Seminar

# Modeling and Simulation of Dynamical Systems

Presented by the
IEEE Control Systems Society
Santa Clara Valley

Sunnyvale, 5 February 2011

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# Numerical Physical Simulation

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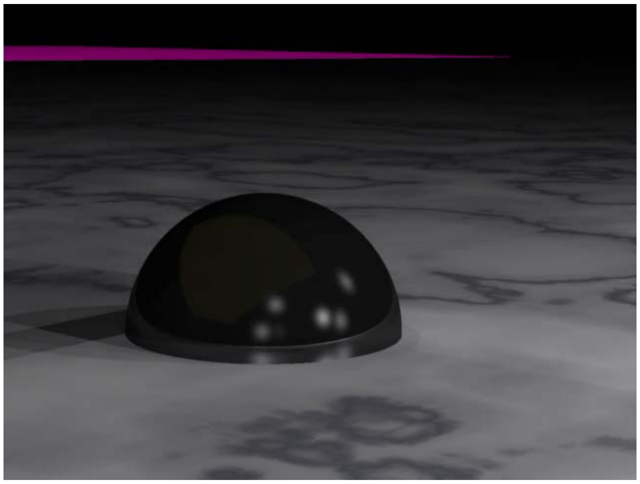
## Numerical Physical Simulation

- Initial Value Problem:
  - Given some initial state q(t)
  - Want the state at some point in the future q(t+dt)
  - The change in state should convergently follow the desired set of physical laws.
    - PDEs, F=ma, Collisions, Constraints, etc...

## Simulation Materials

- Continuum Mechanics
  - Deal with volumes rather than infinitesimal particles
- Range of different materials
  - Rigid <-> Elastic <-> Plastic <-> Fluid
- Different set of challenges with each material

## Cloth Simulation



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## Steps to Simulation

- Spatial Discretization
- Temporal Discretization
- Material Specific Issues
- Solvers

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## Spatial Discretization

- How to represent state (positions, velocities, mass) throughout space
- Eulerian
  - Store a grid fixed in space and change values
  - Mass moves between degrees of freedom
- Lagrangian
  - Discretization moves with material
  - Mass is fixed to degrees of freedom
- Arbitrary Lagrangian Eulerian (ALE)

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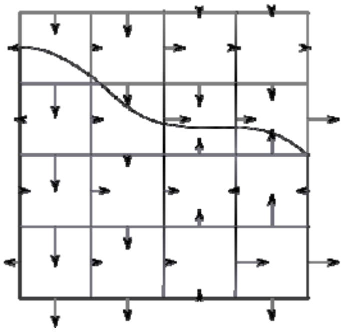
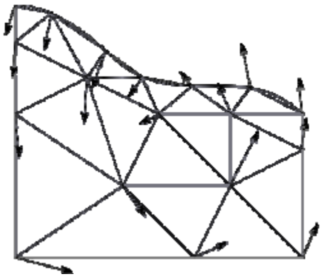
## Eulerian Simulation

- Pros:
  - Accurate numerical derivatives
  - Lower memory usage with better cache coherency
- Cons:
  - Adaptivity is difficult
    - Lose memory and accuracy benefits
    - T-junctions when adjacent cells are misaligned
  - Can't accurately represent boundaries unaligned with grid
    - Represent as implicit surfaces



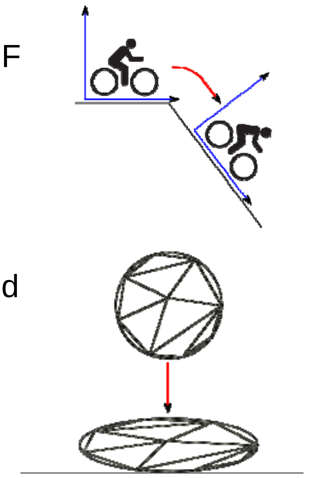
## Lagrangian Simulations

- Pros:
  - Adaptivity is easy
  - Boundaries are easy to resolve
- Cons:
  - Higher memory requirements
  - Remeshing is necessary to guarantee accuracy of derivatives
  - Collisions
    - Topology changes are nontrivial



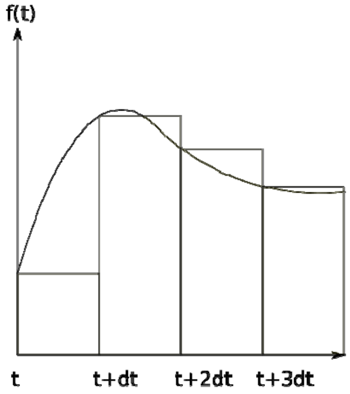
## Common Spatial Discretizations

- Rigid Bodies (lagrangian)
  - 3D positions and orientation (6 DOF space)
  - Geometry stored in local frame
- Elastic Bodies/Plastic Materials (lagrangian)
  - Plastic bodies need to be remeshed when rest state is too deformed
- Fluids (lagrangian/eulerian)
  - Particles in a lagrangian scheme
  - Velocities stored on grid faces in eulerian scheme



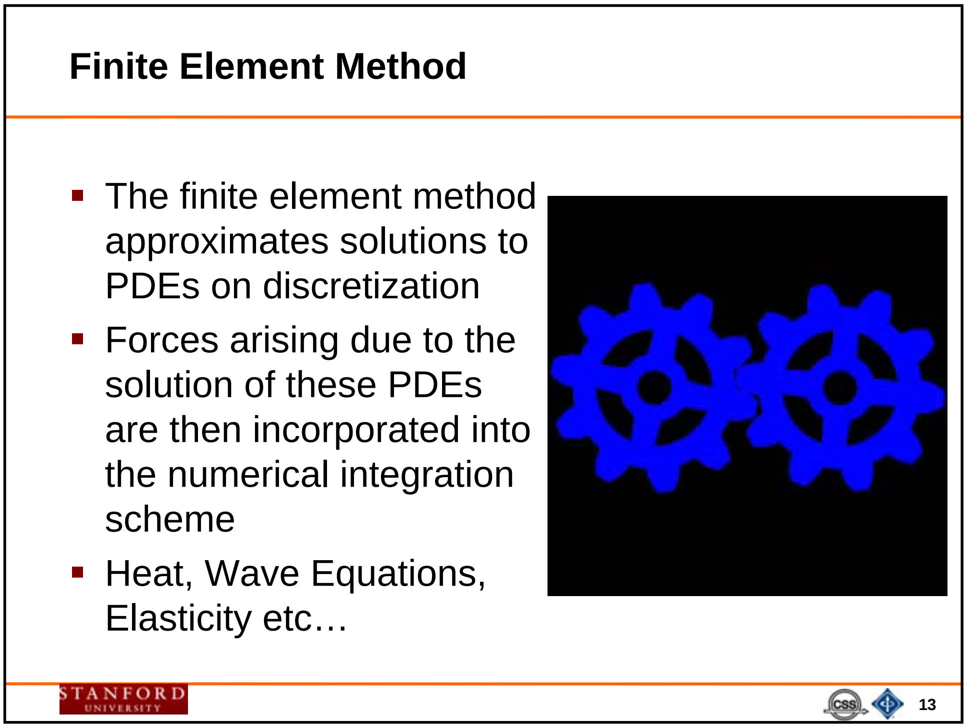
## Temporal Discretization

- Use a numerical integration scheme to compute values at new time
- Many schemes
  - Taylor expansions (forward/backward euler, bdf, etc...)
  - Runge Kutta,...
  - Quadrature
- Explicit/Implicit
- Stability/Accuracy



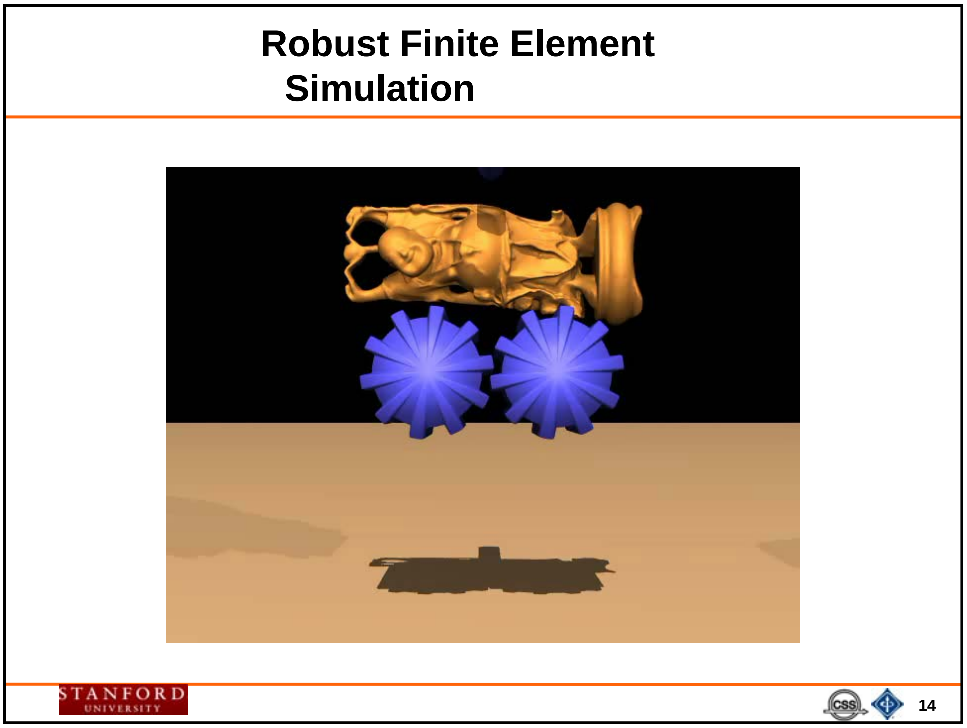
## Rigid and Deformable Body Integration

- Rigid body motion is hard to solve analytically
  - Approximate by assuming rotation is constant over timestep
- Deformable motion needs to be solve implicitly to be stable
  - New positions are used when computing forces
    - Backward euler, trapezoidal rule…
- Elastic and Inelastic collisions are handled as a post process



## Finite Element Method

- The finite element method approximates solutions to PDEs on discretization
- Forces arising due to the solution of these PDEs are then incorporated into the numerical integration scheme
- Heat, Wave Equations, Elasticity etc…



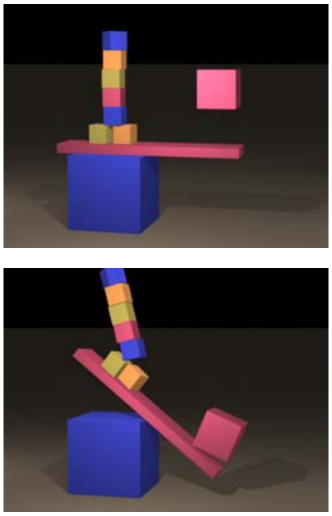
## Robust Finite Element Simulation

## Elastic Collisions

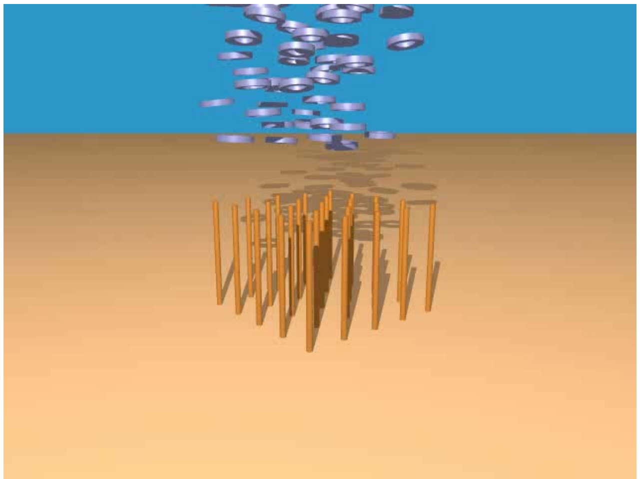
- Iteratively process each collision one at a time
  - New collisions may occur and need to be processed
- Thin shell objects need to be collision free
  - Any objects still colliding after a certain number of iterations are combined and evolved rigidly for the current time step
- For speed allow volumetric objects to penetrate slightly

## Contact (Inelastic Collisions)

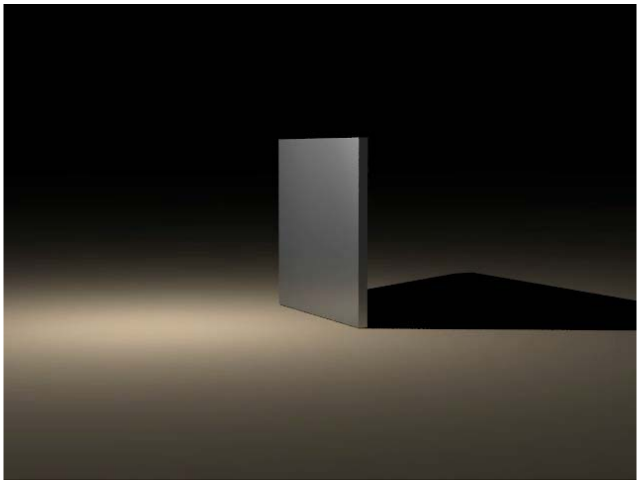
- When many objects are stacked need to solve contacts accurately to prevent penetration and without rigidifying
- Impose inequality constraints in monolithic system
- Generally requires the solution of a linear complementarity problem



## Large Scale Stacking



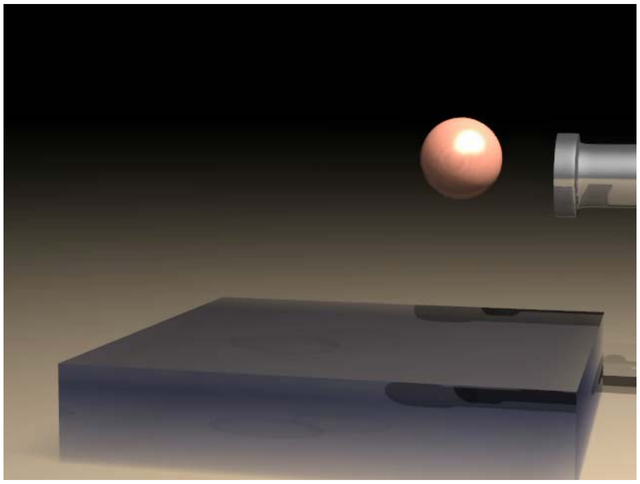
## Plasticity and Fracture



## Fluid Integration

- Typically split integration into two steps
  - Advection – moves velocities and mass around (computed explicitly)
    - Semi-lagrangian advection traces backwards from velocity samples back and interpolates new velocities from locations
  - Pressure and Viscosity Solve (computed implicitly)
    - Incompressible flows project the advected velocity field to be divergence free
- Other field values are also advected
  - Levelset implicit surface, density, temperature, etc…
- Solid fluid coupling
  - Straightforward when purely lagrangian
  - Eulerian frameworks need to handle nonconforming boundaries
    - Usually handled in implicit solve

## Multiphase Reacting Flow



## Solvers

- Large sparse linear systems
  - Direct
    - Large sparse matrix factorizations (Cholesky, LU)
  - Iterative
    - Jacobi, Gauss Seidel, Conjugate gradient
- Collisions (Nonlinear nonsmooth systems)
  - Direct
    - Pivoting methods: Lemke, Dantzig (simplex method)
  - Iterative
    - Projected Gauss Seidel

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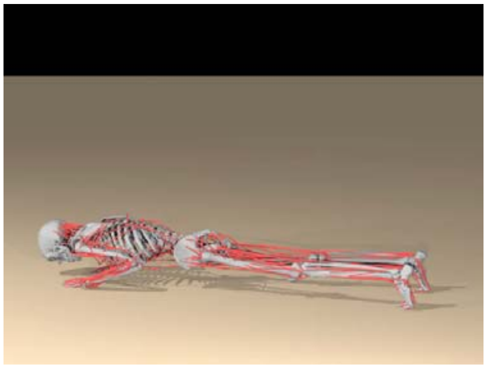
## Controller Driven Systems

- Simulations can have external forces due to controller output
  - Articulated body joints
  - Point attached muscles
- Both mechanical and biomechanical systems
- Numerical optimization of parametric designs
  - Aircraft, automobiles,…



## Ongoing Research

- Energy conservation
  - Not necessarily more accurate but stable (large dt)
  - More meaningful results when not converged
- Resolve all collisions - interpenetration free simulation (continuous collision detection)
- Monolithic integration schemes
  - Solve for both collision and constitutive model forces in the same system
- Scalability – solvers which scale better than O(n^2)
  - Optimal solvers - O(n) or O(n log(n)) time (Multigrid)

## Annotated bibliography

http://physbam.stanford.edu Dr. Ron Fedkiw group website. Contact information, papers, videos, images.

# Q & A