Breaking the Barriers in Wireless Network Information Theory

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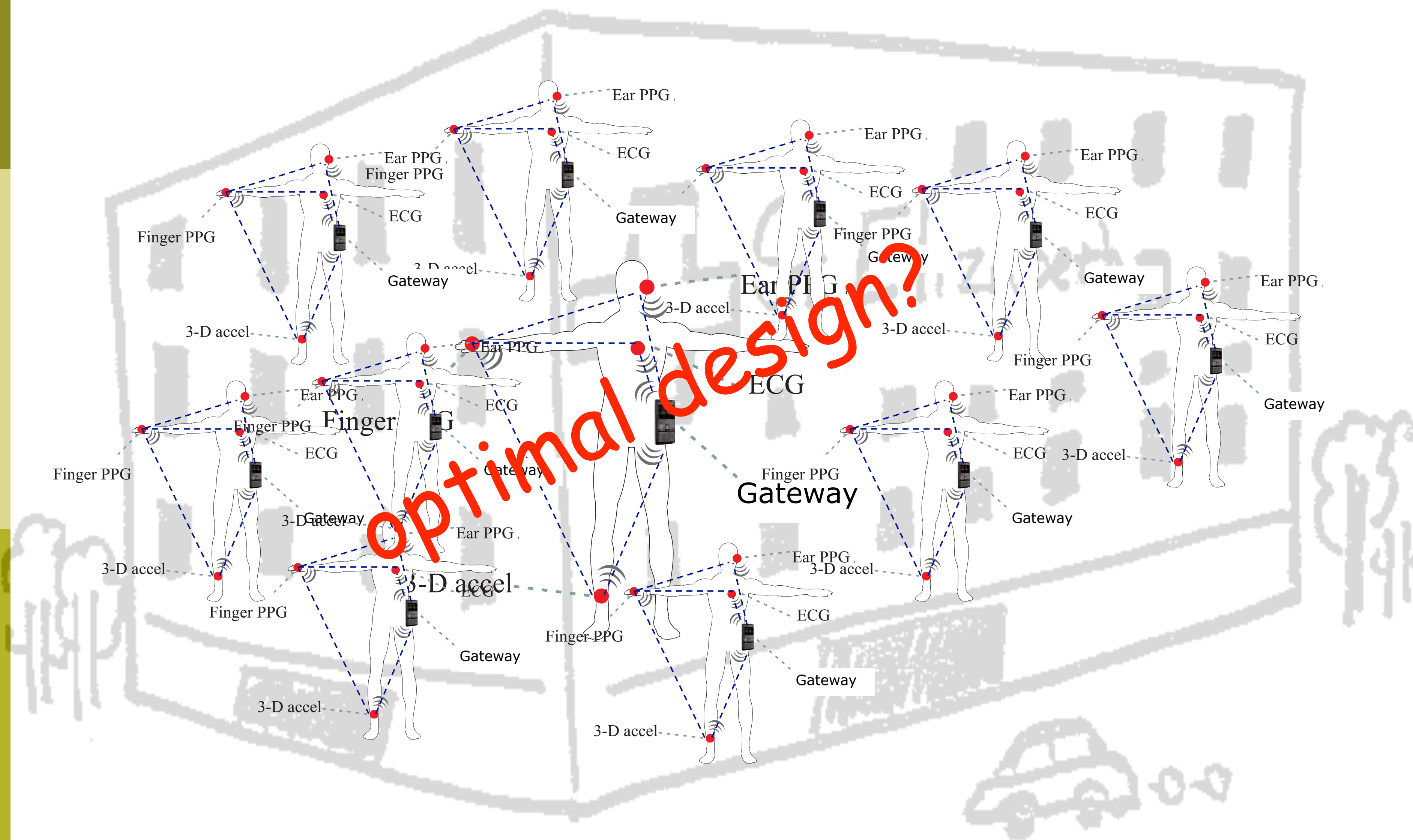
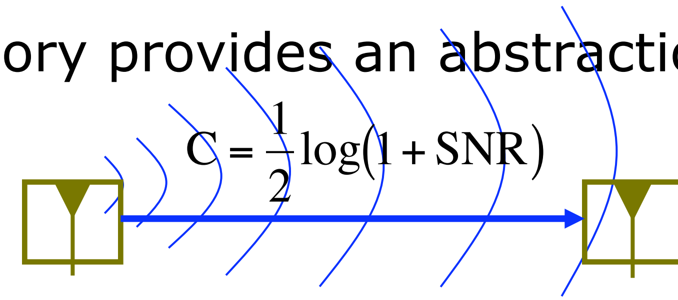
Motivation



Overview

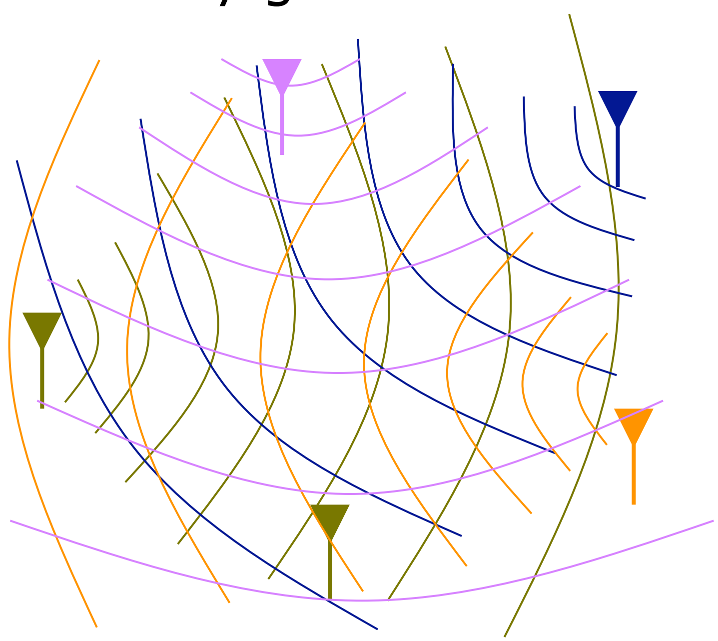
- Point-to-point channel
  - Information theory provides an abstraction



$$C = \frac{1}{2}\log(1 + SNR)$$

Claude Shannon
(1916-2001)

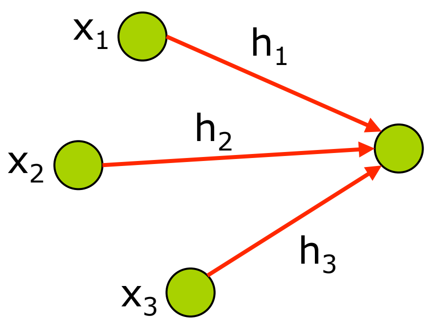
- Wireless network
  - Does information theory give us a similar picture? Not yet.



Basic model for wireless medium

- Key features of wireless medium
  - Broadcast
  - Interference
  - High dynamic range of channel variations

- Basic PHY layer model: additive-Gaussian channel model

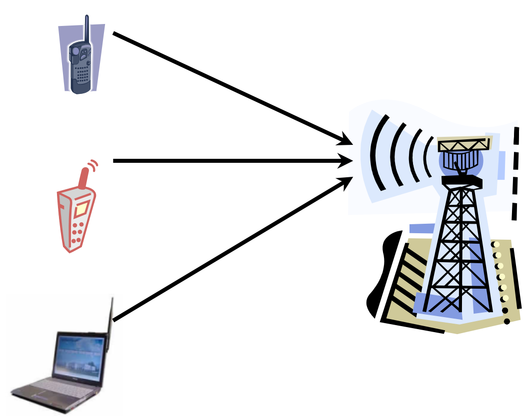


$$y = \sum_i h_i x_i + z$$

What is known?

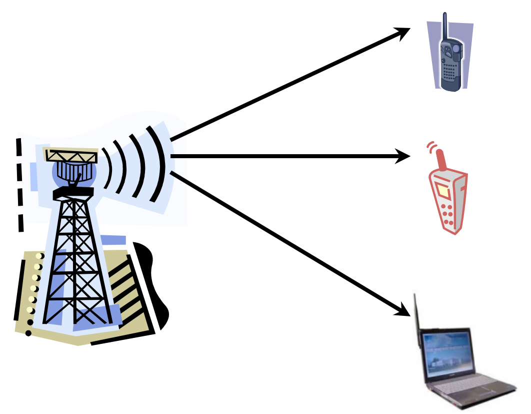
Point to point: $C = \frac{1}{2}\log(1 + \mathrm{SNR})$

(Shannon 1948)



Multiple access

(Ahlswede, Liao 70's)



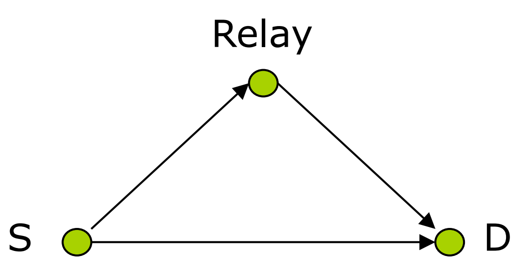
Broadcast

(Cover, Bergmans 70's)

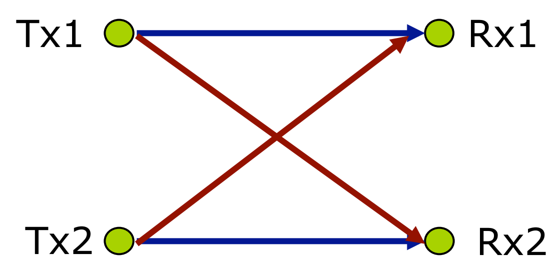
State of the art

- Unfortunately, we don’t know the capacity of most other Gaussian networks

- 3 decades of studying basic networks with 3 or 4 nodes
  - Still the capacity is not known



Relay Channel



Interference channel

- How can we make progress?

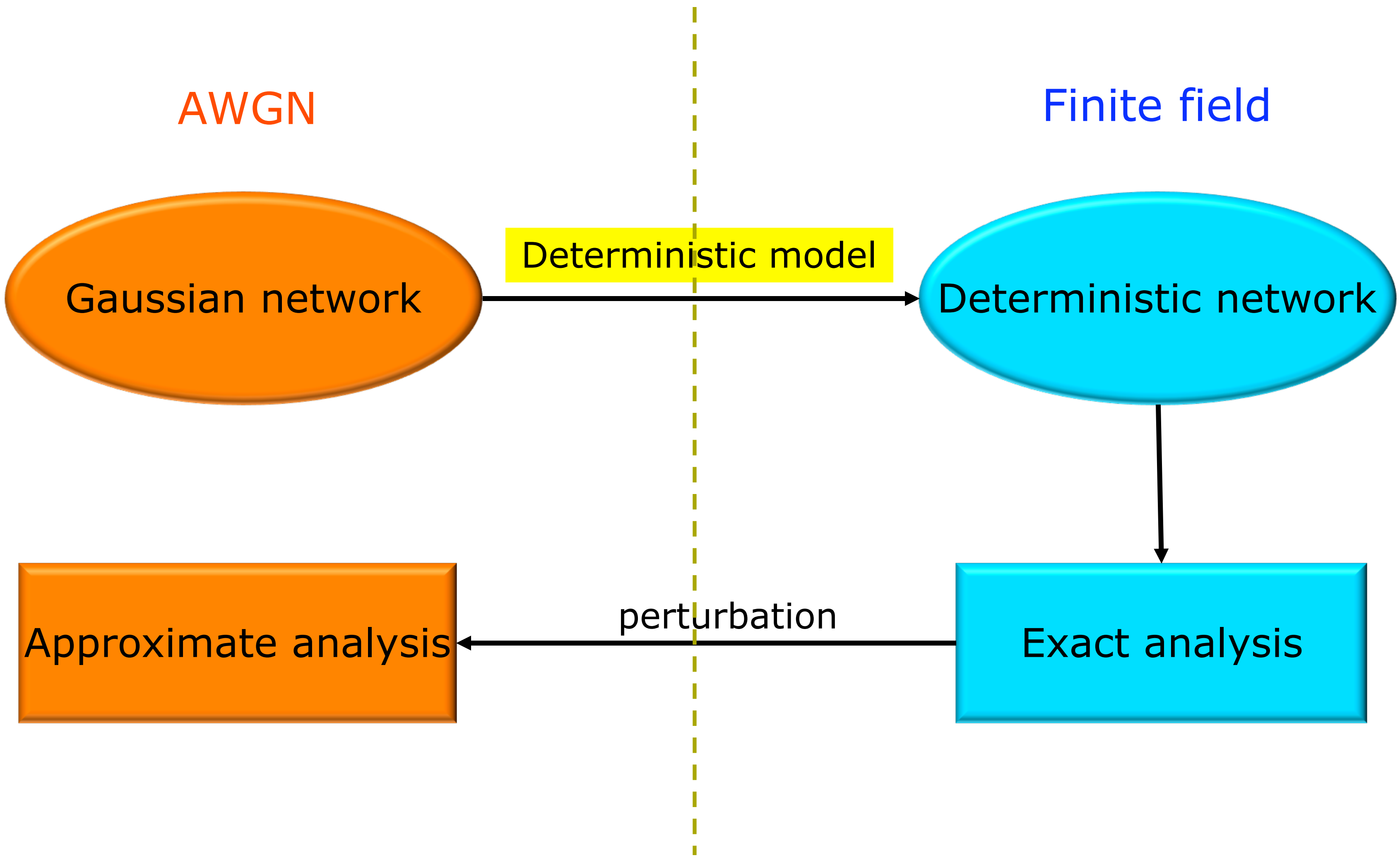
Our approach

- Change the focus to approximation results
  - with hard guarantees on the gap to optimality

- We develop simpler deterministic channel models
  - De-emphasize the background noise
  - Focus on the interaction between users’ signals

- Utilize them systematically to approximate the Gaussian model

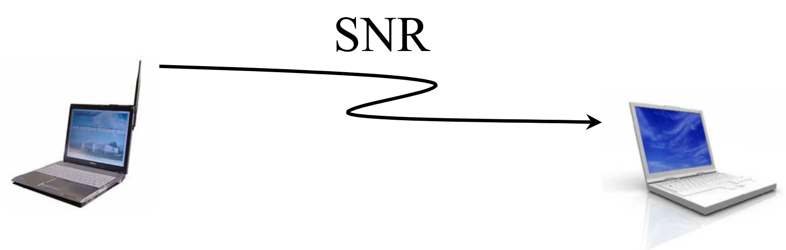
Methodology



In this talk …

- Introduce the deterministic channel model
- Apply it to some examples:
  - Relay network
- Distributed compressive sensing

Deterministic Model (A.-Diggavi-Tse 2007)

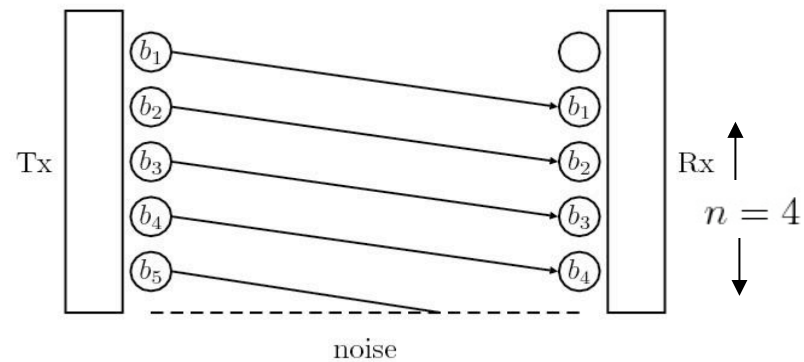


$$y = \sqrt{\mathrm{SNR}}\, x + z$$

$$x = 0.b_1 b_2 b_3 b_4 b_5 \ldots$$

$$\sqrt{\mathrm{SNR}}\, x = b_1 b_2 \cdots b_n . b_{n+1} \cdots$$

$$C = \frac{1}{2}\log(1 + \mathrm{SNR})$$



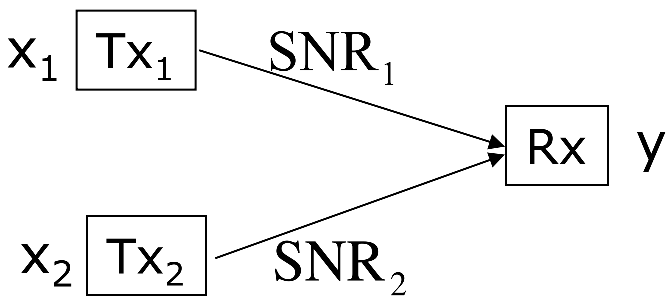
Least significant bits are truncated at noise level

$$n \leftrightarrow \left[\frac{1}{2}\log \mathrm{SNR}\right]^+$$

$$C = \left[\frac{1}{2}\log \mathrm{SNR}\right]^+$$

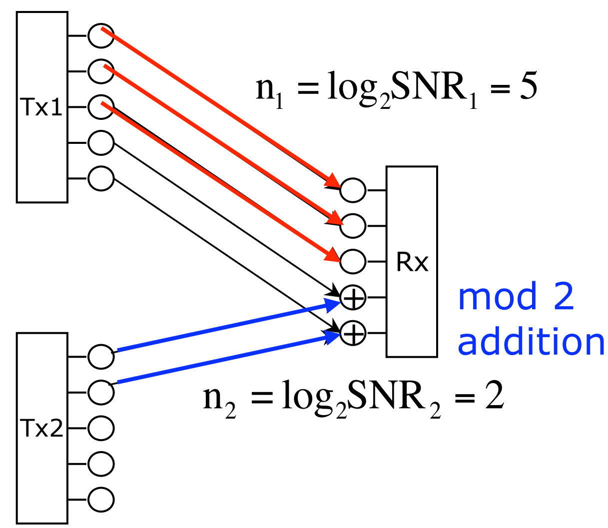
Multiple access

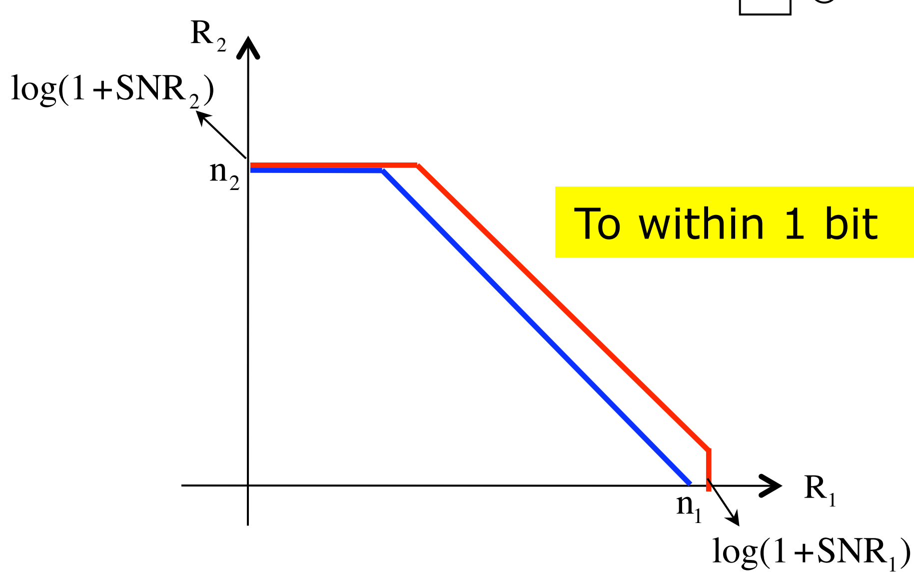
Gaussian



$$y = \sqrt{\mathrm{SNR}_1}\, x_1 + \sqrt{\mathrm{SNR}_2}\, x_2 + z$$

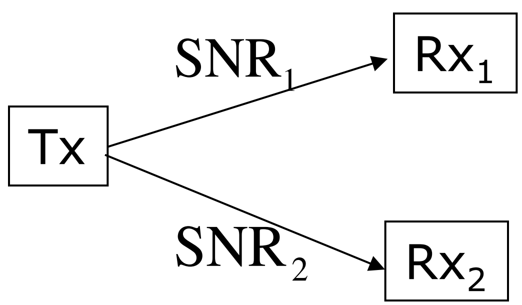
Deterministic



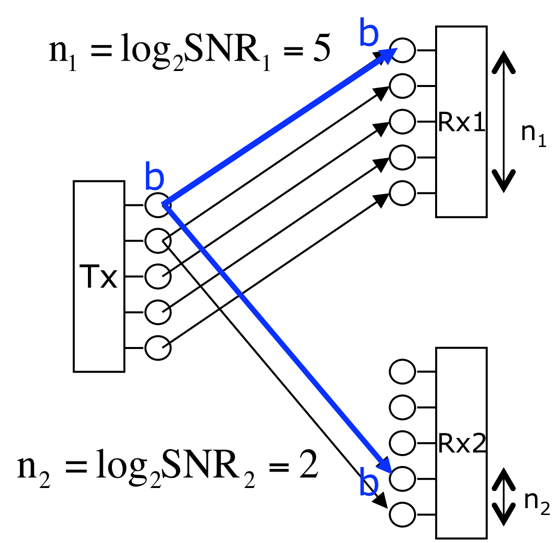


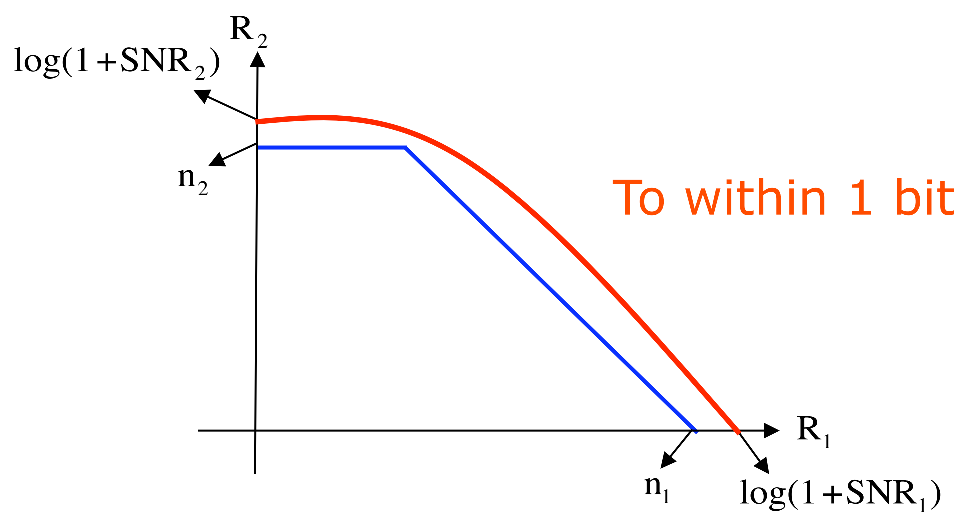
Broadcast

Gaussian



Deterministic

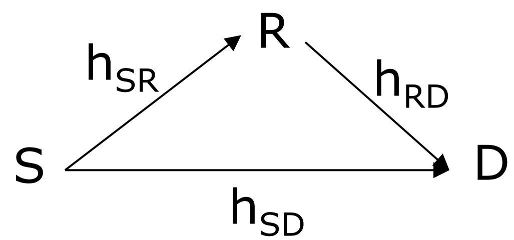




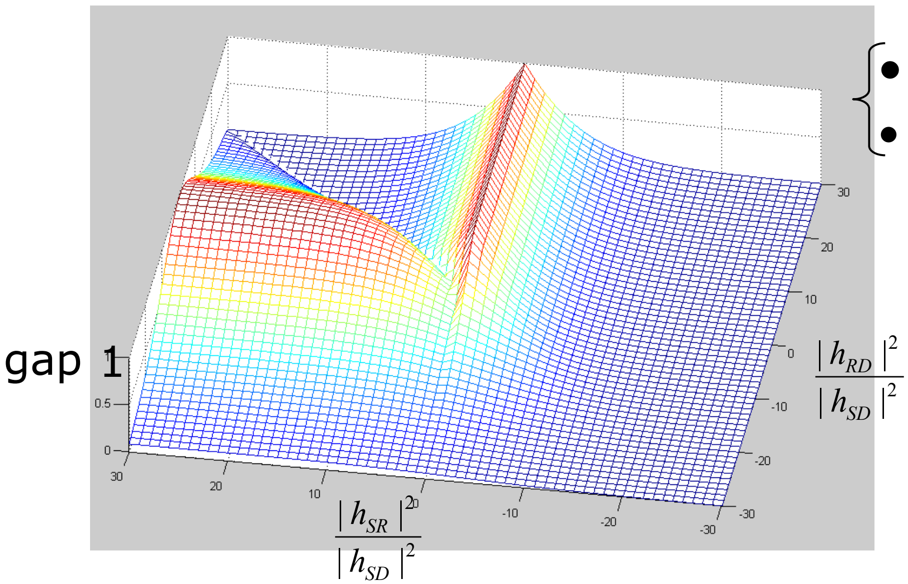
Relay Networks

Example: One relay

Gaussian

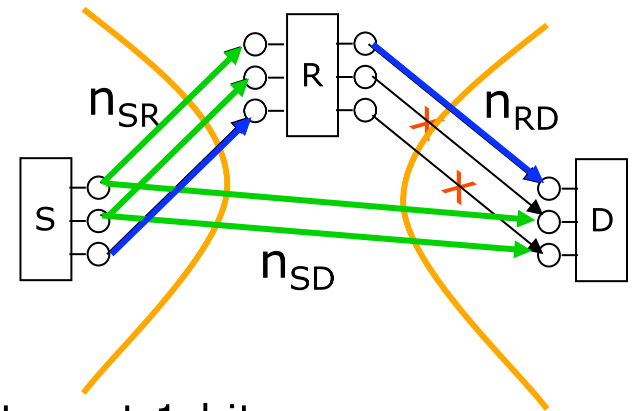


$$\overline{C} - 1 \le C \le \overline{C}$$



Decode-Forward is near optimal

Deterministic



- Gap is at most 1-bit
- On average it is much less than 1-bit

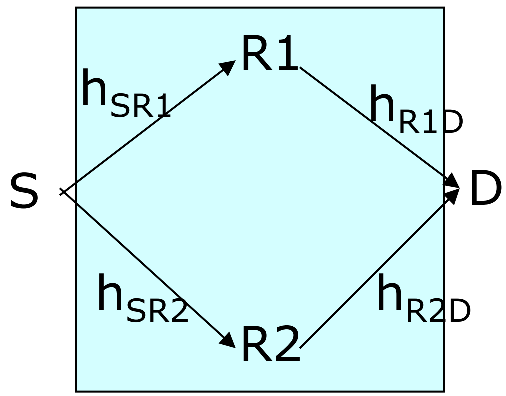
$$C \le \min\Big(\underbrace{\max(n_{SD}, n_{SR})}, \underbrace{\max(n_{SD}, n_{RD})}\Big)$$

$$= n_{SD} + \min\Big((n_{SR} - n_{SD})^+, (n_{RD} - n_{SD})^+\Big)$$

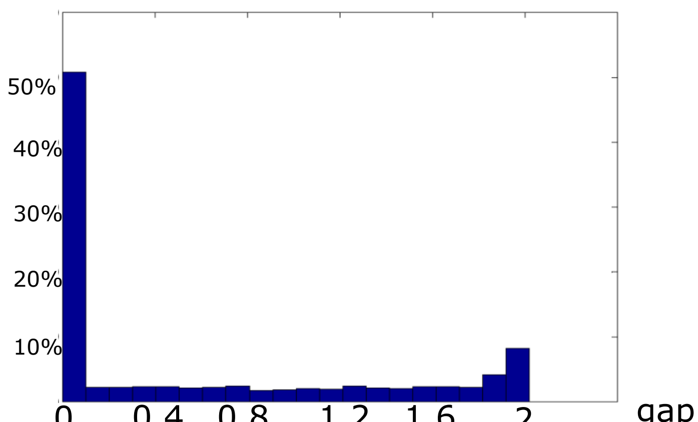
Decode-Forward is optimal

Example: Two relays

Gaussian

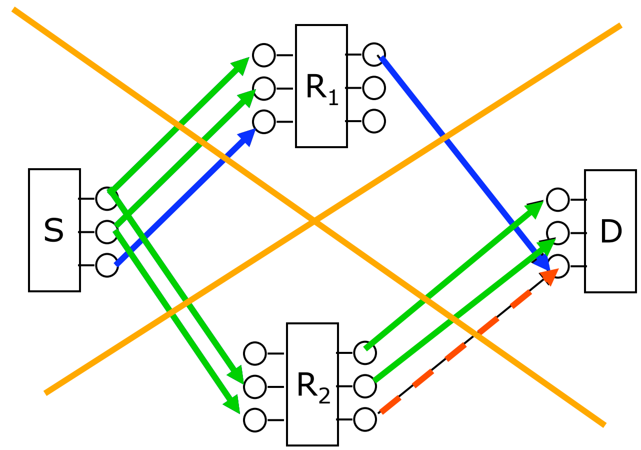


$$\overline{C} - 2 \le C \le \overline{C}$$



Partial Decode-Forward is near optimal

Deterministic



$$C = \min\begin{pmatrix} \max(n_{SR1}, n_{SR2}), \max(n_{R1D}, n_{R2D}), \\ \underbrace{n_{SR1} + n_{R2D}}, \underbrace{n_{SR2} + n_{R1D}} \end{pmatrix}$$

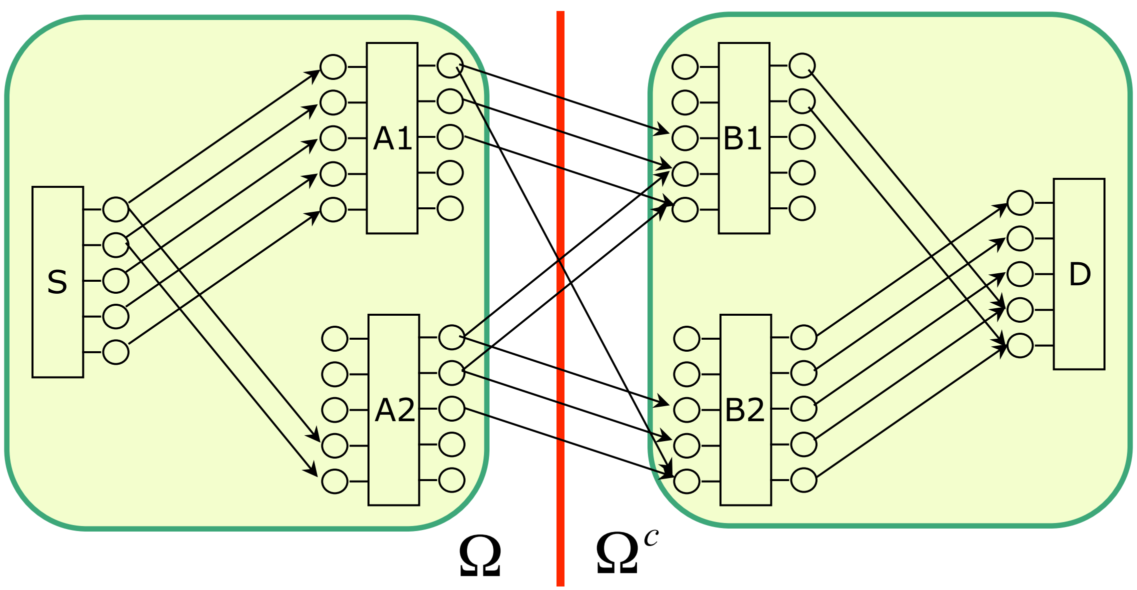
Partial Decode-Forward is optimal

General deterministic relay networks (A.-Diggavi-Tse 2007)

- Theorem: Cutset bound is achievable,

$$C_{\text{relay}} = \overline{C} = \min_{\Omega} \operatorname{rank}(G_{\Omega \to \Omega^c})$$

  - Our theorem is a generalization of Ford-Fulkerson max-flow min-cut theorem



Relaying scheme

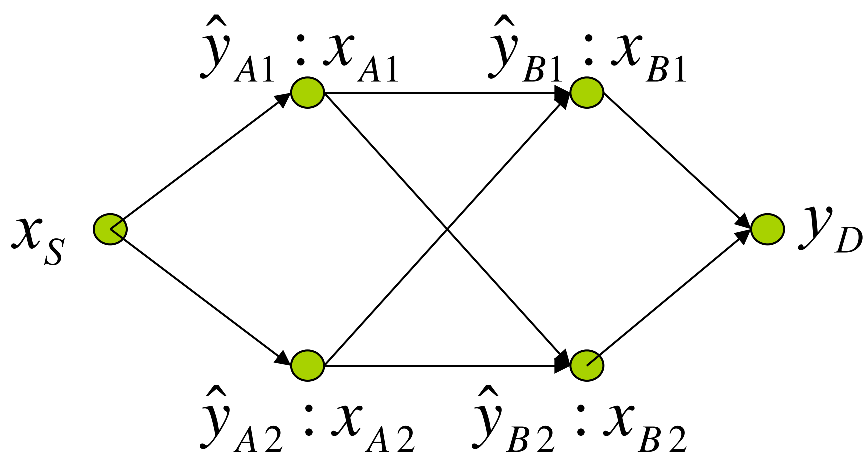
Deterministic

- S encodes the message over T symbol times
- Each relay randomly maps the received signal into a transmit codeword
- D decodes the message

optimal

Gaussian

- S encodes the message over T symbol times
- Each relay,
  - Quantizes the received signal at noise level
  - Randomly maps it into a Gaussian codeword
- D decodes the message



Properties of the scheme

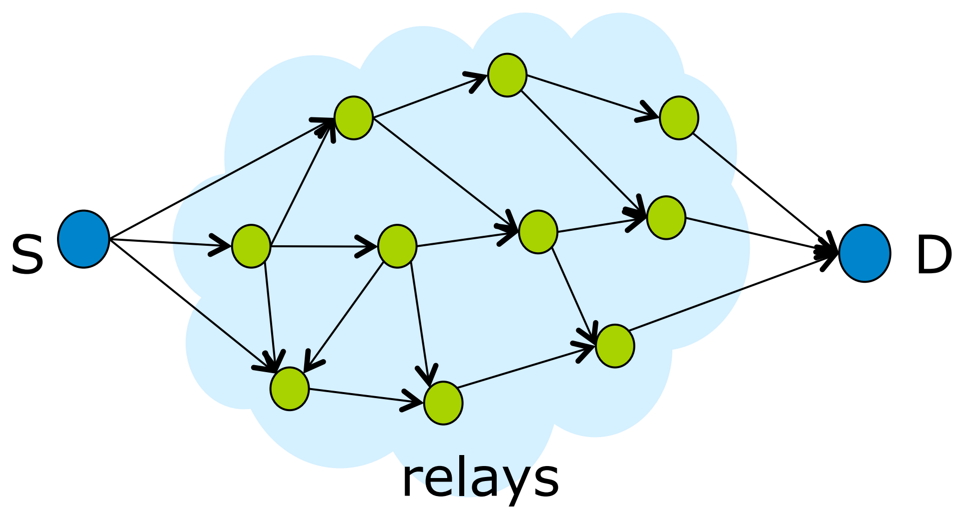
- Simple
  - Quantize
  - Map to a transmit codeword
- Relays don’t need any channel information
- How does it perform?

Capacity of Gaussian relay networks (A.-Diggavi-Tse 2008)

- Theorem: for any Gaussian relay network

$$\overline{C} - \kappa \leq C \leq \overline{C}$$

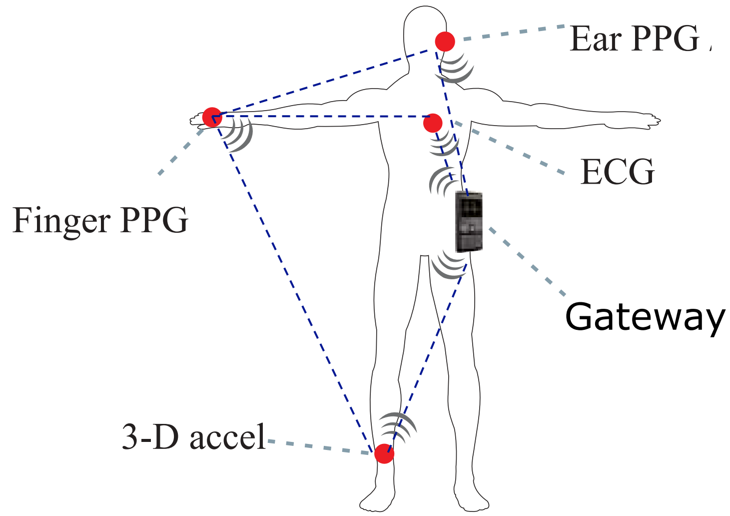
  - $\overline{C}$ is the cut-set upper bound on the capacity
  - κ is a constant that depends on size of the network, but not the channel gains or SNR's of the links

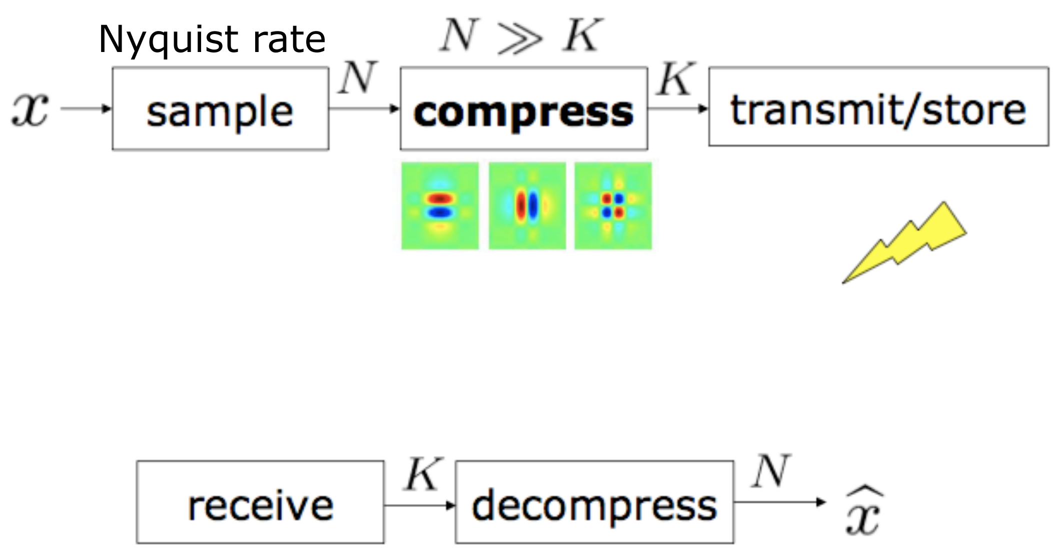


Distributed Compressive Sensing

Compressive sensing

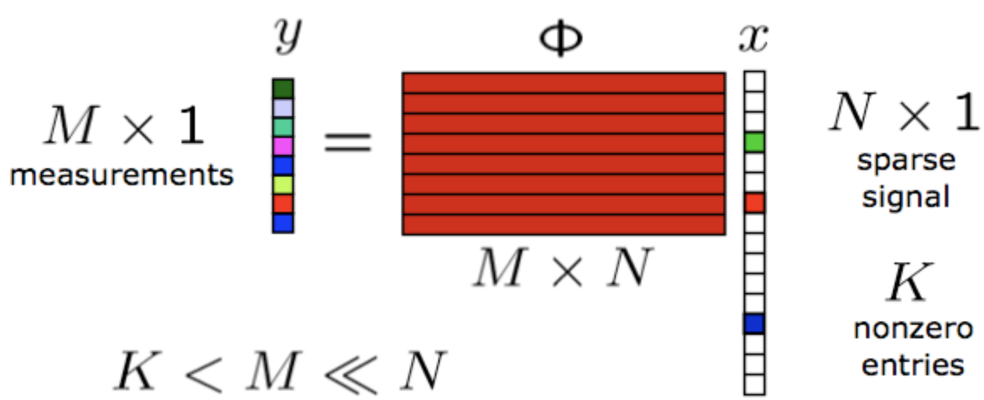
- The measured data is very redundant!
- Almost all current systems:
  - Sample at Nyquist rate
  - Compression
- How can we sense efficiently?

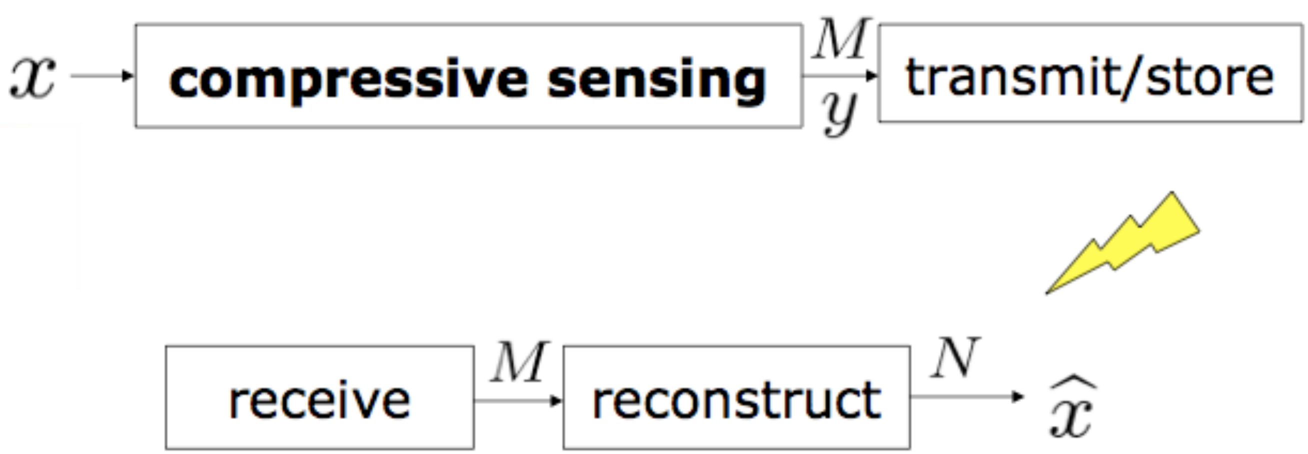




Compressive sensing (cont.)

- Can we recover by having only a few "linear" measurements?



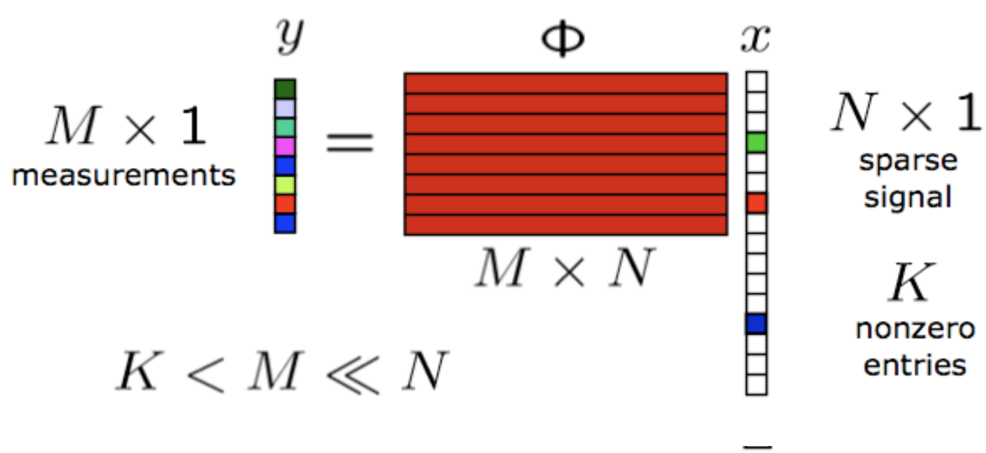


Compressive sensing (cont.)

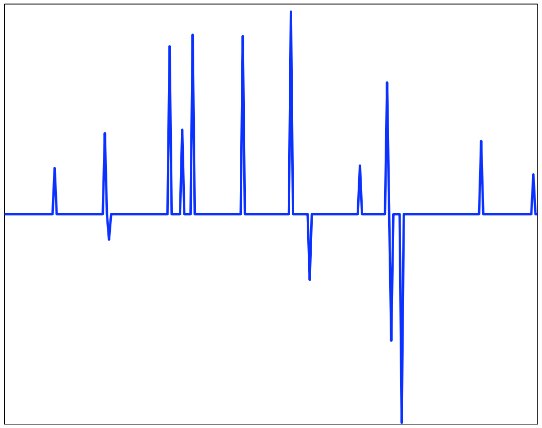
- Can we recover by having only a few "linear" measurements? Yes!
- As long as
  - The signal is sparse (in some domain)
  - And the measurement matrix satisfies the RIP condition

- Decompression is quick (L1 minimization) (Candes-Tao& Donoho, 2006)

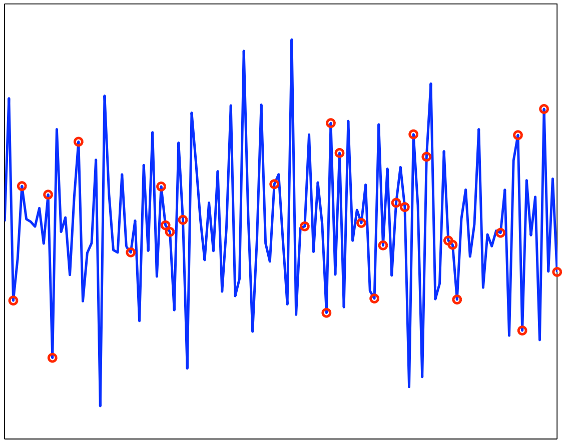
- Random projection works!



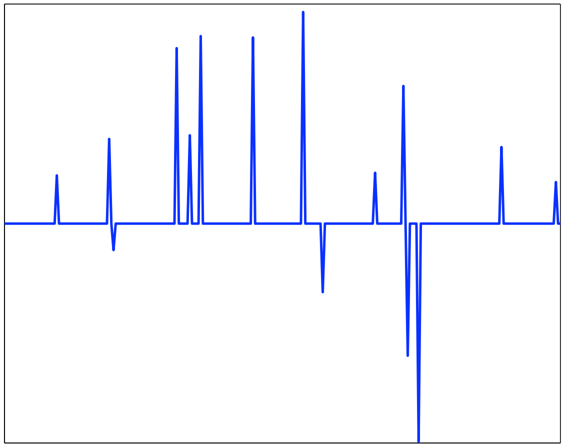
Example



(a) Original signal in the frequency domain $\hat{f}(w)$. It has 15 non-zero components in the frequency domain.



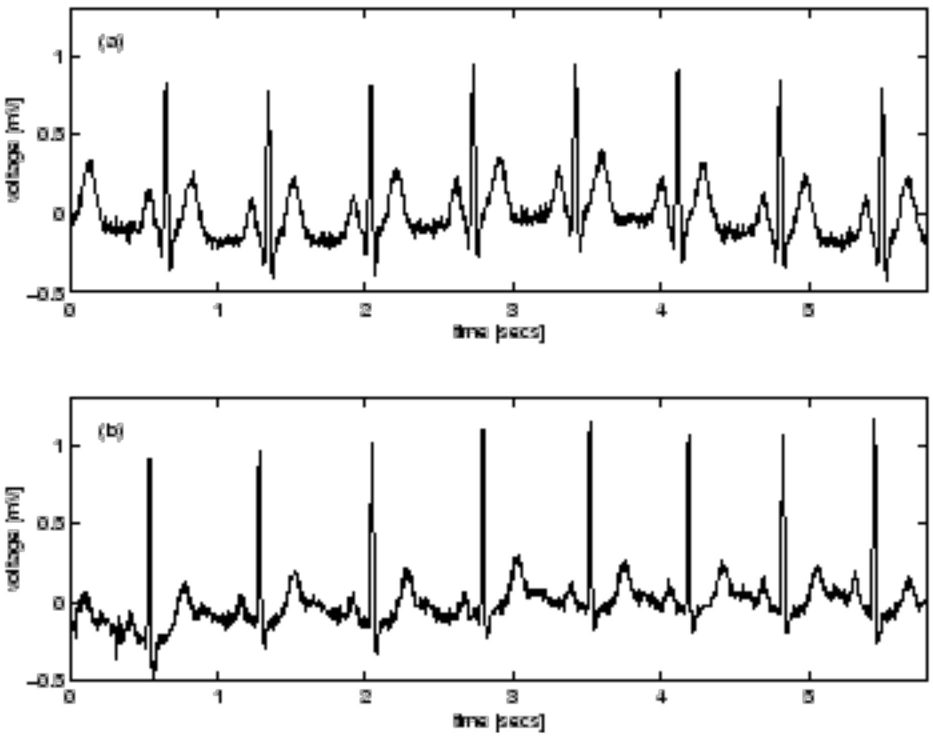
(b) Given $m = 30$ time-domain samples of f(t).

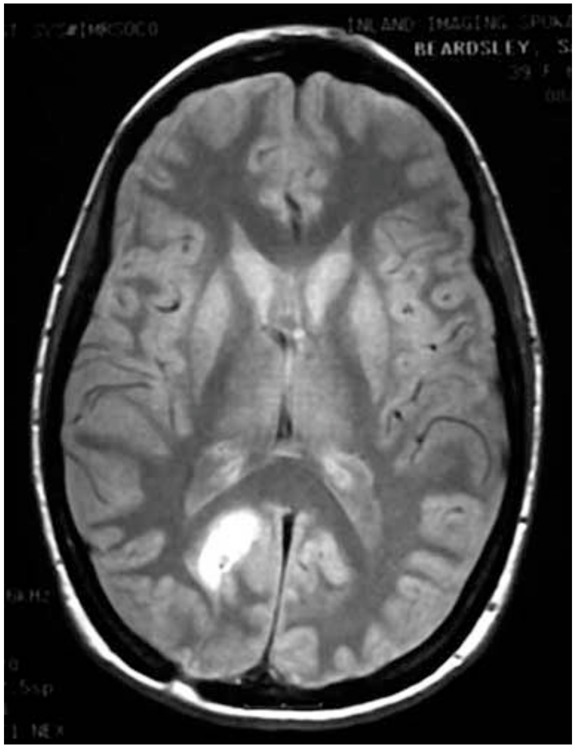


(c) Perfect recovery using ℓ_1 minimization.

Good news!

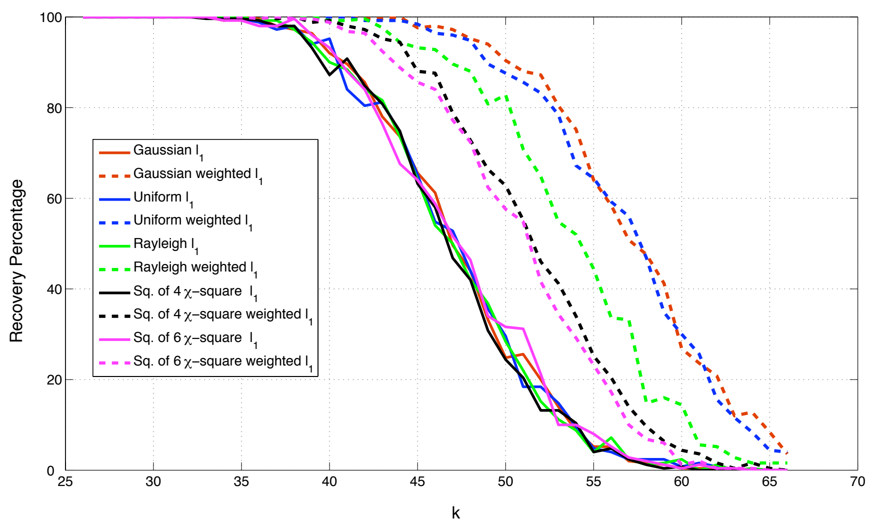
- Many signals are sparse





Even more efficient algorithms

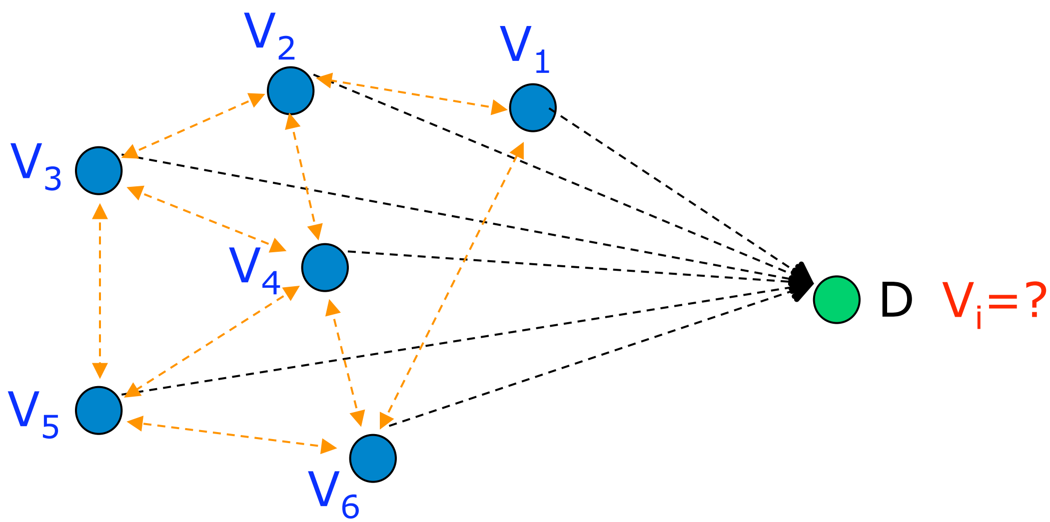
- Can boost the performance by running iterative weighted L1 minimizations



joint work with A. Khajehnejad, W. Xu, and B. Hassibi

This is just the beginning!

- Distributed and collaborative compressive sensing



Summary

- There is a large gap between the current designs and the optimal design
- Recent advances in information theory can help to bridge the gap

Questions?