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Intuitive physics of collision effects on simulated spheres differing in size, velocity, and material

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This is an intuitive physics study of collision events. In two experiments the participants were presented with a simulated 3D scene showing one sphere moving horizontally towards another stationary sphere. The moving sphere stopped just before colliding with the stationary one. Participants were asked to rate the positions which both spheres would have reached after a fixed time if the collision had really occurred. The simulated masses of both spheres and the velocity of the moving one were manipulated. Specifically, in Experiment 1 different implied masses were defined by varying only the size of the spheres; in Experiment 2, different implied masses were defined by varying both the size and the apparent texture (material) of the spheres. Functional Measurement was used to compare the physical laws of collision events with cognitive integration rules. Cognitive rules proved to be more similar to physical laws in Experiment 2, i.e., when both spheres size and apparent texture were manipulated. Surprisingly, in both experiments only half the participants took into account the possibility that the moving sphere could have bounced back after the collision. These and other results are important for teaching elementary physics.

INTRODUCTION

Intuitive physics typically depends on multiple stimulus cues: that the magnitude of one variable may depend on that of another variable is ingrained in everyday thought and action. Functional thinking seems to be a general mode of cognition (Karpp & Anderson, 1997, p. 360). Stimulus integration often obeys simple algebraic rules, such as addition, multiplication or averaging (Anderson, 1981). The basis of intuitive knowledge is thus Cognitive Algebra. Information Integration Theory (IIT) and Functional Measurement (FM) (Anderson, 1981, 1982) offer suitable

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theoretical and methodological frameworks for assessment of these cognitive algebraic rules (see Hofmans, Mairesse, & Theuns, 2007).

Several physical laws (e.g., Newton's laws of motion) are formalized as simple algebraic rules: IIT and FM are powerful tools for directly comparing cognitive and physical rules, thus unifying intuitive and symbolic knowledge.

One of the major determinants of the congruency between intuitive and formal knowledge appears to be familiarity with the task. Although people may fail in solving abstract problems, they may still be able to make accurate predictions of physical events in familiar specifications of such problems (Kaiser, Jonides, & Alexander, 1986; Masin & Rispoli, 2010). Another major determinant of this congruency is the realism of stimuli: when people make predictions concerning dynamic events, the use of dynamic animations as stimuli improves their performance (Kaiser, Proffitt, Whelean, & Hecht, 1992).

In this study, I investigate the intuitive physics of collision effects between simulated 3D spheres differing in size, apparent weight, and material. As the stimuli in my experiments are more familiar and realistic than those used in previous experiments on the intuitive physics of collisions, participants' performance is expected to be closer to formal physics than in previous experiments.

Physics of collisions. Let us presume that a sphere (*A*) is moving horizontally towards another sphere (*B*) which is stationary, and that their centers of mass lie on a horizontal line[1]. If this system is isolated (i.e., not subject to external forces), if the spin of the two spheres is ignored, and if the collision is perfectly elastic, then:

$$v'_A = v_A \ (m_A - m_B) / (m_A + m_B) \qquad (1)$$
$$v'_B = 2\, v_A \ m_A / (m_A + m_B) \qquad (2)$$

where v'_A and v'_B are the post-collision velocities of *A* and *B*, v_A is the pre-collision velocity of *A* ($v_B = 0$ because *B* is stationary before the collision), and m_A and m_B are the masses of *A* and *B*. Equations (1) and (2) are derived from Newton's Third Law of motion (Kittel, Knight, & Ruderman, 1973). Note that according to Equation (1), if $m_A < m_B$, then v'_A is negative, which means that *A* bounces back.

[1] This pattern of stimuli is similar to those used by Michotte (1945/1963) in his classical experiments on the 'launching effect.'

From 2D to 3D stimuli. An early study on the intuitive physics of collisions with stationary 2D stimuli was conducted by Legrenzi & Sonino (1984), who found serious misconceptions about the proposed physical situation. De Sá Teixeira, De Oliveira, & Viegas (2008) recently conducted a study using Functional Measurement and moving 2D stimuli. They showed that participants additively integrate the area and the velocity of a moving square to predict the distance travelled by a stationary square hit by the moving square, instead of the physically correct multiplicative rule[2]. White (2006, 2009) showed that most observers are prone to ignoring the effect that a stationary object exerts on the post-collision behavior of a moving object colliding with it: this is the *causal asymmetry hypothesis.* A common feature of the experiments discussed above is the abstractness of the stimuli presented to participants: 2D objects varying only in velocity and area.

The primary aim of my research was to determine whether these misconceptions are due to the abstractness of the stimuli employed. In ordinary life, we are immersed in a 3D environment where collisions usually take place between 3D moving objects differing in size, specific weight, and velocity. The 2D figures used as stimuli in the above experiments are highly simplified representations of people's everyday experience. Considering that familiarity with the task is one of the major determinants of the congruency between intuitive and formal physics (Kaiser et al., 1986; Masin & Rispoli, 2010), it is not surprising to find incongruity in experiments carried out with unfamiliar stimuli. Do these misconceptions still occur when people are presented with more naturalistic simulations of collisions? In my experiments, by means of computer graphics, I created a 3D scenario with moving spheres of different size, texture, and velocity. My prediction was that, in such situations, participants' intuitive knowledge would be more congruent with formal physics than found in previous experiments. However, I did not predict that participants' performance would be perfectly isomorphic to physics: I intended to use FM and IIT as means to assess the degree of similarity between Equations (1) and (2) and cognitive algebraic integration rules.

[2] The comparison between the additive integration rule of area and velocity and the physically correct multiplicative rule of mass and velocity makes sense only under the assumption that manipulations in area are conceived of as manipulations of implied mass: this assumption was made by De Sá Teixeira et al. (2008), and is common in intuitive physics experiments.

EXPERIMENT 1

Participants. The participants were 7 male and 13 female paid students of Psychology, aged between 20 and 26. They all had normal or corrected-to-normal visual abilities.

Stimuli and Apparatus. The stimuli were presented on a personal computer equipped with a 37.5 cm × 30 cm screen, a mouse, and a keyboard. Figure 1 shows the scenario as it appeared to participants.

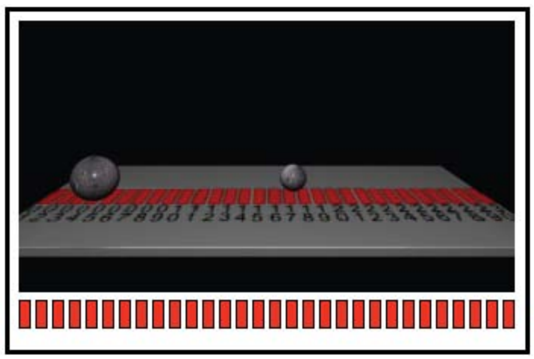


Figure 1: Drawing of one of the stimuli of Experiment 1.

Participants sat at a distance of about 50 cm from the screen, the background of which was white. A (35.5 cm × 22 cm) 3D animation was displayed in the upper part of the screen, leaving an 8-cm white space under the animation itself. This white space contained a horizontal graduated scale (*response scale*), composed of 30 red rectangular steps, separated by white edges. Numbers from 1 to 30 (from left to right) appeared below the steps of the response scale.

Animation. The animation was created by 3D Studio Max. It represented two 3D spheres on a 3D gray horizontal rectangular table. The background of the animation was black. The spheres were simulated as slightly raised above the table, so that they did not appear to touch its surface. Participants had the impression of being in front of a table and viewing it in perspective. A horizontal graduated scale (*table scale*) composed of 30 red rectangular steps appeared in the middle of the table. Numbers from 1 to 30 (from left to right) appeared below the steps of the table scale. The table scale appeared to be so similar to the response scale that the correspondence between them was obvious. The response scale was intended to be a 2D representation of the table scale. The instructions given to participants also emphasized this correspondence.

At the beginning of the animation and during the subsequent horizontal motion, the two spheres were simulated as located above the table scale without touching the table itself. The spheres had a photographic texture of a material like iron, and their reflectance was regulated according to the pattern of reflectance typical of metals. All participants were asked if they clearly recognized two iron spheres and said they did. At the beginning of the animation, one sphere (*A*) appeared close to the left side of the table and the other sphere (*B*) in a central position. Then, 360 milliseconds after the appearance of the animation, *A* began to move horizontally from left to right towards *B*, and stopped about 2 mm (measured on the screen) from it. *A* moved at 8.9, 14.4, or 38.3 cm/s. At the end of the motion, *A* was located between steps 15 and 16 on the *table scale*, and *B* between steps 17 and 18, depending on the size of the two spheres. A visual warning signal appeared 500 milliseconds after *A* had stopped moving. This signal was 2D a yellow rectangular bar (23.5 cm × 0.7 cm measured on the screen) which appeared in the middle of the black background of the animation for 2 seconds.

The simulated material (iron) of the two spheres was kept constant, so that variations in their implied mass (*IM*) were only obtained by manipulating their size. Their apparent volumes[3] were 4.2, 8.2, or 17.2 cm^3. The velocity of *A* was uniform throughout the motion. i.e., the motion of *A* did not appear uniform, but slightly decelerated (Runeson, 1974). The two spheres moved without spin.

Procedure and Experimental Design. Participants were told that they would be presented with a video showing an iron sphere moving horizontally towards another iron sphere which was stationary, and that the video originally showed a collision between the two, but the video had been cut just before the collision took place. They were asked to pay attention to the yellow bar which appeared in the middle of the dark background after the moving sphere had stopped. Lastly, they should remember that the scale represented on the table (the table scale) corresponded exactly with that below the video (the response scale). Two stimuli were randomly chosen and presented, in order to familiarize participants with them.

Participants were then told that their task was to imagine that the collision between the spheres had really occurred, and to predict the positions they would have reached on the table scale (as if the video had not

[3] The volume of each sphere was calculated by measuring its diameter on the screen.

been cut) when the yellow bar appeared. They could watch the sequence as many times as they wanted by pressing SPACE on the keyboard. When they felt ready to answer, they could press ENTER, after which the cursor of the mouse appeared on the response scale below the animation. Participants had to rate the position (on the response scale) of *B* with a first click of the mouse, and the position of *A* with a second click. Four randomly chosen stimuli were then presented as examples.

After these practice trials, all participants stated that they understood the task. The experiment followed a 3 (IM_A) × 3 (IM_B) × 3 (v_A) factorial design. The stimuli were presented in random order twice.

RESULTS

The rated positions of *A* (second click of the mouse) and *B* (first click) were analysed separately.

Position of A. As a paired sample *t*-test showed that the effect of replication was not statistically significant ($t(539) = 0.897$, $p = 0.37$), the two responses were averaged across replications.

An important preliminary consideration is that, according to Equation (1), *A* can move after the collision in the same direction as its motion before the collision or, if $m_A < m_B$, it should bounce back. Surprisingly, eight participants did not take this possibility into account, never placing *A* any step backwards from step 15 of the response scale (i.e., the position of *A* when it stops moving).

The top left panel of Figure 2 shows the mean rated position of *A*, averaged over its three velocities, as a function of the implied mass of *A* (horizontal axis) for each implied mass of *B* (different lines). The pattern of lines seems to be somewhat indefinite, since they initially converge and then diverge. Although the position of *A* is proportional to the difference between IM_A and IM_B, no integration rule can be deduced from this pattern of data.

The left panel of Figure 3 shows the mean rated position of *A* as a function of the implied masses of *A* and *B* (horizontal axis) for each velocity of *A* (different lines). A family of diverging curves fits the data, supporting a multiplicative rule for the integration of the combined effect of the implied masses and v_A. Since the implied masses of the two spheres were integrated according to an indefinite rule (top left panel of Figure 2), the left hand panel of Figure 3 supports this overall integration rule:

$$Position\ A = v_A \times f\,(IM_A, IM_B) \tag{3}$$

where f is an unknown. Equation (3) may be called the *multiplicative-indefinite* integration rule.

An analysis of variance was performed to test Equation (3). Two main effects of two factors were significant: IM_A ($F(2,38) = 14.36$, $p = 2.27 \times 10^{-5}$), and IM_B ($F(2,38) = 19.76$, $p = 1.31 \times 10^{-6}$). v_A was not significant ($F(2,38) = 2.4$, $p = 0.1$). The $IM_B \times v_A$ interaction was significant ($F(4,76) = 3.54$, $p = 0.01$), the linear-by-linear trend component of the interaction being the only significant one ($F(1,76) = 13.56$, $p = 0.0004$). The $IM_A \times v_A$ interaction was marginally significant ($F(4,76) = 2.43$, $p = 0.055$), the linear-by-linear trend component of the interaction being the only significant one ($F(1,76) = 8.96$, $p = 0.004$). No other interaction effects were significant. This pattern of statistical results supports Equation (3) (see Anderson, 1982, p. 117).

Individual data were plotted in the same manner as group data, and visual inspection of the graphs indicated that only three participants integrated the variables in accordance with Equation (3). Among the remaining participants, six used an *IM*-only integration rule, ignoring v_A, four used a v_A-only integration rule, ignoring the implied masses of the two spheres, four always placed *A* on the same step of the scale, and three seemed to respond at random.

Position of B. As a paired sample *t*-test showed that the effect of replication was not statistically significant, ($t(539) = -0.516$, $p = 0.57$), the two responses were averaged across replications.

The top right panel of Figure 2 shows the mean rated position of *B*, averaged over the three velocities of *A*, as a function of the implied mass of *A* (horizontal axis) for each implied mass of *B* (different lines). The lines converge upwards-right. The unequal weights averaging model may account for this pattern of deviation from parallelism (Anderson, 1981, p. 67). Although it was impossible to compute the relative weights associated with each level of the variables[4], the subjective weight associated with variable *A* increases as the implied mass of *A* increases.

The right hand panel of Figure 3 shows the mean rated position of *B* as a function of the implied masses of *A* and *B* (horizontal axis) for each velocity of *A* (different lines). A family of diverging curves fit the data,

[4] All sub-designs are needed in order to estimate the weights attached to each level of each variable (Zalinski & Anderson, 1991). However, the stimuli that would correspond to all sub-designs were meaningless in the context of Experiment 1, so that estimation of the weights was impossible.

supporting a multiplicative integration rule between the implied masses of the spheres and the velocity of *A*. Since the implied masses of the two spheres were integrated according to an averaging rule (top right panel of Figure 2), the right panel of Figure 3 supports this overall *multiplicative-averaging* integration rule:

$$Position\ B = v_A \times (w_0 IM_0 + w_{Ai} IM_{Ai} + w_{Bj} IM_{Bj}) / (w_0 + w_{Ai} + w_{Bj}) \quad (4)$$

where IM_{Ai} is the implied mass of level i of A, IM_{Bj} is the implied mass of level j of B, w_{Ai} is the subjective weight associated with IM_{Ai}, w_{Bj} is the subjective weight associated with IM_{Bj}, and w_0 and IM_0 are default values.

An analysis of variance was performed to test Equation (4). All the main effects of all factors were statistically significant: IM_A ($F(2,38) = 34.7$, $p = 2.67 \times 10^{-9}$), IM_B ($F(2,38) = 57.58$, $p = 3.15 \times 10^{-12}$), and v_A ($F(2,38) = 75.38$, $p = 5.94 \times 10^{-14}$). All two-factor interactions were significant: $IM_A \times IM_B$ ($F(4,76) = 7.19$, $p = 5.71 \times 10^{-5}$), $IM_A \times v_A$ ($F(4,76) = 5.69$, $p = 0.0005$), and $IM_B \times v_A$ ($F(4,76) = 8.53$, $p = 9.75 \times 10^{-6}$). The three-factor interaction $IM_A \times IM_B \times v_A$ was also significant ($F(8,152) = 4.59$, $p = 4.96 \times 10^{-5}$). This pattern of statistical results supports Equation (4) (see Anderson, 1982, p. 117).

Individual data were plotted in the same manner as group data, and visual inspection of the graphs indicated that only seven participants consistently integrated the variables according to Equation (4). Of the remaining participants, six used an *IM*-only integration rule, ignoring v_A, four used a v_A-only integration rule, ignoring the implied masses of the two spheres, and three seemed to respond at random.

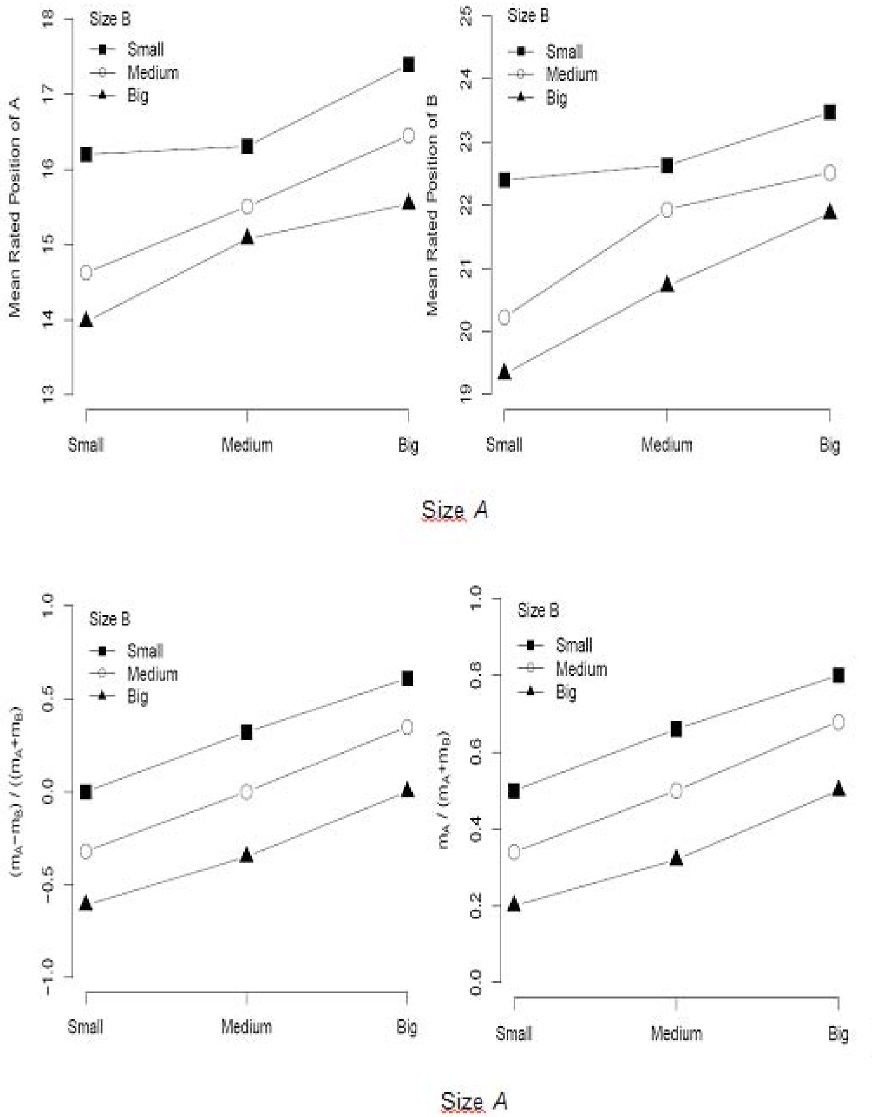


Figure 2: Top panels: Mean rated positions of *A* (top left) and *B* (top right) in Experiment 1, averaged over 3 velocities of *A*, as a function of size of *A* for each size of *B*. Bottom panels: Simulations of Equation (1) (bottom left) and Equation (2) (bottom right) as a function of m_A (horizontal axis) for each m_B (different lines) with v_A=1.

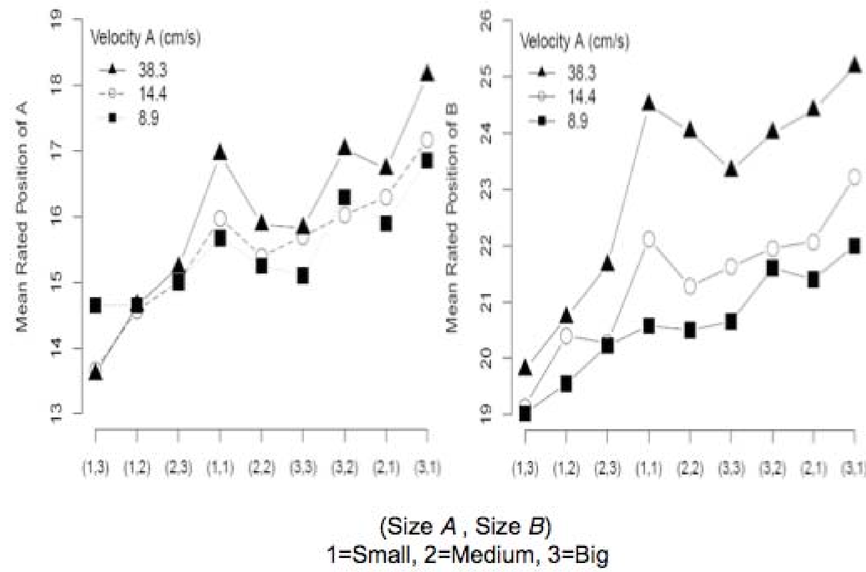


Figure 3. Mean rated positions of *A* (left) and *B* (right) in Experiment 1 as a function of sizes of *A* and *B* for each velocity of *A*. Since mean rated positions of *A* and *B* were both proportional to difference between Size *A* and Size *B*, ordered pairs (Size *A* , Size *B*) are plotted on abscissa in ascending order of difference between sizes.

DISCUSSION

Participants were asked to rate the positions that both spheres would reach after a fixed time interval (500 ms) from the imagined collision (i.e., when the yellow bar appeared). This procedure is the easiest way of estimating the imagined post-collision velocities of the spheres. It was reasonably assumed that the rated positions were linear functions of imagined velocities: this would support the linearity of the response and facilitates comparisons between Equations (1) and (2) and cognitive integration rules. This assumption rests on the hypothesis that the fixed time interval from the imagined collision and the appearance of the yellow bar was always perceived as being the same during the course of the experiment. There is no clear reason to believe that this hypothesis is not true.[5]

The bottom left and right panels of Figure 2 show simulations of Equations (1) and (2), respectively as functions of m_A (horizontal axis) for each m_B (different lines) with $v_A = 1$, as if the two spheres were real material spheres with density 8 g/cm^3 (mean physical density of iron), with volumes of 4.2, 8.2, and 17.2 cm^3. Note that the two bottom panels are very similar to each other, both having a slightly slanted barrel pattern. This suggests that Equations (1) and (2) are substantially similar. The only notable difference between them is that Equation (1) accounts for negative values: if $m_A < m_B$, then the post-collision velocity of *A* is negative, i.e., *A* bounces back. When $v_A = 1$, Equations (1) and (2) may both be considered as instances of the general ratio integration rule (Anderson, 1981, p. 77).

Figure 2 allows us to compare the cognitive integration rules for the implied masses of *A* and *B* (top left and top right panels, respectively) with the physically correct ratio integration rules as formalized by Equations (1) and (2) (bottom left and bottom right panels respectively).

The most striking differences appear between the cognitive and the physical integration rules for *A*. While Equation (1) predicts a slight upwards-right convergence of the lines according to a slanted barrel pattern (bottom left panel of Figure 2), the lines of the functional graph in the top left panel initially tend to converge and then to diverge. In addition to the notable differences concerning the integration rule, eight participants never considered the possibility that *A* could bounce back after the collision.

Some differences also appear between the cognitive and physical integration rules for *B*. Equation (2) predicts a slight upwards-right convergence of the lines according to a slanted barrel pattern (bottom right panel of Figure 2). This also appears in the top right panel, but the slanted barrel does not appear.

In sum, it seems that the greatest misconceptions about collision effects concern the post-collision behavior of *A*. Research on the perception of collision effects supports this tenet: O'Sullivan (2005) and Reitsma & O'Sullivan (2009) presented 3D collisions between simulated spheres to their participants, and reported that they are less sensitive to post-collision anomalies of the initially moving sphere with respect to those of the initially

[5] This does not mean that the imagined post-collision velocity is a linear function of the theoretically correct physical velocity. If we presume that that the participants imagined the spheres were subject to friction, the imagined velocity would be a non-linear negatively accelerated function of physical velocity. This is not essential for a discussion of the results.

stationary sphere. White (2009) reported that perceived forces in collisions are asymmetrical: we perceive the force exerted by the moving object on the stationary one, but not vice versa.

Despite these misconceptions, the intuitive physics of collisions as shown by the participants in Experiment 1 is definitely more consistent with normative physics than that of the participants in previous experiments. Both cognitive integration rules concerning the predicted positions of A and B (as expressed by Equations (3) and (4)) show a multiplicative integration rule between the combined effect of the implied masses and v_A, whereas De Sá Teixeira et al. (2008) found that area and velocity were combined additively (see Note 2). Thus, 3D (realistic) stimuli rather than 2D (abstract) ones improved participants' overall performances.

One explanation for the discrepancies found in Experiment 1 between cognitive and physical integration rules is the relatively small range of variation of the implied masses of the two spheres. In Experiment 1, the variations of the implied masses were only obtained by varying the sizes of the two spheres. To test this hypothesis, a second experiment used spheres differing in both size and simulated material (texture).

EXPERIMENT 2

Participants. Participants were 5 male and 15 female paid students of Psychology, aged between 20 and 26. They all had normal or corrected-to-normal visual abilities. None had participated in Experiment 1.

Stimuli and Apparatus. The stimuli and apparatus were the same as in Experiment 1, except that manipulation of the implied masses of the two spheres was carried out by varying both size and simulated material (texture) according to a 2 (*Texture*) × 2 (*Size*) factorial design. Two possible photographic textures were attached to each sphere, one depicting iron (the same as in Experiment 1) and the other depicting polystyrene. In both cases, the reflectance of the spheres was manipulated to increase the realism of the photographic texture. When asked, all participants clearly identified the simulated material of the spheres.

The apparent volumes of the spheres were either 4.2 or 17.2 cm^3. The pre-collision velocity of A was either 12.2 or 25.9 cm/s. In sum, there were four different implied masses of the spheres and two different pre-collision velocities of A.

Procedure and Experimental Design. The procedure was the same as in Experiment 1, except that participants were told that the spheres in the video could be made of either iron or polystyrene.

The experiment obeyed a 4 (IM_A) × 4 (IM_B) × 2 (v_A) factorial design. The stimuli were presented in random order twice.

RESULTS

The rated positions of *A* (second click of the mouse) and *B* (first click) were analysed separately.

Position of A. As a paired sample *t*-test showed that the effect of replication was not statistically significant ($t(639) = 0.4$, $p = 0.69$), the two responses were averaged across replications.

As in Experiment 1, an important preliminary consideration was the number of participants – ten – who did not consider the possibility of *A* bouncing back.

The top left panel of Figure 4 shows the mean rated position of *A*, averaged over the three velocities of *A*, as a function of the implied mass of *A* (horizontal axis) for each implied mass of *B* (different lines). The slanted barrel pattern supports a ratio integration rule for the implied masses of the two spheres (see Anderson, 1981, p. 77).

The top panel of Figure 5 shows the mean rated position of *A* as a function of the implied masses of *A* and *B* (horizontal axis) for each velocity of *A* (different lines). Two diverging curves fit the data, supporting a multiplicative integration rule between implied masses and the velocity of *A*.

Since the implied masses of the two spheres were integrated according to a ratio rule (top left panel of Figure 4), the left hand panel of Figure 5 supports this overall *multiplicative-ratio* integration rule:

$$Position\ A = v_A \times IM_A / (IM_A + IM_B) \qquad (5)$$

An analysis of variance was performed to test Equation (5). All the main effects of all factors were statistically significant: IM_A ($F(3,57) = 24.0$, $p = 3.58 \times 10^{-10}$), IM_B ($F(3,57) = 26.8$, $p = 6.06 \times 10^{-11}$), and v_A ($F(1,19) = 16.14$, $p = 7.35 \times 10^{-4}$). All two-factor interactions were significant: $IM_A \times IM_B$ ($F(9,171) = 4.02$, $p = 1.08 \times 10^{-4}$), $IM_A \times v_A$ ($F(3,57) = 3.32$, $p = 0.026$), and $IM_B \times v_A$ ($F(3,57) = 10.43$, $p = 1.44 \times 10^{-5}$). The three-factor interaction $IM_A \times IM_B \times v_A$ ($F(9,171) = 0.91$, $p = 0.52$) was not significant. According to Anderson (1982, p. 117) the three-factor interaction is indispensable for statistical validation of Equation (5). This incongruence for the *multiplicative-ratio* model was probably due to the use of a wide range of variation of implied masses and a relatively narrow range of variations of velocity of *A*. Despite this statistical flaw, Equation (5) seems the best way to represent the data.

Individual data were plotted in the same manner as group data, and visual inspection of the graphs indicated that eight participants integrated the variables according to Equation (5). Among the remaining participants, seven used an implied masses-only integration rule, ignoring v_A, and five seemed to respond at random. Interestingly, the integration rule adopted by each participant was independent of considering the possibility of *A* bouncing back. Some participants did consider it, but responded without applying a definite integration rule; others did not consider the possible bouncing back of *A* but used the *multiplicative-ratio* rule of Equation (5).

Position of B (initially stationary). As a paired sample *t*-test showed that the effect of replication was not statistically significant, ($t(639) = -0.893$, $p = 0.37$), two responses were averaged across replications.

The top right panel of Figure 4 shows the mean rated position of *B*, averaged over the three velocities of *A*, as a function of the implied mass of *A* (horizontal axis) for each implied mass of *B* (different lines). The slanted barrel pattern supports a ratio integration rule for the implied masses of the two spheres (see Anderson, 1981, p. 77).

The bottom panel of Figure 5 shows the mean rated position of *B* as a function of the implied masses of *A* and *B* (horizontal axis) for each velocity of *A* (different lines). Two diverging curves fit the data, supporting a multiplicative integration rule between implied masses and the velocity of *A*.

Since the implied masses of the two spheres were integrated according to the ratio rule (top right panel of Figure 4), the right panel of Figure 5 supports this overall *multiplicative-ratio* integration rule:

$$Position\ B = v_A \times IM_A / (IM_A + IM_B) \qquad (6)$$

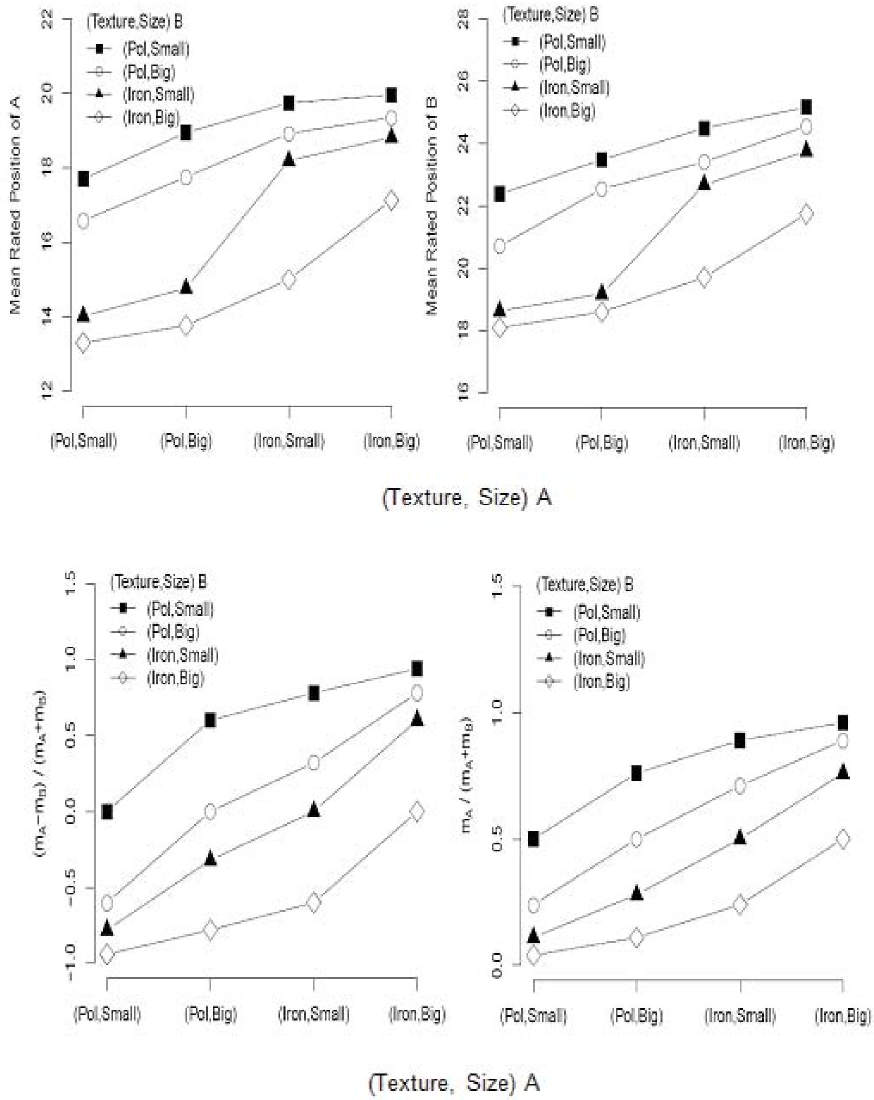
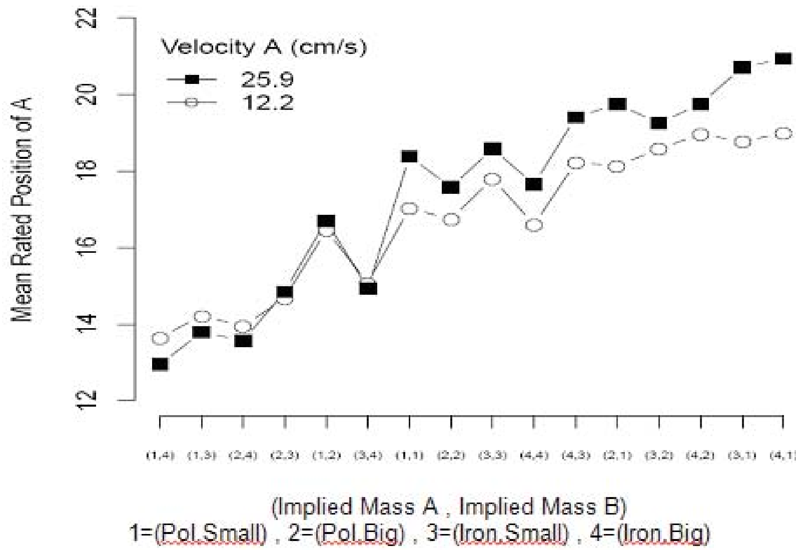


Figure 4: Top panels: Mean rated positions of *A* (top left) and *B* (top right) in Experiment 2, averaged over 3 velocities of *A*, as a function of implied mass of *A* for each implied mass of *B*. Bottom panels: Simulations of Equation (1) (bottom left) and Equation (2) (bottom right) as a function of m_A (horizontal axis) for each m_B (different lines) with $v_A = 1$



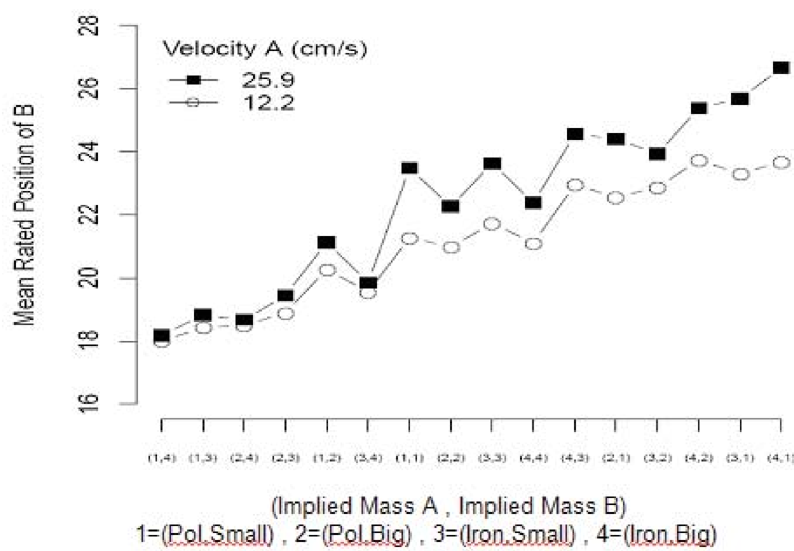


Figure 5. Mean rated positions of *A* (top) and *B* (bottom) in Experiment 2 as a function of implied masses of *A* and *B* for each velocity of *A*. Since the mean rated positions of *A* and *B* were both proportional to difference between implied mass of *A* and the implied mass of *B*, ordered pairs (Implied Mass *A* , Implied Mass *B*) are plotted on abscissa in ascending order of difference between Implied Mass of *A* and Implied Mass of *B*.

An analysis of variance was performed to test Equation (6). All the main effects of all factors were statistically significant: IM_A ($F(3,57) = 79.9$, $p < 2.2 \times 10^{-16}$), IM_B ($F(3,57) = 94.6$, $p < 2.2 \times 10^{-16}$), and v_A ($F(1,19) = 32.07$, $p = 1.85 \times 10^{-5}$). All two-factor interactions were significant: $IM_A \times IM_B$ ($F(9,171) = 7.73$, $p = 1.66 \times 10^{-9}$), $IM_A \times v_A$ ($F(3,57) = 3.14$, $p = 0.032$), and $IM_B \times v_A$ ($F(3,57) = 19.87$, $p = 6.12 \times 10^{-9}$). The three-factor interaction $IM_A \times IM_B \times v_A$ ($F(9,171) = 1.63$, $p = 0.11$) was not significant. Like the statistical validation of Equation (5), the lack of the three-factor interaction is probably due to the wide range of variation of implied masses and relatively narrow range of variations of velocity of *A*. Despite this flaw, this pattern of statistical results supports Equation (6) (see Anderson, 1982, p.117)

Individual data were plotted in the same manner as group data, and visual inspection of the graphs indicated that twelve participants integrated the variables according to Equation (6). Of these twelve, six had only a slight effect of variable v_A. Of the remaining participants, five used an implied-masses integration rule, ignoring the velocity of *A*, and three seemed to respond at random.

DISCUSSION

As in Experiment 1, it was assumed that the rated positions of the two spheres is a linear function of their imagined post-collision velocity (see note 5).

The bottom left and right panels of Figure 4 show simulations of Equation (1) and (2), respectively as functions of m_A (horizontal axis) for each m_B (different lines) with v_A=1, as if the two spheres were real material spheres of density 8 g/cm^3 (the mean physical density of iron) or 1 g/cm^3 (the mean physical density of polystyrene), with volume of 4.2 or 17.2 cm^3. Figure 4 allows us to compare the cognitive integration rules for the implied masses of *A* and *B* (top left and top right panels, respectively) with the physically correct ratio integration rules as formalized by Equations (1) and (2) (bottom left and bottom right panels respectively). All four panels show a slanted barrel pattern, supporting the idea that participants used a physically correct ratio integration rule to integrate the implied masses of the spheres in order to predict the positions of *A* (top left panel) and *B* (top right panel) (see Anderson, 1981, p.77). However, some deviations do appear.

The most conspicuous difference between the top and bottom panels is the non-parallelism of the second and third curves (from the top) in the top panels. In particular, the rate of growth of these curves is not constant, as predicted by the ratio models shown in the bottom panels. The two curves are steeper when the implied mass of *A* (horizontal axis) is less than or equal to the implied mass of *B* (different lines), but are flatter when the implied mass of *A* is greater than that of *B*. Note that the physically correct ratio rules (Equations (1) and (2)) predict that the effect of a constant increase in the mass of *A* on the post-collision velocity of *B* decreases as the absolute difference between the masses of *A* and *B* increases. Thus participants have emphasized the physically correct ratio rule.

GENERAL DISCUSSION

The main findings of the above experiments are be summarized as follows:

(1) The data of the present experiments strongly indicate that, whether cognitive integration rules are isomorphic to physical rules or not, people are generally able to integrate various stimulus cues (e.g., velocity and implied masses) to make predictions about physical situations (Anderson, 1983). Proffitt and Gilden (1989, p. 384) argued that "…people make judgments about natural object motions on the basis of only one parameter of information that is salient in the event...". Instead, the results of the present experiments show that people are able to take into account different sources of information in making predictions about dynamic events.

(2) The extent of the misconceptions found in previous experiments on intuitive physics of collision effects (Legrenzi & Sonino, 1984; De Sá Teixeira et al., 2008) is connected to abstract 2D stimuli. Although the participants in the present experiments show some remarkable misconceptions, their overall performance (particularly in Experiment 2) is definitely more aligned with normative physics than that of participants in earlier experiments (note in particular the physically correct multiplicative integration rule between the velocity of *A* and the combined effect of the implied masses which was found for both spheres in both experiments).

(3) The general cognitive integration rule for the post-collision position of *A* changed from the physically wrong multiplicative-indefinite rule of Experiment 1 to the physically correct multiplicative-ratio rule of Experiment 2. In addition, the number of participants who used the physically correct multiplicative-ratio rule to rate the post-collision positions of both spheres increased in Experiment 2. This sounds like a warning to researchers on intuitive physics: functional knowledge varies as the nature of the stimuli varies. Some misconceptions about physical situations in which the masses of stimuli are important, may be due to the

narrow range of variations in implied mass induced by variations in the area of 2D stimuli.

(4) The results of both Experiments 1 and 2 showed that rating the position of *A* was the harder task for participants. Previous research on the perception of collision effects is consistent with this finding (O'Sullivan, 2005; White, 2009).

(5) One striking misconception that cannot be avoided using realistic 3D stimuli is the failure to consider (by about half the participants) the possibility of *A* bouncing back. Surprisingly, some of the participants who ignored the possible bouncing back of *A* still used the physically correct *multiplicative-ratio* rule of Equation (6). This suggests that the possibility of *A* bouncing back is independent of the cognitive integration rule.

(6) With few exceptions, in these experiments both participants who ignored the possibility of *A* bouncing back and those who did consider it used some algebraic rules involving the properties of both spheres to predict the post-collision position of *A*. One of the main tenets of White's *causal asymmetry hypothesis* (2006, 2009) is that, in a collision event, we are generally prone to ignoring the effect that the stationary sphere (*B* here) exerts on the post-collision behavior of the moving sphere (*A* here). The results of the present experiments suggest that this was not the case.

CONCLUSIONS

One of the main challenges in teaching elementary physics is closing the gap between what is taught and what is learned (McDermott, 1991). An unavoidable requirement for this is to identify the actual status of students' knowledge. Differences between cognitive integration rules and normative physical rules should be the starting point to modify students' status of knowledge, until their functional knowledge becomes reasonably similar to the rules of physics.

FM and IIT provide a unique contribution in this regard, for they allow assessment of the functional knowledge of each single student (Karpp & Anderson, 1997). The data of the experiments presented here, indicate that the assessment of functional knowledge of the physical world is facilitated by using naturalistic stimuli. They also provide useful insights for teaching the physics of collisions. One of these is that physics teachers should focus on the post-collision behavior of the moving sphere (*A* here), and in particular on the possibility of its bouncing back. Participants who apply the correct *multiplicative-ratio* rule but ignore the possibility of *A* bouncing back probably only need to be informed about this fact, whereas participants applying a physically wrong integration rule probably need more practice in order to improve their functional knowledge.

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