Seminar

Modeling and Simulation of Dynamical Systems

Presented by the
IEEE Control Systems Society
Santa Clara Valley

Sunnyvale, 5 February 2011

Session 5

Simulation with Software Tools

Karl Mathia

ZiTech Engineering

Session 5: Software Tools

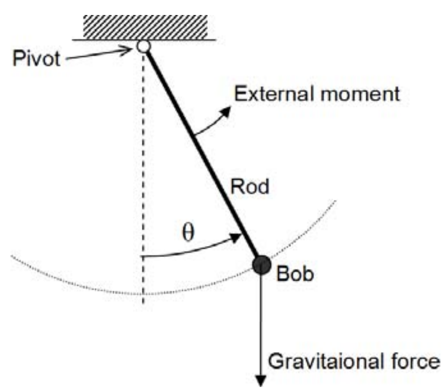
Topics:

- Example: Simple Pendulum
- Dynamical model:
  - Differential equation
  - Discrete time
  - Dynamics: numerical integration
- Software:
  - Commercial
  - Open source

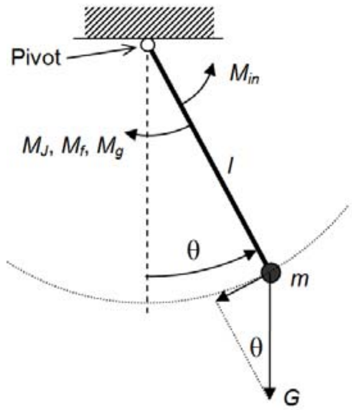
Example: Simple Pendulum

Dynamical model: differential equation

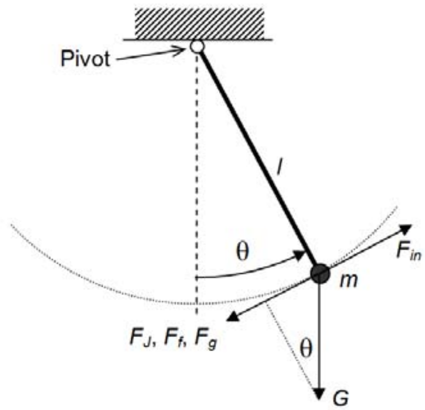
- Several methods available:
  - Moment equation
  - Force equation
  - Lagrange's method
  - Euler's method



Example: Simple Pendulum



Moment equation



Force equation

Example: Simple Pendulum

Moment equation:

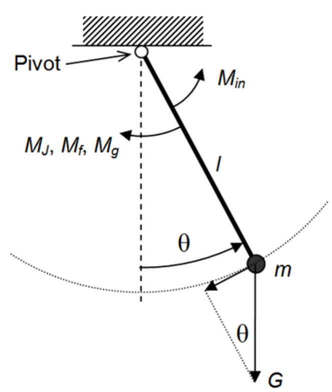
$$M_J + M_f + M_g - M_{in} = 0$$

With:

$$M_J = J \cdot \ddot{\theta} = m \cdot l^2 \cdot \ddot{\theta}$$

$$M_f = k \cdot l \cdot \dot{\theta} \cdot l = k \cdot l^2 \cdot \dot{\theta}$$

$$M_g = m \cdot g \cdot \sin(\theta) \cdot l$$



Example: Simple Pendulum

Differential equation:

=0 (unforced)

$$m \cdot l^2 \cdot \ddot{\theta}(t) + k \cdot l^2 \cdot \dot{\theta}(t) + m \cdot g \cdot \sin(\theta(t)) \cdot l - M_{in} = 0$$

As state equation:

$$\ddot{\theta}(t) = \frac{1}{m \cdot l^2}(-k \cdot l^2 \cdot \dot{\theta}(t) - m \cdot g \cdot \sin(\theta(t)) \cdot l + M_{in}(t))$$

$$= -\frac{k}{m} \cdot \dot{\theta}(t) - \omega_0^2 \cdot \sin(\theta(t)) + \frac{1}{m \cdot l^2} M_{in}(t), \quad \omega_0 = \sqrt{\frac{g}{l}}$$

Example: Simple Pendulum

State equation: vector-valued, nonlinear function

$$x_1 = \theta, \; x_2 = \dot{x}_1, u = M_{in}$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ -\omega_0^2 \cdot \sin(x_1(t)) - \frac{k}{m} \cdot x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m \cdot l^2} \end{bmatrix} \cdot u(t)$$

Example: Simple Pendulum

Simulation of dynamics: num. integration

$$\dot{x} = f(t, x)$$

Simple approximation:

$$\dot{x} = \frac{dx}{dt} \approx \frac{x(n+1) - x(n)}{\Delta t}$$

$$\boxed{x(n+1) = x(n) + \Delta t \cdot f(x(n))}$$

Example: Simple Pendulum

More sophisticated: Runge-Kutta 4th order

$$\dot{x} = f(t, x)$$

$$x(n+1) = x(n) + \frac{1}{6}\Delta t \cdot \left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$k_1 = f(t(n), x(n))$$

$$k_2 = f(t(n) + \frac{1}{2}\Delta t,\ x(n) + \frac{1}{2}\Delta t k_1)$$

$$k_3 = f(t(n) + \frac{1}{2}\Delta t,\ x(n) + \frac{1}{2}\Delta t k_2)$$

$$k_4 = f(t(n) + \Delta t,\ x(n) + \Delta t k_3)$$

Session 5: Software Tools

Good news:

- We don’t need to worry about numerical integration.
- There are many software packages available to perform such computations.
- Commercial SW and Open Source SW

Session 5: Software Tools

Commercial Software:

- LabView (presented earlier)
- MathCad (presented earlier)
- Matlab (general)
- Simulink (graphical programming)
- Mathematica (general)
- Maple (general)
- SPICE (simulates electrical circuits)

Session 5: Software Tools

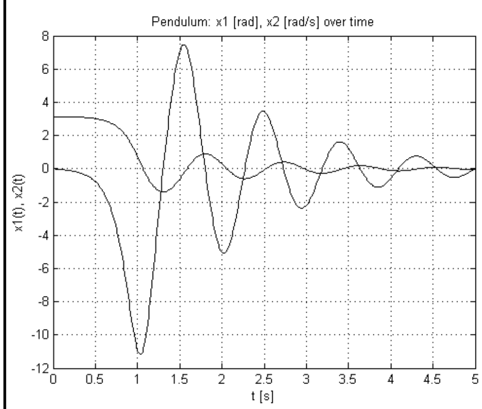
Pendulum in Matlab:

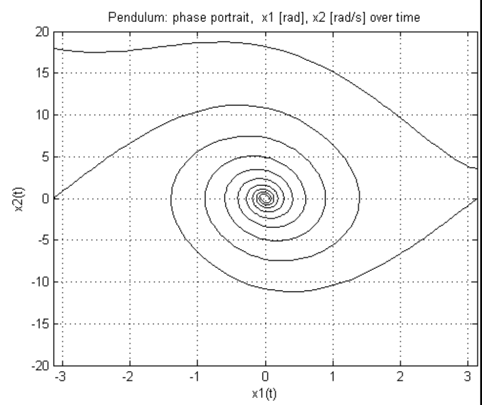
```
function xdot = pend(t, x)
xdot(1,1) = x(2);
xdot(2) = -k/m*x(2) - w0_2*sin(x(1)) + 1/(m*l^2)*u;

------Calling Function--------
…
t = linspace (0.0, 5.0, 501);     % time axis in [s]
x0 = [pi-0.01; 0.0];              % initial condition
[tt,xa] = ode45(@pend,t,x0);
…
```


Session 5: Software Tools

Pendulum in Matlab: representations



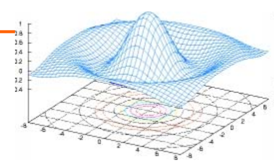


Session 5: Software Tools

Open Source Software:

- GNU Octave (emulates Matlab)
- Scilab (emulates Matlab)
- Maxima (symbolic processing)
- Python (scripting language)
- Bullet Physics
- Open Dynamics Engine
- …and many other packages

Session 5: Software Tools



Pendulum in GNU Octave:

```
function xdot = pend(x, t)
xdot(1,1) = x(2);
xdot(2) = -k/m*x(2) - w0_2*sin(x(1)) + 1/(m*l^2)*u;

------Calling Function--------
…
t = linspace (0.0, 5.0, 501);     % time axis in [s]
x0 = [pi-0.01; 0.0];              % initial condition
xa = lsode("pend", x0, t);
…
```

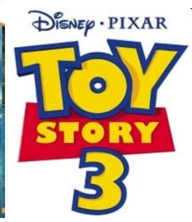
Bullet Physics (Open Source)

Software Development Kit (SDK), Library:

- Collision detection, rigid body dynamics, cloth
- Open source (Zlib lic.)
- C++
- PlayStation 3, Xbox 360, Multi-core CPUs

Animation Movies

Computer Games



Bullet Physics: SDK, Library

“2012”: Destroying Los Angeles

Remember: this is open source software!

Open Dynamics Engine

Open source SDK:

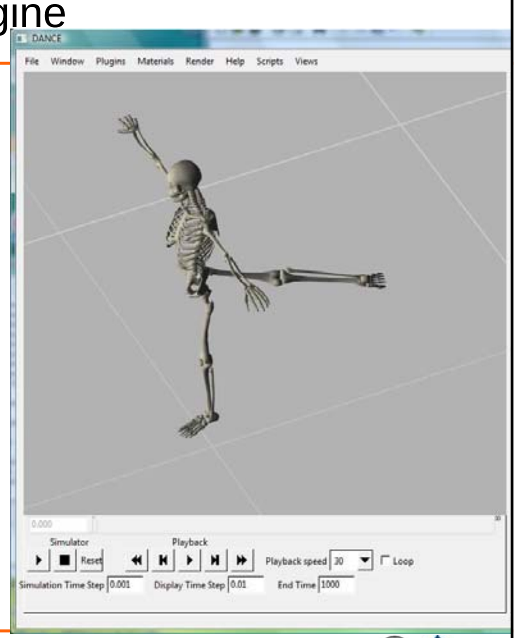
- Simulating rigid body dynamics
- Advanced joints, collision detection with friction
- Simulating vehicles, objects in virtual reality environments, virtual creatures
- C/C++ API

Open Dynamics Engine

Physics-based skater animation

Youri Dimitrov, Cheng Zhang, 2008

(http://www.cse.ohio-state.edu/~zhangche/788parent.htm)



Session 3: Simulation

Annotated bibliography

- http://www.mathworks.com. Home page of Matlab, a "technical computing language."
- http://www.wolfram.com. "Application for computations."
- http://www.gnu.org/software/octave. GNU Octave is a high-level language, primarily intended for numerical computations.
- http://www.scilab.org. Home page of Scilab, free and open source software for numerical computation.
- http://bulletphysics.org/wordpress. Bullet Physics' website with downloadable distribution, tutorials, documentation, demos.
- http://www.ode.org. ODE is an open source, high performance library for simulating rigid body dynamics.

Session 5: Software Tools

Q & A

Program

Welcome	08:45 – 09:10am	Coffee and bagels, Seminar kickoff at 9:00am
Session 1	09:10 – 10:00am	Mathematical models of dynamical systems Dr. P.K. Menon, Optimal Synthesis
Session 2	10:10 – 11:00am	System Identification - Theory and Practice Dr. Mark B. Tischler, Ames Research Center
Session 3	11:10 – 12:00am	Visualization and Virtual Environments Dr. Hadi Aggoune, Cogswell Polytech. College
Lunch	12:00 – 12:40pm	Sandwiches, sodas, discussions and product demos
Session 4	12:40 – 01:30pm	Applications of Hardware-in-the-Loop Simulators Christoph Wimmer, National Instruments
Session 5	01:40 – 2:30pm	Simulation with Software Tools Elliot English, Dr. Martin Aalund, Dr. Karl Mathia

Sponsors

http://www.adi.com

http://www.applimotion.com

http://www.cogswell.edu

http://www.ni.com

http://www.willowgarage.com