Statistical Early Warning Signs of Instability in Synchrophasor Data

IEEE GMS/PES
Synchrophasor Meeting
October 12, 2016

Goodarz Ghanavati,
Taras Lakoba,
Paul Hines*
*To whom all blame is due

Funding gratefully acknowledged:
NSF Awards ECCS-1254549, DGE-1144388,
DOE Award DE-OE0000447

US Northeast and Canad

August 14, 2003

50 million people

California, Arizona, Mexico
September 8, 2011
5 million people

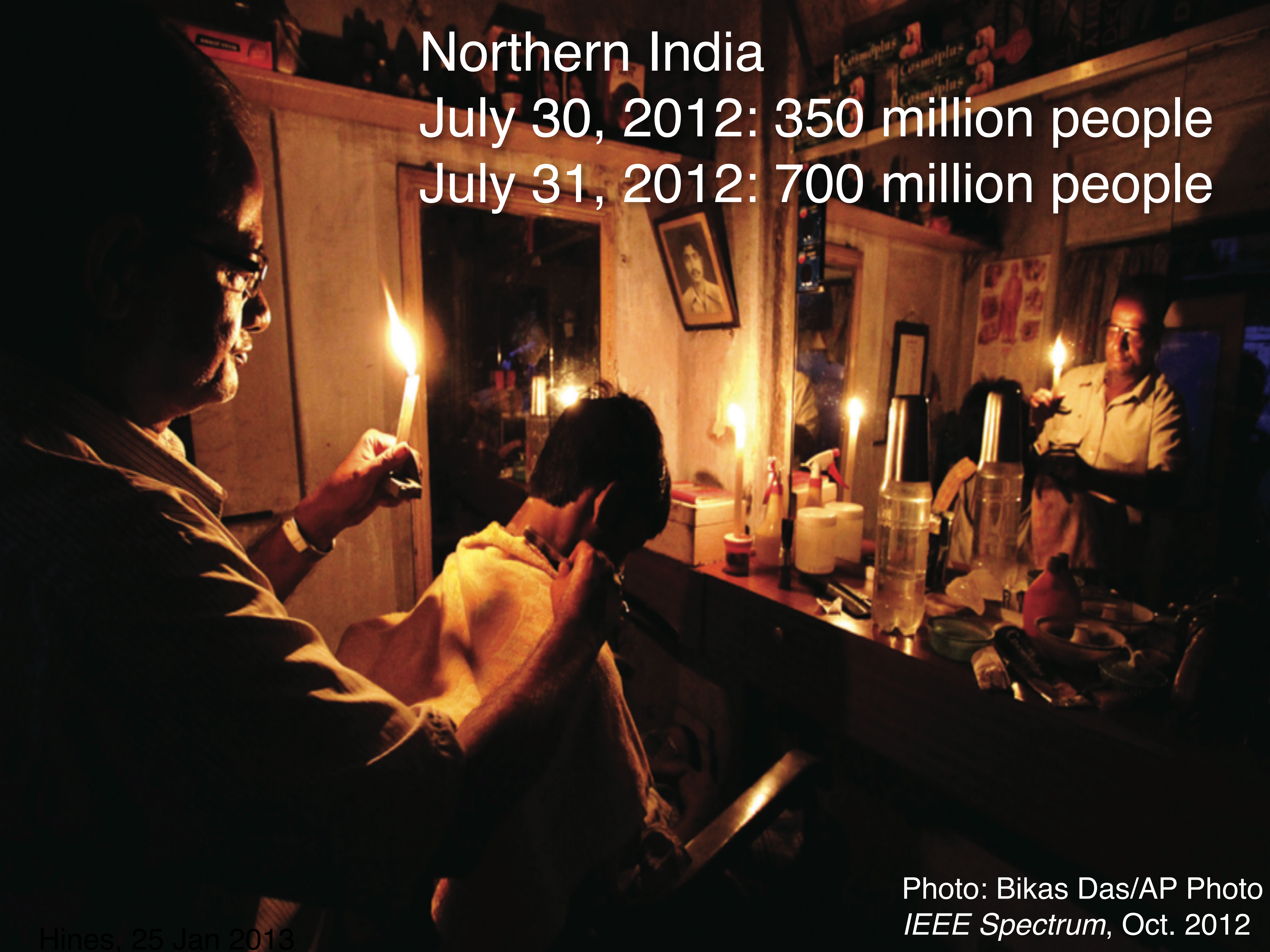


Photo: Bikas Das/AP Photo
IEEE Spectrum, Oct. 2012

Bangledesh. 1 November 2014

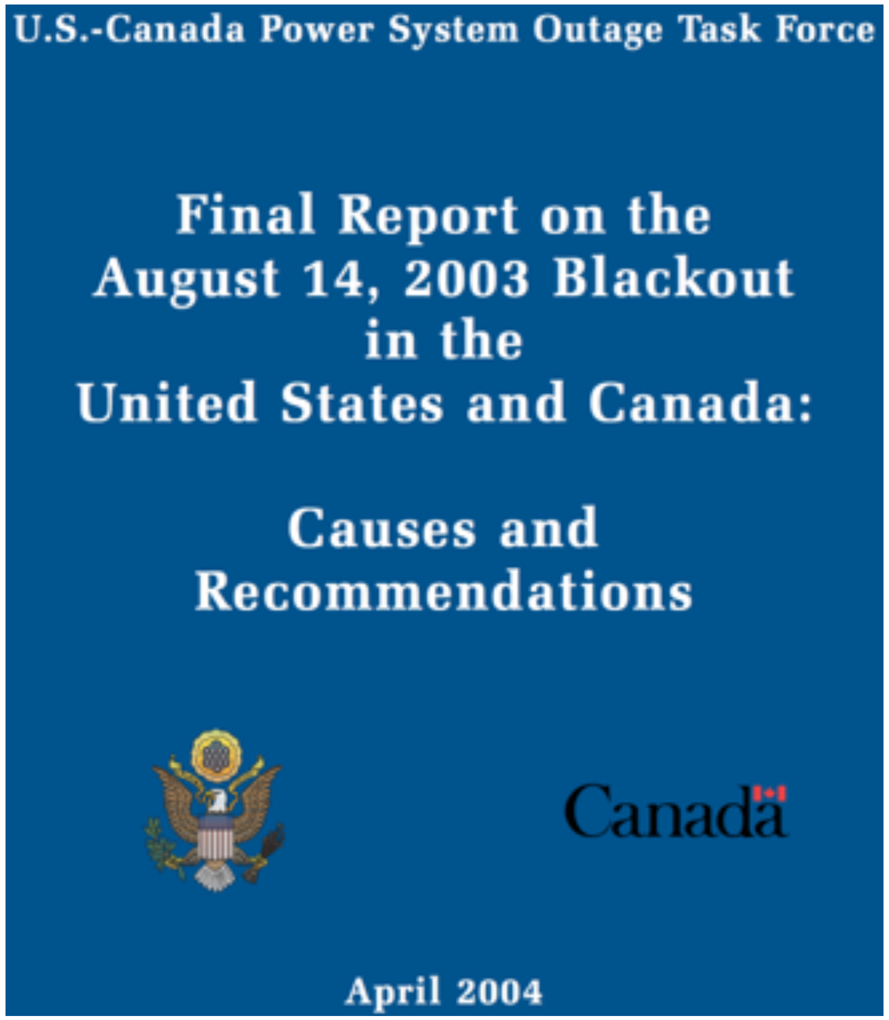
Officials said it would take at least 12 hours to repair the system and restore power to the capital Dhaka [AP]

Washington DC, April 7, 2015

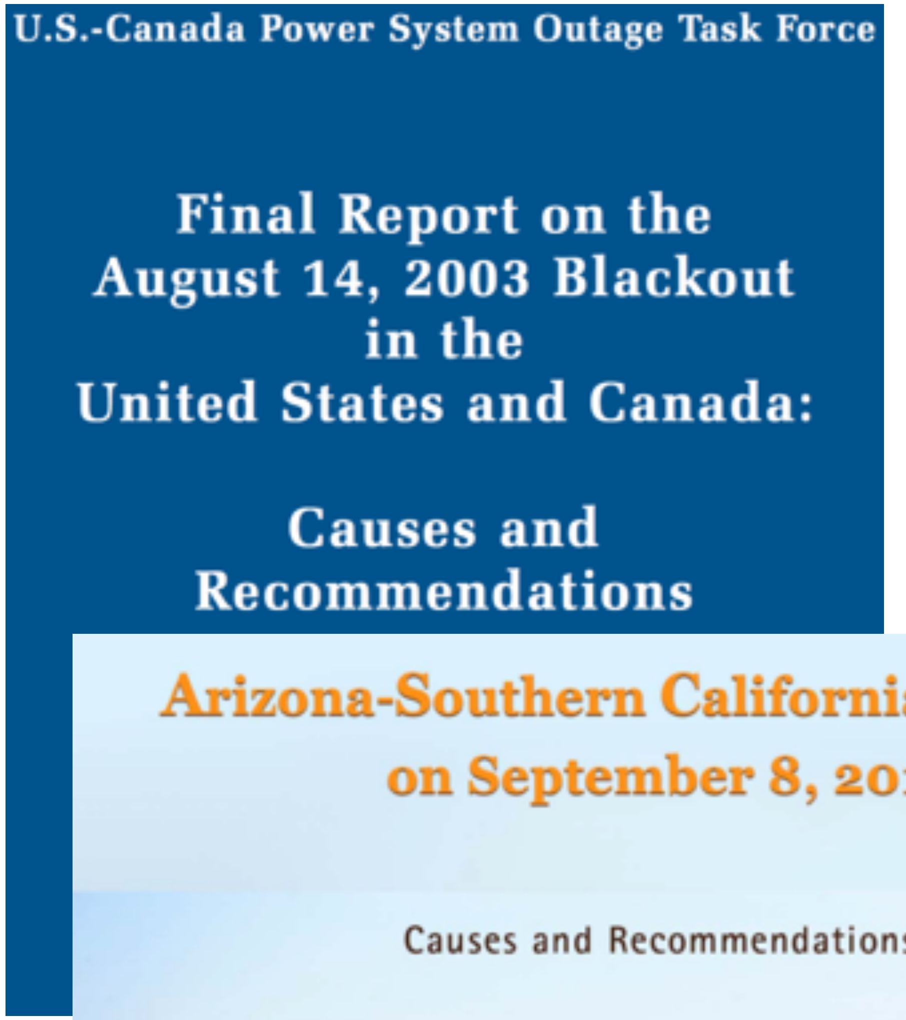
washingtonpost.com

Situational Awareness

Situational Awareness



Situational Awareness



Situational Awareness

U.S.-Canada Power System Outage Task Force

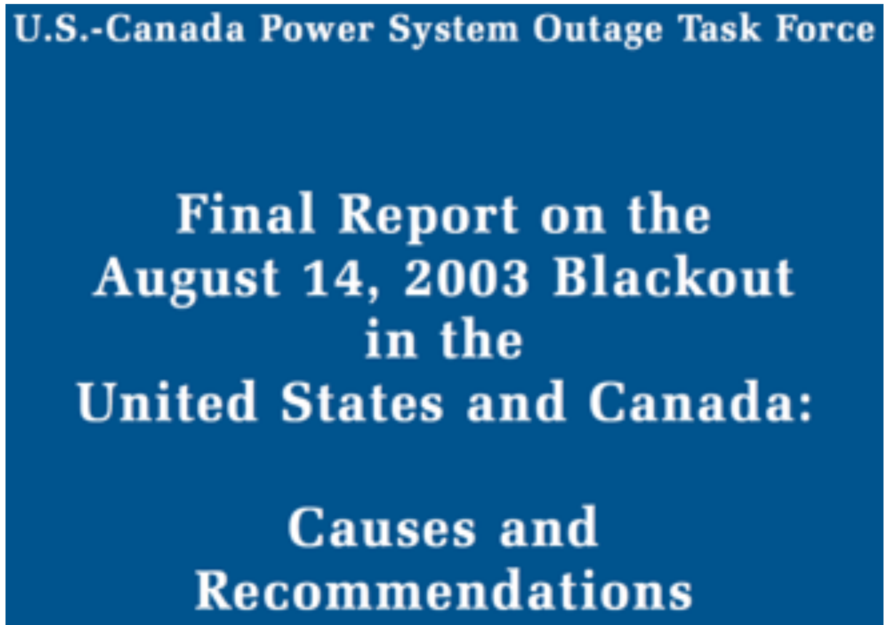
Final Report on the August 14, 2003 Blackout in the United States and Canada:

Causes and Recommendations

Inadequate Situational Awareness

The 2003 Blackout Report stated, "A principal cause of the August 14 blackout was a lack of situational awareness, which was in turn the result of inadequate reliability tools and backup capabilities."[109] Similarly, the instant inquiry determined that inadequate real-time situational awareness contributed to the cascading outages. In

Situational Awareness


U.S.-Canada Power System Outage Task Force

Final Report on the August 14, 2003 Blackout in the United States and Canada:

Causes and Recommendations

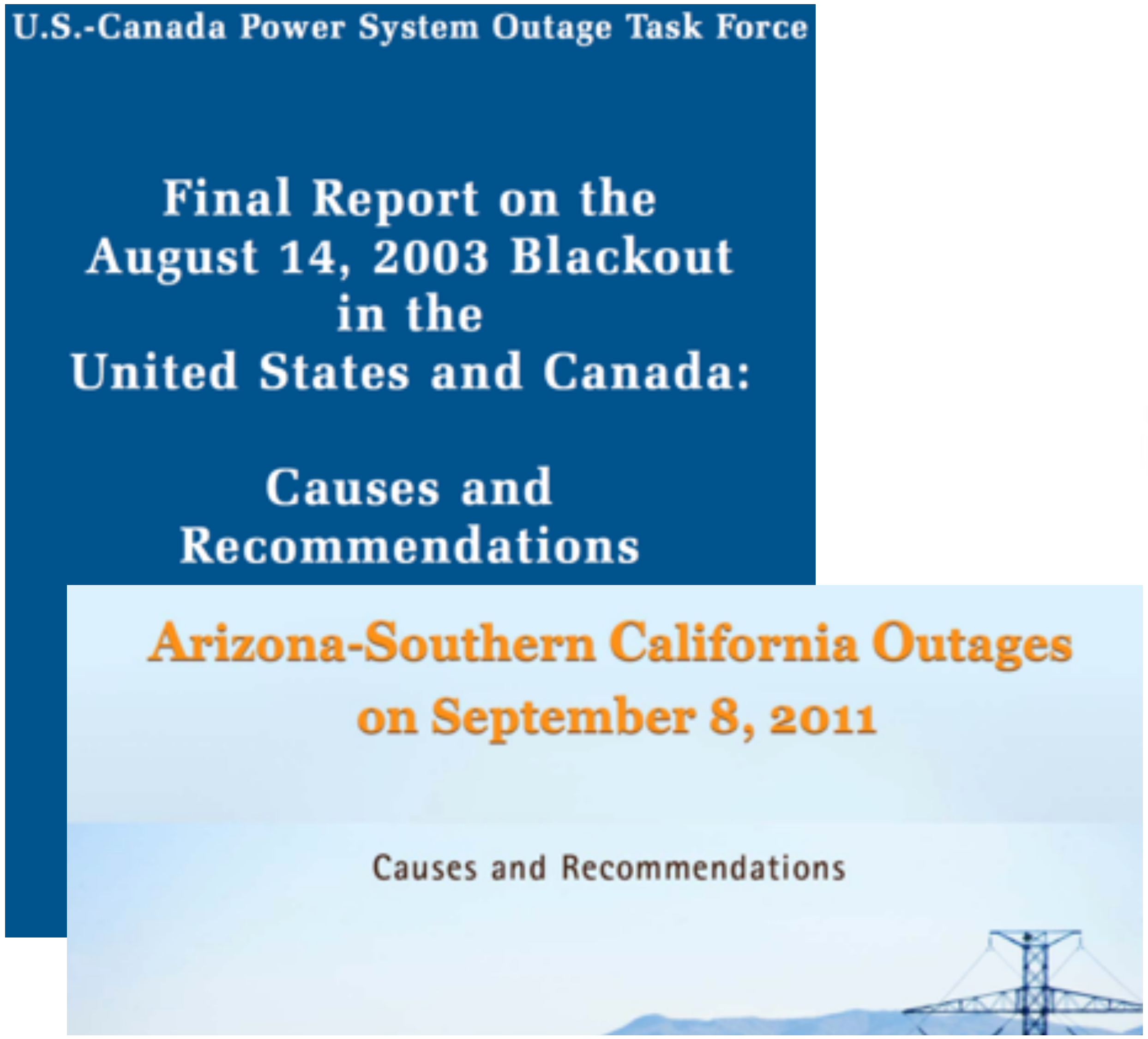

Arizona-Southern California Outages on September 8, 2011

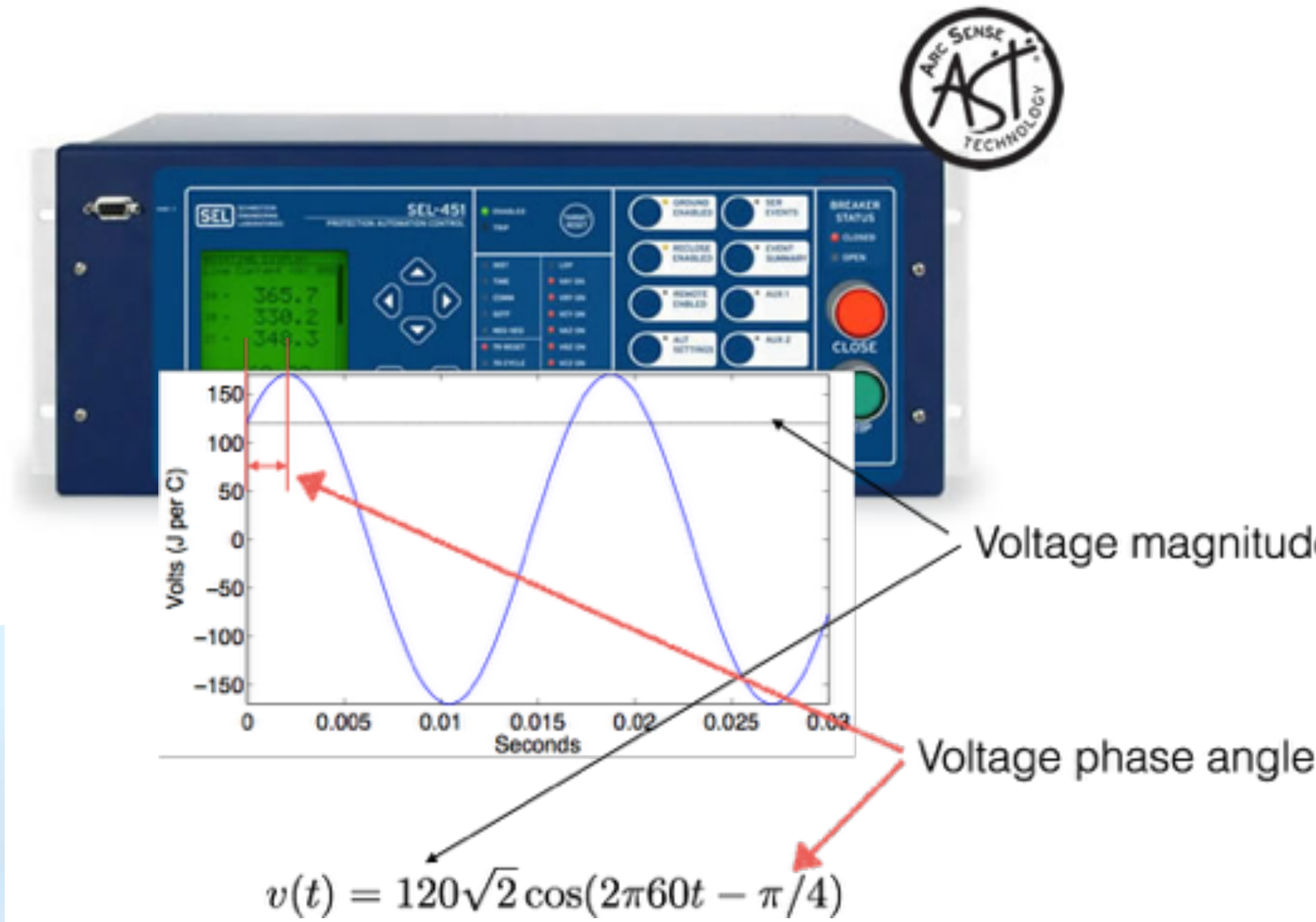
Causes and Recommendations

Inadequate Situational Awareness

The 2003 Blackout Report stated, "A principal cause of the August 14 blackout was a lack of situational awareness, which was in turn the result of inadequate reliability tools and backup capabilities."[109] Similarly, the instant inquiry determined that inadequate real-time situational awareness contributed to the cascading outages. In

Situational Awareness





Inadequate Situational Awareness

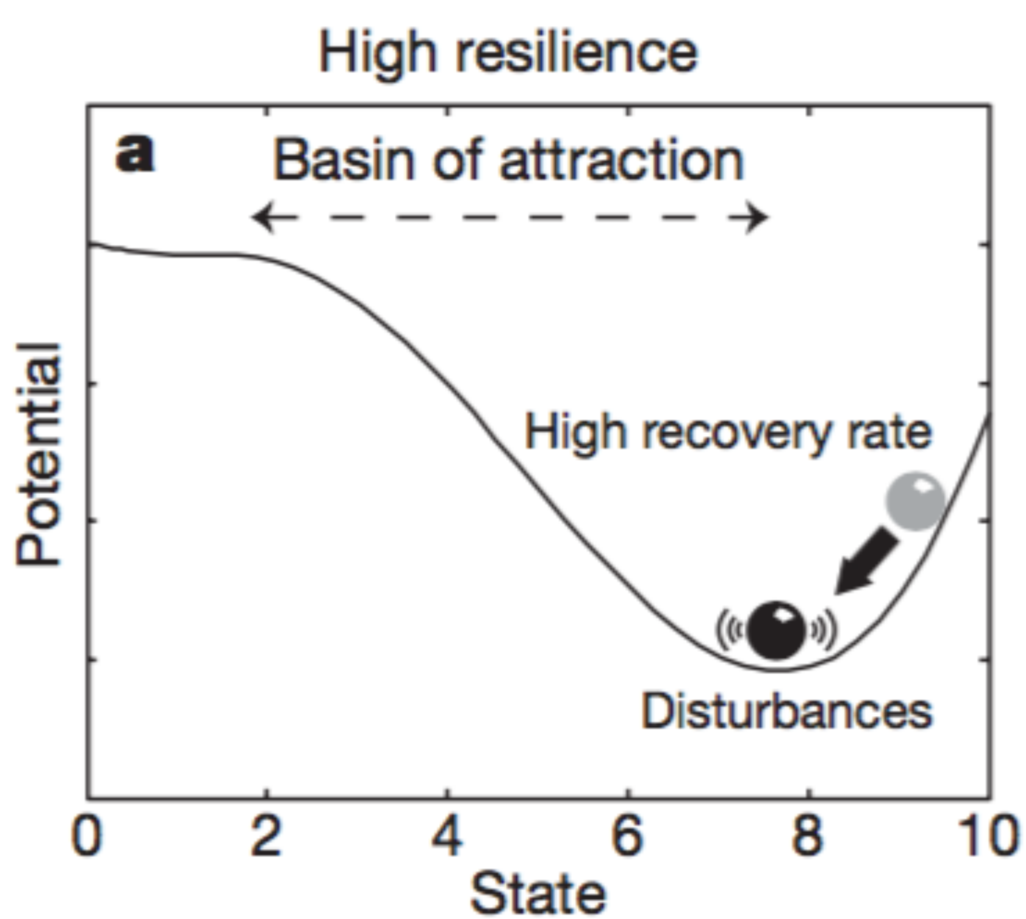
The 2003 Blackout Report stated, "A principal cause of the August 14 blackout was a lack of situational awareness, which was in turn the result of inadequate reliability tools and backup capabilities."[109] Similarly, the instant inquiry determined that inadequate real-time situational awareness contributed to the cascading outages. In

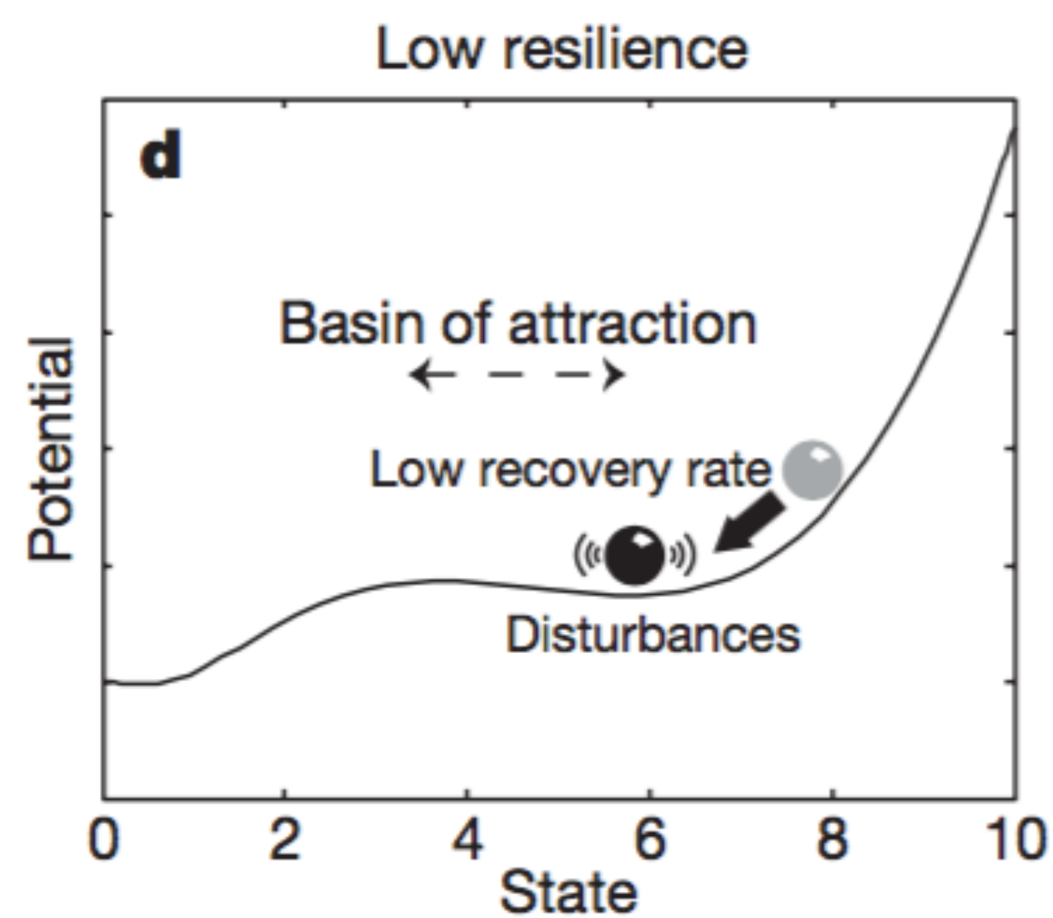
Vol 461|3 September 2009|doi:10.1038/nature08227 nature

REVIEWS

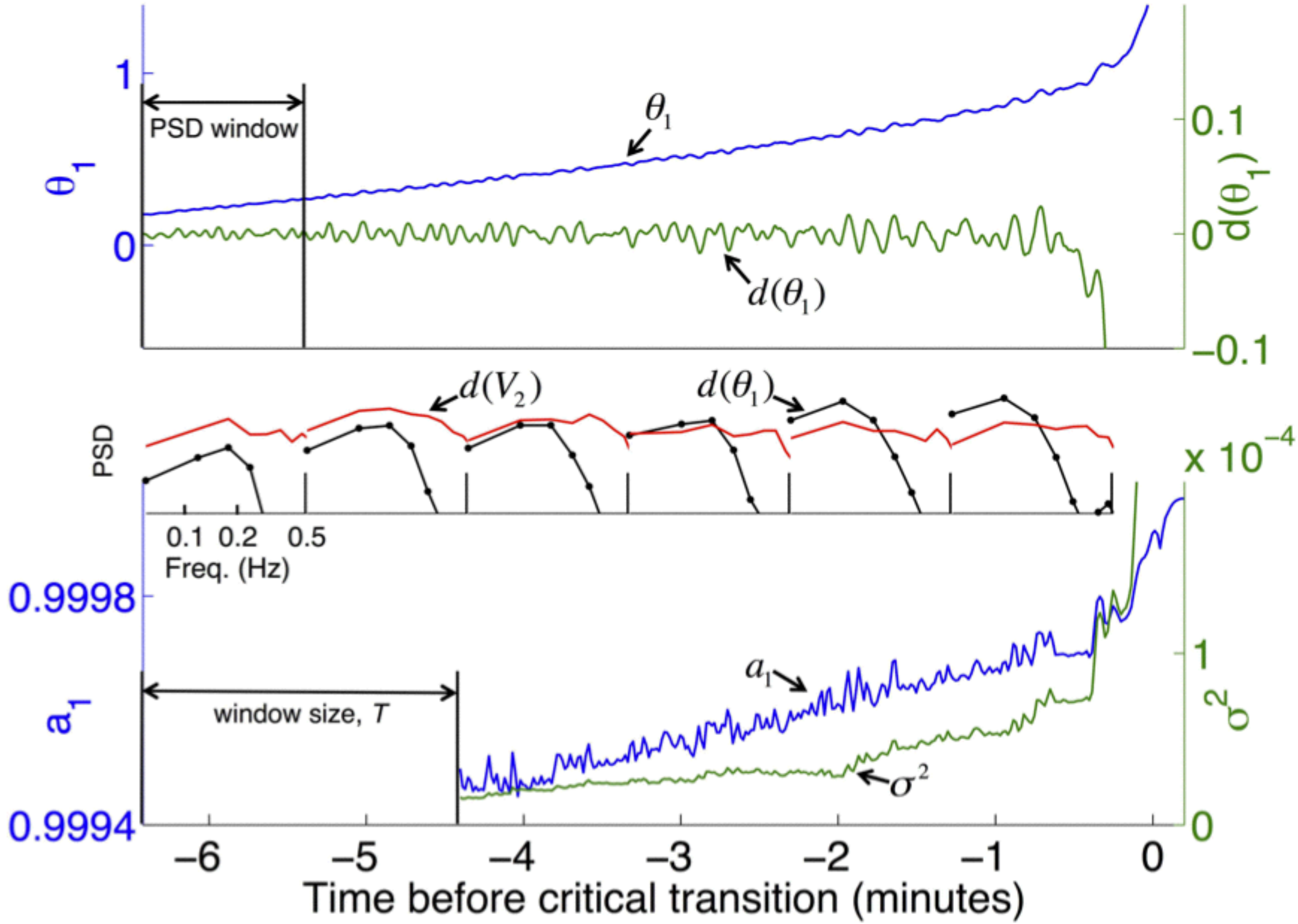
Early-warning signals for critical transitions

Marten Scheffer[1], Jordi Bascompte[2], William A. Brock[3], Victor Brovkin[5], Stephen R. Carpenter[4], Vasilis Dakos[1], Hermann Held[6], Egbert H. van Nes[1], Max Rietkerk[7] & George Sugihara[8]

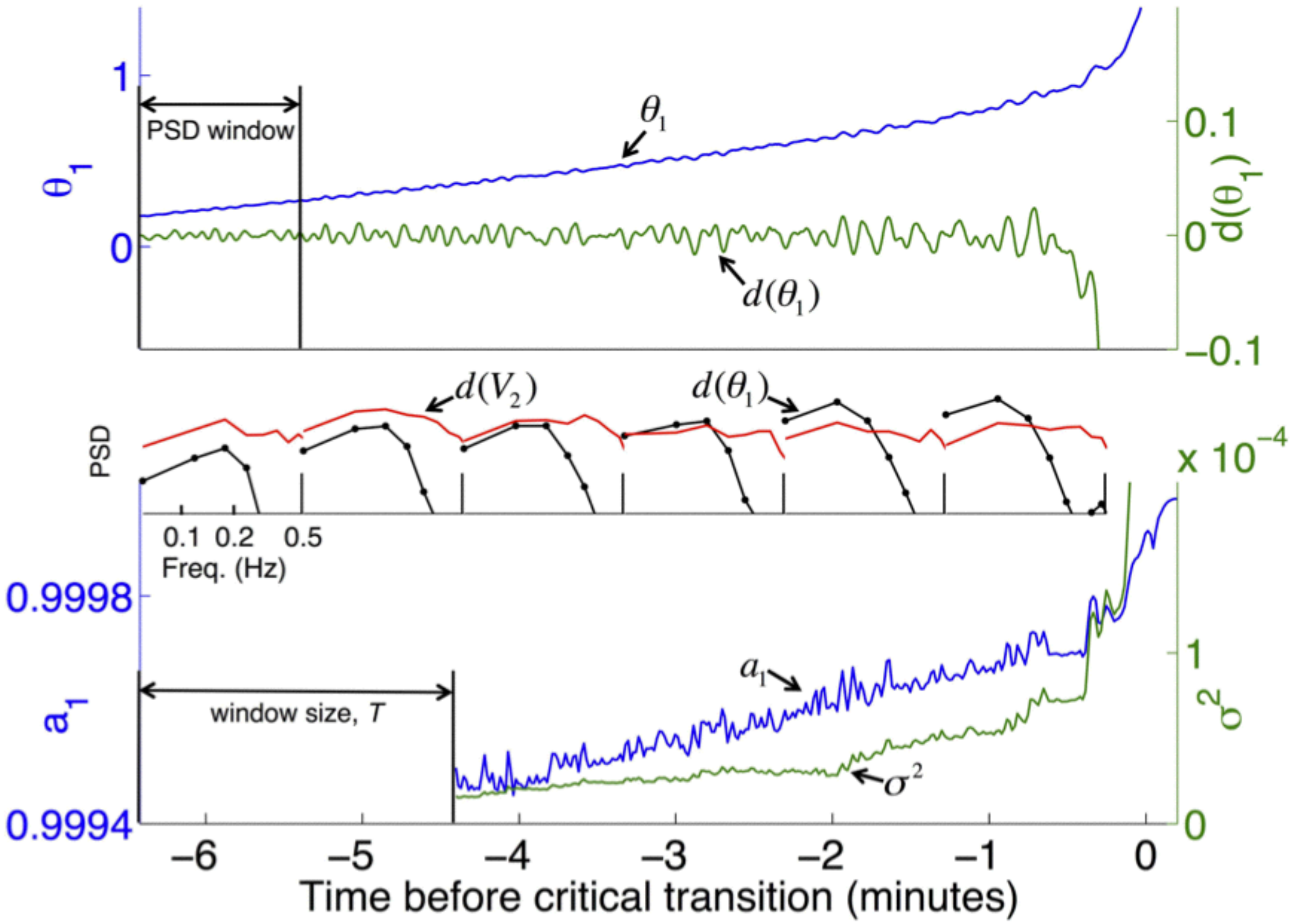




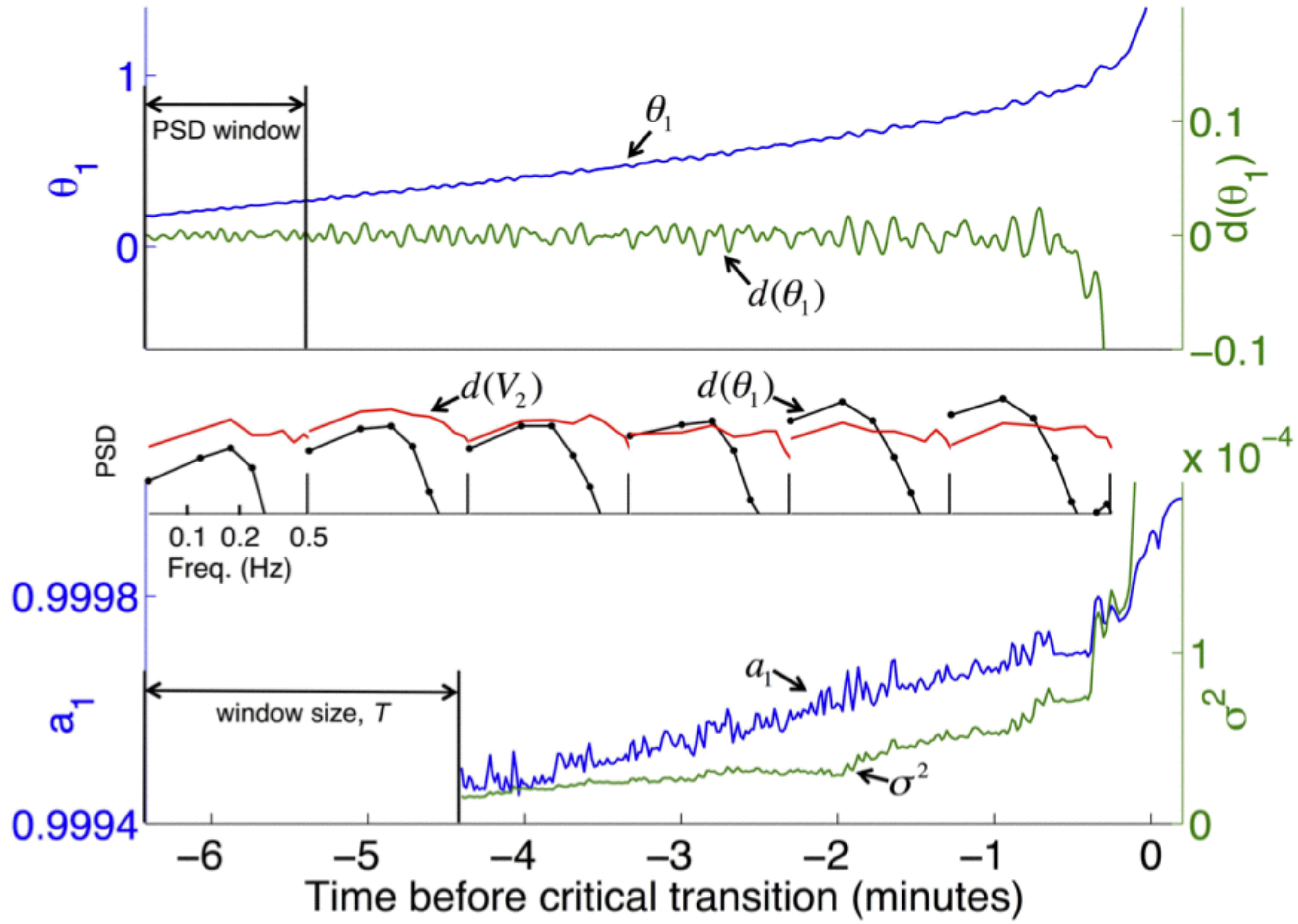
Sure enough…statistics can be useful indicators



Sure enough…statistics can be useful indicators



Sure enough…statistics can be useful indicators



Cotilla-Sanchez, Hines, Danforth, IEEE Trans Smart Grid, 2012.
See also:
DeMarco and Berge, IEEE Trans on Ckt & Sys, 1987.
Dhople, Chen, DeVille, Domínguez-García, IEEE Trans on Ckt Sys, 2013
Podolsky and Turitsyn, arXiv:1307.4318, Jul. 2013.
Susuki and Mezic, IEEE Trans. Power Syst., 2012 (and others)

How can we find the useful* statistical early warning signs?

**Useful*: A sign that shows up early enough that we might actually be able to do something about it, even if there is measurement noise

First let's define our SDEs

First let's define our SDEs

$$\dot{\underline{x}} = f\left(\underline{x}, \underline{y}\right)$$

Differential equations. (swing eqs., governors, exciters, etc.)

$$0 = g\left(\underline{x}, \underline{y}, \underline{u}\right)$$

Algebraic equations

First let's define our SDEs

$$\dot{\underline{x}} = f\left(\underline{x}, \underline{y}\right)$$

Differential equations. (swing eqs., governors, exciters, etc.)

$$0 = g\left(\underline{x}, \underline{y}, \underline{u}\right)$$

Algebraic equations

r.v. for stochastic load perturbations

First let's define our SDEs

$$\underline{\dot{x}} = f\left(\underline{x}, \underline{y}\right)$$

Differential equations. (swing eqs., governors, exciters, etc.)

$$0 = g\left(\underline{x}, \underline{y}, \underline{u}\right)$$

Algebraic equations

r.v. for stochastic load perturbations

$$\underline{\dot{u}} = -E\underline{u} + C\underline{\xi}$$

Loads modeled as Ornstein–Uhlenbeck process

First let's define our SDEs

$$\dot{\underline{x}} = f\left(\underline{x}, \underline{y}\right)$$

Differential equations. (swing eqs., governors, exciters, etc.)

$$0 = g\left(\underline{x}, \underline{y}, \underline{u}\right)$$

Algebraic equations

r.v. for stochastic load perturbations

$$\dot{\underline{u}} = -E\underline{u} + C\underline{\xi}$$

Loads modeled as Ornstein–Uhlenbeck process

Encodes corr. time of load fluctuations

First let's define our SDEs

$$\dot{\underline{x}} = f\left(\underline{x}, \underline{y}\right)$$

Differential equations. (swing eqs., governors, exciters, etc.)

$$0 = g\left(\underline{x}, \underline{y}, \underline{u}\right)$$

Algebraic equations

r.v. for stochastic load perturbations

$$\dot{\underline{u}} = -E\underline{u} + C\underline{\xi}$$

Loads modeled as Ornstein–Uhlenbeck process

Ind. Gaussian r.v.s, 1% std. dev.

Encodes corr. time of load fluctuations

Choose an operating point, and linearize around that point

$$\Delta \underline{y} = \begin{bmatrix} -g_y^{-1} g_x & -g_y^{-1} g_u \end{bmatrix} \begin{bmatrix} \Delta \underline{x} \\ \Delta \underline{u} \end{bmatrix}$$

$$\begin{bmatrix} \Delta \dot{\underline{x}} \\ \Delta \dot{\underline{u}} \end{bmatrix} = \begin{bmatrix} f_x - f_y g_y^{-1} g_x & -f_y g_y^{-1} g_u \\ 0 & -E \end{bmatrix} \begin{bmatrix} \Delta \underline{x} \\ \Delta \underline{u} \end{bmatrix} + \begin{bmatrix} 0 \\ C \end{bmatrix} \underline{\xi}$$

Choose an operating point, and linearize around that point

$$\Delta \underline{y} = \begin{bmatrix} -g_y^{-1} g_x & -g_y^{-1} g_u \end{bmatrix} \begin{bmatrix} \Delta \underline{x} \\ \Delta \underline{u} \end{bmatrix}$$

$$\begin{bmatrix} \Delta \underline{\dot{x}} \\ \Delta \underline{\dot{u}} \end{bmatrix} = \begin{bmatrix} f_x - f_y g_y^{-1} g_x & -f_y g_y^{-1} g_u \\ 0 & -E \end{bmatrix} \begin{bmatrix} \Delta \underline{x} \\ \Delta \underline{u} \end{bmatrix} + \begin{bmatrix} 0 \\ C \end{bmatrix} \underline{\xi}$$

Jacobian matrix: *df/dx*

Choose an operating point, and linearize around that point

$$\Delta \underline{y} = \begin{bmatrix} -g_y^{-1} g_x & -g_y^{-1} g_u \end{bmatrix} \begin{bmatrix} \Delta \underline{x} \\ \Delta \underline{u} \end{bmatrix}$$

$$\begin{bmatrix} \Delta \underline{\dot{x}} \\ \Delta \underline{\dot{u}} \end{bmatrix} = \begin{bmatrix} f_x - f_y g_y^{-1} g_x & -f_y g_y^{-1} g_u \\ 0 & -E \end{bmatrix} \begin{bmatrix} \Delta \underline{x} \\ \Delta \underline{u} \end{bmatrix} + \begin{bmatrix} 0 \\ C \end{bmatrix} \underline{\xi}$$

Choose an operating point, and linearize around that point

$$\Delta \underline{y} = \begin{bmatrix} -g_y^{-1} g_x & -g_y^{-1} g_u \end{bmatrix} \begin{bmatrix} \Delta \underline{x} \\ \Delta \underline{u} \end{bmatrix}$$

$$\begin{bmatrix} \Delta \dot{\underline{x}} \\ \Delta \dot{\underline{u}} \end{bmatrix} = \begin{bmatrix} f_x - f_y g_y^{-1} g_x & -f_y g_y^{-1} g_u \\ 0 & -E \end{bmatrix} \begin{bmatrix} \Delta \underline{x} \\ \Delta \underline{u} \end{bmatrix} + \begin{bmatrix} 0 \\ C \end{bmatrix} \underline{\xi}$$

Which gives us a system of SDEs in Ornstein–Uhlenbeck form:

Choose an operating point, and linearize around that point

$$\Delta \underline{y} = \begin{bmatrix} -g_y^{-1} g_x & -g_y^{-1} g_u \end{bmatrix} \begin{bmatrix} \Delta \underline{x} \\ \Delta \underline{u} \end{bmatrix}$$

$$\begin{bmatrix} \Delta \dot{\underline{x}} \\ \Delta \dot{\underline{u}} \end{bmatrix} = \begin{bmatrix} f_x - f_y g_y^{-1} g_x & -f_y g_y^{-1} g_u \\ 0 & -E \end{bmatrix} \begin{bmatrix} \Delta \underline{x} \\ \Delta \underline{u} \end{bmatrix} + \begin{bmatrix} 0 \\ C \end{bmatrix} \underline{\xi}$$

Which gives us a system of SDEs in Ornstein–Uhlenbeck form:

$$\dot{\underline{z}} = A \underline{z} + B \underline{\xi}$$

Choose an operating point, and linearize around that point

$$\Delta \underline{y} = \begin{bmatrix} -g_y^{-1} g_x & -g_y^{-1} g_u \end{bmatrix} \begin{bmatrix} \Delta \underline{x} \\ \Delta \underline{u} \end{bmatrix}$$

$$\begin{bmatrix} \Delta \dot{\underline{x}} \\ \Delta \dot{\underline{u}} \end{bmatrix} = \begin{bmatrix} f_x - f_y g_y^{-1} g_x & -f_y g_y^{-1} g_u \\ 0 & -E \end{bmatrix} \begin{bmatrix} \Delta \underline{x} \\ \Delta \underline{u} \end{bmatrix} + \begin{bmatrix} 0 \\ C \end{bmatrix} \underline{\xi}$$

Which gives us a system of SDEs in Ornstein–Uhlenbeck form:

$$\dot{\underline{z}} = A\underline{z} + B\underline{\xi}$$

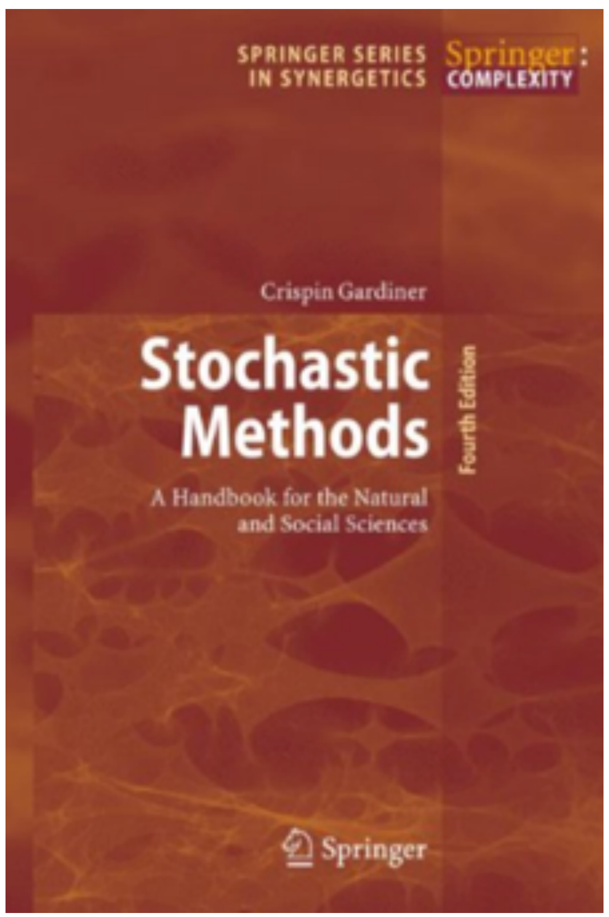
Now solve the SDEs

Now solve the SDEs

I'd like to tell you that we came up with new, elegant mathematics to solve. In reality…

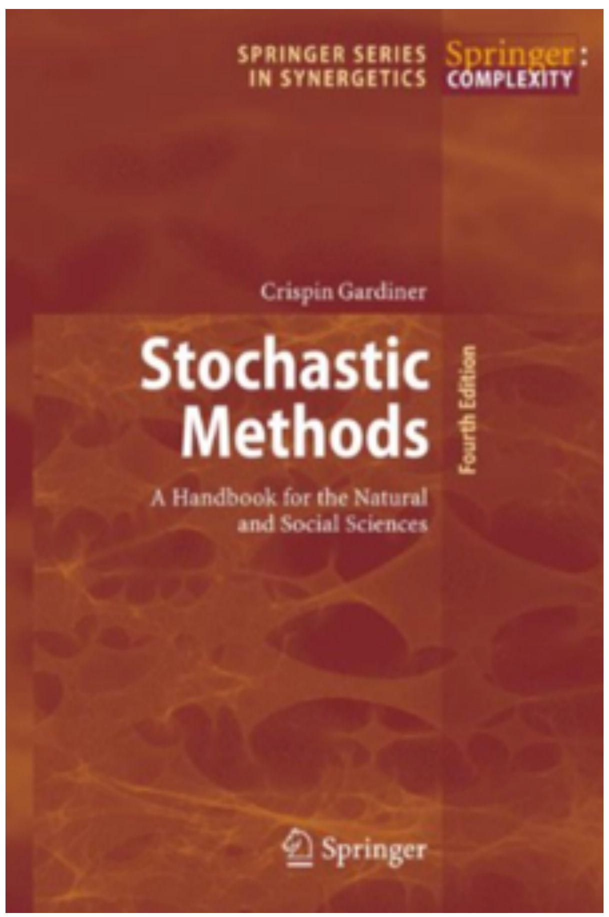
Now solve the SDEs

I'd like to tell you that we came up with new, elegant mathematics to solve. In reality…



Now solve the SDEs

I'd like to tell you that we came up with new, elegant mathematics to solve. In reality…

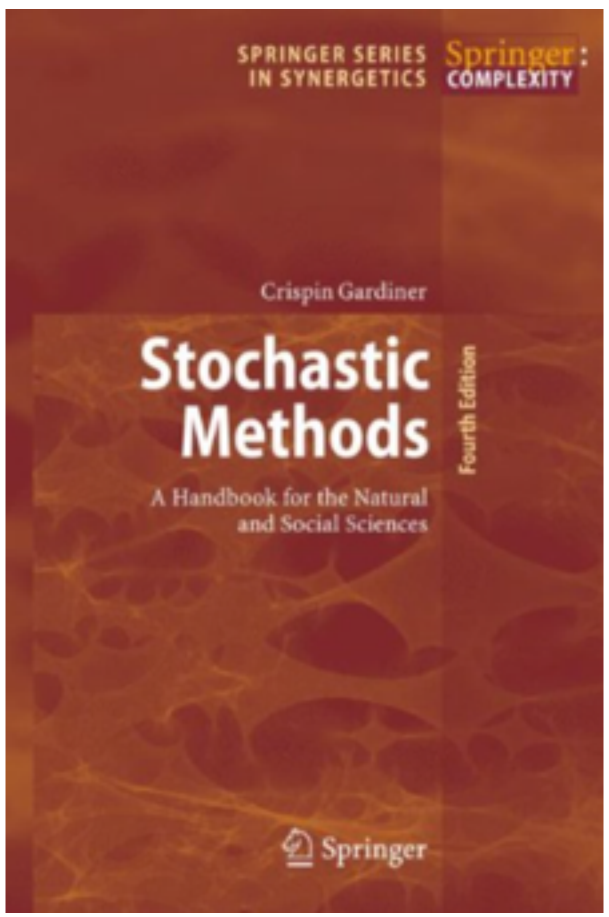


$$A\sigma_{\underline{z}} + \sigma_{\underline{z}} A^T = -BB^T$$

$$\mathrm{E}\left[\underline{z}\,(t)\,\underline{z}^T\,(s)\right] = \exp\left[-A|t-s|\right]\sigma_{\underline{z}}$$

Now solve the SDEs

I'd like to tell you that we came up with new, elegant mathematics to solve. In reality…

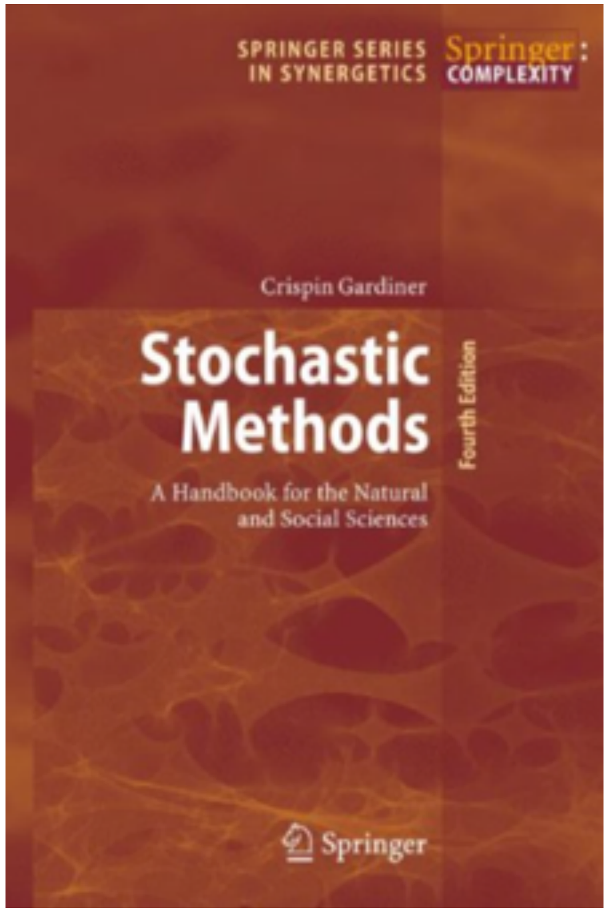


$$A\sigma_{\underline{z}} + \sigma_{\underline{z}} A^T = -BB^T$$ Lyapanov eq.

$$\mathrm{E}\left[\underline{z}\left(t\right)\underline{z}^T\left(s\right)\right] = \exp\left[-A|t-s|\right]\sigma_{\underline{z}}$$

Now solve the SDEs

I'd like to tell you that we came up with new, elegant mathematics to solve. In reality…



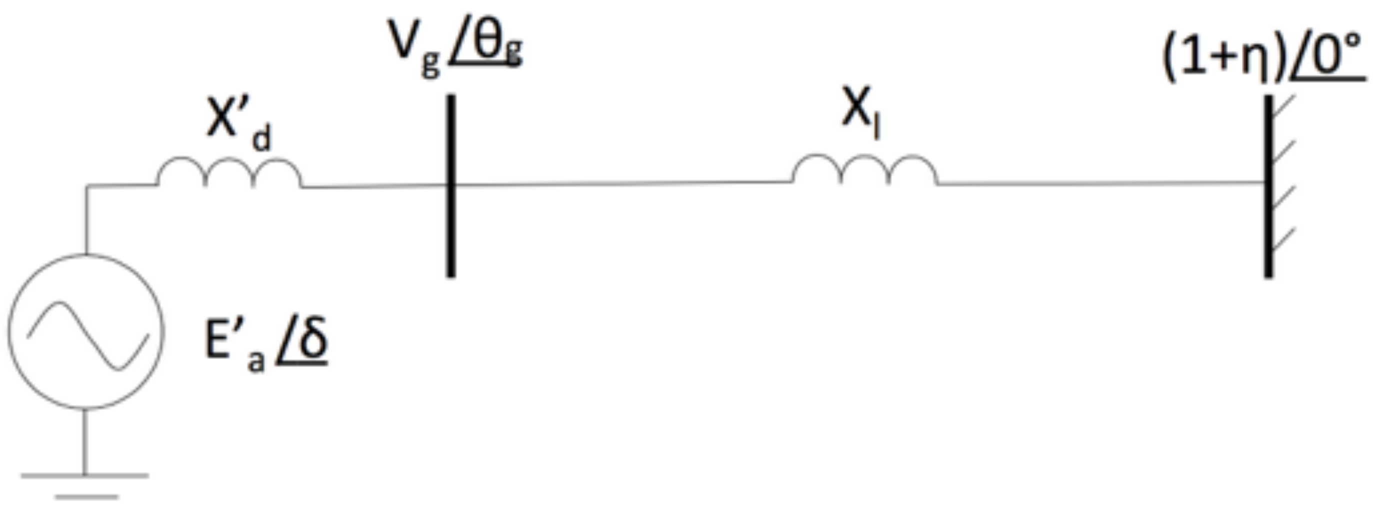
$$A\sigma_{\underline{z}} + \sigma_{\underline{z}} A^T = -BB^T$$ Lyapanov eq.

$$\mathrm{E}\left[\underline{z}\left(t\right)\underline{z}^T\left(s\right)\right] = \exp\left[-A|t-s|\right]\sigma_{\underline{z}}$$

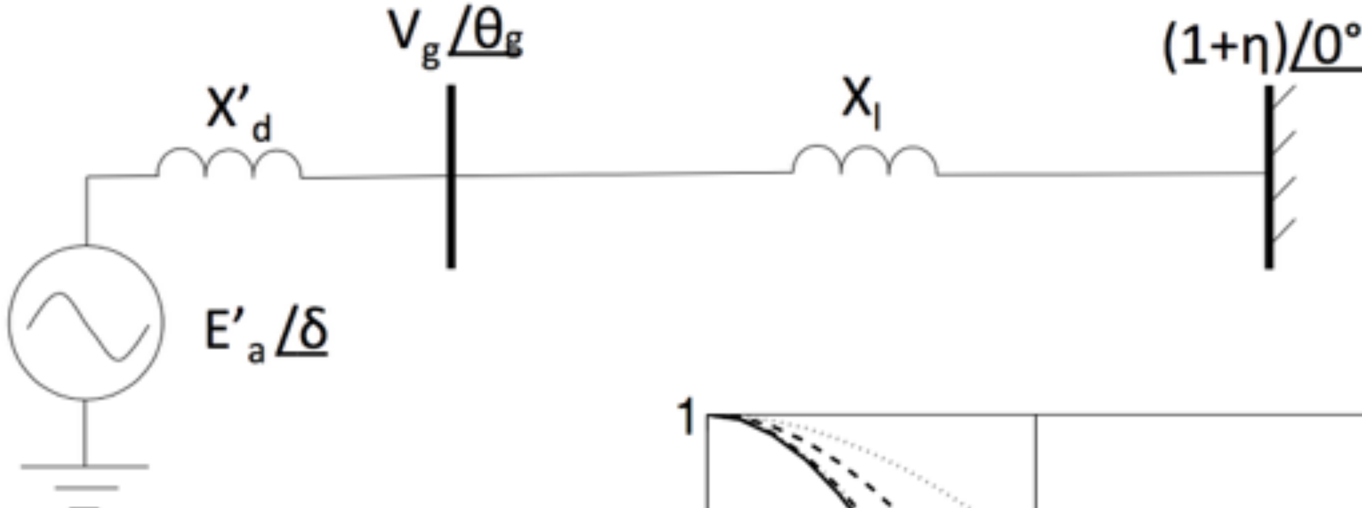
And then reverse the Kron reduction to compute the variance and autocorrelation of voltage and current magnitudes.

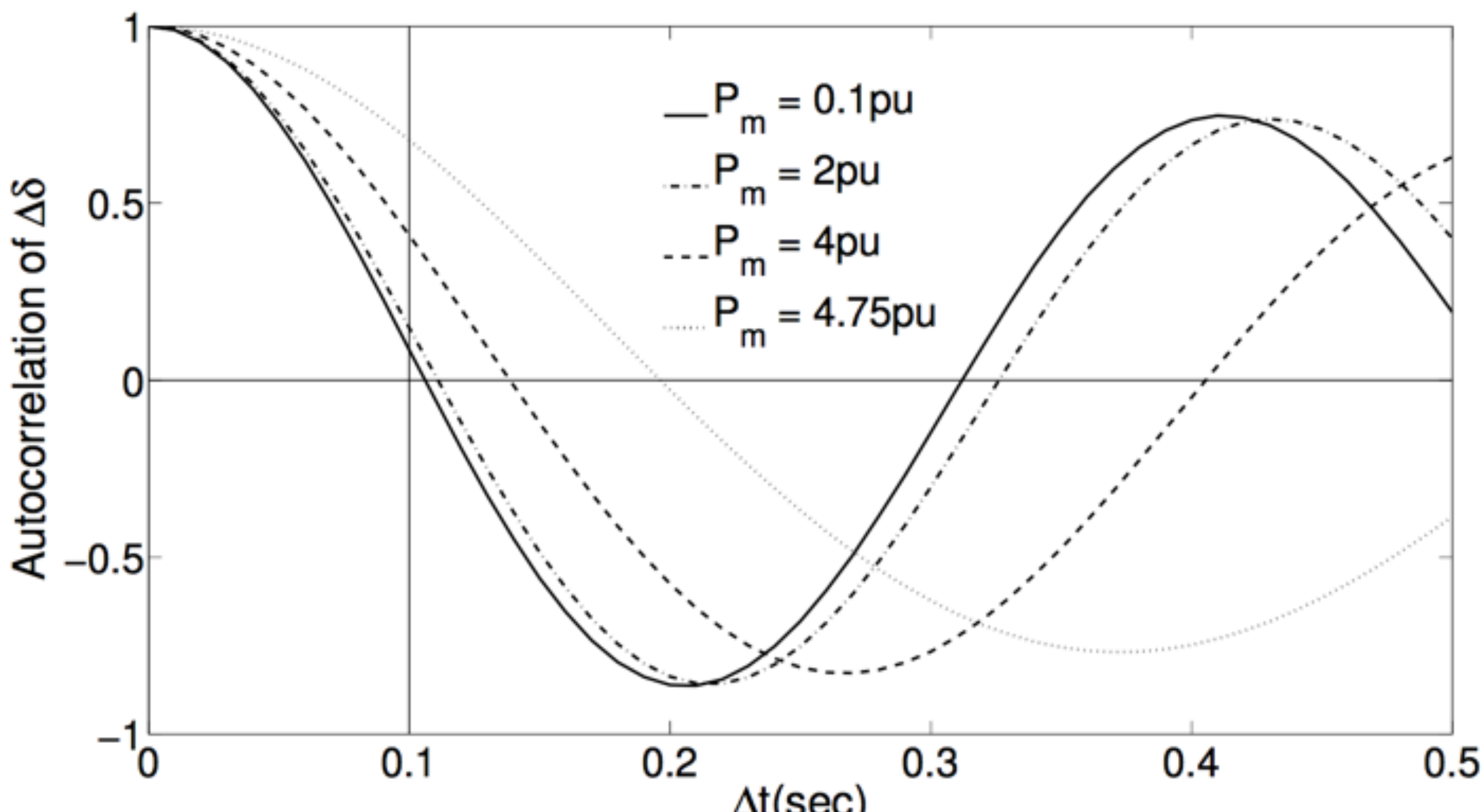
and choose a time delay for autocorrelation measurements

and choose a time delay for autocorrelation measurements



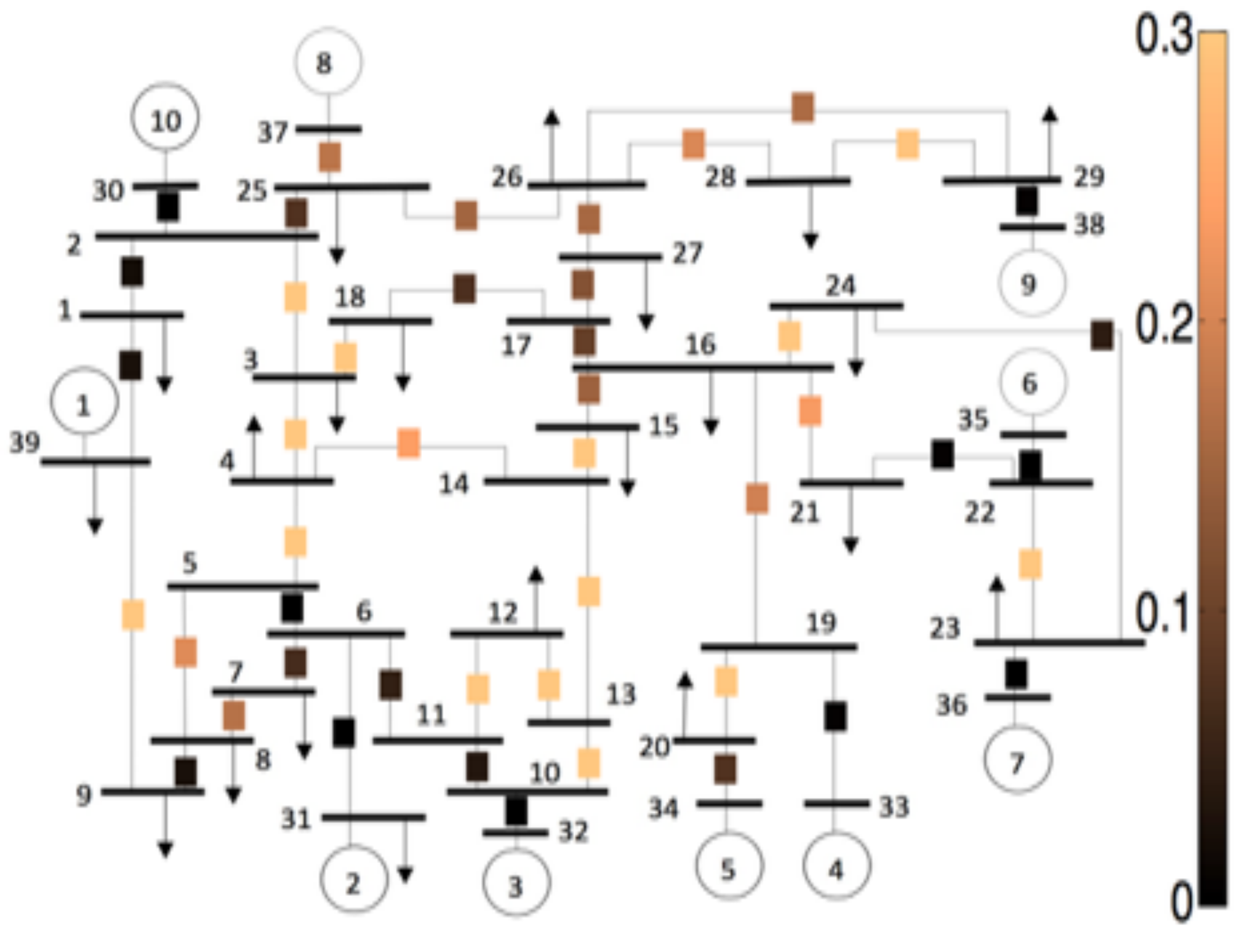
and choose a time delay for autocorrelation measurements



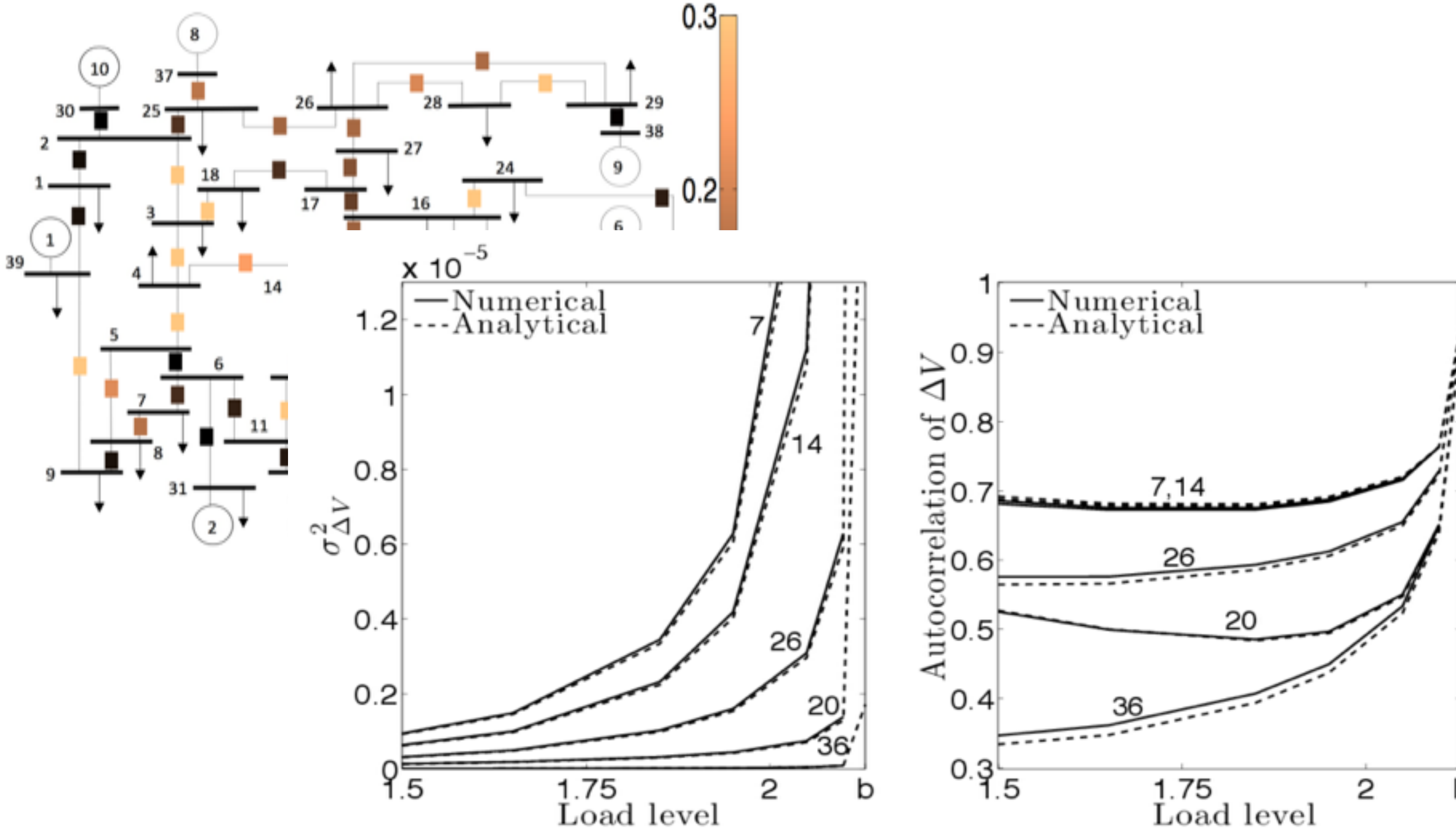


Check to make sure that the analytical and numerical line up

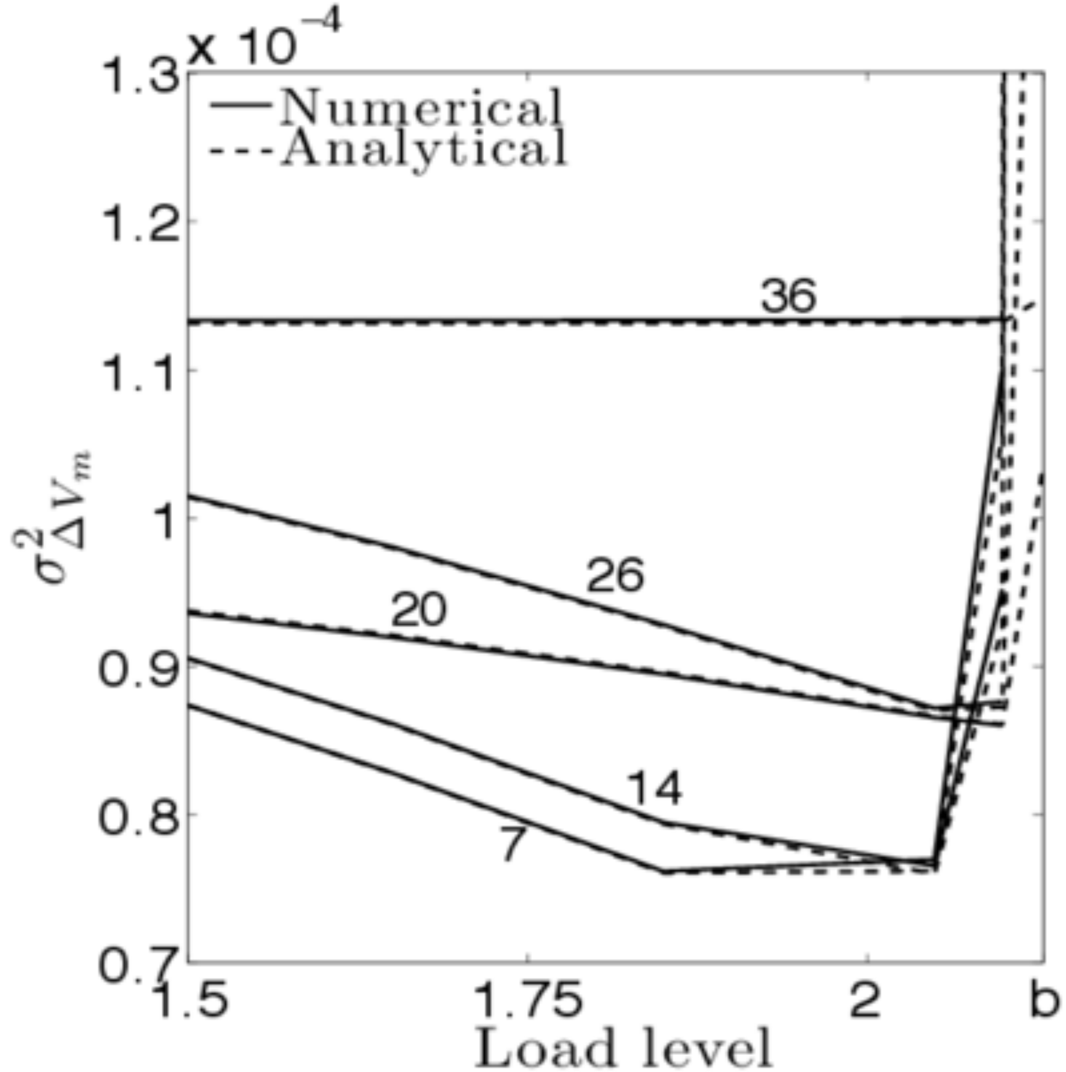
Check to make sure that the analytical and numerical line up

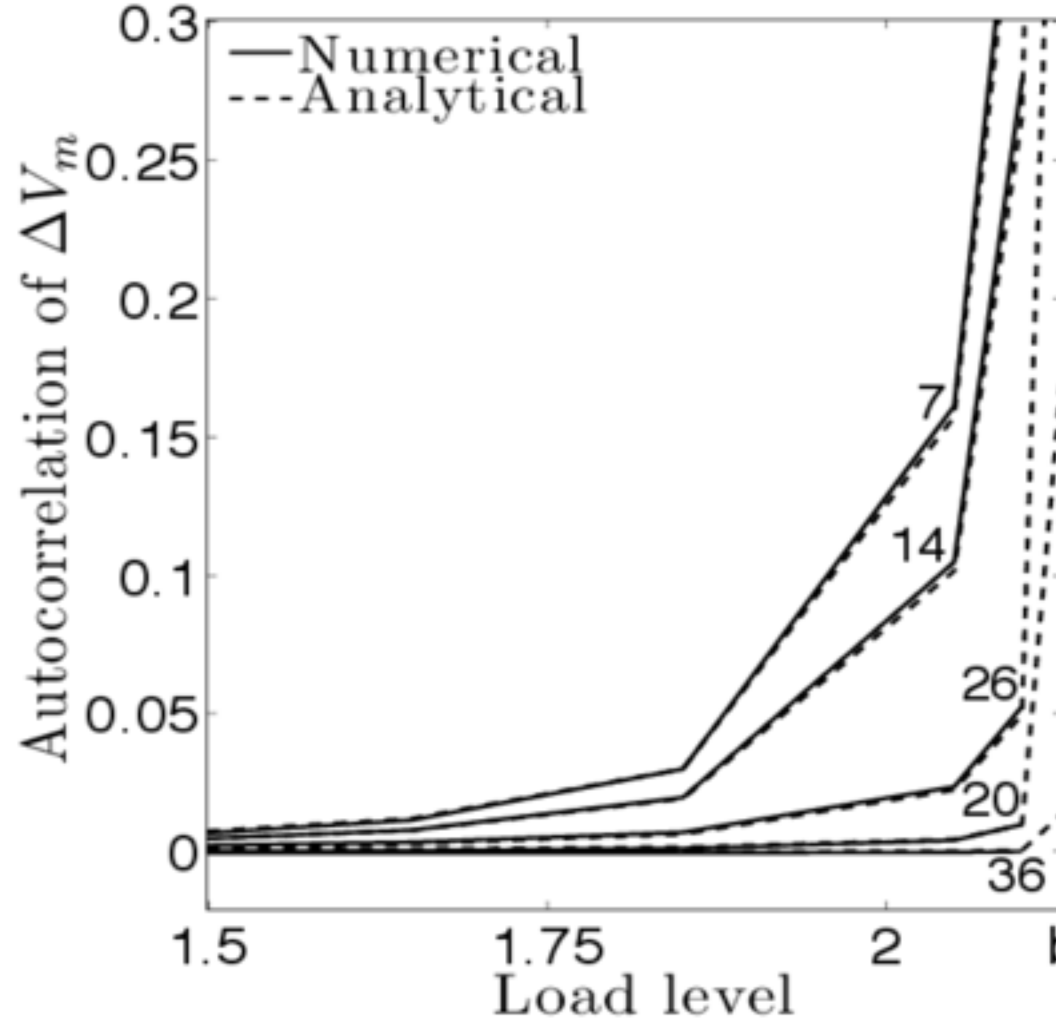


Check to make sure that the analytical and numerical line up

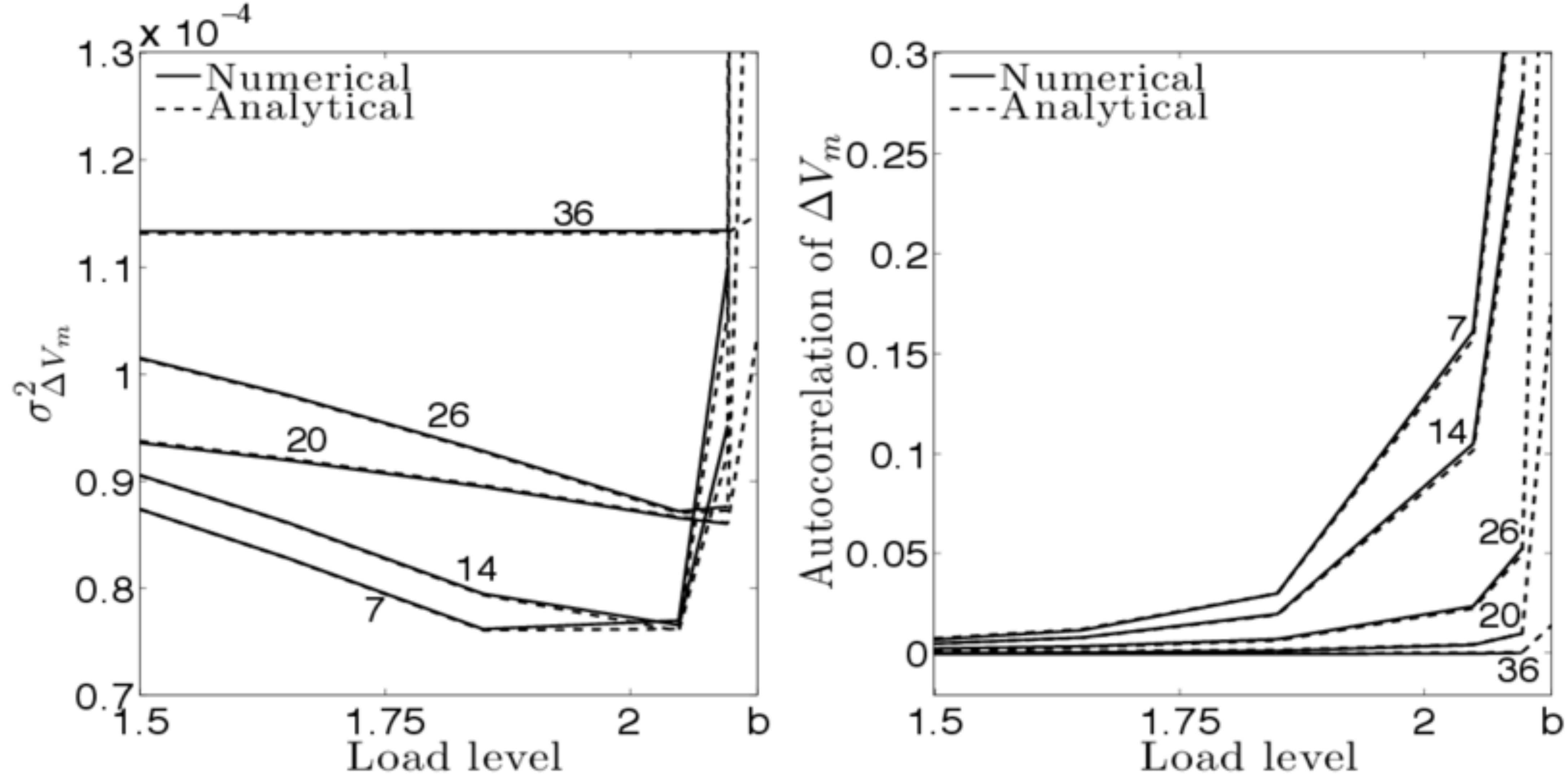


And add measurement noise





And add measurement noise



Which we can subsequently filter to
largely regain our original signal,
with the interesting side-effect that some of
the variance now appears as autocorrelation.

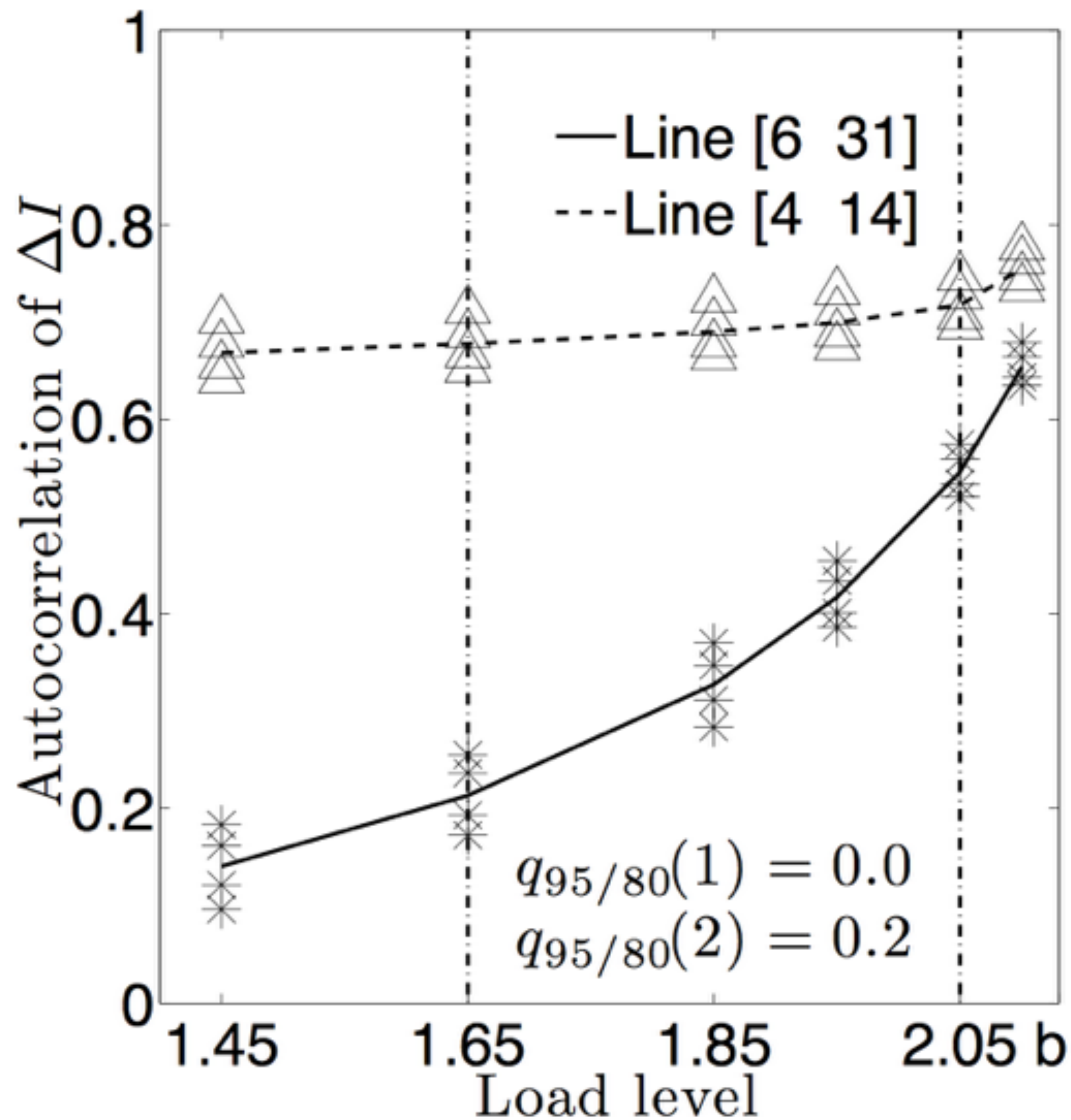
At key locations, we can see clear signs of instability in Autocorrelation and Variance

At key locations, we can see clear signs of instability in Autocorrelation and Variance

How do we measure
"detectability"
to distinguish
useful statistical signals
from non-useful ones?

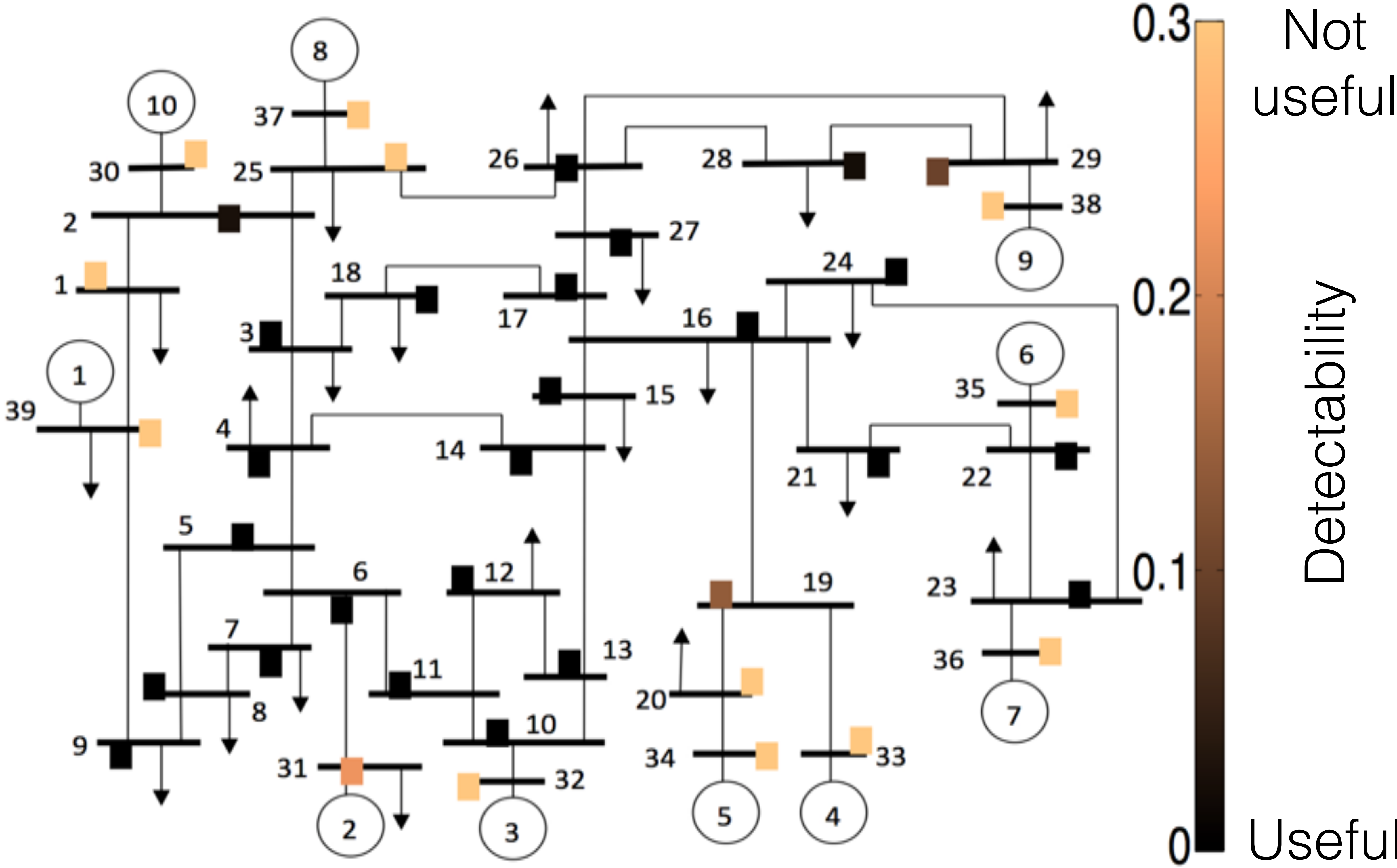
At key locations, we can see clear signs of instability in Autocorrelation and Variance

How do we measure
"detectability"
to distinguish
useful statistical signals
from non-useful ones?

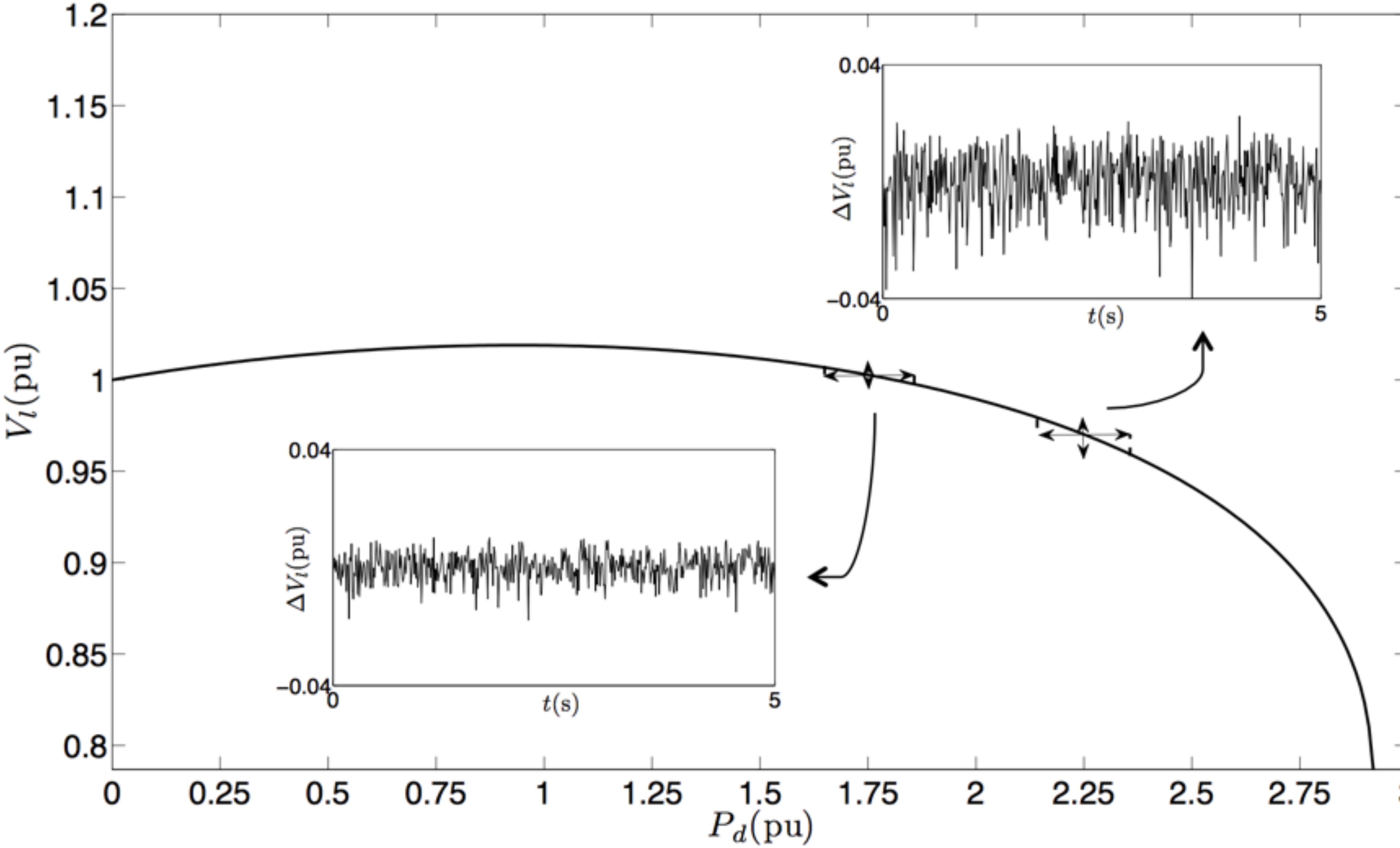


Which statistics provide useful (detectable) early warning?

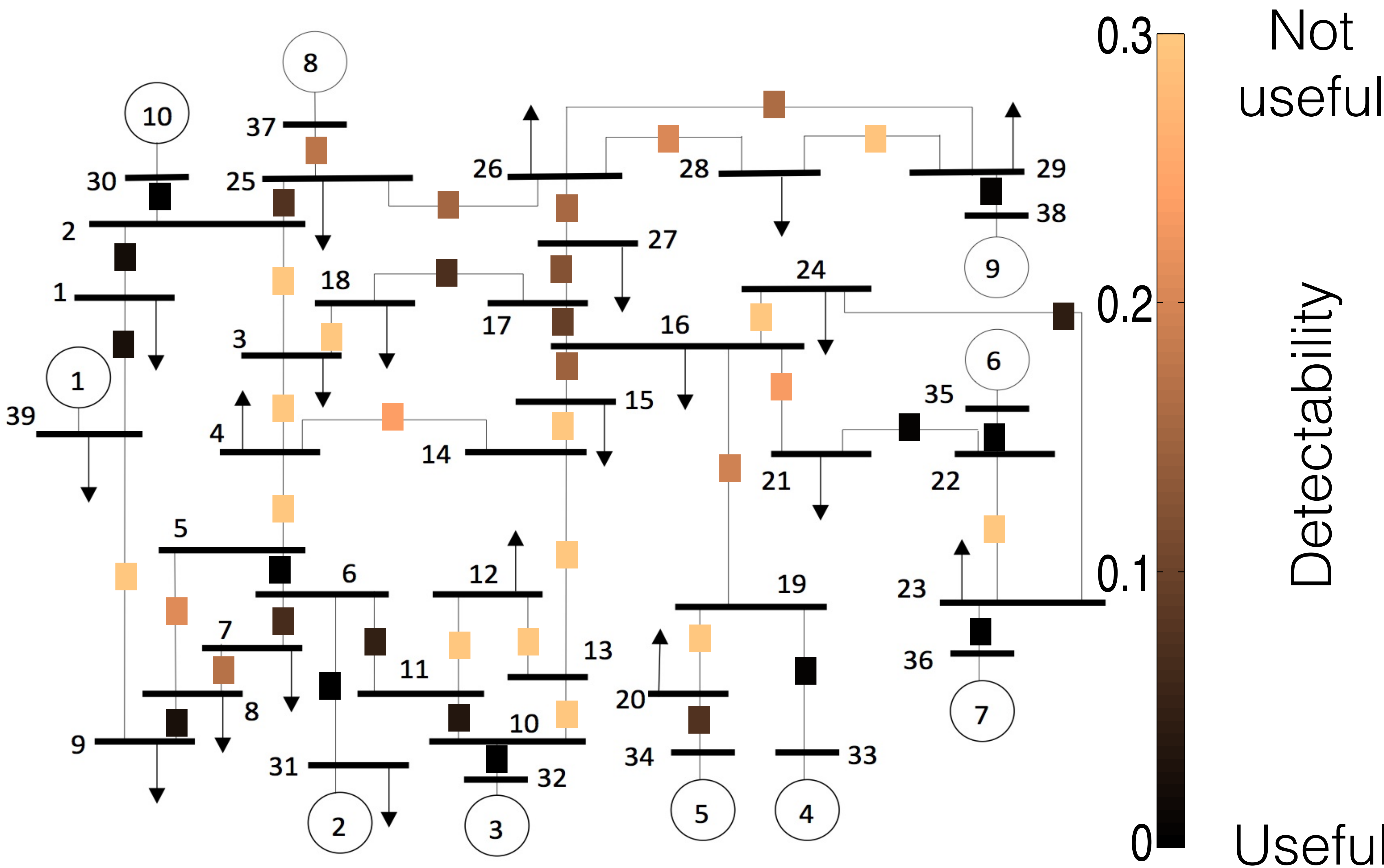
Variance of voltages



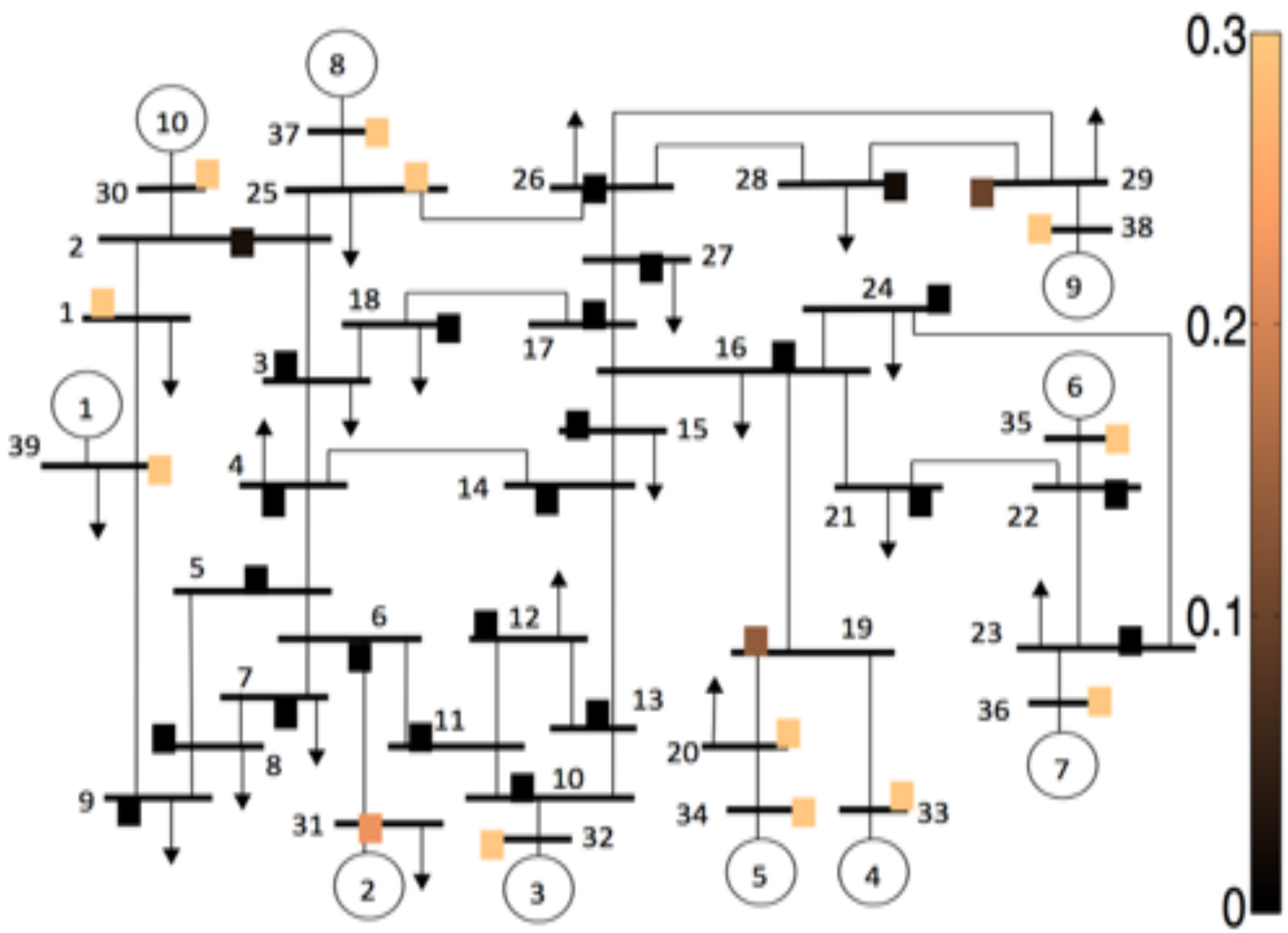
Why is variance in voltage useful?



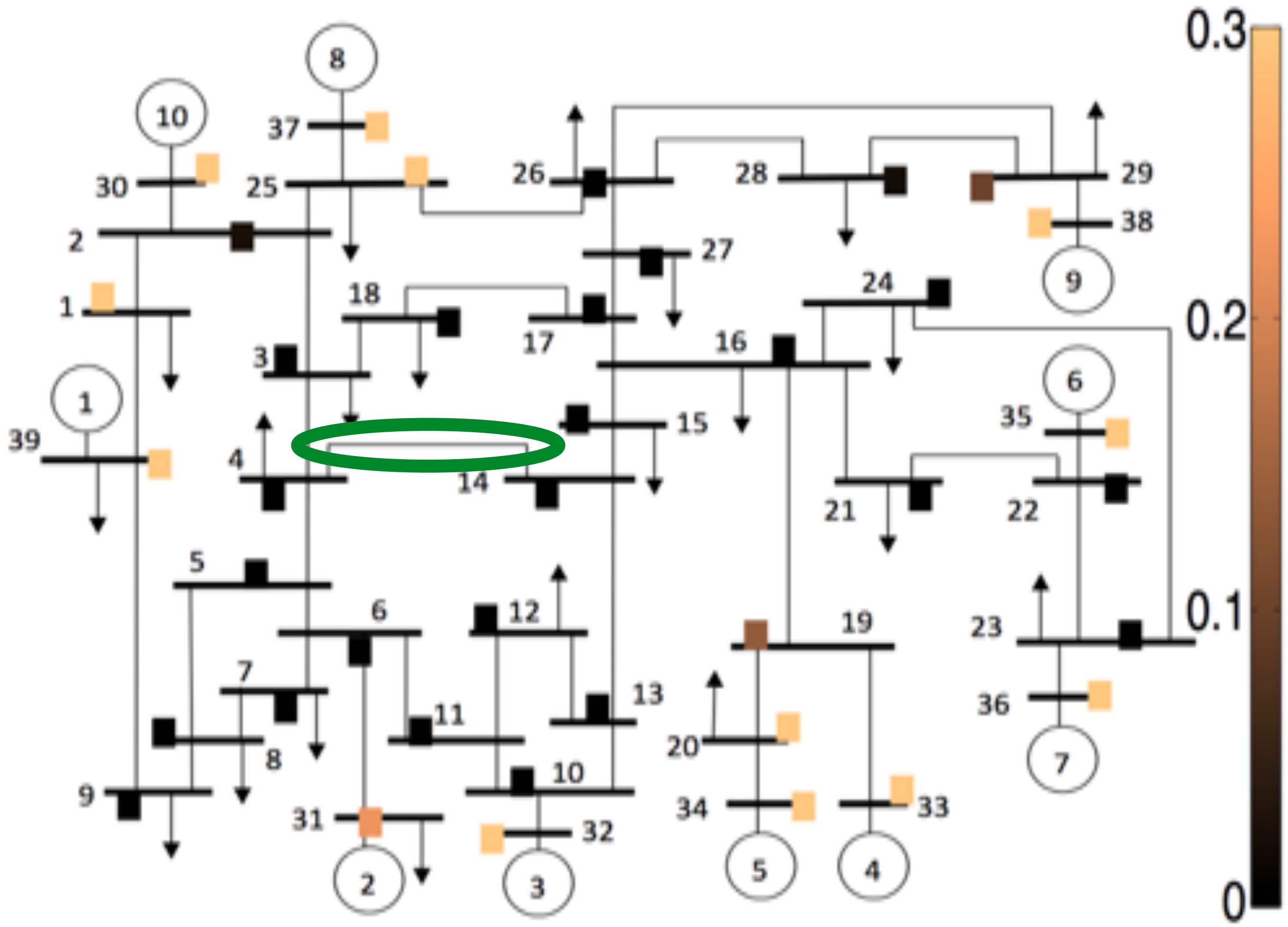
Autocorrelation of currents



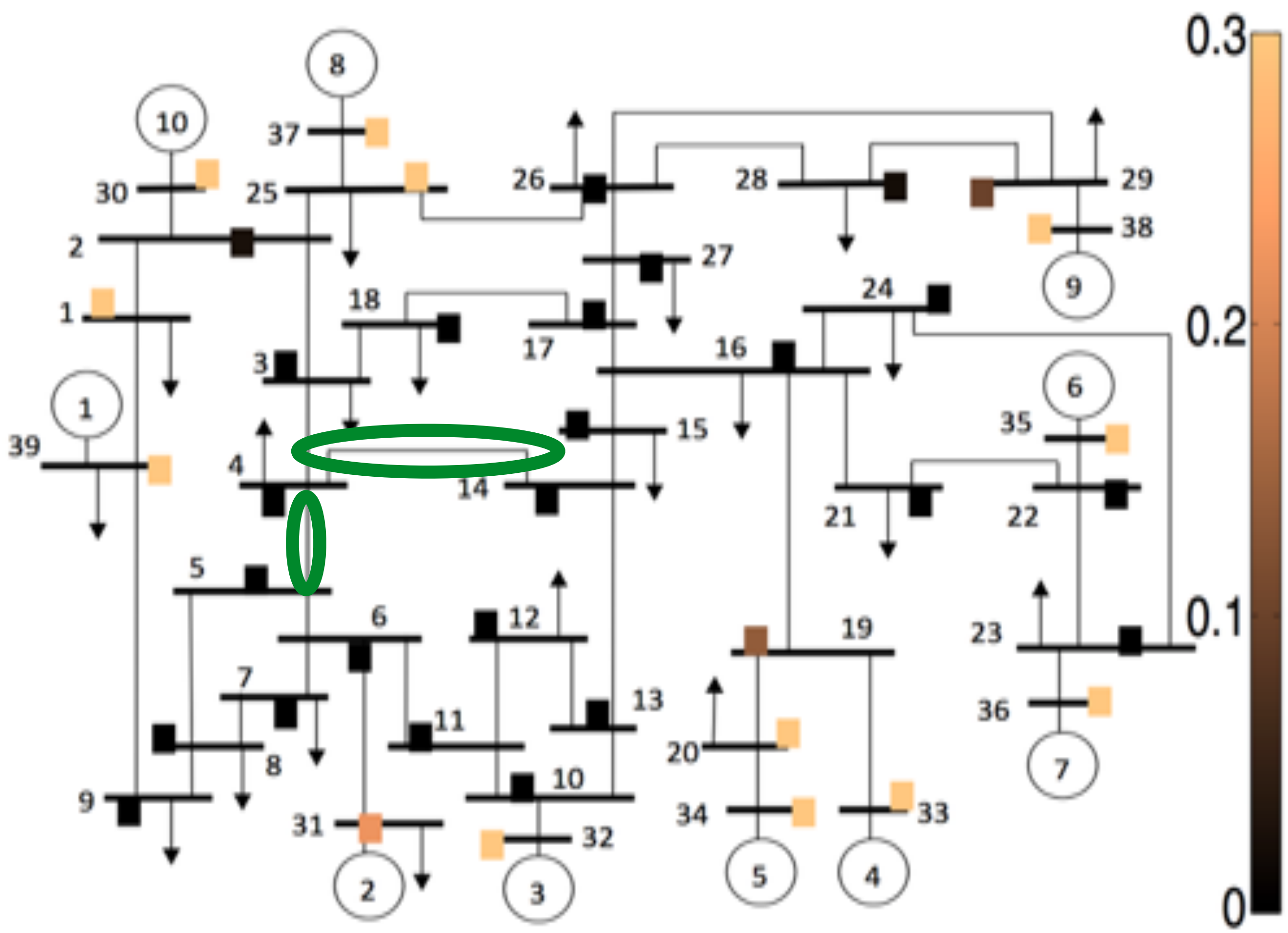
Can we find the location/source of a problem given the statistics?



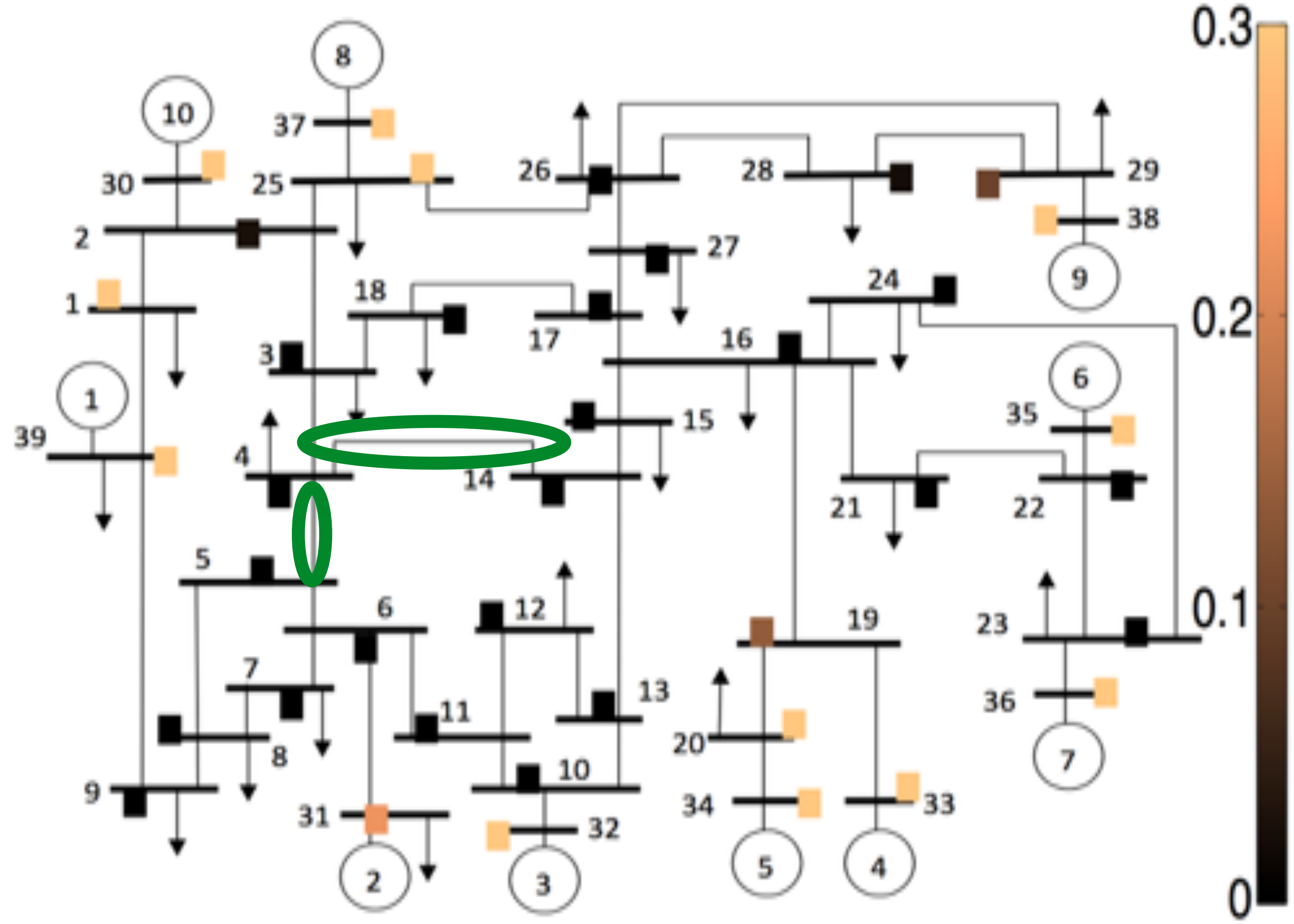
Can we find the location/source of a problem given the statistics?

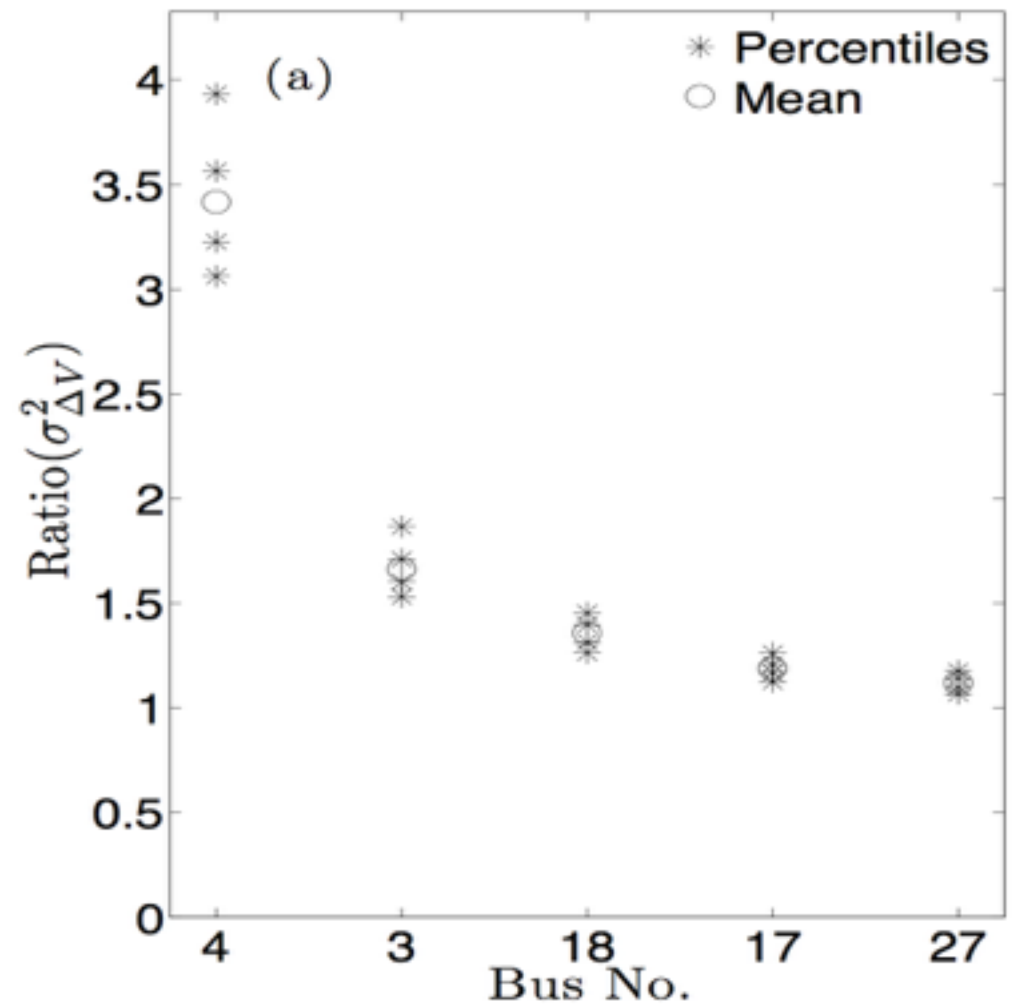


Can we find the location/source of a problem given the statistics?

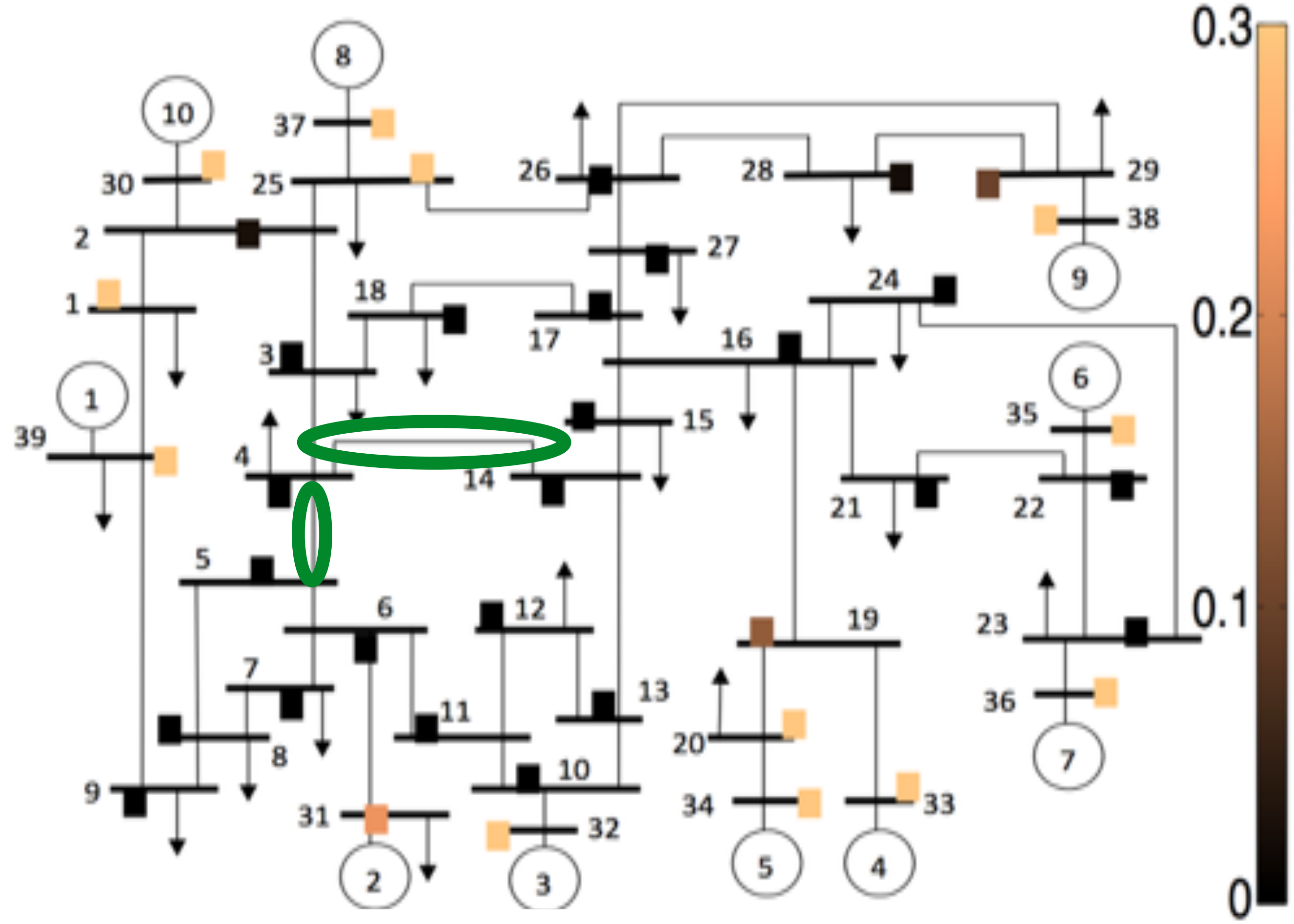


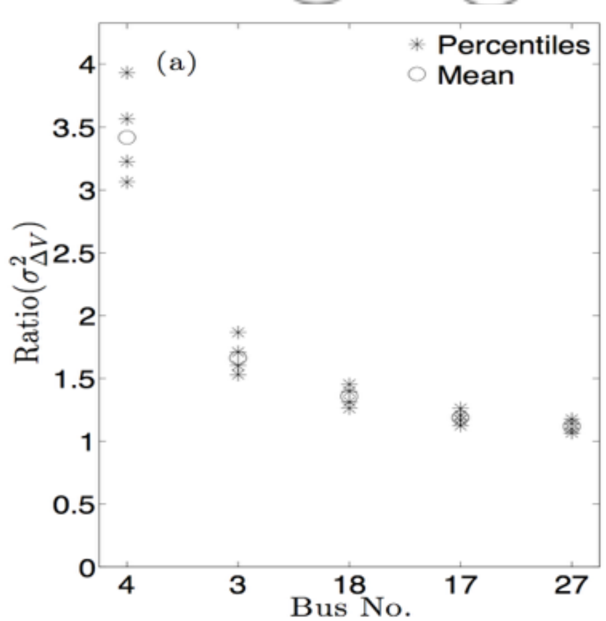
Can we find the location/source of a problem given the statistics?



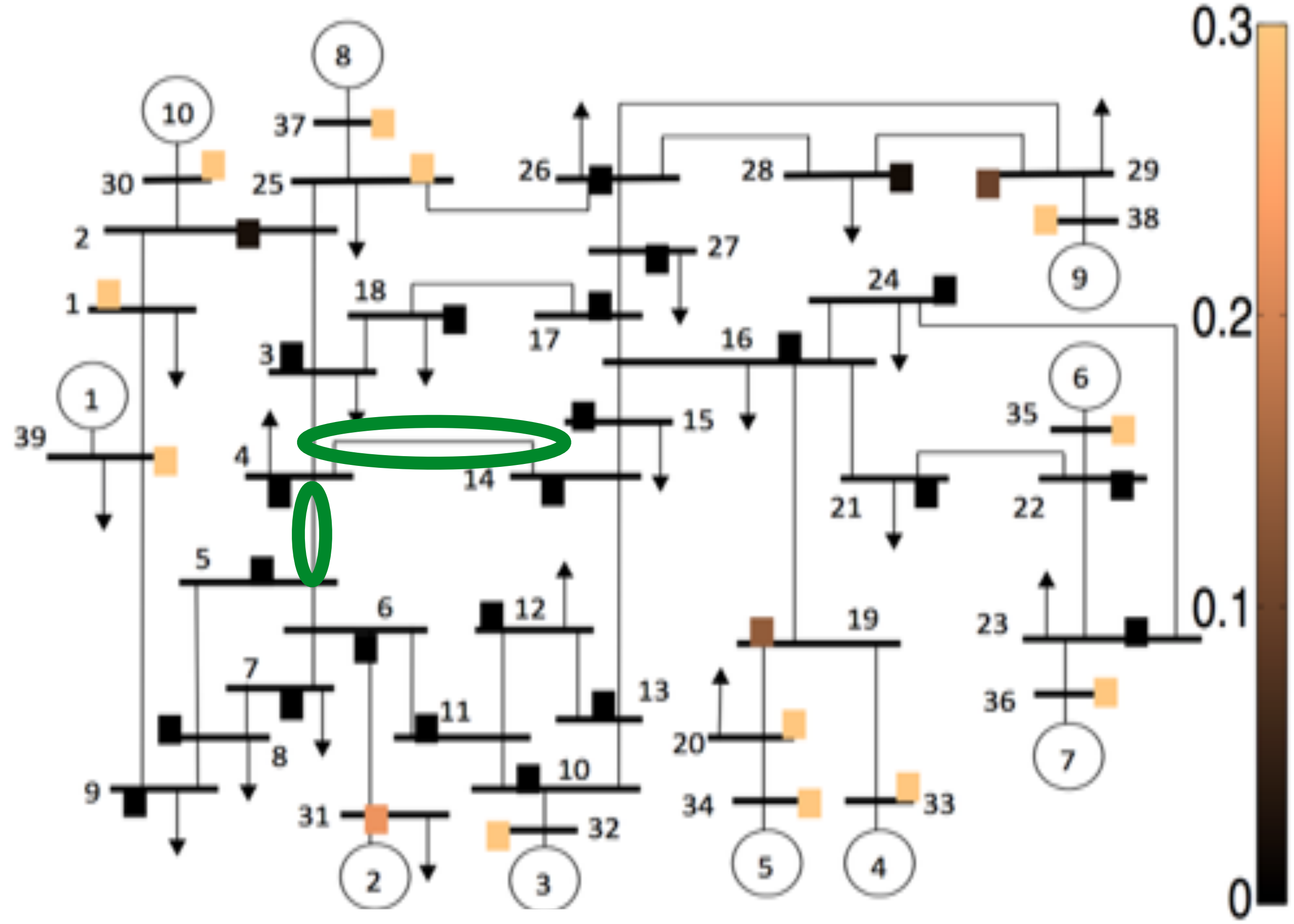


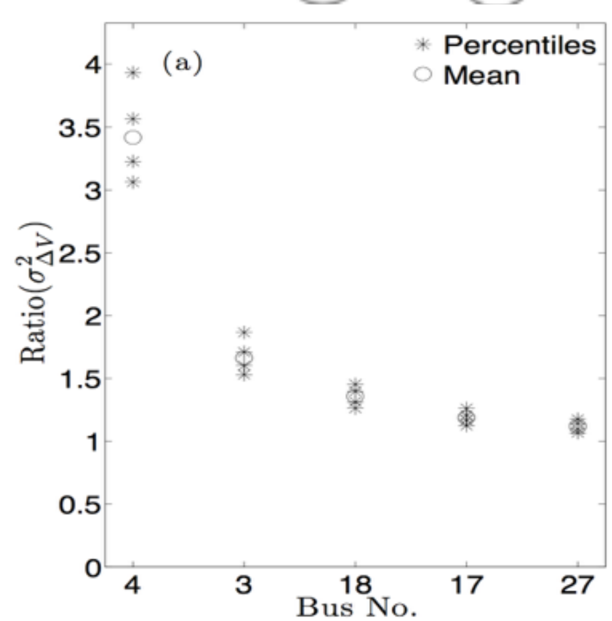
Can we find the location/source of a problem given the statistics?

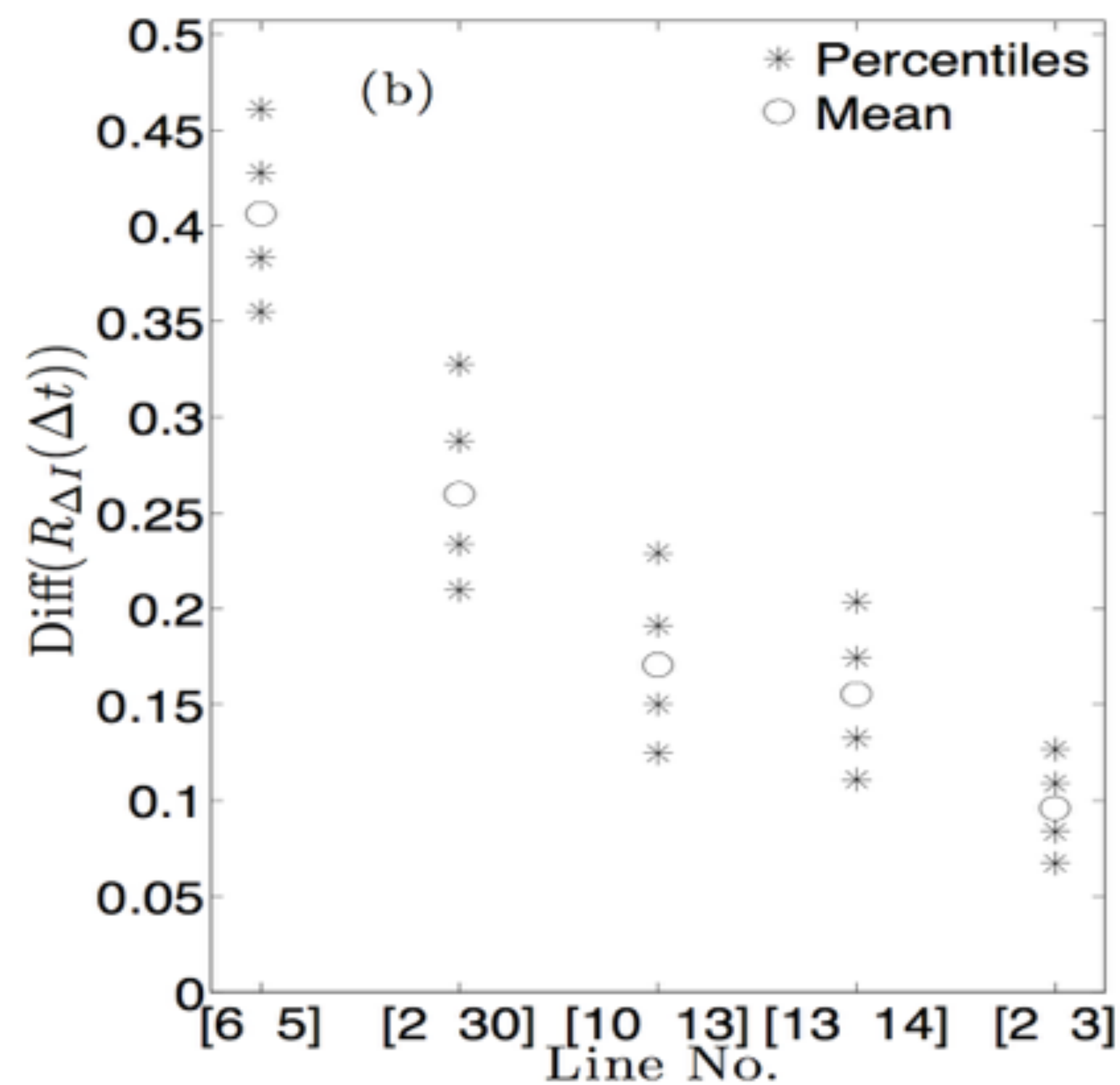




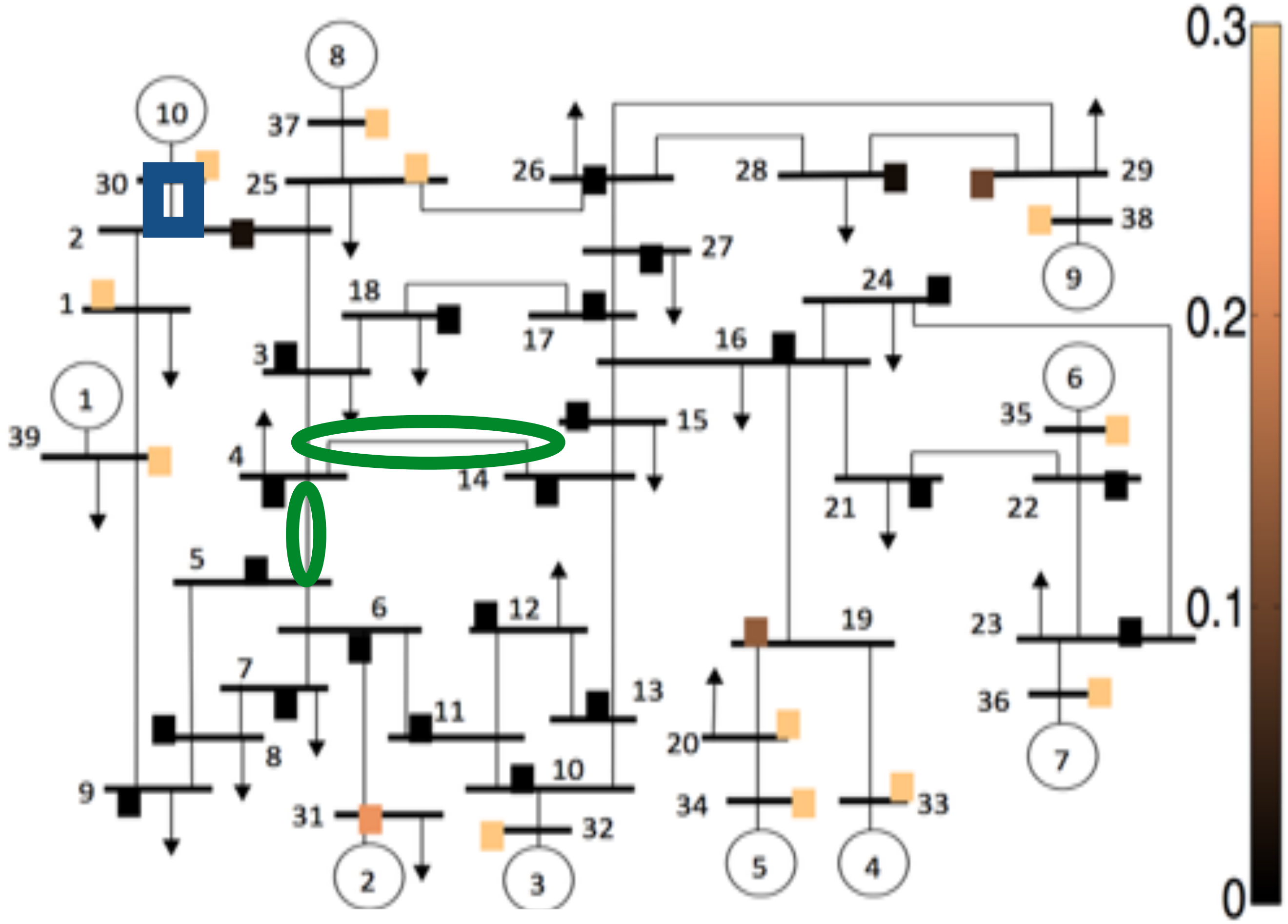
Can we find the location/source of a problem given the statistics?

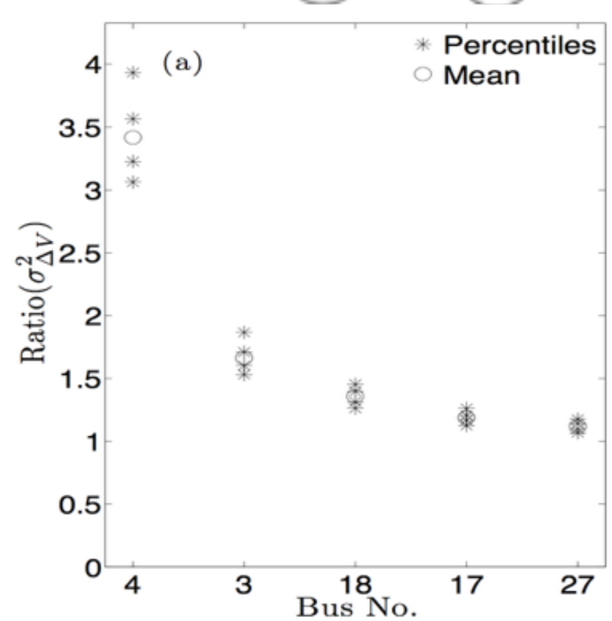


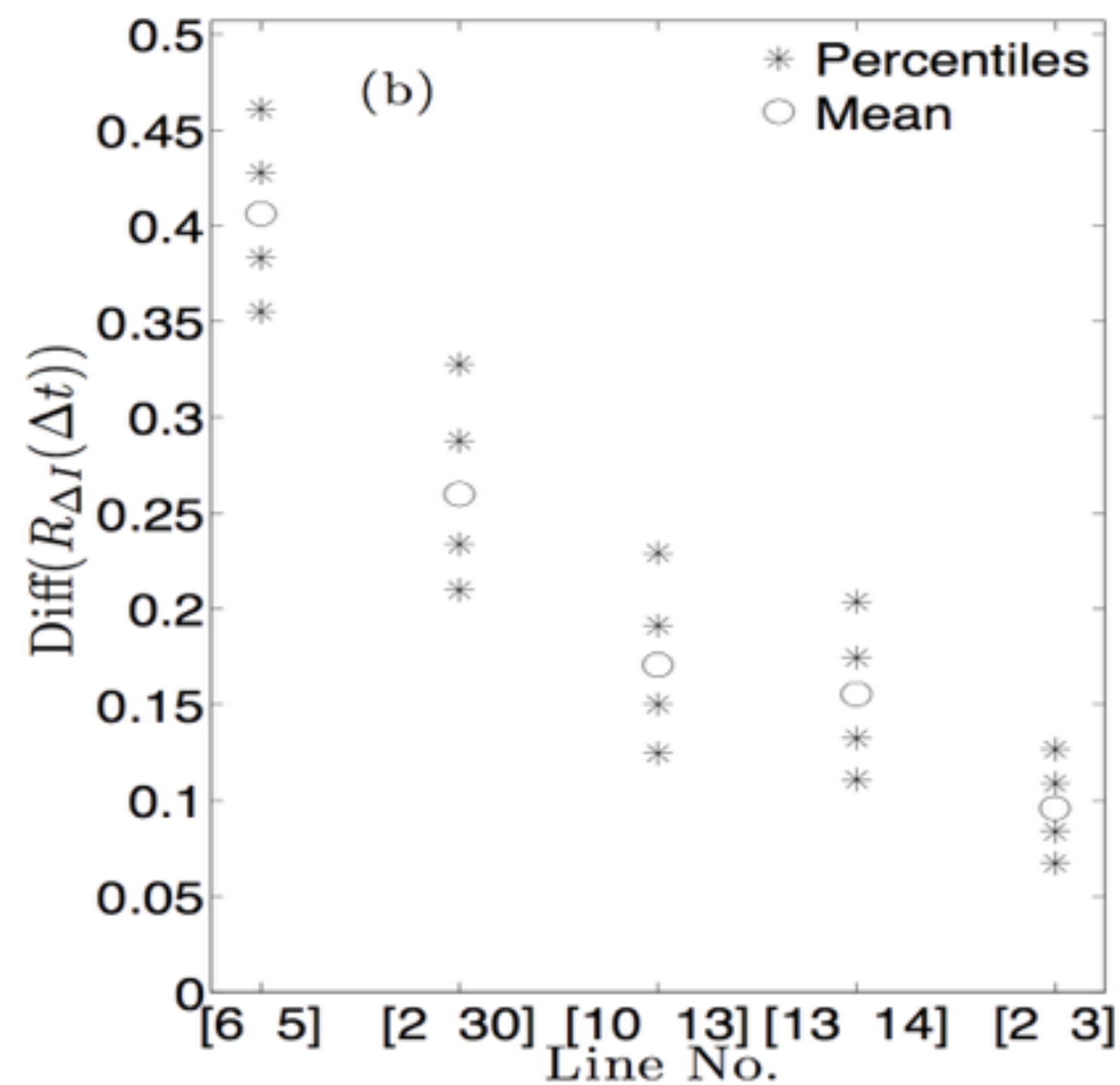




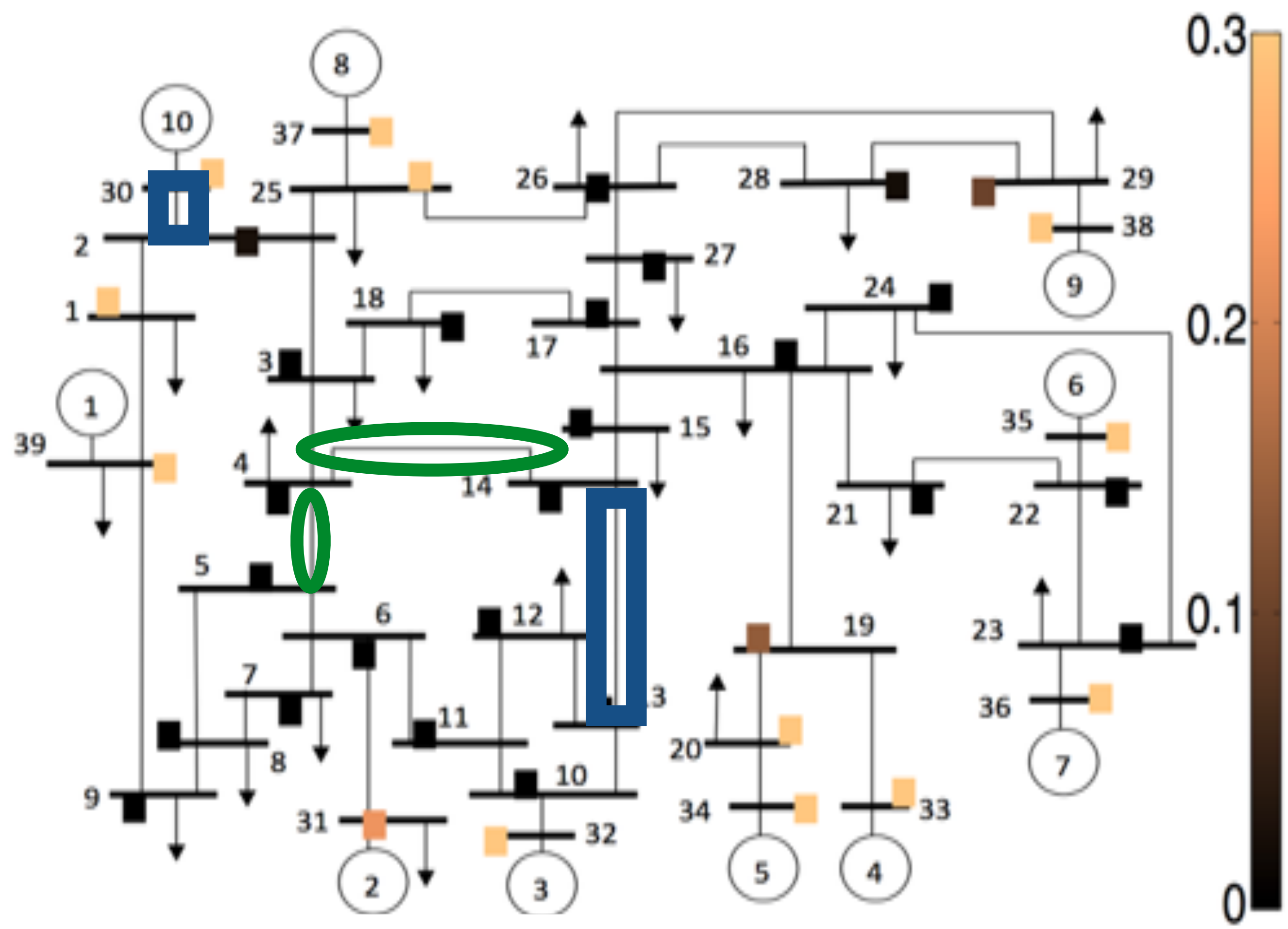
Can we find the location/source of a problem given the statistics?

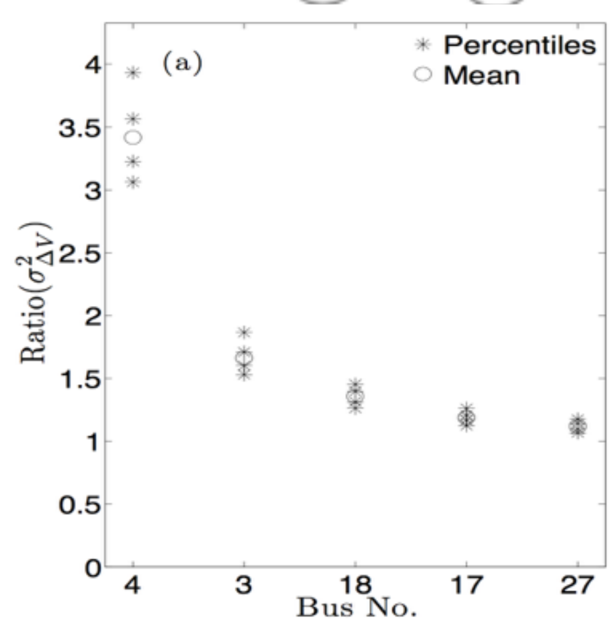


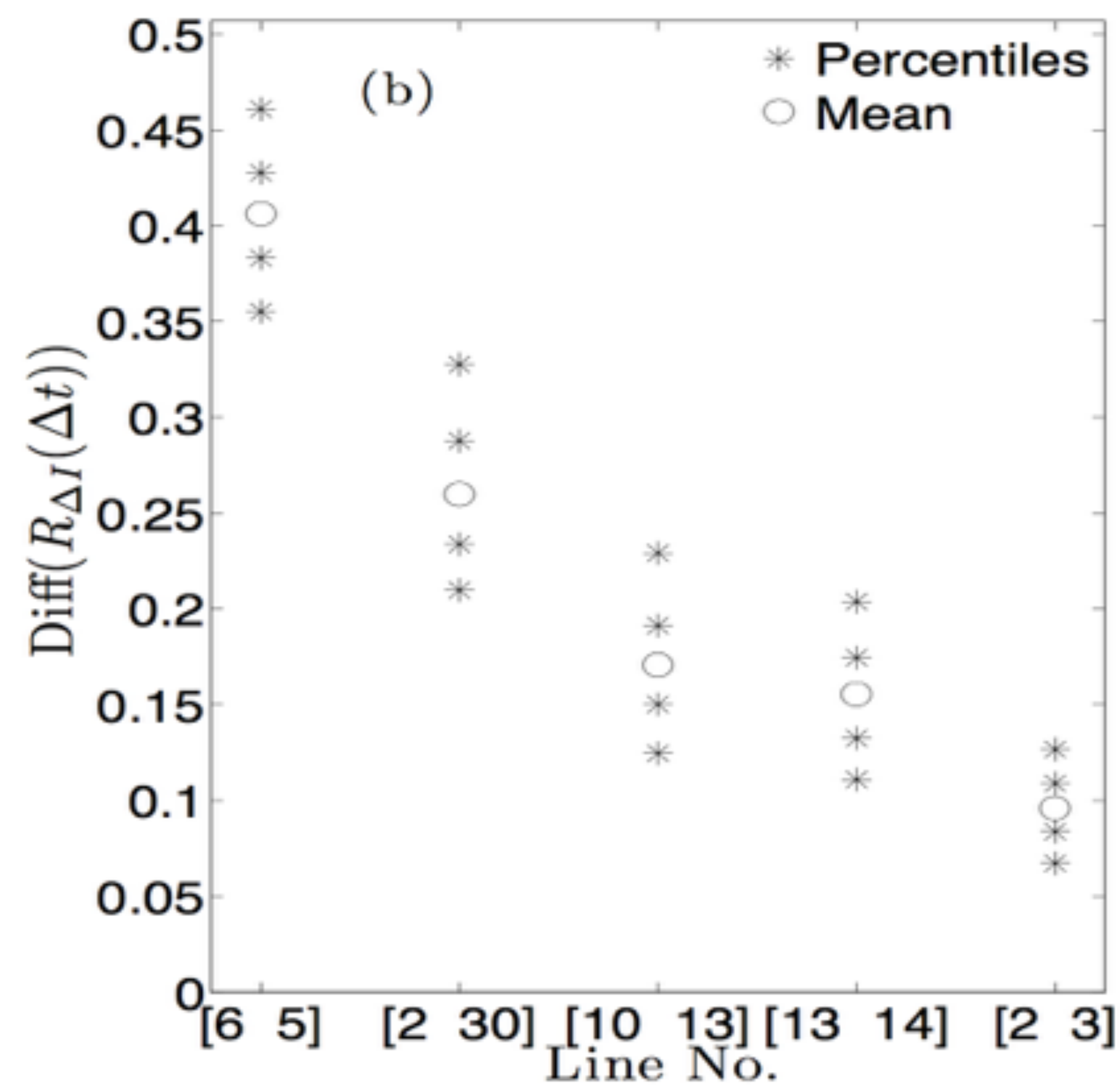




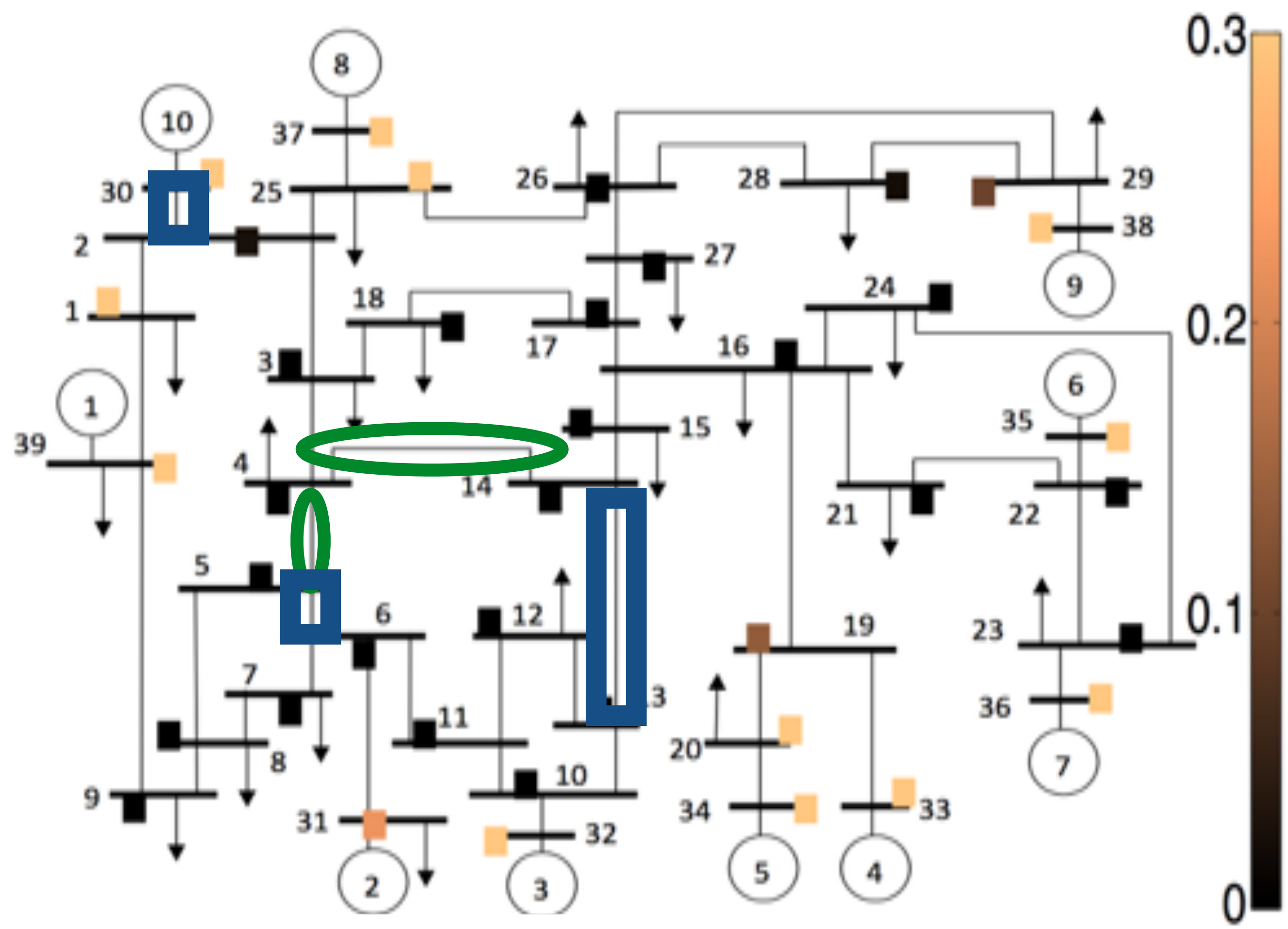
Can we find the location/source of a problem given the statistics?

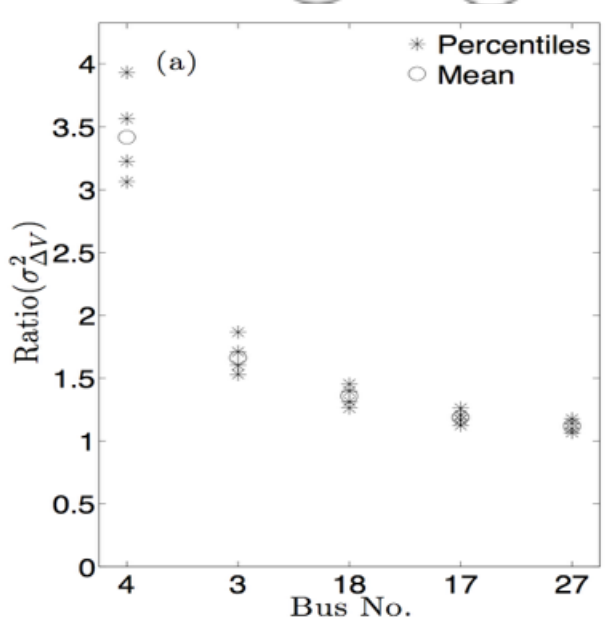


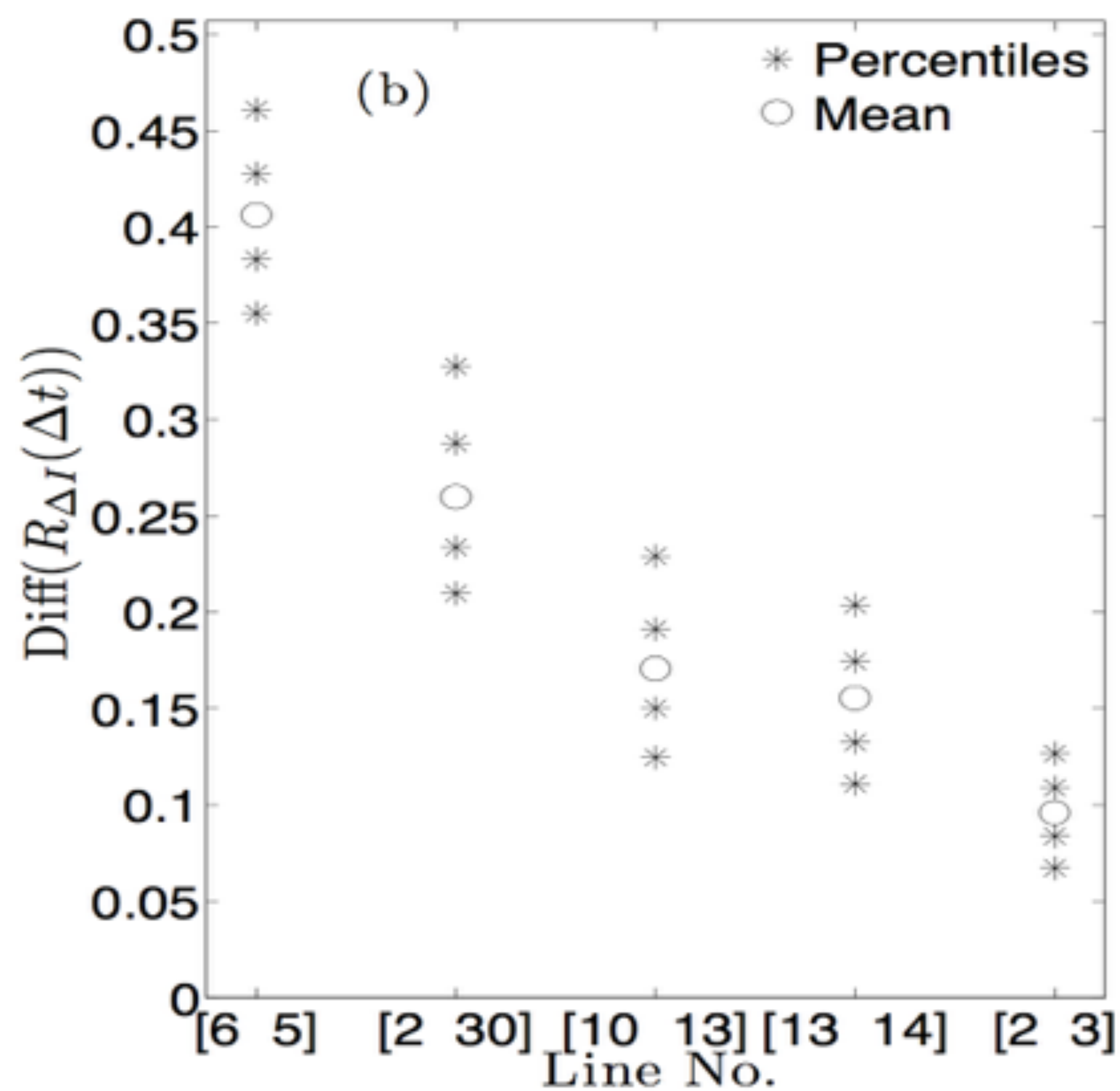




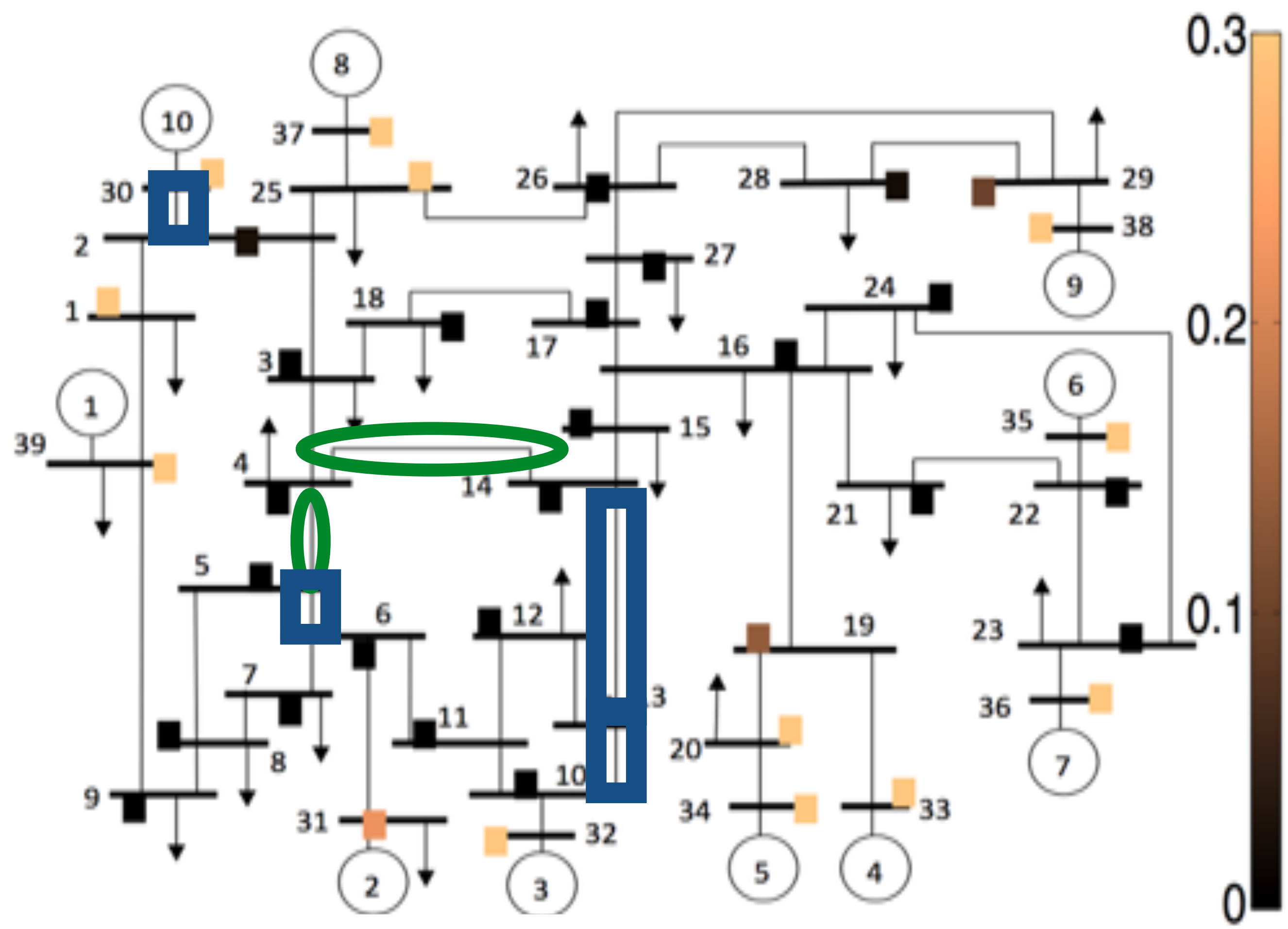
Can we find the location/source of a problem given the statistics?

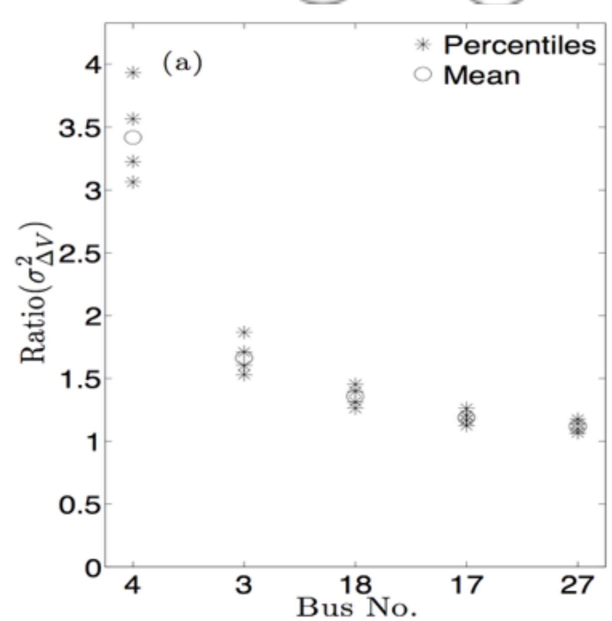


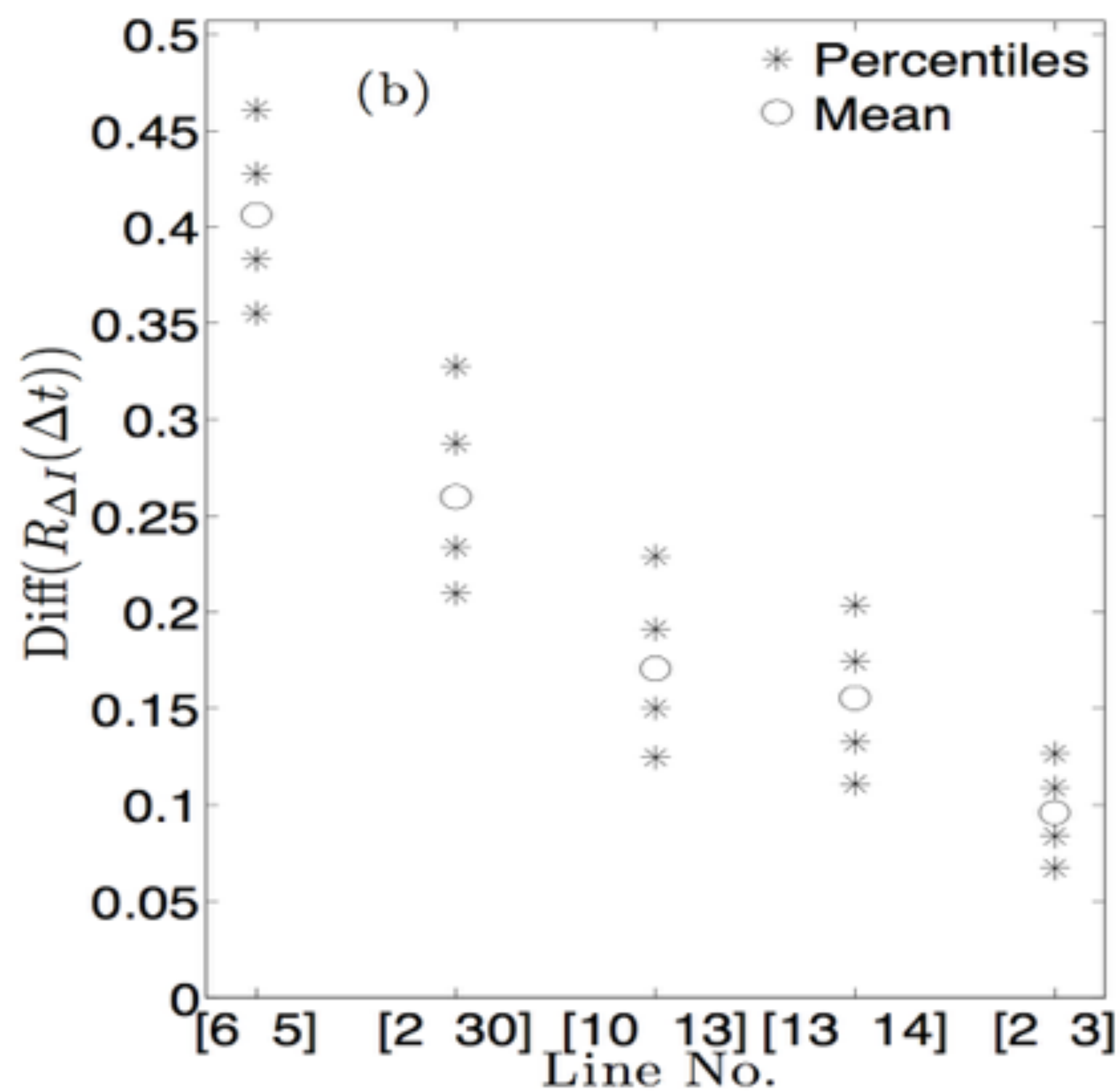




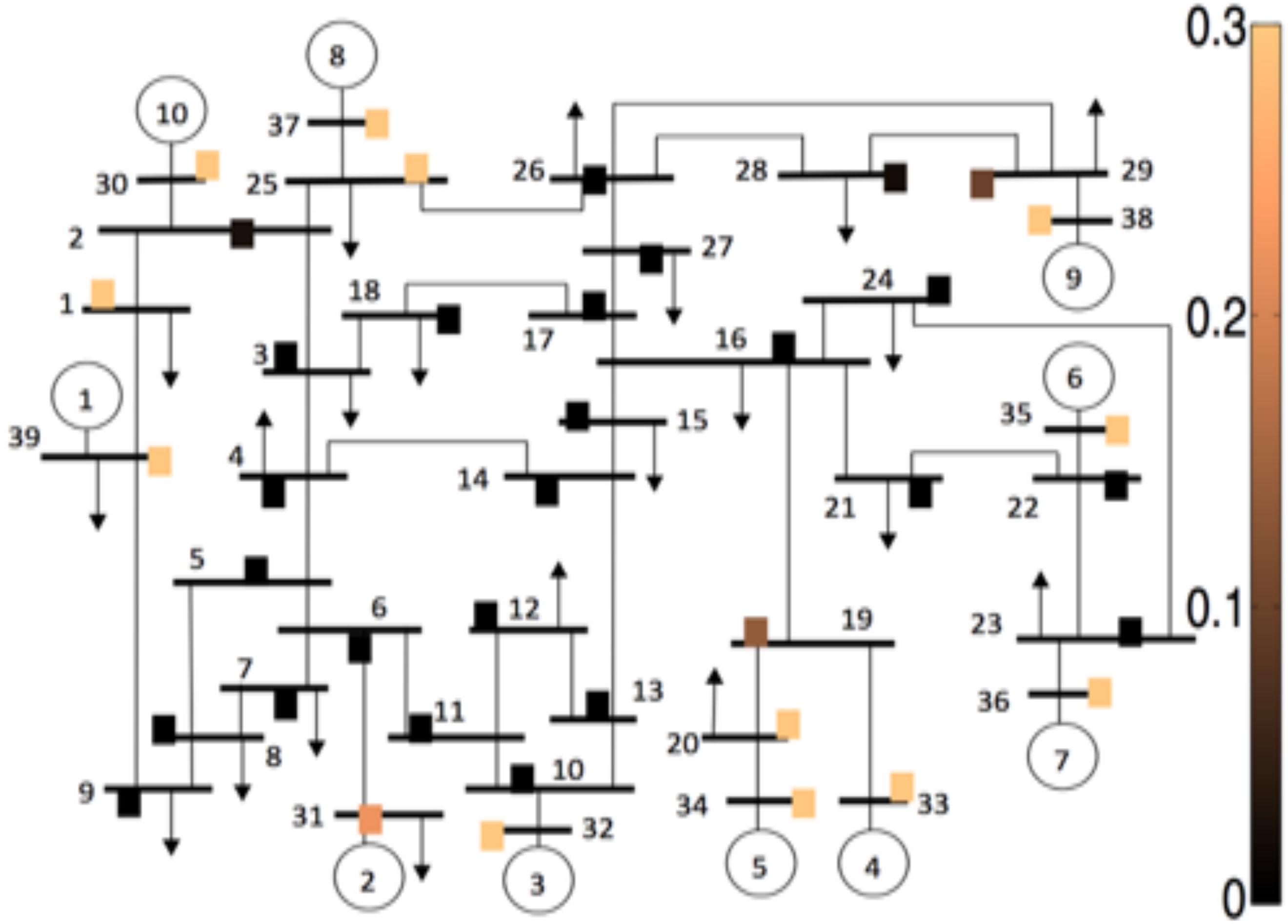
Can we find the location/source of a problem given the statistics?



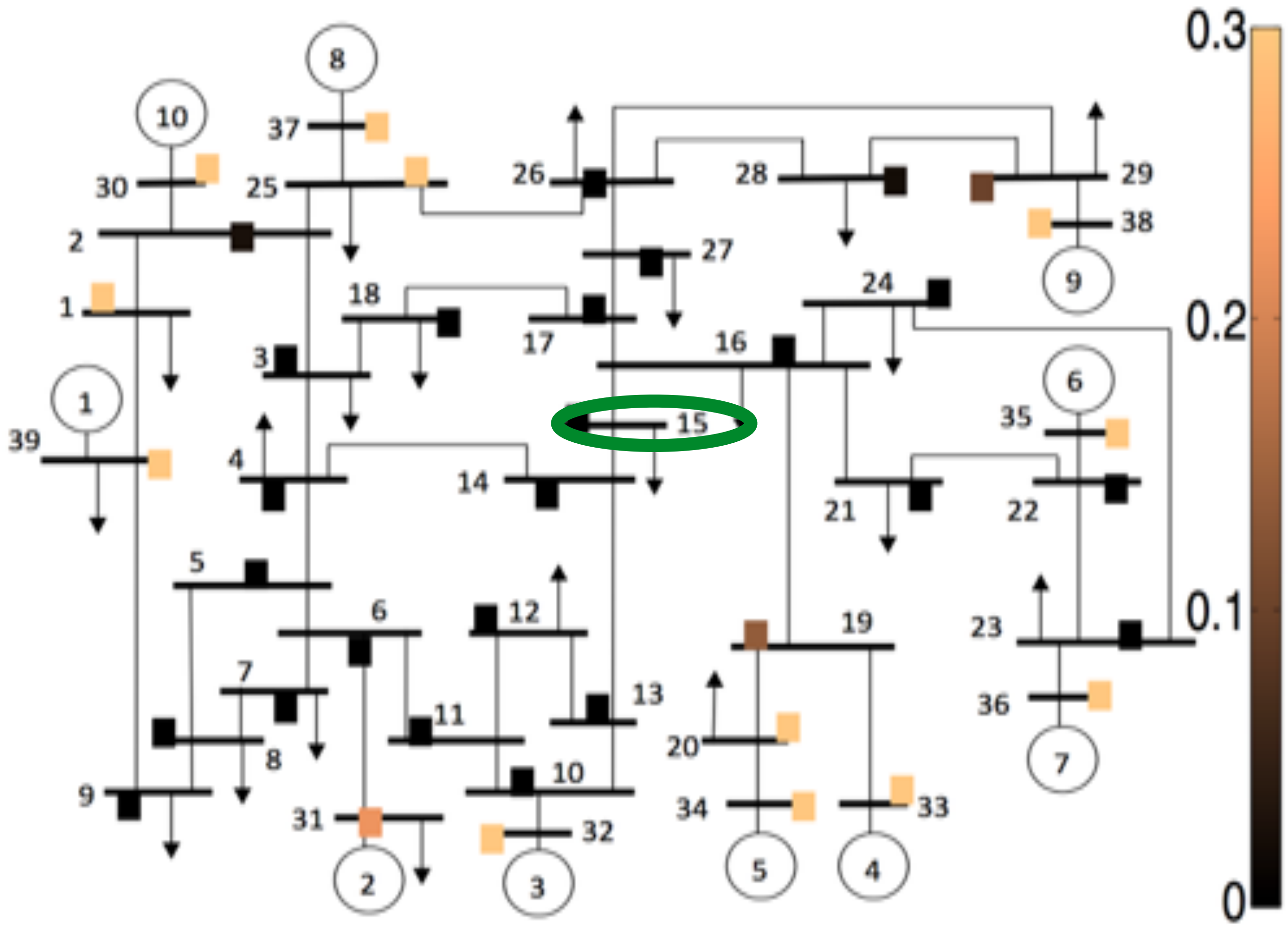




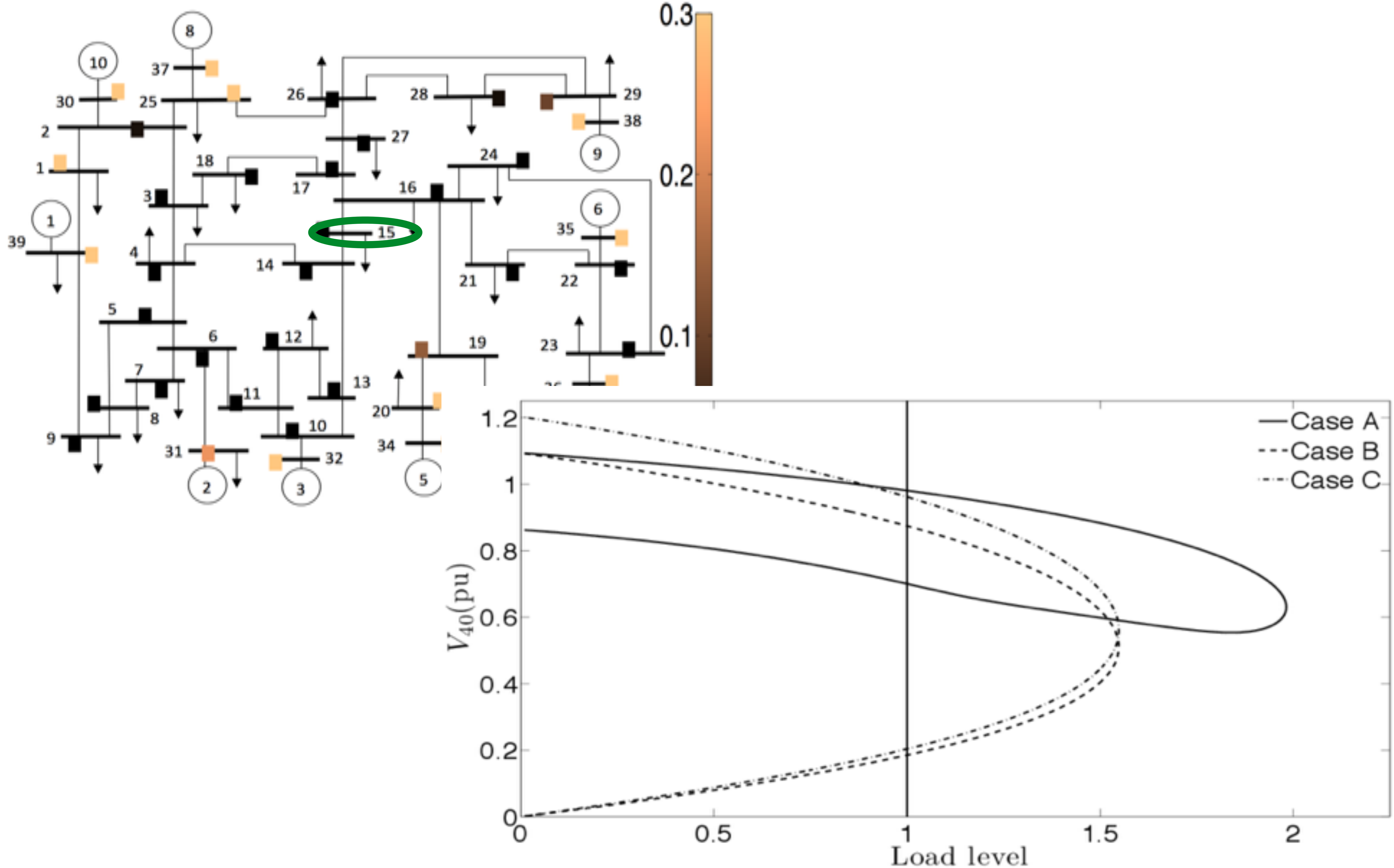
Can we find trends that would not show up in mean values?



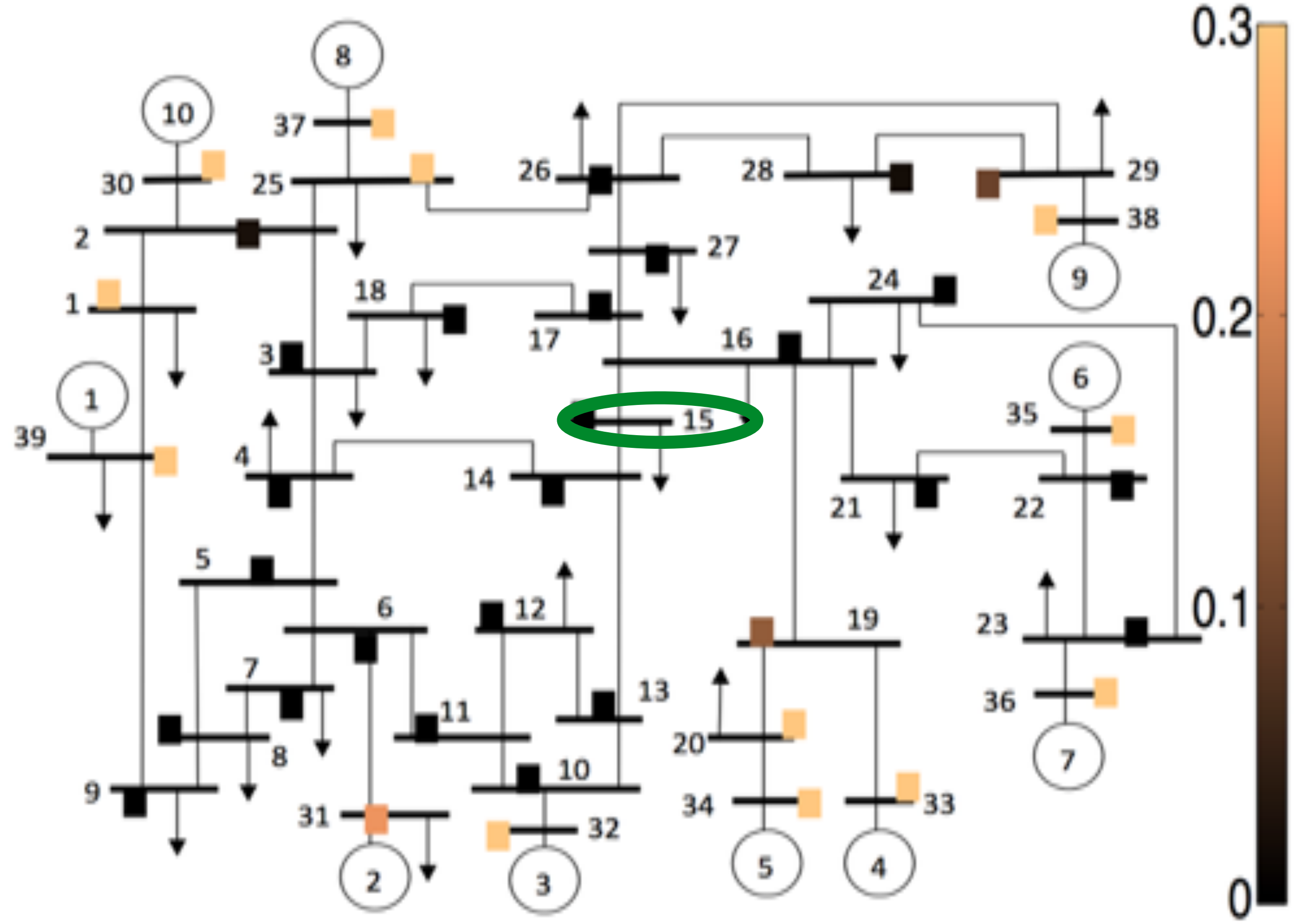
Can we find trends that would not show up in mean values?

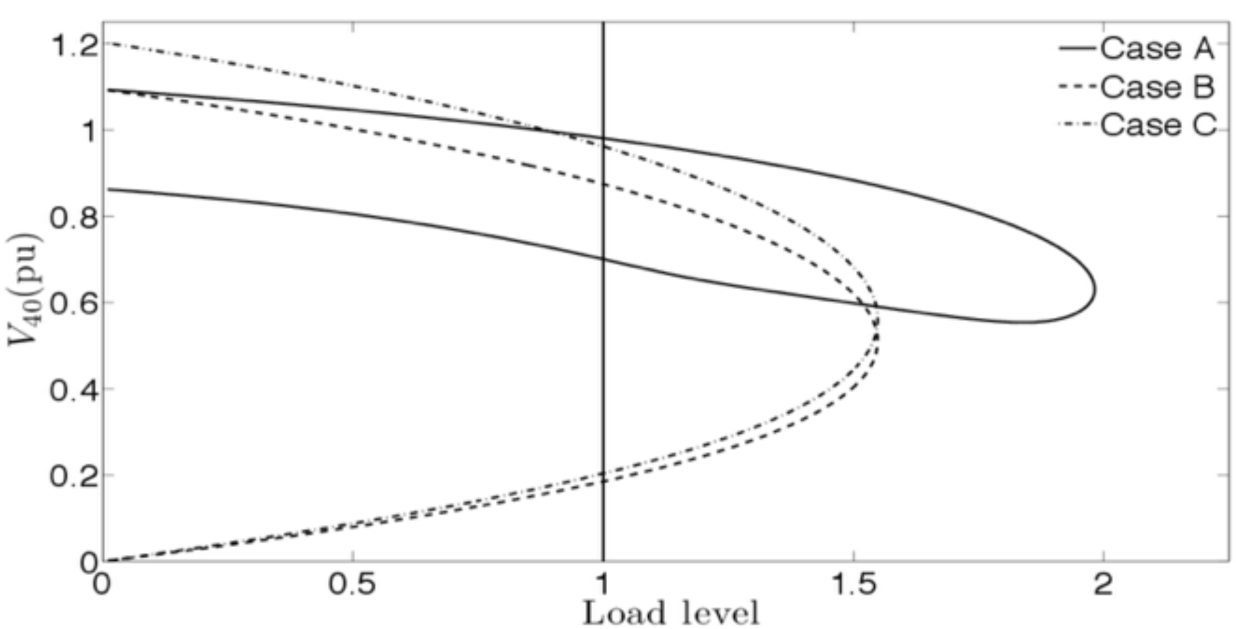


Can we find trends that would not show up in mean values?

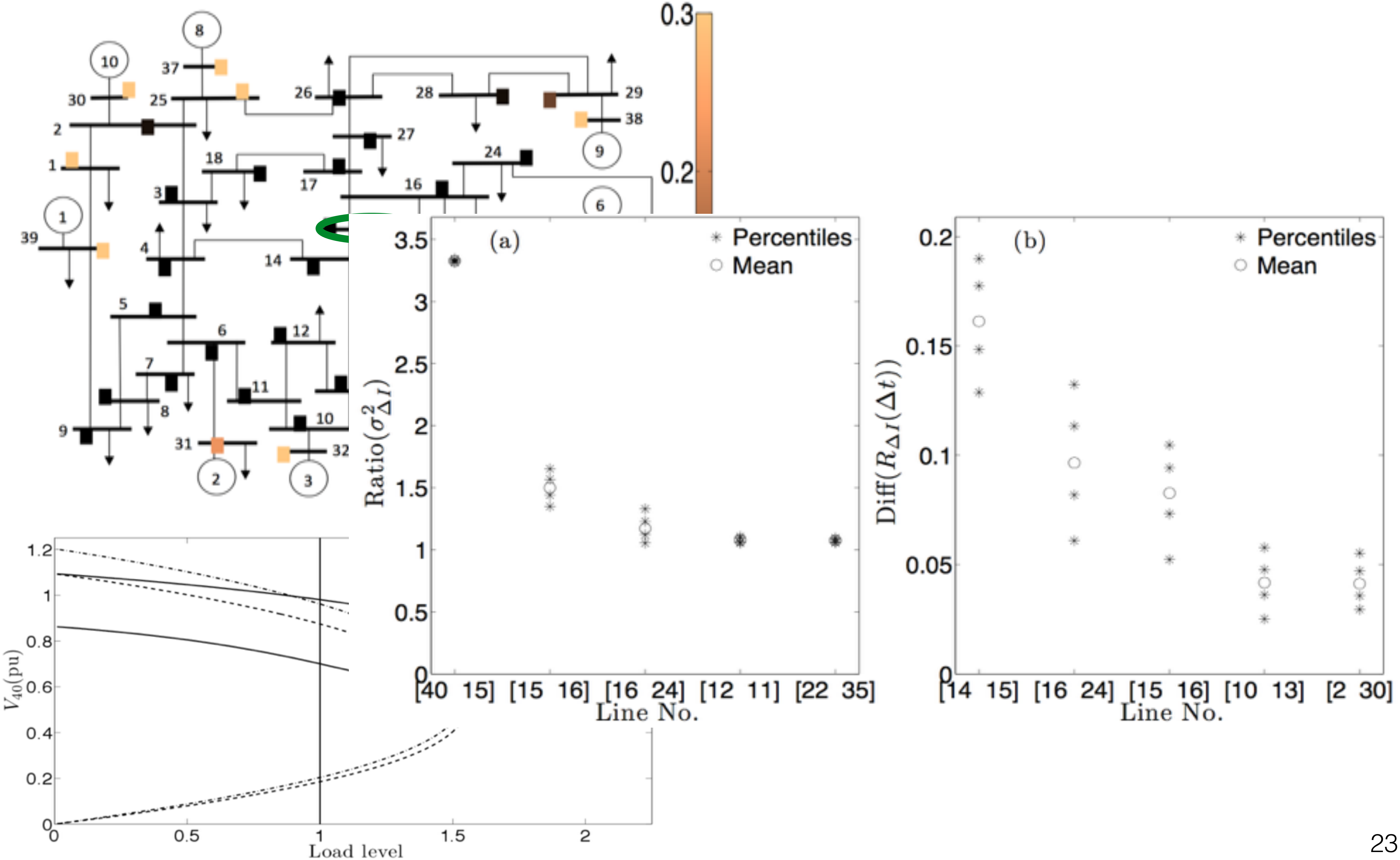


Can we find trends that would not show up in mean values?



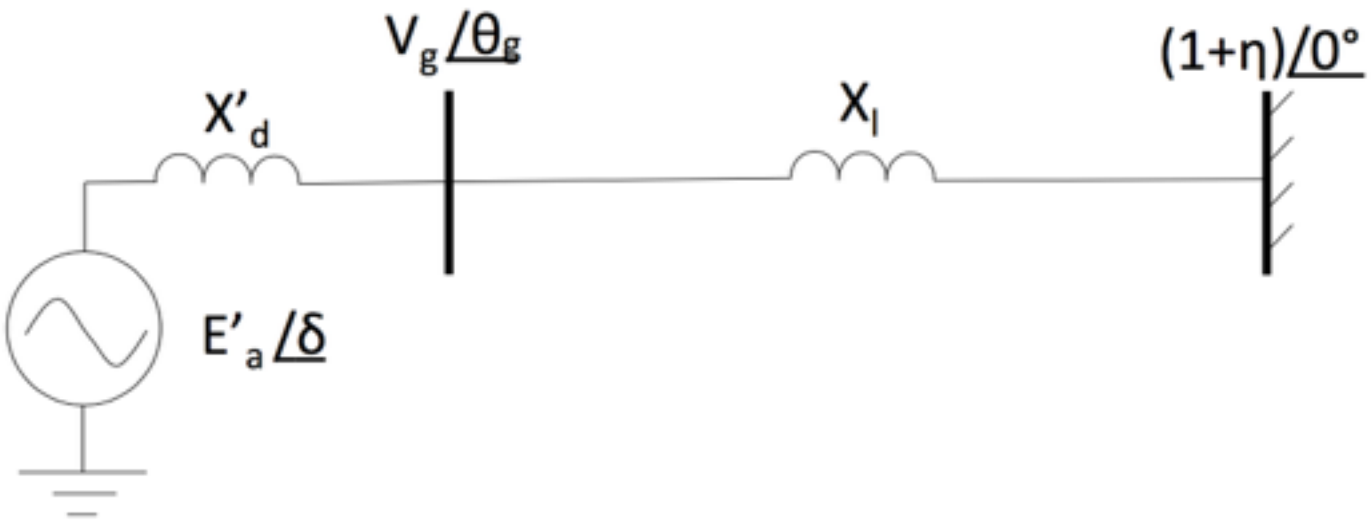


Can we find trends that would not show up in mean values?

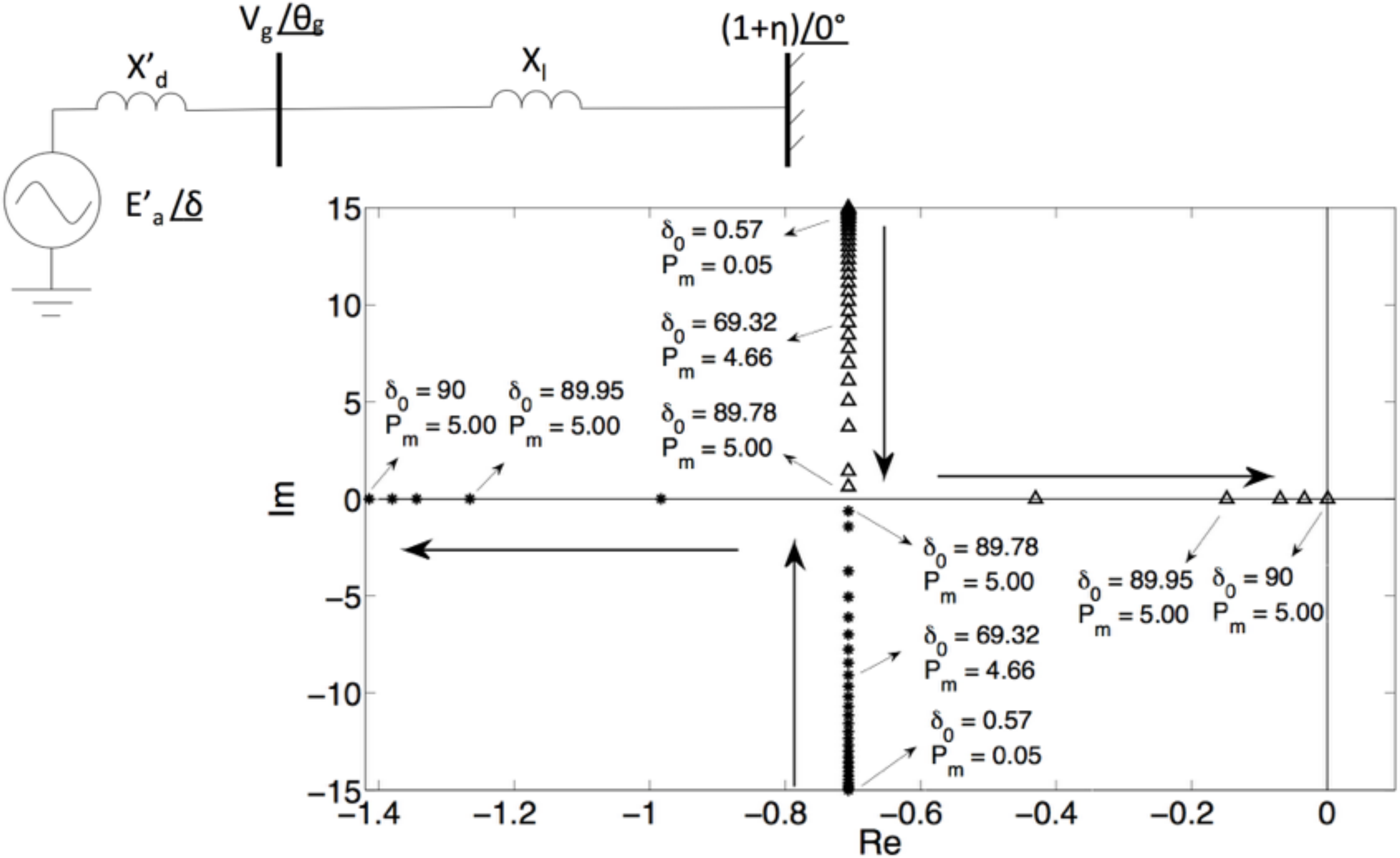


Why not just monitor critical modes/eigenvalues?

Why not just monitor critical modes/eigenvalues?



Why not just monitor critical modes/eigenvalues?



In summary

In summary

- **Autocorrelation** and **variance** are, ***sometimes***, useful indicators of proximity to instability.

In summary

- **Autocorrelation** and **variance** are, ***sometimes***, useful indicators of proximity to instability.

- Variances of **voltages near loads** are consistently good indicators of proximity to voltage collapse, even when voltage magnitudes are not.

In summary

- **Autocorrelation** and **variance** are, ***sometimes***, useful indicators of proximity to instability.
- Variances of **voltages near loads** are consistently good indicators of proximity to voltage collapse, even when voltage magnitudes are not.
- Autocorrelations of **currents near generators** (particularly smaller ones) are generally good indicators of system-wide stability issues (e.g., inter-area oscillations—Hopf bifurcation)

In summary

- **Autocorrelation** and **variance** are, ***sometimes***, useful indicators of proximity to instability.
- Variances of **voltages near loads** are consistently good indicators of proximity to voltage collapse, even when voltage magnitudes are not.
- Autocorrelations of **currents near generators** (particularly smaller ones) are generally good indicators of system-wide stability issues (e.g., inter-area oscillations—Hopf bifurcation)
- Frequently, fluctuations can **identify the locations** of emerging problems in the network

Statistical Early Warning Signs of Instability in Synchrophasor Data

IEEE GMS/PES
Synchrophasor Meeting
October 12, 2016

Goodarz Ghanavati,
Taras Lakoba,
Paul Hines*
*To whom all blame is due

Funding gratefully acknowledged:
NSF Awards ECCS-1254549, DGE-1144388,
DOE Award DE-OE0000447

NY city, Nov. 9, 1965