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LEARNING MATHEMATICS NEEDED FOR TEACHING THROUGH DESIGNING, IMPLEMENTING, AND TESTING LEARNING OBJECTS

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Abstract

This paper discusses the results of a pilot study that explored how prospective secondary school teachers are shaped by learning experiences during their undergraduate mathematics education. The collaborative study, which was conducted by a mathematician and a mathematics educator, drew from the experiences of prospective teachers in a non-traditional undergraduate mathematics program that makes extensive use of technology. Analysis of data collected from detailed questionnaires, journals, and focus group discussions strongly suggests that designing, implementing, and testing Learning Objects promotes prospective teachers' learning of the mathematics needed for teaching. Furthermore, the analysis shows that prospective teachers' experiences of ownership, engagement, and pride are key to positive learning experiences.

Keywords: learning mathematics for teaching; secondary school prospective teachers; learning by designing, implementing and testing Learning Objects; awareness; innovative undergraduate mathematics program; mathematicians and mathematics educators collaboration

Introduction

> The idea [is] that you are providing students with an interactive connection tool between math and learning. You are not simply giving kids a bunch of 'junk' and forcing them to read it and understand it; through computers we have the capability of interacting between machine and human, which is what creates the learning in the child.
> —Excerpt from the journal of a prospective mathematics teacher (MICA I student 1)

Prospective secondary school teachers can typically perform school mathematics tasks without difficulty. However, when asked how they would explain a mathematics concept or skill to someone who is learning for the first time, most prospective teachers respond with a rule-based explanation (Kinach, 2002). What do prospective teachers need to know in order to offer meaningful explanations? What kinds of experiences should be offered to support their learning?

The journal excerpt at the beginning of this section was written during a pilot study for an ongoing collaborative research project, aimed at exploring how prospective teachers of secondary school mathematics are shaped by their learning experiences during their undergraduate mathematics education. The project draws from prospective teachers' experiences in a non-traditional core undergraduate mathematics program called Mathematics Integrated with Computers and Applications (MICA) (Ben-El-Mechaiekh, Buteau, & Ralph, 2007)

Our research is informed by the following ideas. Learning how to teach mathematics requires learning the mathematics needed for teaching (Ball & Bass, 2002). Mathematics teacher education programs need to take into account mathematics needed for teaching (Canadian Mathematical Society [CMS], 2003; Conference Board of Mathematical Sciences [CBMS], 2001). In most of English-speaking North America, secondary mathematics teacher education programs prospective teachers learn mathematics content in departments of mathematics and learn mathematics didactics in faculties of education. From this split, a specious assumption has emerged that secondary school teachers learn mathematics well enough in their undergraduate studies to teach the subject (Cooney, 2002). However research shows that prospective mathematics teachers find it difficult to bridge the gap between acquired academic knowledge, with its axiomatic emphasis, and school mathematics, with its emphasis on meaning, pre-formal thinking, and contexts (Leufer & Predger, 2006). In response, mathematics departments have made ongoing and emerging attempts to reform their programs (CMS, 2003; Leufer & Predger, 2006; Muller & Buteau, 2006; Pesonen & Malvera, 2000). However, very little research has been done to find out whether and how these attempts impact prospective teachers' learning (Bednarz, 2001) as well as progress in research continues to be thwarted by a lack of collaboration between mathematicians and mathematics educators (Even & Ball, 2003). Our research attempts to address the gaps in research and collaboration (Mgombelo & Buteau, 2006).

Our research utilizes some elements of a grounded theory approach (Glaser & Strauss, 1967) to generate several conceptual categories based on practitioners' observations and an in-depth exploratory pilot study. Glaser and Strauss describe *grounded theory* as "the discovery of theory from data systematically obtained from social research" (p. 2). Grounded theory emphasizes a bottom-to-top, iterative, inductive approach to theory development, as opposed to theory generated by logical deduction from a priori assumptions (Patton, 2002). Our adaptation of the grounded theory approach to our research project proceeded in the following stages:

1. We started by reflecting on practitioners' observations of prospective teachers' learning experiences in the department of mathematics at our institution, in order to see what research questions and categories emerged (Buteau & Muller, 2006; Muller & Buteau, 2006; Muller, Buteau, Ralph, & Mgombelo, 2009). We focused our attention on one learning activity: prospective teachers' designing, implementing (i.e., computer programming), and testing of Learning Objects (LOs).
2. We refined and formulated specific categories and research questions with the goal of generating a postulate that would guide our research. We decided to concentrate on one learning experience aspect (development of a personal relationship with the activity) and formulated a tentative postulate: Prospective teachers' relationship with the activity – in terms of dedication, pride, ownership, and engagement with the mathematical and didactical work in designing, implementing, and testing LOs – could be a key to positive experiences of learning mathematics needed for teaching.
3. We designed and conducted a pilot study whose purpose was to test the viability of and to refine our tentative postulate, as well as to inform the methodology and conceptual framework of our research. More specifically, we aimed at gathering first evidence of prospective teachers' experiences of learning the mathematics needed for teaching through the selected task. (Mgombelo & Buteau, 2009, p. 1052)

After this three-stage process, we refined our postulate and moved to developing a conceptual framework (Mgombelo & Buteau, 2009). Later, specific research tools will be refined (e.g., coding schemes will be developed, tested and revised), and a large-scale study will be designed.

In this paper we describe our work on these first three stages. We begin our discussion by providing a practical context through a description of the MICA program, focusing on the learning experiences of students in designing, implementing, and using/testing Learning/Exploratory Objects (stage 1). We then describe how we formulated research questions, categories, and a postulate through our reflections on previous work, which was based on practitioners' experiences of teaching MICA courses and assessing students' projects (stages 1 and 2). Lastly, we provide a description of the pilot study (stage 3).

Mathematics Integrated with Computers and Applications (MICA) Experience

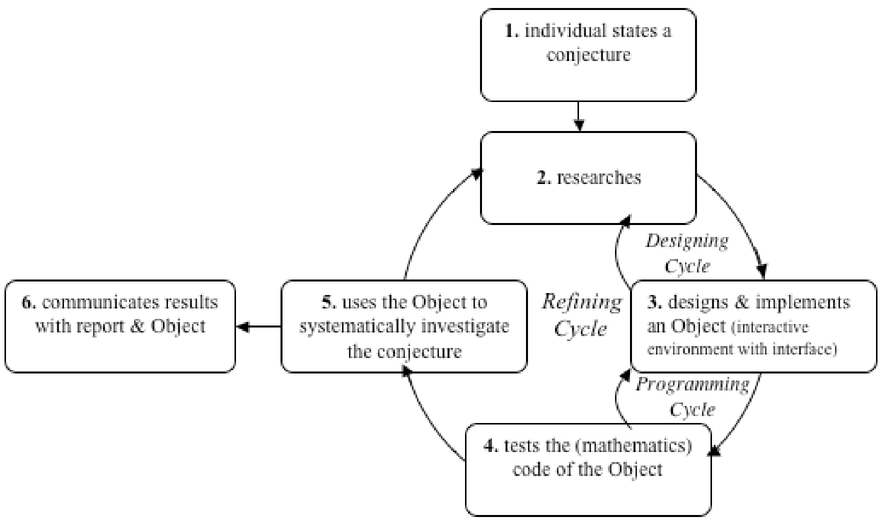
MICA, (launched in 2001), provides opportunities for students to make extensive use of technology to support their growth in mathematics through the integration of computers, applications, and modeling (Ralph & Pead, 2006). Two of the guiding principles of the program are to encourage creativity and intellectual independence, and to develop mathematical concepts in the context of computers and applications. MICA also strives to strengthen the concurrent mathematics teacher education programs. MICA exposes prospective teachers to a broad range of mathematical experiences rather than to a deep concentration in one or two areas. In their mathematics courses, prospective teachers make extensive use of software – such as Maple, Journey Through Calculus (Ralph, 1999), Geometer's SketchPad, and Minitab – that may nurture their understanding of the teaching and learning of mathematics.

In addition to a revision of all traditional courses under these guiding principles, three innovative core project-based courses, called *MICA I – III*, have been introduced, in which all students learn from year one to investigate mathematics concepts and conjectures or to simulate real-world situations by designing, implementing, and using interactive computer environments (VisualBasic.NET, Maple, C++), which are called Exploratory Objects (EO). An EO is an "interactive and dynamic computer-based model or tool that capitalizes on visualization and is developed to explore a mathematical concept, conjecture, or real-world situations" (Muller, Buteau, Ralph, & Mgombelo, 2009b, p. 64).

MICA approach echoes a shift in higher education from a traditional emphasis on the mere transmission of knowledge to approaches that encourage students to understand, investigate, and solve problems (e.g. Learning by Design, Project-Based Science, Problem-Based Learning, and Knowledge Building (Bereiter & Scardamalia, 2003)). Learning by Design is reflected in MICA courses. According to Han and Bhattacharya (2001), Learning by Design emerged from constructionist theory and emphasized the value of learning through creating, programming, or participating in other forms of designing. The design process creates a rich context for learning. Accordingly, Learning by Design values both the process of learning and its outcomes or products. Han and Bhattacharya contend that the essence of Learning by Design is in the construction of meaning and that designers (learners) create objects or artifacts representing a learning outcome that is meaningful to them.

In MICA courses, students experience learning mathematics by creating and using EOs. The MICA courses (two hours lecture and two hours lab per week) have been designed to provide opportunities for students to (a) conjecture and raise mathematics questions and (b) design, implement, and use an interactive computer environment that would support a systematic exploration of the conjecture or questions. For example, MICA I students experience the following: a lecture in which they phrase conjectures concerning prime numbers in small groups, followed by a class discussion on *all* the conjectures, their possible validity, relations among them, and their testability with technology; a project assignment to design and implement the RSA encryption method to encode and decode secret messages; and a lab in which students are guided through a systematic exploration of the stability of a dynamical system by using the EO they designed and implemented themselves (Muller & Buteau, 2009). Figure 1 summarizes the main steps involved in the development of an EO project about a conjecture. The project entails the Object and a written report that consists of a statement of the conjecture/concept/theorem/real-world situation, the mathematical background, results of the exploration including an interpretation of the data and graphs, a discussion of the results, and a conclusion (Muller & Buteau, in press).

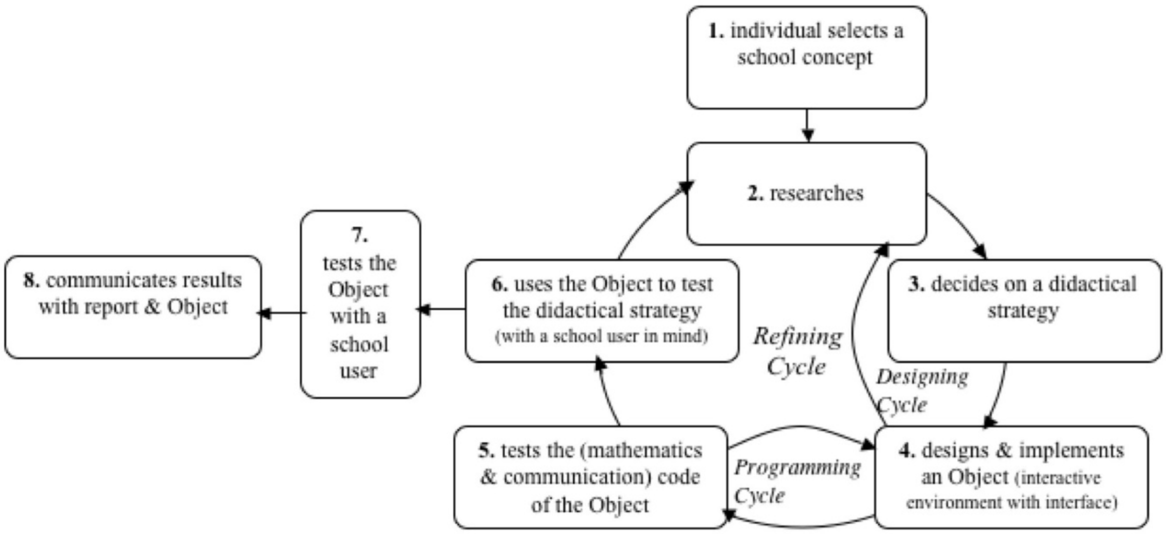
Figure 1



Final assessment in these MICA courses is structured around students' final projects, in which students, individually or in pairs, create in the last two to three weeks of the course an original interactive computer program on a topic *of their own choosing*. These projects can be (a) exploratory, e.g., testing his/her own conjecture; see "Structure of the Hailstone Sequence" (Brock University, 2010); (b) an application, e.g., modeling or simulation; see "Running in the Rain" project (Brock University, 2010); or (c) didactic, i.e., so-called *LO*s. According to Muller et al. (2009b), a LO is an "interactive and dynamic computer-based environment that engages a learner through a game or activity and that guides him/her in a stepwise development towards an understanding of a mathematical concept" (p. 64).

LOs are innovative, interactive, highly engaging, and user-friendly computer environments that teach one or two mathematical concepts. Generally, prospective teachers complete a LO about a school mathematics concept for their final MICA projects. The reader is invited to navigate through the original MICA students' interactive environments (Brock University, 2010). Our research focuses on these prospective teachers' experiences of creating LOs. Figure 2 summarizes the main steps involved in completing a LO project. The project entails the Object and a written report that presents the didactical purpose, the target audience, the mathematical background of the target audience, a brief account of their observations of a school pupil experience using their LO, and a discussion (Buteau & Muller, 2009).

Figure 2



LOs have often been mentioned and discussed in the literature, although “it is challenging to extract a concise and agreed upon definition of LOs” (Christiansen & Anderson, 2004, p. 2). Wiley (2002) describes LOs as “any digital resource that can be reused to support learning” (p. 6). The IEEE Learning Technology Standards Committee (2002) offers a broader definition: “any entity, digital or non-digital, that may be used for learning, education or training” (p. 6). According to these definitions, what we call EOs could also be identified as LOs. However, for the purpose of describing and analyzing the work of MICA students, we provided more focused definitions to distinguish between our two Object types – Exploratory and Learning – according to user. In EOs, the student designer develops his/her Object for him/herself as the user, and as such a different user may not experience any mathematics learning and may not even understand the mathematics at stake without reading the written report. In LOs, the student designer develops his/her Object for a different user, and as such the Object explicitly guides the user step by step to experience mathematics learning.

Buteau and Muller (2009) explain that an EO final project written report is somewhat comparable to a science laboratory report; the EO could be compared to the student’s self-designed “virtual laboratory” for the investigation of the self-stated conjecture or real-world application. In LO final projects, the written report is similar to a lesson plan, including a post-lesson reflection, but the “written” description of the lesson is replaced by “an interactive self-directed lesson (with a virtual learner), i.e., by the [Learning] Object.” Note that there is no class implementation involved in a LO final project, but rather a reflection on and observation of use of the LO by one elementary or secondary student at the appropriate grade level. These considerations serve as a guideline for students when designing their LOs. In other words, the main didactical objective of the (LO) final project is to promote a first reflection on the learning of a mathematics concept. It does not address orchestration in the classroom.

Toward researching prospective teachers' learning experiences OF designing, implementing, and testing Learning Objects

A crucial and rather uncommon aspect of our research is that we are focussed on Learning by Design, through which prospective teachers create (as opposed to use) LOs. Although there is interest among higher education researchers and practitioners in the Learning by Design approach there is very little literature on prospective teachers learning mathematics by creating LOs. Most research on LOs concentrates on users' rather than designers' experiences. A few studies address designers' experiences, but most of these discuss instructors' experiences of designing LOs for their own courses. This limited literature and research might be the result of most LOs being designed by instructors or by technicians in the software industry. In the case of technicians, they may perceive "design for education as just another field of application of generalized design processes" (Kynigos, 2009, p. 1). Kynigos's (2009) work on mathematics education graduate students designing "half-baked" mathematical microworlds provides an example of students learning by designing LOs. Our research focuses on experiences of prospective teachers learning (mathematics needed for teaching) by the designing, implementing, and testing of LOs rather than by their use.

In the first stage of our research, we reflected on previous work by practitioners/instructors, which was based on their observations, experiences of assessing students' computer projects, and interactions with students in the MICA program (Muller et al., 2009b; Buteau & Muller, 2006; Muller & Buteau, 2006). This previous work suggests that MICA students are provided with positive experiences in learning mathematics. Muller et al. (2009b), describe and exemplify each of these experiences with concrete MICA student EOs and LOs. In Muller, Buteau, Klincsik, Perjési-Hámori, & Sárvári (2009a), Muller and Buteau note the following skills that MICA students develop: to express their mathematical ideas in an exact way; to self-assess their mathematics; to realize their creativity in mathematics and in communicating their understanding of mathematics; and to become independent in mathematical thinking. Furthermore, students are provided with opportunity to concretize personalized original mathematics work, and to identify with their future profession. Students develop a personal relationship with the activity of designing and implementing an EO or LO. They demonstrate a strong engagement and ownership in the activity and exhibit much pride of their EO or LO.

It became evident from our reflection on the previous work that the experience of designing, implementing, and testing a LO requires considerable didactic-sensitive mathematical work, that is, work that requires attending to the mathematics as well as to the learning and teaching of mathematics. This experience points to the mathematics needed for teaching. Furthermore, the previous work suggests that prospective teachers experience ownership, engagement, and pride in designing, implementing, and testing LOs. This finding led us to postulate that these experiences (ownership, engagement, pride) might be a key to the positive experiences of learning the mathematics needed for teaching (stage 2 of our research). In other words, although the skills and opportunities observed from the previous work point to other important aspects of learning the mathematics needed for teaching, we decided to focus on the development of prospective teachers' personal relationship with the activity in terms of ownership, engagement, and pride (Muller et al., 2009a). This decision resulted in the need for an in-depth investigation of prospective teachers' development of a personal relationship with the LO activity in the MICA courses to better understand their impact.

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Researching prospective teachers' learning experiences of creating and testing Learning objects

Given the complexity of our research and the limited literature and research on students' Learning by Designing LOs in mathematics teacher education, we conducted a pilot study (stage 3 of our research) to gather first evidence and inform the methodology and conceptual framework of our research. In the following sections we present the conceptual framework, methodology, and findings of the pilot study. The study was guided by the following questions:

1. Does designing, implementing, and testing LOs promote prospective teachers' learning of the mathematics needed for teaching?
2. In what ways does designing, implementing, and testing LOs provoke prospective teachers' awareness of their own learning of mathematics and of what it means for students to learn mathematics?

Conceptual framework

The conceptual framework of our pilot study is guided by our postulate (that ownership, engagement in the activity, and pride are key for prospective teachers' positive learning experiences) and the research questions. We draw from the work of Mason and Spence (1999) on "knowing-to act," as a kind of knowing that requires awareness, and Mason's (1998) work on levels of awareness in mathematics teacher education. These works provided us with a provisional theory as well as an initial grounding on the development of a conceptual framework of our research. The works helped us to conceptualize prospective teachers' knowing of the mathematics needed for teaching, from a dynamic, active and evolving perspective of knowing-to as opposed to the static, passive and possessive perspective of (traditional) accumulated knowledge. Mason and Spence (1999) elaborate on the difference between these two perspectives.

Furthermore, Mason's forms of awareness provide us with a way of conceptualizing the mathematics needed for teaching as both a development of sensitivity to mathematics (awareness in discipline) and being able to support this development from a learner's perspective. Building on Gattegno's (1970) idea of awareness as that which enables powers that have been integrated into one's functioning to be employed, Mason describes three forms of awareness: *awareness-in-action*, which involves a human being's powers of construal and of acting in the material world; *awareness-in-discipline*, which is awareness of awareness-in-action, emerging when awareness-in-action is brought into explicit awareness and formalized; and finally *awareness in counsel*, which is awareness of awareness-in-discipline and involves enabling others to work on their awareness-in-discipline.

To put this into a mathematics perspective, awareness-in-action might be exemplified by an act of counting numbers (one, two, three) without being aware of the underlying notions such as one-to-one correspondence. Awareness-in-discipline emerges when one becomes aware of this one-to-one correspondence in counting. Finally, awareness-in-counsel emerges when one is able to let others develop their awareness of counting as one, two, three, as well as develop their awareness of the notion of one-to-one correspondence. As Mason (1998) notes, "Awareness of awareness-in-discipline provides access to sensitivities which enable us to be distanced from the act of directing the actions of the others in order to provoke them into becoming aware of their own awareness in action and awareness-in-discipline" (p. 261).

Methodology: Participants and Data Collection

The data presented here were collected from detailed questionnaires, journals, and focus group discussions involving four prospective teachers enrolled in the MICA program, four teacher candidates (K-12) in the pre-service department, and one practicing teacher. In order to probe deeply into prospective teachers' experiences in terms of awareness, questions and prompts in the questionnaires and journals were open ended. Special care was taken to avoid any direct or leading questions that might bias data toward our postulate regarding ownership, pride, and engagement. All participants were informed only that this study was about mathematics teacher education and LOs. Participants were provided with the questionnaires and guidelines to take home in order to write the responses at their own pace. They were encouraged to take their time in answering questions in detail, and to be as honest as possible. A small honorarium was given to the participants to acknowledge their commitment to the research. The focus group discussions helped to uncover more about prospective teachers' experiences as they discussed their LOs with the pre-service students and the teacher.

Given the student-instructor relationship between students participating in the study and the investigators, we (after receiving ethics clearance) extended a verbal invitation to student volunteers who might be interested in participating in the study. All students enrolled in MICA I and MICA II, and all pre-service students enrolled in a mathematics methods course, were invited to participate in the research. Four MICA student volunteers and four pre-service student volunteers were then recruited. After recruitment, an information session was conducted at which the invitation letters and consent forms were shared, and questions asked and answered. One practicing teacher from a local school was approached and invited to participate in the study. All the data collected from students were stored by our research officer. We did not have access to the data until course grades were assigned.

MICA Participants. The participants in the project were two prospective teachers in their first year (attending the MICA I course) and two in their third year (attending the MICA II course). They filled out detailed questionnaires, wrote journals during the design of their LOs, and provided their finished MICA projects (computer programs and write-ups). The data reported in this study involved three LOs: one on parabola (designed by the two MICA I students); one on multiplication (designed by MICA II student 1 during MICA I course) and one on slope (designed by MICA II student 2 during MICA I course). The two MICA II students also created two MICA II final projects, both exploratory. We decided not to include these projects in our pilot study, however, because of the differences in the learning experiences involved in exploratory vs. didactic projects (we briefly comment on this in the conclusion section). We did include the two MICA II students' a posteriori reflections on their MICA I experience.

Prospective teachers filled out a detailed questionnaire at the beginning and at the end of both MICA I and MICA II regarding the use of technology in teaching and learning; their experiences of designing, implementing, and testing a LO; and their learning, both in terms of mathematics and the mathematics needed for teaching. Also, they kept a journal regarding their experiences during the process of designing, implementing, and testing the LO. Guidelines for writing the journal were provided. Finally, all MICA I and MICA II prospective teachers filled out a post-project questionnaire focusing on their reflection on the design, implementation, and testing of LOs.

In addition to these research tasks, MICA II prospective teachers met with four pre-service students (enrolled in an elementary school mathematics method course in the faculty of education) for a one-hour focus group discussion, and then with a practicing teacher for a one-hour focus group discussion, to discuss the use and improvement of their existing LOs. The purpose of these focus group discussions was to further probe the learning experiences (in terms of mathematics didactics) of the prospective teachers from a user perspective. All the MICA II prospective teachers, the pre-service students, and the practicing teacher responded to a questionnaire both before the discussion (to prepare) and after (to reflect on what had happened). The focus group discussions were moderated, video-taped, and transcribed by a research assistant (graduate student).

Pre-Service Students. We should note that the role of the pre-service students in the research was not to act as research subjects but rather to facilitate data collection. The four pre-service participants were required to use and didactically analyze the two LOs (one on multiplication and one on slope) designed by MICA II participants. They then met in a focus group discussion for one hour with the MICA II students to discuss their experiences of using the LOs. Pre-service students responded to a one-hour questionnaire before the discussion with the MICA students (to prepare) and after (to reflect on what had happened).

Practicing Teacher. As with the pre-service students, the role of the teacher was not to act as a research subject but to facilitate data collection. One practicing teacher was invited to use the two LOs designed by MICA II participants (one on multiplication and one on slope), reflect on them, assess their pedagogical quality, and offer her thoughts on the difficulties and advantages of using them in class. The teacher responded to a questionnaire about her background and experience, both in teaching and in learning mathematics with technology. She then met for one hour with the MICA II students to discuss the LOs. She also responded to two other questionnaires: one before the meeting (to prepare) and one after (to reflect on what had happened).

Data Analysis and Results

Given the small scale and qualitative nature of the pilot study, all data from questionnaires, LOs, and transcripts from videos were analyzed according to the interpretation of themes. The analysis was guided by the postulate that ownership, engagement in the activity, and pride were key for positive learning experiences; and by Mason's (1998) three forms of awareness as outlined in the conceptual framework. Our analysis proceeded as follows. First, we looked for words and phrases in the data that indicated evidence of prospective teachers' personal relationship with the activity, discerning whether these words and phrases indicated prospective teachers' experiences of ownership, engagement, and pride. Then, using Mason's levels of awareness, we identified which levels of awareness were engaged, as well as ways in which they related to experiences of ownership, engagement, and pride. The analysis led to an elaboration of three qualities required for a successful learning experience via the design, implementation, and testing of LOs. In the following sections, we present these three qualities: ownership, engagement and pride. In Mgombelo and Buteau (2009) we further elaborate on the three qualities in relation to other literature.

Ownership. The four prospective teachers started their MICA I course without any knowledge of programming and without having attended any mathematics didactics courses in the faculty of education. For example, one prospective teacher observed, "[...] I had no clue how mathematics was involved in computer programming" (MICA I student 1, questionnaire). Yet, after an introduction to programming during the MICA I course, these prospective teachers accomplished the design, implementation, and testing of a LO on a topic of their own choice as a final project. The remarkable finding from our study is that as part of the process of designing, implementing, and testing LOs, all four prospective teachers began by attending to the act of learning of mathematics on the topic they chose. As we have noted earlier in this paper, this is remarkable, because typically prospective teachers begin by attending to the content of their learning (rules, etc.) and not the action. The prospective teachers attended to the learning first in terms of their own learning and then in terms of their audience's (students') learning. This is significant because it is through attending to their learning as an action that these prospective teachers could educate their awareness (because awareness involves action) of their own awareness-in-action of mathematics and in turn to educate their awareness of awareness-in-discipline. From this analysis we elaborated on the experience of ownership as involving attention to the act of learning, as opposed to the content of learning. This is exemplified by the following prospective teacher's response to the questionnaire question on why she chose the topic for her LO:

> My MICA I LO [...] dealt with explaining and practicing multiplication. It was aimed towards elementary school students, but when I tested it on my roommate from first year, she learned from it as well. I chose this topic because in grade four I was very, very behind on my multiplication. I could not do the calculations in my head, and I was stuck on the first sheet of questions my teacher would give us. I think we had one minute to complete the page or something, and if you did and got them all right then you would move on to the next page. I was stuck on page one so long that I just memorized the order of the answers. Since it is something I struggled with and something that I have to overcome to become a Math major, I thought it would be a great idea to develop a program that could allow students to practice without just doing the same questions over and over. I also included different ways of thinking about what multiplication means. (MICA II student 1 questionnaire)

From this response, we clearly see how this prospective teacher attended to her own learning of multiplication or her own awareness-in-action of multiplication. But to attend to one's own learning does not necessarily guarantee an emergence of awareness of awareness-in-action (awareness in discipline). As we have noted above, in order for awareness to emerge, one has to attend to the act of learning, not the content. As Mason (1998) notes, the behaviours in which awareness-in-action plays a role can be trained to some degree without explicit allusion to awareness. Ownership, as we define it here, has to engender awareness. Put differently, ownership engenders awareness of awareness-in-action. The distinction between ownership and the lack thereof is that ownership involves the act of learning, which requires personal connection and meaning, whereas lack of ownership involves only the content of learning, which might be confined to memorizing or to following rules from an outside authority.

The prospective teacher in the above response did not want to design a program based on multiplication routines and rules. Instead, she wanted to include the different ways of thinking about what multiplication means, and this involves awareness. Another prospective teacher further elaborated upon this aspect of ownership: "Technology and programming has made me a better problem solver and more capable of figuring out concepts on my own" (MICA I student 2, questionnaire). It is clear that figuring out concepts and being a better problem solver involves awareness of awareness-in-action, which indicates ownership on the parts of prospective teachers with regard to mathematics.

By attending to their users' (students') learning of mathematics through the designing, implementing, and testing of a LO, prospective teachers thought about not only their own awareness of awareness-in-action, but also their students' awareness of awareness-in-action – students' relationships to mathematics in terms of ownership. This process is revealed in the following questionnaire response from a prospective teacher: "Through the design and implement [sic] of my project, I put my knowledge of certain aspects of mathematics to practical use for the basis of helping younger students learn easier" (MICA I student 1, questionnaire). It is interesting to note that this response refers not to teaching students but to helping students learn. In other words, this prospective teacher views his role as a guide to his students' learning, thus attending to students' ownership. The opening quote in this paper illustrates the same aspect of ownership. In that quote, the prospective teacher notes that the idea behind designing a LO is to provide an interactive tool that provokes students' learning of mathematics and not just "giving kids a bunch of junk." In summary, our analysis of the data shows that ownership engenders awareness and involves prospective teachers attending to their own act of learning.

Engagement. Awareness-in-discipline arises when we become aware of awareness-in-action. According to Mason, the term *discipline* means encountering facts, techniques, habits of thought, types of meaningful questions, and methods of resolving those questions. Our analysis of the data indicates that through the design, implementation, and testing of LOs, prospective teachers engage with mathematics in terms of the aspects of discipline outlined by Mason. From our analysis, we elaborated on the experience of engagement with mathematics as another aspect of learning the mathematics needed for teaching, involving awareness-in-discipline and awareness-in-counsel. In terms of awareness-in-discipline, engagement is recognized in the way prospective teachers use games, graphics, and colours in their LOs in order to engage students in a meaningful way. Different representations or meanings of mathematics concepts (such as grid or area models of multiplication) were attended to, as revealed in excerpts from the journal entry of a prospective teacher:

> I set my goals too high before I started. I had some pretty neat ideas of cool games to include and stuff like that. One of them was actually the table where they get to push the numbers and it fills in the grid. I worked on that for a very long time but, I had expected it to be fairly easy. I did not get as much accomplished as I would have thought and it was not as engaging as I would like it to be [...] I researched a few sites about how to teach multiplication and then I felt better about what I could put on the teaching section. (MICA II student 1, journal)

As with ownership, engagement also involves both prospective teachers' own engagement with mathematics and their audience's (students') engagement. We see from the above excerpts that the respondent noticed that her LO "was not engaging" enough and that she therefore decided to research different ways of teaching multiplication. The following response from a questionnaire supports this analysis: "I think it [LO] would be useful because it gives students a chance to visually experience the mathematical knowledge they've been taught verbally" (MICA I student 1, questionnaire). This idea – that engagement involves engagement of both the prospective teachers and their audience – is also revealed in the following questionnaire response by a prospective teacher:

> I learned how to keep instructions short and simple, and how to gear a lesson towards your audience. I learned to think about the audience I was trying to reach and what would be engaging to them, I added in Bart Simpson and made it as bright and colorful as I could. I learned multiple ways of explaining multiplication. (MICA II student 1 questionnaire)

The last sentence is very important, because it indicates how prospective teachers engage with mathematics by learning to explain mathematics in multiple ways, which is an important element of mathematics for teaching (Davis & Simmt, 2006).

Our suggestion for prospective teachers' engagement is supported by observations on their LOs by the teacher, as well as observations by the pre-service students. Indeed, with regard to the use of LOs in the classroom, the teacher thought that the LOs would be useful for teaching mathematics:

> I think that the LOs could be used for students to discover concepts on their own, before I teach them new ideas [...] the students [could use the LOs] when I am away so that they would be self-directed. (Teacher, questionnaire)

A pre-service student candidate, who self-identified as having mathematics anxiety, found that the LOs were useful for his own learning of mathematics. "I did learn basic multiplication and used practice tests to remind me of what I know. I also learned more about a slope. I do not remember learning this and so it was helpful" (Pre-service student, questionnaire).

Pride. Sustaining ownership requires one to invest some personal energy and to have a sense of purpose. In our discussion of the data analysis, we elaborated on the experience of pride as a third aspect of learning the mathematics needed for teaching. Our analysis shows that pride is demonstrated by prospective teachers' sense of accomplishment or satisfaction in their work, as indicated in the following journal excerpts: "I am proud of my program" (MICA II student 1, journal); "You're always thinking about ideas and ways to improve your project while you are in class, watching television" (MICA I student 1, journal). The latter excerpt clearly shows how much personal energy the prospective teacher invested in the project. The same prospective teacher also indicated how proud he was of his learning, as marked by his use of the word *superbly* to describe the usefulness of his LO: "I think it's a helpful tool that could be very beneficial. After using [my partner's] sister as a test subject and observing the results of her reactions, I am confident that our program would be superbly useful" (MICA I student 1, questionnaire).

As we have mentioned earlier, questions and prompts were carefully designed to avoid biasing the data toward what we had postulated. We did not ask direct questions regarding pride. The above quotes are a result of prospective teachers' spontaneous responses and reactions to questionnaires and journals. We are currently working on finding ways to prompt for students' experiences of pride.

Conclusion

The goal of the pilot study was to gather first evidence of prospective teachers' experiences of learning the mathematics needed for teaching, through the design, implementation, and testing of LOs. In addition, we hoped to inform the methodology and conceptual framework of our research. Our pilot study strongly suggests that the experience of designing, implementing, and using LOs promotes prospective teachers' learning of the mathematics needed for teaching. The experience of designing, implementing, and testing LOs seems to prompt prospective teachers to attend to their own act of learning in order to attend to their students' act of learning. Through this process, prospective teachers bring to awareness their awareness-in-action of the mathematics, which in turn helps them to be aware of this awareness when designing, implementing, and testing LOs. This is what we refer to as ownership.

In addition, our analysis indicates that prospective teachers' experiences of designing, implementing, and testing LOs tend to elicit the need to explain and attend to different representations and meanings of mathematics concepts, a very important aspect of teaching (Ball & Bass, 2002; Davis & Simmt, 2006). The experience seems to trigger awareness of awareness-in-action (awareness in discipline) and in turn awareness of awareness-in-discipline (awareness-in-counsel), which enables the prospective teachers to design meaningful mathematics contexts. We refer to this experience of explaining and attending to different representations and meanings of mathematics concepts and skills as engagement – another aspect of prospective teachers' learning of the mathematics needed for teaching. In order to sustain ownership and engagement in mathematics activities in relation to awareness, prospective teachers (and their students) have to invest themselves in the activity (in terms of energy, emotion, interest, etc.).

In order to sustain activity so that awareness is encountered in mathematics activities, prospective teachers (and their pupils/users) have to invest themselves in the activity in terms of energy (Mason, 1998). According to Mason, the notion of "discipline" in awareness-in-discipline means being systematic, concentrating, and persisting, and all of these depend on emotional energy. Emotional energy can be dissipated (e.g., when prospective teachers feel uninterested) or creative. Creative energy allows prospective teachers to make significant mathematical choices and to sustain relevant activity so that (a) awarenesses can be encountered, and (b) activity is drawn to a close by prompting reflection and digestion of what has taken place. It is this process and accomplishment that are demonstrated by prospective teachers' experiences of pride, the third aspect of prospective teachers' learning of the mathematics needed for teaching.

Further empirical questions emerged from the study: What aspects of designing, implementing, and testing LOs prompt such a positive experience? In what ways does prospective teachers' learning in these tasks differ from the use of other, more traditional learning tasks? These questions have led to a larger-scale, collaborative research project involving some 30 MICA future teacher candidates, each followed over two years. The project will thoroughly investigate the students' "repositioning" (in terms of engagement, ownership, and pride, with respect to mathematics and mathematics didactics) when realizing their MICA final projects (the LOs) compared to more traditional mathematics activities. We are also interested in exploring which characteristics or features of the learning activity promote learning. To this end, a conceptual framework has been developed to guide this comprehensive study – the methodology (Mgombelo & Buteau, 2009).

The pilot study also taught us that the experience of designing, implementing, and using EOs may also promote prospective teachers' learning of the mathematics needed for teaching, as the following response of a prospective teacher (MICA II student) to a question suggests:

> I believe that any project that involves extensive research, followed by the implication of the program or presentation, will help one in teaching about that same topic in the future. I believe this because in having to explain and demonstrate something, it requires one tries to do so in ways that the audience will understand. This project helped me to do that, which

may be helpful in reaching individuals when I am a teacher. (MICA II student 2, questionnaire)

Moreover, observations in focus group discussions from this project support the ongoing use of MICA LOs to provide "safe" non-judgmental opportunities for pre-service students to learn the mathematics needed for teaching by using and analyzing (non-professional) MICA LOs. This collaboration between the department of mathematics and the faculty of education contributes to progress in the field of mathematics teacher education, in terms of both research and practice.

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Figure Citations

Figure 1. Development process of an Exploratory Object for the purpose of investigating a conjecture. (Source: Buteau & Muller, 2009)

Figure 2. Development process of a Learning Object. (Source: Buteau & Muller, 2009)