

## IMPROVING 8<sup>TH</sup> GRADES SPATIAL THINKING ABILITIES THROUGH A 3D MODELING PROGRAM

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### ABSTRACT

Implementation of emerging technology in sub disciplines of mathematics education provides a potential for educators to elaborate the capacity of digitized learning for human being. Spatial thinking is considered as a factor of scientific deduction from a multi disciplinary point of view. This paper reports a study aimed at exploring the effect of a 3D Modeling program on the spatial ability of the 8th grade students through an experimental research design. The study also focuses on the relation between the gender difference and spatial thinking. The study population was consisted of 82 8th grade students and divided into the control group (n=40) and the treatment group (n=42). The data in the study were collected through a qualitative research method. According to the findings of the research, the success rate of post test increased after the application in terms of differential aptitude, mental rotation and spatial visualization. On the other hand, irrespective of the relevant literature, female pupils were observed as better performers comparing to the males on post application of the measurement instruments.

**Keywords:** Spatial ability, spatial visualization, mental rotation, concrete manipulatives, 3D modeling, Google SketchUp

### INTRODUCTION

All aspects of mathematics teaching and learning are being pervaded by the tremendous improvements in technology. As the computer technology progressed, it improved mathematics by allowing the emergence of further disciplines of the field; for instance, fractal geometry did not exist until the advent of high-speed computers (Tooke & Henderson, 2001). Tall, (2009) summarizes the impact of computers in mathematics in three parts: *symbolism of numeric computation*, *graphical display*, and *enactive interface* allowing selection and manipulation of objects drawn onscreen. The implementation of technology in mathematics instruction has been recommended on many occasions by the National Council of Teachers of Mathematics (NCTM, 1996, 2000) and the Conference Board of the Mathematical Sciences (2001). Since computers have changed the ways that mathematics are taught and learned, mathematics *educators* should know the realities as well as the possibilities for human learning in an age of information technology.

Recent discussions of teaching and learning approaches have emphasized the role of visualizations and graphical representations to enhance students' learning experiences (Frank, 2005). With 3D visualizations, students can experiment with different 'what-if' scenarios and actively discover unique solutions to the problems (Messner et al, 2003). The National Council of Teachers of Mathematics recommends that the mathematics curriculum for grade 5-8 should include the study of the geometry of one, two, and three dimensions in a variety of situations, so that students can visualize and represent geometric figures with special attention to developing spatial sense (NCTM, 1989). Also, the initiative establishing learning trajectories and achievement targets for Dutch primary school identified several spatial activities as being important in primary school education (Van den Heuvel-Panhuizen & Buys, 2005). Another study promoting the use of technology in geometry education was conducted by González and Herbst (2009) by analyzing how students solved geometry problems over four days, with two days spent using static diagrams and the other two with dynamic diagrams drawn using a calculator with dynamic geometry software. Dynamic Geometry Software (DGS) are computer programs which allow one to create and then manipulate geometric constructions, primarily in plane geometry. In most DGS, one starts construction by putting a few points and using them to define new objects such as lines, circles or other points. After some construction is done, one can move the points one started with and see how the construction changes. González and Herbst (2009) say when students used dynamic geometry software they were more successful in discovering new mathematical ideas than when they used static, paper-based diagrams. The dynamic geometry software really helped them make connections that they hadn't made before.

As the ability to construct, retain, retrieve, and manipulate visual images of two- and three-dimensional objects (Lohman, 1993), spatial thinking is one of the essential traits for scientific thought. Linn and Petersen (1985) identified three categories of spatial ability: spatial perception, mental rotation, and spatial visualization. Spatial perception is defined as the ability to “determine spatial relationships with respect to the orientation of their own body” (p. 1482). The mental rotation category includes both two-dimensional and three-dimensional mental rotation tasks, such as Cards, Figures, and Flags. Spatial visualization comprises spatial tasks that involve multistep, analytic procedures, and require flexibility in strategy selection. Clements and Battista (1992) underline the use of spatial thinking in representing and manipulating information in learning and problem solving. Spatial visualization (SV) provides another accurate predictor of success in a variety of academic areas (Humphreys, Lubinski, & Yao, 1993). As the ability to imagine shapes rotated into a new orientation (Shepard & Cooper, 1982), mental rotation (MR) is an important spatial ability for two reasons. Firstly, mental rotation is a simple, relatively atomic sub-skill, used extensively in more complex spatial skills such as spatial visualization. Secondly, mental rotation is the spatial skill that shows the largest and most persistent gender differences, with males performing better. The learning and transfer of spatial skills SV and MR are important to mathematics. There are many potential between-country cultural factors that might mediate spatial skills training and one of them is electronic media (Olkun, Altun, & Smith, 2005). The computer has a very important effect on student spatial visualization.

The use of technology may help learners to gain sufficient geometric reasoning (Battista & Clements, 1996). Students’ conceptualizations of three-dimensional buildings can be based on faces; they do not consider the figure as three-dimensional nor do they consider the interior cubes in the building. Hirstein (1981) underlines that students tend to confuse volume with surface area while they are finding the number of cubes in rectangular buildings. At a higher level of conceptualization, learners are fully aware of the three-dimensionality and space-filling properties of the cubes and of the whole building. Secondly, they may conceptualize a cubic building as being organized into columns, rows, and layers so that they account for both visible and hidden cubes systematically. Olkun, (1999) maintains that learners often return to a primitive strategy when they are exposed to advance level complex buildings.

Students’ use of visual spatial imagery while solving math problems is positively and significantly correlated with mathematics problem-solving performance (Van Gardener & Montague, 2003). Through re-testing and practice, people can sometimes improve spatial skills within a narrow context, but such improvements have not transferred to other contexts globally (Olkun, 2003). Battista and Clements (1998) reported that elementary students are unable to coordinate the different orthogonal views of the cube configuration. In fact, to construct the three-dimensional cube building correctly and to explore the invisible cubes, mental configuration of the orthogonal views is necessary. Understanding two-dimensional representations of three-dimensional buildings is also a part of spatial visualization which includes mental integration of different views, such as orthogonal and isometric views.

The topic of gender differences in spatial ability is highly recurring in literature. It has been proposed that sex differences in spatial ability are the result of culturally based differences in socialization patterns and sex-typed activities (Goldstein, Haldane, & Mitchell, 1990). Males typically outperform females on several spatial tasks (Halpern & Collaer, 2005). In accordance with this research tradition, an additional aim of the current study was to examine boy-girl differences in spatial ability. Although male advantages have been found on various spatial tests, between different spatial tasks large variation exists in the size of these gender differences (Linn & Petersen, 1985; Voyer et al., 1995). From the experiential explanation of the gender differences in spatial ability, one can expect spatial training to have a greater effect on females’ spatial test performance than on males’. According to this theory, males already have received a lot of training by experience and thus already perform close to their maximum potential, whereas females have more room to improve (Baenninger & Newcombe, 1989).

As one of the dynamic geometry software, Google SketchUp is a powerful, sophisticated, user-friendly Computer Aided Design (CAD) program (Fleron, 2009). It was developed initially by Last Software in 2000. They were acquired by Google in 2006 and the first free version of this software –Google SketchUp (GSU) – was released on 27 April, 2006. There are some pragmatic Issues that Make Google SketchUp Useful in Education. First and foremost, Google SketchUp (GSU) is free. Not only does this allow schools access to powerful software without the hassle of licensing and the budgetary issues that are so severe right now, but it also means students can download it and use it at home. Technically, GSU is robust. It is supported on PCs and Macs equally. Download times with a cable modem are about 2 minutes. There are relatively meager hardware requirements. This software has a very gentle learning curve, especially considering its remarkable power. The tool provides active, substantive, and curricular appropriate connections of mathematics to art, architecture, engineering, regional

planning, construction trades, graphic design, animation, graphics, and many other areas. Thus, it provides wonderful opportunities for collaboration between and among students, teachers, and professionals. Our goal here is to help provide a bridge for its curricular integration.

Integrating technology supported environments into Mathematics curriculum makes it possible to examine pupils' actions and thinking processes in detail, which allows to assess their strategies in more precise ways than can paper-and-pencil formats. Clements (1998) and Singleton (2004) point out that computer environment may reduce cognitive demand while working on a task and help teachers gain a deeper understanding of key difficulties. The potential of computer environments to provide insight into pupils' cognitive processes makes them a fruitful setting for research on how this learning takes place (Kolovou, Van den Heuvel-Panhuizen, Bakker, & Elia, 2008). In sum, it is crucial that teachers who deal with pupils in mathematics should have a good understanding of technology supported learning beside their pupils' capabilities, ie, their thinking skills.

## METHODOLOGY

This study aims to determine how Google SketchUp program influences 8<sup>th</sup> grades spatial thinking ability and how the gender difference affects learners' spatial thinking traits through an experimental research design. First, all students were pre-tested by using geometry and spatial visualization tests (DAT-MRT-SV). Then one of the groups was assigned as control (n=40) and other as experimental (n=42). Two treatments, with and without the computer, were administered with programs that potentially develop spatial ability. An important question for a study on the effects of a learning material on spatial ability is whether spatial ability can be trained. In a meta-analysis, Baenninger and Newcombe (1989) found that spatial test scores typically improve by both practice (test-retest) and training. Furthermore, for training to be more effective than mere practice, the training has to be of at least medium duration, consisting of more than one training session during more than three weeks. Thus, a six weeks laboratory application conducted in the current study. Two treatments, with and without the computer, were administered with programs that potentially develop spatial ability. The computer treatment involved using Google SketchUp (GSU) labs that involve creating buildings from two-dimensional plans. The computer group did all the activities that control group had, firstly, and then they took their lessons with Google SketchUp (GSU) labs for 5 weeks. The treatment time ranged from 60 to 80 minutes for each lessons. Control group did not have computer experience. During the treatment, students in the experiment group had their lessons with computer experience for six weeks and solved computer-based problems. During the labs, learners used GSU to unfold the sides of a three-dimensional building to determine its two-dimensional plan. In addition, they used an internet site that rotates three-dimensional figures orthogonally and isometrically. The students in the control group continued on in their regular classes and were not shown any of the treatment materials. The control group also created buildings from two-dimensional plans on paper and drew two-dimensional plans of their dimensional shapes of which lengths were given. However, they did participate in the pre- and posttests. The pretests were administered in the fall semester in September 2010. Treatments were carried out until the end of November. After the treatment, the experiment and control groups were post-tested in a separate room within the same day.

Two research questions delineated for the study are as follows,

1. Does the use of Google SketchUp have any effects on 8<sup>th</sup> grade learners' performance in spatial visualization as measured by the
  - a. Mental Rotations Test (MRT),
  - b. Differential Aptitude Test-Spatial Relations (DAT),
  - c. Spatial Visualization (SV) Test,
 in comparison with the learners who did not utilize any 3D modeling programs or computers during the education?
  
2. Does the gender difference have any effects on 8<sup>th</sup> grade learners' performance in spatial visualization as measured by the
  - a. Mental Rotations Test (MRT),
  - b. Differential Aptitude Test-Spatial Relations (DAT),
  - c. Spatial Visualization (SV) Test?

## Participants

The participants of the study consisted of 82 eight grade students attending a primary school in Kirikkale in Central Anatolia of Turkey. The participants were administered treatment of the computer (n=42) and control group (n=40). For each group both males and females were recruited for participation. A signed consent form was obtained from the education division of the Kirikkale Proconsulate. Parents of the participants were also

informed via the administration of the laboratory school. In control group, the males (n = 25) and females (n = 15) and in the experimental group males (n = 26) and females (n = 16).

**Materials**

Participants took the Mental Rotation Test (MRT, Peters et al. 1995), Differential Aptitude Test (DAT, Bennett, 1947) and Spatial Visualization (SV, Winter et al. 1896) instruments in terms of measurement. The MRT is a twenty-four problem set. Each problem has a target figure shown on the left and four stimulus figures on the right. Two of these stimulus figures are rotated versions of the target figure, and two of the stimulus figures cannot be matched to the target figure. Students receive one point for both correct answers. The DAT/Space Relations (SP) measures capability of analyzing three-dimensional figures with a sixty-item aptitude test that analyses the ability to visualize 2D or 3D drawings, an expectation for jobs in engineering, architecture or design. The SV is a measure of the ability to construct three-dimensional buildings using manipulatives. The nineteen-item assessment requires translation of information from two-dimensional objects and to visually distinguish between a left-right or front-back orientation in both two and three dimensions. The question that the study sought for an answer was whether the students were only able to perform the assessment when they had experience in building and looking at three-dimensional objects. The overall Alpha value of the variables was found out as .80 as a reliability indicator. Besides, the Kolmogorov-Smirnov and Levene’s test results revealed a normal distribution of the data as a pre-assumption of mean comparison tests.

**RESULTS AND INTERPRETATION**

Data gathered from 82 eight grade learners at a primary school in Kirikkale were analyzed through statistical package for social sciences (SPSS) version 15.0 for quantitative analysis. Since the main purpose of this research was to understand whether the utilized media (Google SketchUp) did have any effect on learners’ performances in spatial visualization comparing to control group, descriptive statistics and independent samples t-test scores were calculated to summarize the data.

Participants responded to the tests prior to the program, and again after the program. Differences in scores of the pupils before and after participating in the program identified any change in performance in spatial ability as a result of the treatment. When we compared post-treatment and pre-treatment results there is an increase in spatial abilities for all students in both treatment programs. Based on this data, the pre-post differences nearly on each instrument, DAT, MRT, and SV were statistically significant with  $p < .05$ . This section will summarize the data in accordance with the research questions.

Table 1 depicts how experiment and control groups performed at pre and post tests of three different achievement tests of spatial visualization.

*Table 1. Group Differences in terms of measurement instruments*

Instrument	Group (42 / 40)	Mean	p-value
DAT 1	Experiment / Control	18.28 / 18.07	.866
DAT 2	Experiment / Control	23.33 / 19.65	.017*
MRT 1	Experiment / Control	4.85 / 3.60	.052
MRT 2	Experiment / Control	5.86 / 3.70	.001*
SV 1	Experiment / Control	5.30 / 5.12	.678
SV 2	Experiment / Control	6.95 / 5.62	.010*

Significance level is defined as .05

As for the Differential Aptitude Test, which forms the content of the first research question, while there is no significant difference between groups’ performances prior to the treatment (18.28 / 18.07), experiment group utilized Google SketchUp program did significantly better at posttest comparing to those who did not use computers during the education (23.33 / 19.65). Similarly, total performances of both group members came out as very close to each other at the pre application of Mental Rotation Test (MRT) but the experiment group outperformed at post application of MRT in comparison to control group. The independent samples t-test results related to the first application of SV test indicated that there is no significant difference within research groups which means that a homogenous level of performance was obtained before the treatment. Respectively, post test results showed a significant variance in favor of experiment group exploiting Google SketchUp Software.

Table 2 is portraying the effect of gender difference on primary level learners’ spatial visualization performances through three different tests.

Table 2. *Gender Differences in terms of measurement instruments for the participant groups*

Instrument	Group (32 / 50)	Mean	p-value
DAT 1	Female / Male	19.09 / 17.60	.259
DAT 2	Female / Male	23.50 / 20.28	.041*
MRT 1	Female / Male	4.31 / 4.36	.936
MRT 2	Female / Male	4.75 / 4.36	.576
SV 1	Female / Male	5.84 / 4.82	.034*
SV 2	Female / Male	6.90 / 5.92	.058

Significance level is defined as .05

As for the second research question, independent two-tailed test of the means were conducted between the scores of females and males. The mean values of the both applications of Differential Aptitude Test for females and males are as follows: DAT1 (F: 19.09 / M: 17.60), DAT2 (F: 23.50 / M: 20.28). The t-test results also imply that while the pre application did not indicate any significant differences (.259), post test of DAT showed a significant difference between girls and boys. The descriptive statistics portrayed the difference in favor of girls (F: 23.50 / M: 20.28). The results of Mental Rotation Test applied both before and after the treatment revealed no significant difference (.936 / .576). However, a positive variance was observed in favor of females at the post test of MRT (F: 4.750 / M: 4.36). On the other hand, while the pre-test of Spatial Visualization revealed a significant difference between the genders (.034), no significant difference was observed at the post application of the test (.058). The mean score of the participants at SV test pointed out that although both girls and boys achieved better at post test, girls' total performances at post test was higher in comparison with boys.

The results of the current study indicate that the use of computer programs such as Google SketchUp and other mental rotations programs may improve spatial ability. Besides, students may be able to improve this important ability more if they have additional computer experiences integrated within their course curriculum, in particular, by using Google SketchUp. The study should be repeated at various learner levels and should be supported by various covariates such as learners' features.

A study by La Ferla et. al, (2009) they made an international comparison of the effect of using computer manipulatives on middle grades students' understanding of three-dimensional buildings and they demonstrated significant differences on MRT and SV tests. The data indicates that the use of computer programs such as Google SketchUp and other mental rotations programs may improve spatial ability. The DAT measure may not have been significant since more time may have been needed for students to work with the various features of Google SketchUp in order to attain the benefits.

A study by Rafi (2008), for instance, examined the effect of Web-based activities and animation aided computer applications on the spatial visualization abilities of two test groups of primary school 2nd Grade students. The same study also included a control group taught through traditional teaching methods. Rafi's study finally concluded that the two test groups had higher levels of spatial ability than that of the control group.

## CONCLUSIONS AND FUTURE CONSIDERATIONS

The results of this study showed that the use of a dynamic geometry tool had a positive effect on learners' spatial progressions. This effect was found, even though the pupils in the experiment group had never used the tool before. The fact that pupils found ways in which they could benefit from using the GSU to fold and unfold three-dimensional shapes. One of the more significant findings to emerge from this study is that working with the students improved their overall spatial ability which may mean that the amount of time needs to be increased and the specific types of labs and activities need to be repeated over time to obtain significant results. Besides, the benefits of learning Google SketchUp (GSU) in primary school will pay benefits for students as they move through the rest of their formal education as well. In particular, GSU can help provide connections and coherence with the secondary curriculum.

Finally, a number of caveats need to be noted regarding the present study. The most important limitation lies in the fact that the small sample size of the study prevents the transferability of the outcomes. One of the main weaknesses of this study was the paucity of information on ICT/software use backgrounds of the participants. One source of weakness in this study which could affect the measurements was that the variation of socio-economic status of the participants might affect the potential computer use skills of them. The current research has thrown up some questions in need of further investigation. More broadly, research is also needed to investigate various types of spatial abilities such as object visualization. If the debate is to be moved forward, a better understanding of other types of training programs that increase spatial visualization such as three-dimensional virtual reality programs needs to be developed. We may want to revise our treatment program.

Students may require more time outside of the classroom to work with GKU. One possibility would be to have students use an iPad with GKU both in and out of the classroom and have them log the amount of time they spend using the software. Also, for the MRT, students may need to work with manipulatives such as pop cubes to assist with the mental rotations as well as practice using the orbit portion of the GKU feature. Again, it may be linked to amount of time using the program and adjusting to a new software package that may lead to gains after a critical amount of time. If students are provided with a computer for use, we can log the amount of time they use the software and for what purpose.

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