniques

The CD player system



Specs for the CD

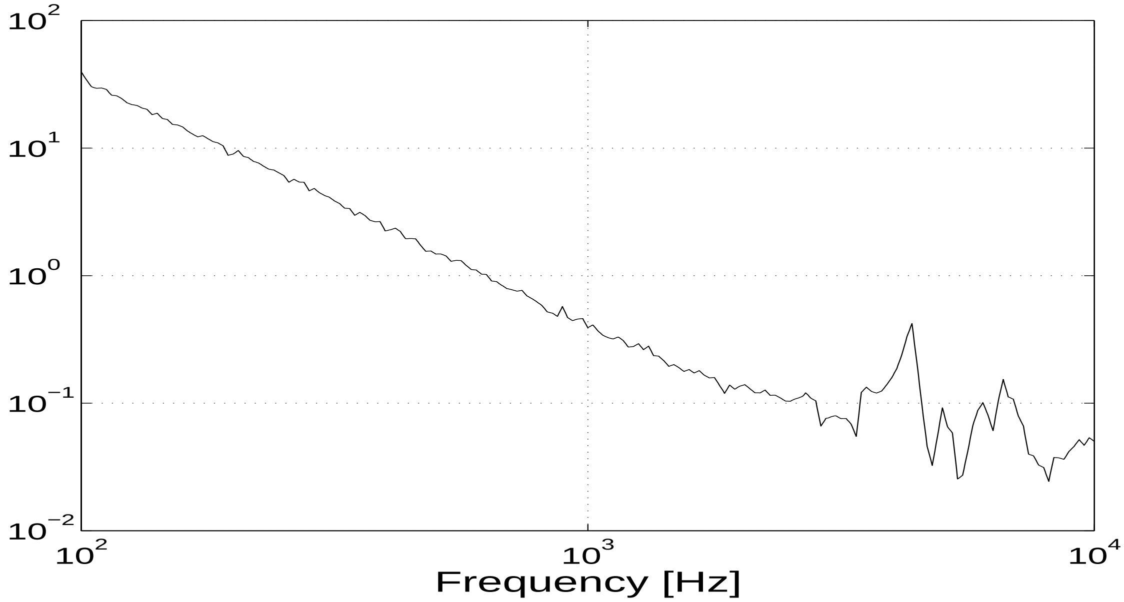


- Disturbance suppression
  max. track eccentricity: 100 μm
  max. allowable position error: 0.1 μm
  $\Rightarrow$ Factor 1000 time-domain attenuation

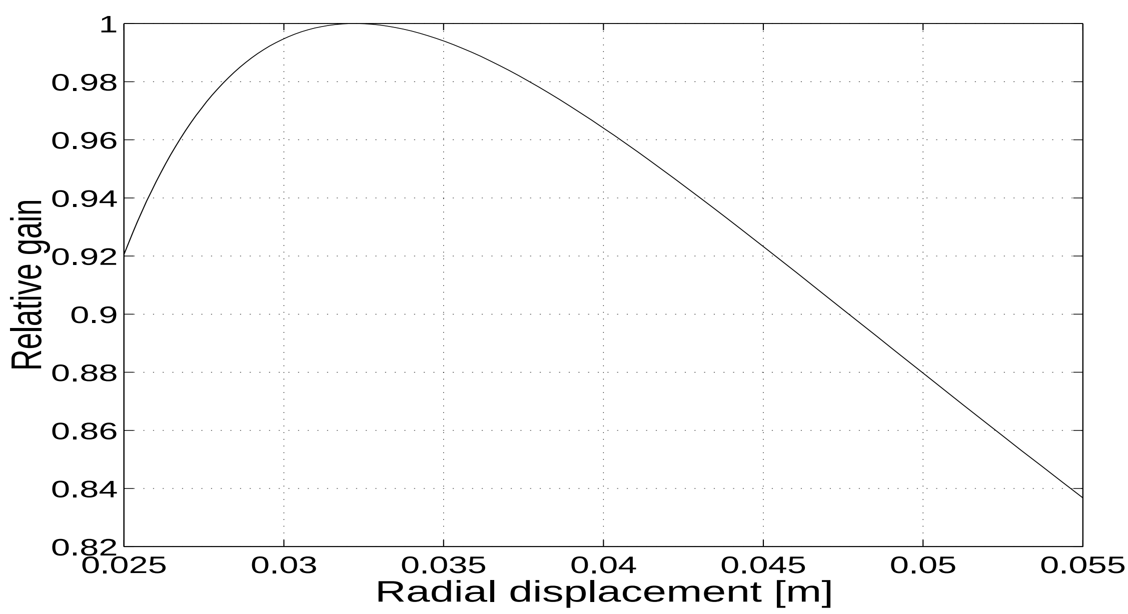
- Relatively small bandwidth
  avoid high power consumption
  no amplification of audible noise
  robustness

Plant characteristics

Amplitude of P at a fixed track position

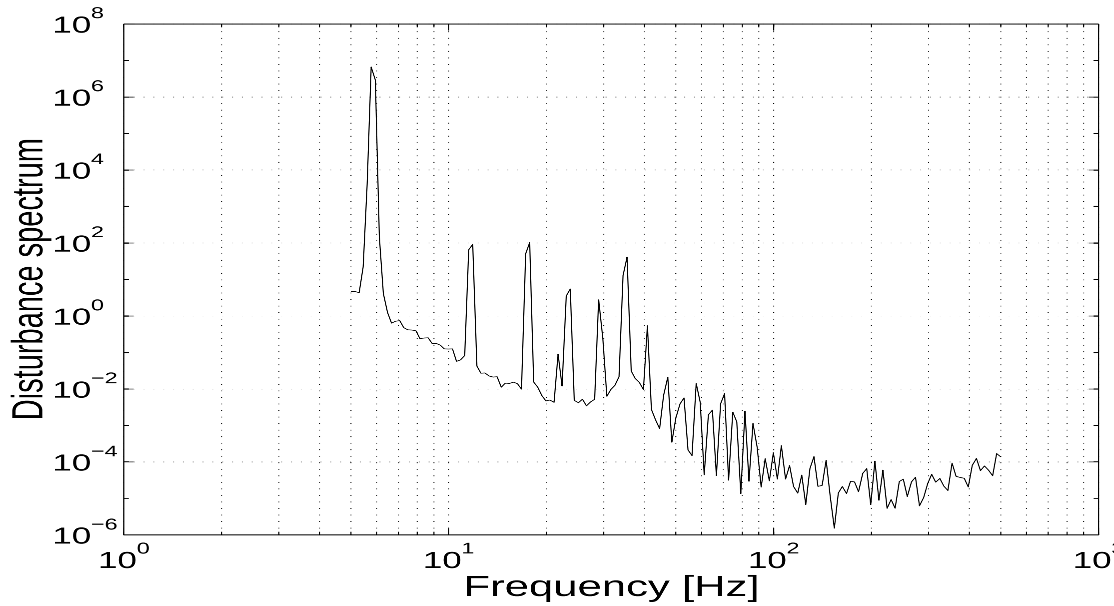


The gain of P varies with track position:



Disturbance characteristics

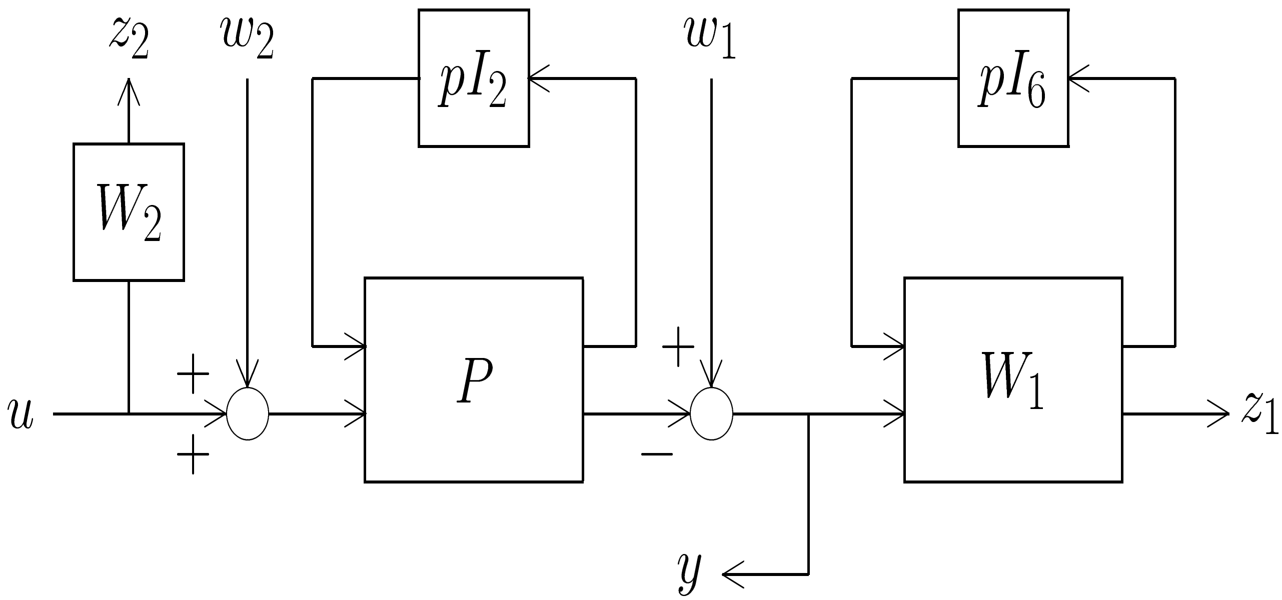
Spectrum of the disturbance at a fixed track



Location of harmonics vary with rotational frequency.

New high-performance applications require higher rotational frequency (30 Hz)
$\Rightarrow$ necessity of adaptive selective disturbance suppression.

LPV model of the CD



Scheduling parameter is $p = 2\pi f_{\text{rot}}$, with 25 Hz $\leq f_{\text{rot}} \leq$ 35 Hz.

P is scheduled for gain variations.

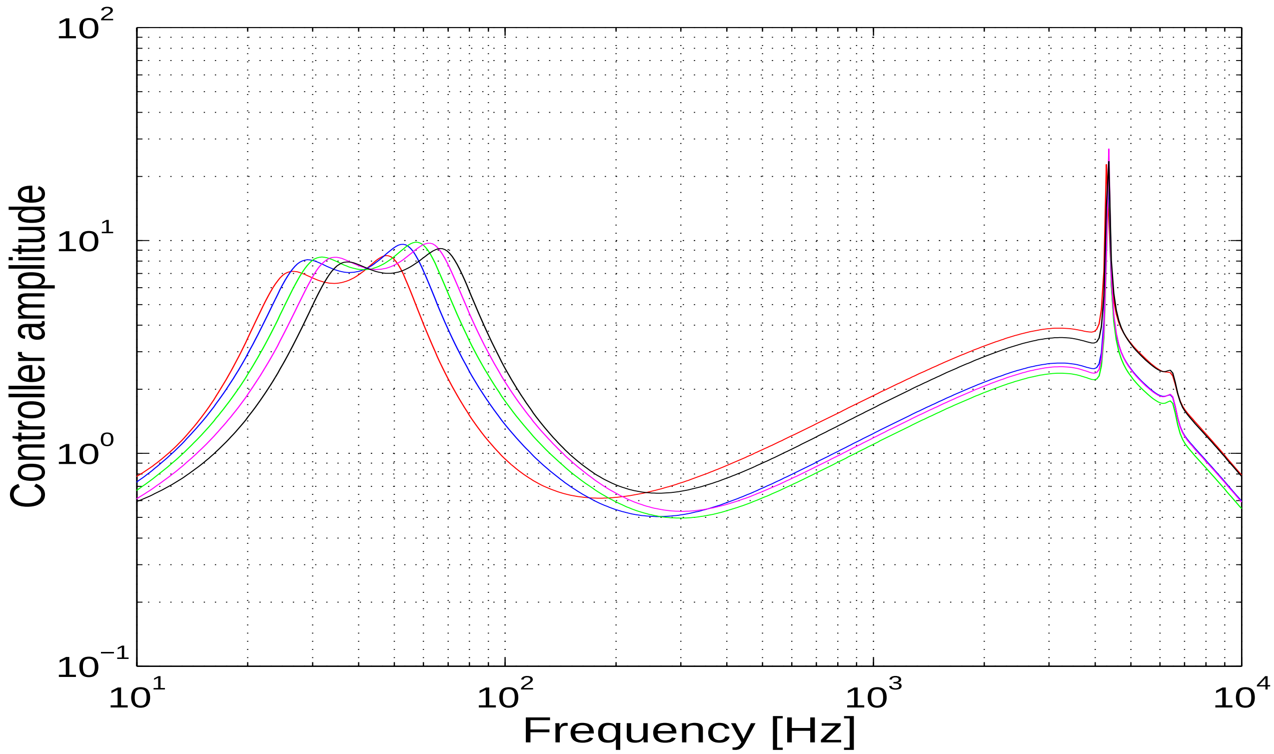
Performance filter W_1 models frequency-varying notches

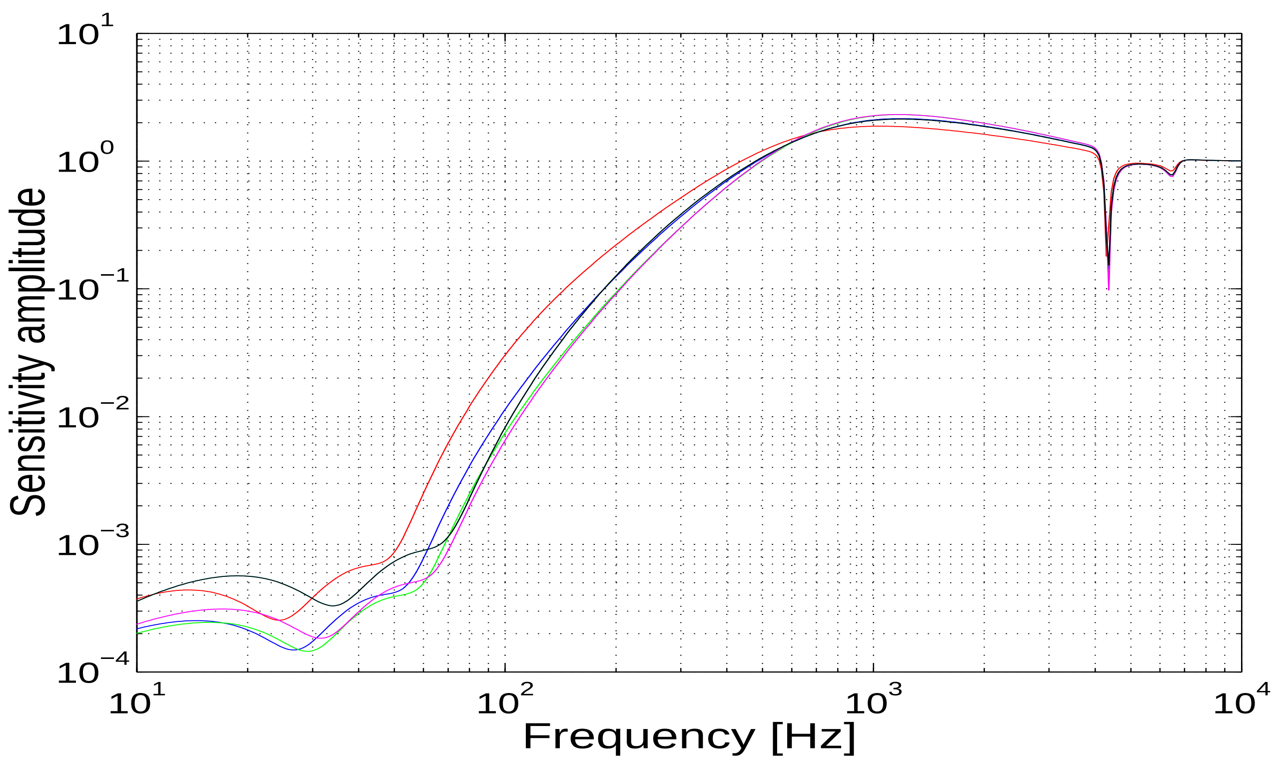
$$W_1(s, p) = \frac{s^2 + 2\zeta_z ps + p^2}{s^2 + 2\zeta_p ps + p^2} \, \frac{s^2 + 4\zeta_z ps + 4p^2}{s^2 + 4\zeta_p ps + 4p^2}$$

Filter W_2 for robustness issues.

Results: "Frequency domain"

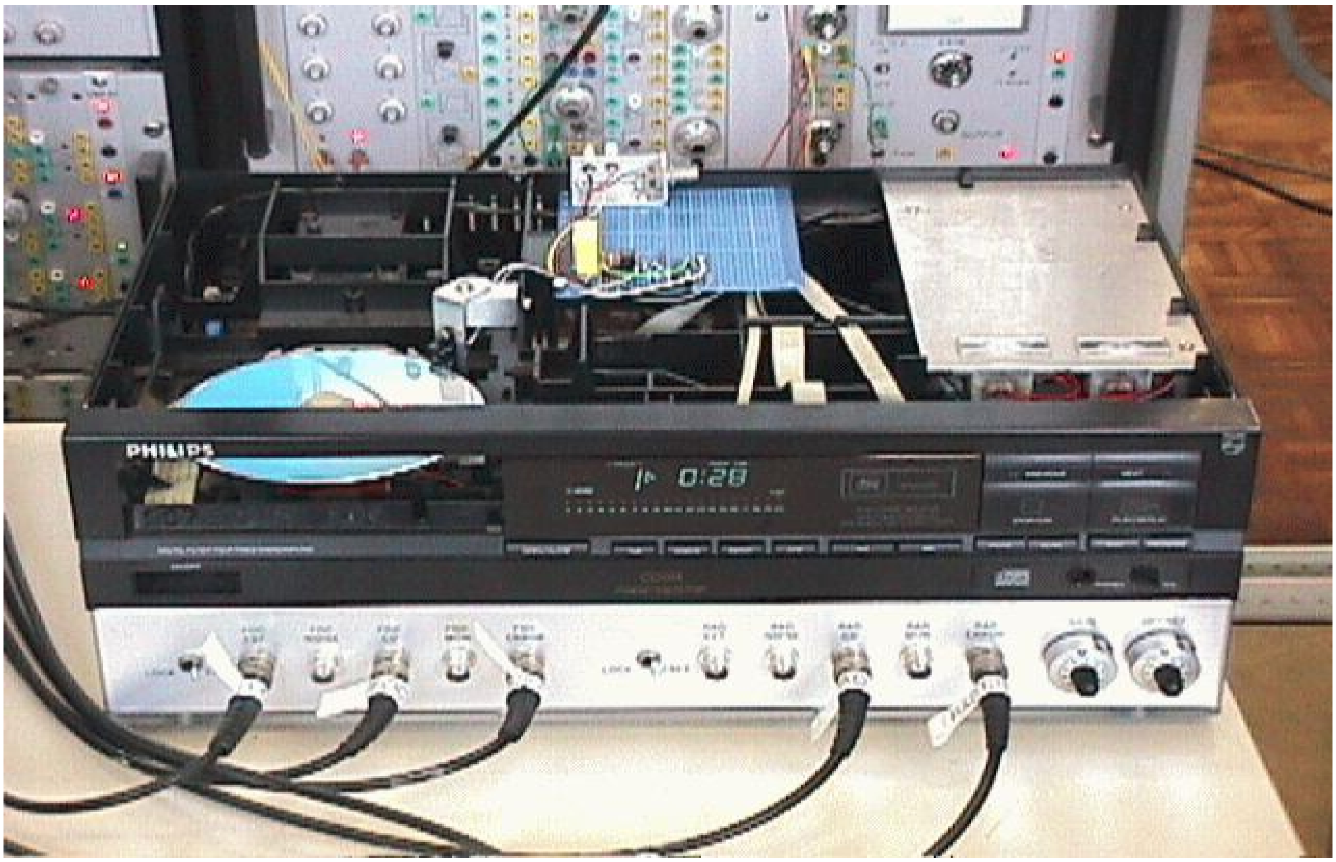
Amplitude of controller and sensitivity for the five "frozen" values f_{rot}=25, 27.5, 30, 32.5, 35 Hz:





Experimental set-up

"Old" Philips audio CD Player



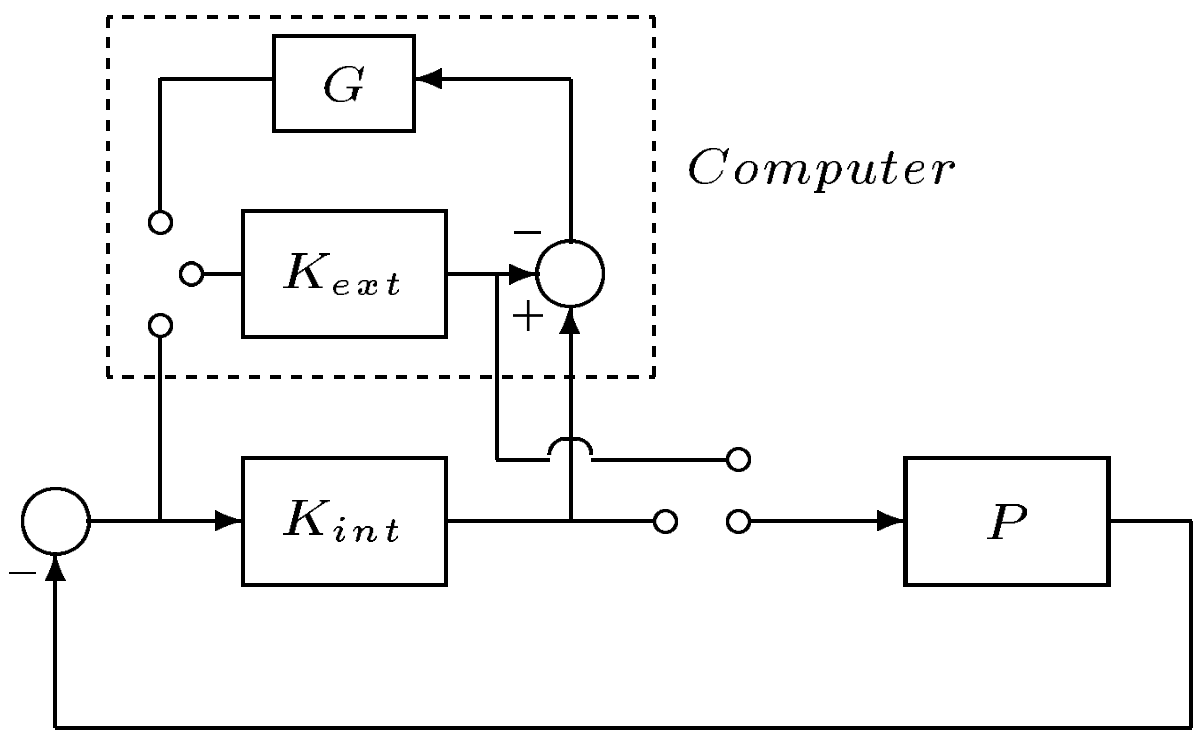
Set-up with two dSpace systems:

- C40 to measure rotational frequency.
- Multiprocessor (C40 and Alpha) to implement the controller.

Implementation scheme

Problems:

- Implementation of unstable controllers
- Bumpless switch from internal to external controller



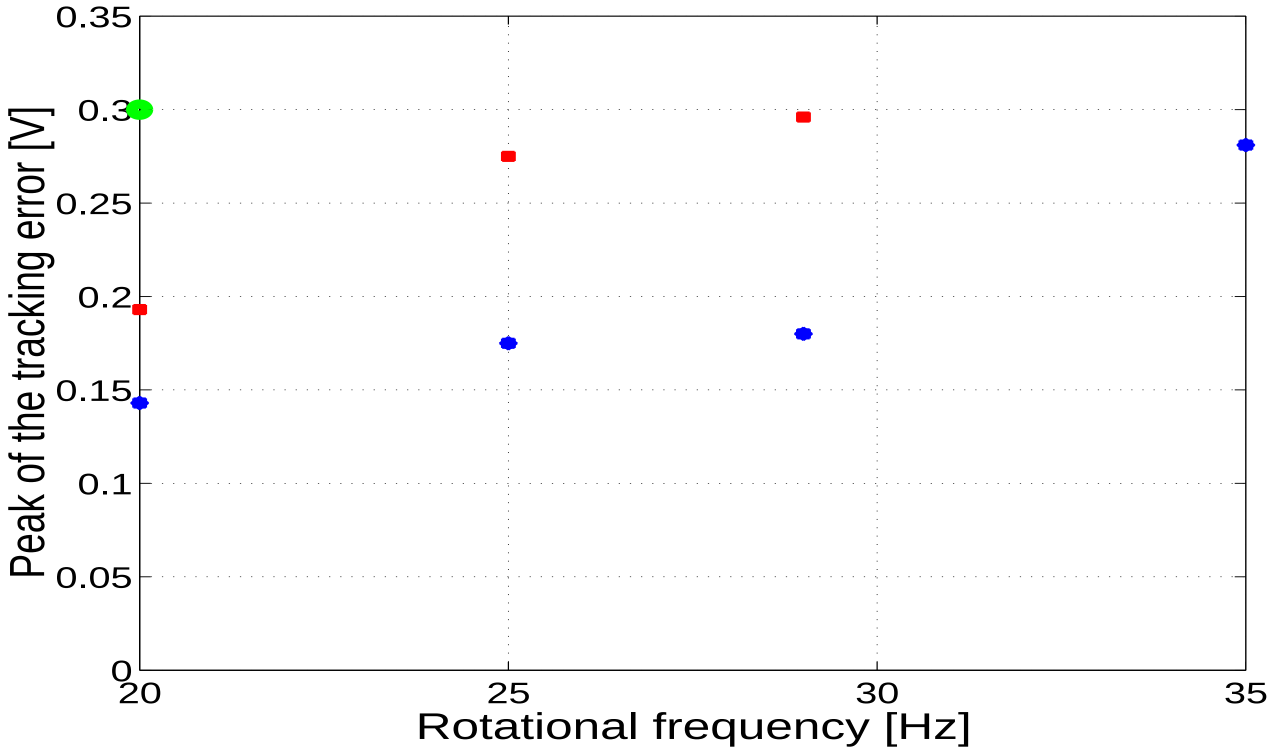
G stabilizes K_{ext} during the start-up procedure

- High order controllers
- High sampling rate (20 kHz)

Experimental results

Comparison with a high performing LTI controller.

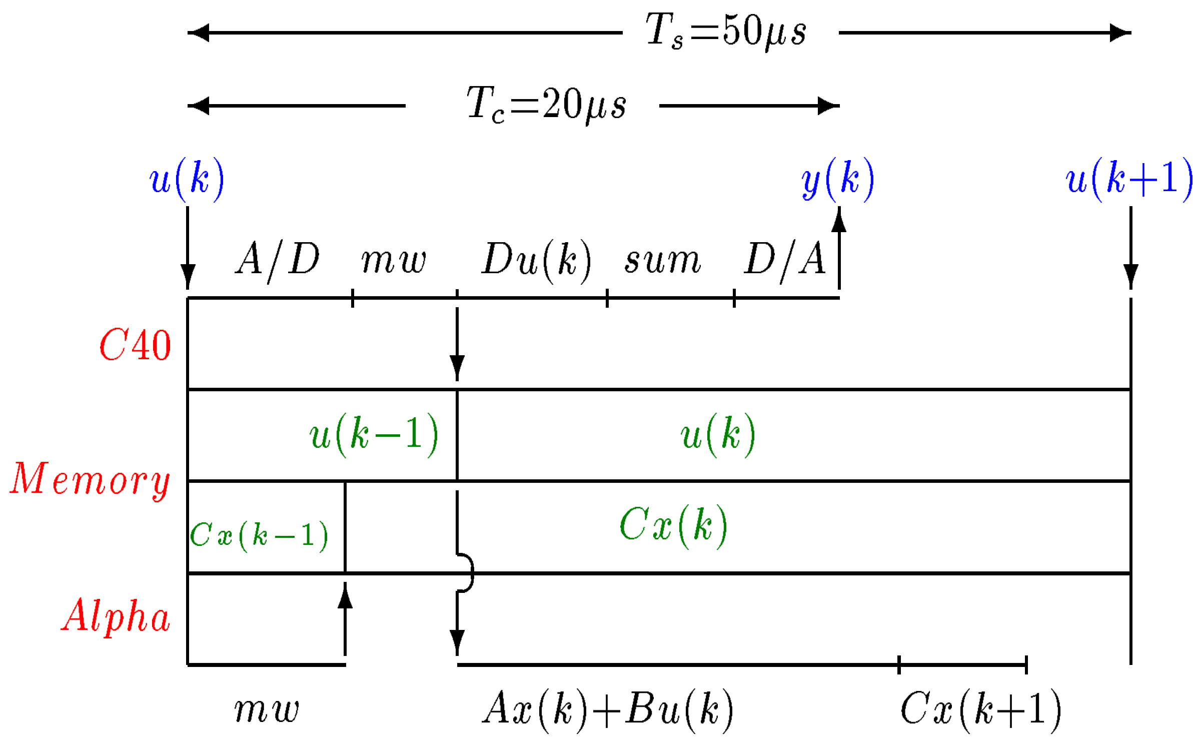
Peak of tracking error for several fixed values of the rotational frequency



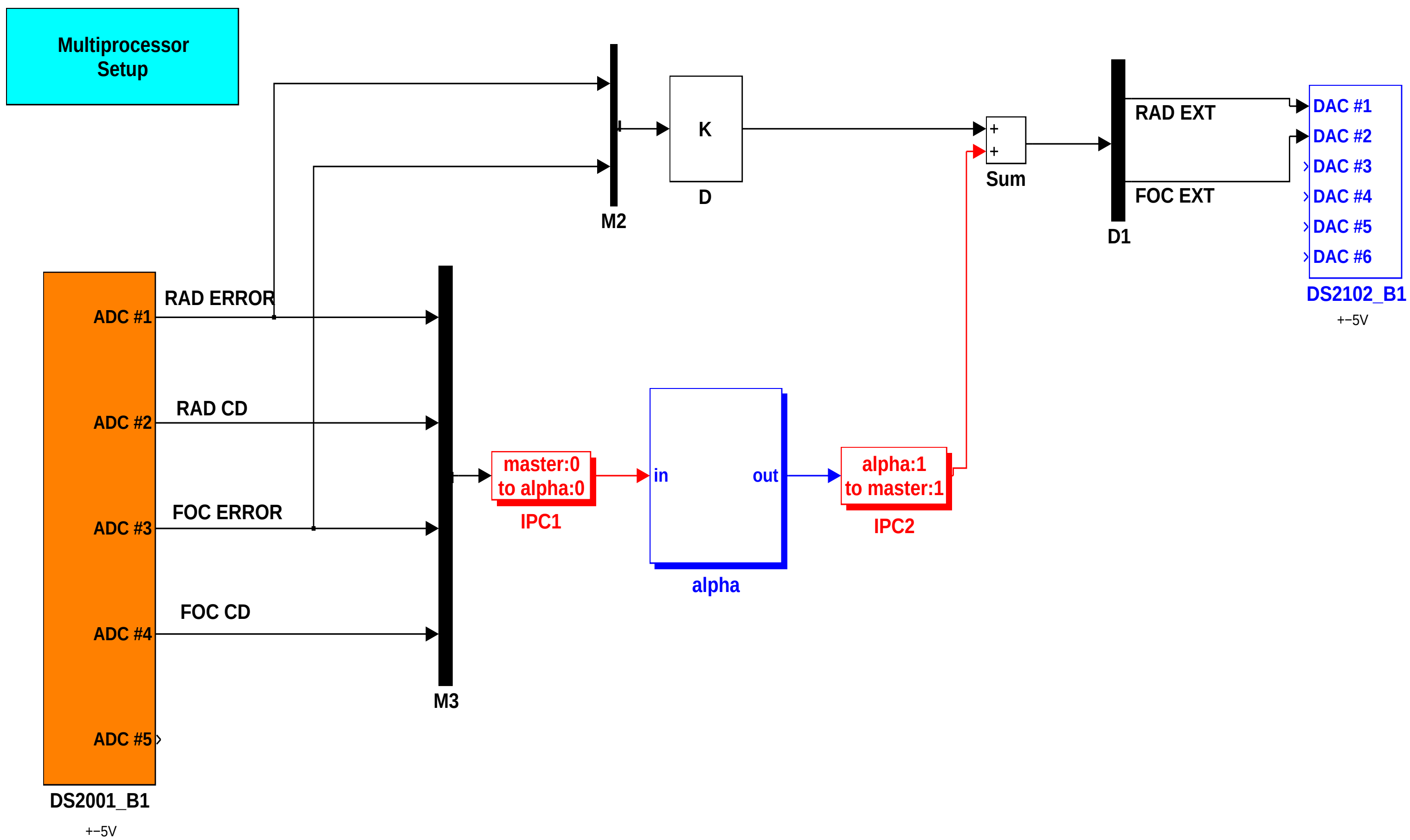
LPV controller

LTI controller

Internal controller

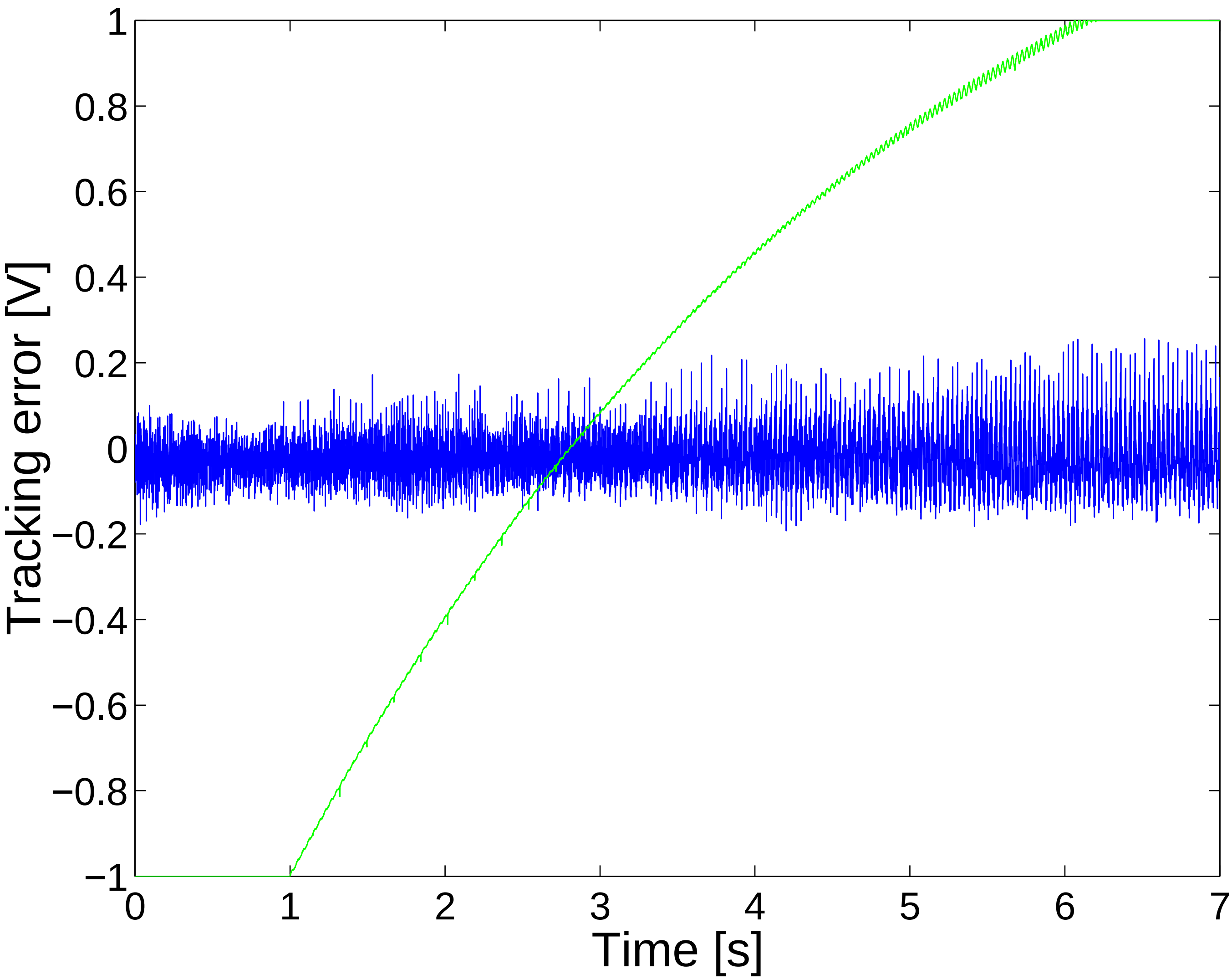


Ts=50μs
Tc=20μs
u(k)
y(k)
u(k+1)
A/D
mw
Du(k)
sum
D/A
C40
u(k−1)
u(k)
Memory
Cx(k−1)
Cx(k)
Alpha
mw
Ax(k)+Bu(k)
Cx(k+1)



Multiprocessor Setup
M2
K
D
Sum
D1
RAD EXT
FOC EXT
DAC #1
DAC #2
DAC #3
DAC #4
DAC #5
DAC #6
DS2102_B1
+-5V
RAD ERROR
ADC #1
RAD CD
ADC #2
FOC ERROR
ADC #3
FOC CD
ADC #4
ADC #5
DS2001_B1
+-5V
M3
master:0
to alpha:0
IPC1
in
out
alpha
alpha:1
to master:1
IPC2

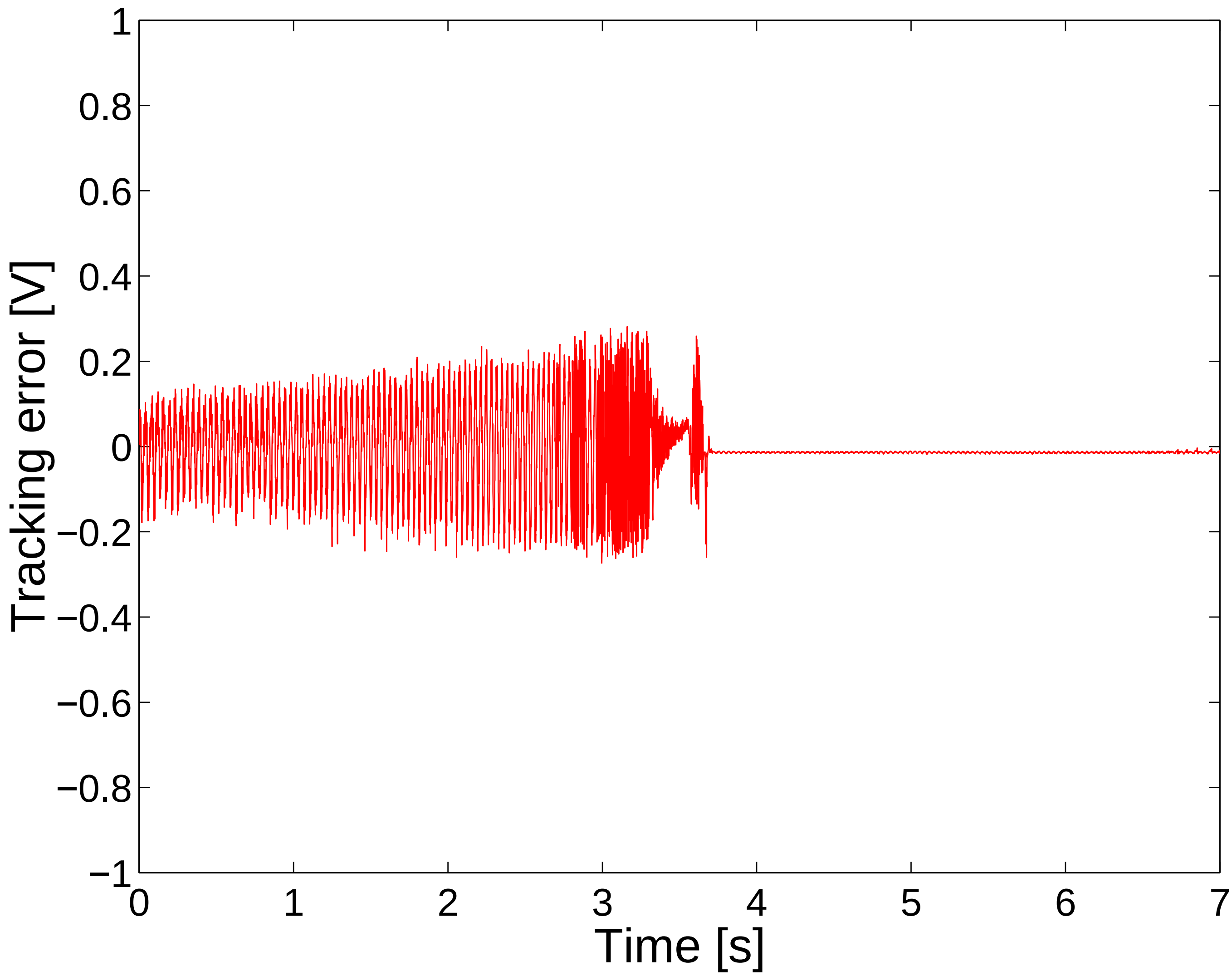
Experimental results: fast parameter transition



Parameter transition

LPV tracking error:

Experimental results: fast parameter transition



LTI tracking error

Conclusions

- LPV techniques allow to design controller with stability and performance guarantees over the whole operative range of the plant.

- Experimental set-up based on dSpace systems to implement LPV controllers.

- LPV controller improves performance of the CD player servosystem as required by high-demanding applications.