

Quantitative Analysis of Pre-service Elementary Mathematics Teachers' Opinions about Doing Mathematical Proof*

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Abstract

Meaning and importance of proof in mathematics and education increases gradually. Therefore levels of doing proof, proof-related opinions and perceptions of the teachers and pre-service teachers who will train the students in future are of importance. Accordingly, this study aims to determine the proof-related opinions of pre-service elementary mathematics teachers who still study at different grade levels. In line with this purpose, a questionnaire developed under the title "Questionnaire for Constructing Mathematical Proof" was used to determine the pre-service teachers' opinions about proof. The questionnaire comprises 27 items based on 5 point Likert-type. In the study, developmental research method was conducted and the questionnaire was applied to 187 pre-service elementary mathematics teachers from different grade levels. As a result of the study, it was revealed that pre-service teachers have positive views about proof. Also, the study revealed that confidence of pre-service teachers in proving is lower than mental process, self-assessment and belief, and attitude factors.

Key Words

Proof, Mathematics Education, Pre-service Elementary Mathematics Teacher, Questionnaire for Constructing Mathematical Proof.

Proofs constitute the most important part of the essential things making mathematics mathematics (Padula, 2006). Because, it provides the accuracy and inaccuracy of each case in mathematics (Tall & Mejia-Ramos, 2006). Proof also demonstrates not only the accuracy and inaccuracy of a case but also explains the reason why it is accurate (Hanna,

2000). Besides, proofs enable students to see the mathematical truths on their own by preventing them to rely on their teachers or books (Knuth, 1999, 2002a). Proof thereby plays an important role in developing and changing mathematical thinking of students (Flores, 2002). Doing proof is defined as a mental act performed to eliminate doubts of an individual or a community regarding the accuracy of a claim (Harel, 2008; Harel & Sowder, 1998, 2007). Hence, proof is of high importance to mathematics (Coe & Ruthven, 1994; Martin & Harel, 1989).

According to Bell (1976) mathematical proof is a defense, explanation (why) and systematization (how). As for that, proving is completed at three stages. The first stage is confirmation; second one is explanation and the last one is abstraction (Baki, 2008). Teaching and developing proof and reasoning skills for students depend on the teacher (Altıparmak & Öziş, 2005; Riley, 2004).

The questions directed (Martino & Maher, 1999) and materials used in the classroom, along with

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teachers influence the proving capacities of students (Stylianides, 2007b). Besides, teachers' perceptions about proof, experiences and skills are effective in the process of gaining proving skills by students (Almeida, 2003; Galbraith, 1995; Knuth, 1999, 2002b; Moralı, Uğurel, Türnüklü, & Yeşildere, 2006). Therefore, it is essential for mathematics teachers to consider what it means to them to know mathematics and what they understand from important mathematical ideas (Masingila, 1998). In this process, it is also important which values teachers teach in the mathematics education, as well as which values students learn from their teachers (Bishop, 2001). Because the core source of verifying and proving experiences of students' is elementary school teachers and the proving, albeit very limited, is included in the elementary school curriculum (Martin & Harel, 1989). National Council of Teachers of Mathematics (NCTM) (2000) indicates that educational programs are required to develop proofs and include evaluation. Proving and reasoning criteria are available within the NCTM (2007) standards. In Turkey, elementary mathematics education curriculum involves no acquisition directly in respect with the proof and proving, however involves the development of some skills such as reasoning and associating, and explaining and defending solutions (Millî Eğitim Bakanlığı [MEB], 2005a, 2005b)

Research shows that students at any grade level have great difficulties in understanding, loving and creating mathematical proof (Güven, Çelik, & Karataş, 2005; Harel & Sowder, 1998; Jones, 2000; Martin & Harel, 1989; Moore, 1994). There exist serious challenges in providing proofs at the undergraduate level, although proving is an important activity in mathematics (Almeida, 2000). It is seen that university students (Almeida, 2000; Dreyfus, 1999; Harel & Sowder, 1998; Jones, 2000; Knapp, 2006; Moore, 1994; Recio & Godino, 2001; Senk, 1983; Stylianides, Stylianides, & Philippou, 2005, 2007; Weber, 2001, 2004) and mathematics pre-service teachers have difficulty in making proofs (Moralı, Köroğlu, & Çelik, 2004; Weber, 2001). Because, proof and their experiences do not generally make sense to students (Galindo, 1998; Stylianides, 2005, 2007a; Szombathelyi & Szarvas, 1998). As a result, students' perception forms regarding proof influence making proof (Moore, 1994; Tatar & Dikici, 2008). It may affect their classroom activities when they become a teacher in future. Because, teachers' beliefs and ideas significantly affect their own behaviors (Erickson, 1993).

Purpose

Although many studies have been conducted on proof abroad, there is a limited number of studies in this field in our country and it is seen that sufficient research has not been conducted on this subject (Özer & Arıkan, 2002). However, the meaning and importance of mathematics and proof in the education thereof is gradually increasing (Aydoğdu İskenderoğlu, 2003; Aydoğdu, Olkun, & Toluk, 2003; Baki, İskenderoğlu, & İskenderoğlu, 2009; İskenderoğlu, 2010; Moralı et al., 2006; Üzel & Özdemir, 2009).

Therefore levels of doing proof, proof-related opinions and perceptions of the teachers and pre-service teachers who will train the students are of significance (Dickersen, 2006; Moralı et al., 2006). However it is seen that mathematics pre-service teachers have difficulties in reasoning and doing proof (Moralı et al., 2004). One of such difficulties experienced is that pre-service teachers regard the proof only as an explanation. This indicates that the pre-service teachers do not understand the function of the term of proof (Dane, 2008) and they underrate the proofs (Ginsburg & Seo, 1999; Yıldız, 2006). Yet, proofs used by pre-service teachers, their perspectives to the proof and route followed in the process of doing proof will influence the classroom activities to be applied in respect of the proof when they become a teacher in future. Issues such as proof-related confidences, attitudes and beliefs, mental processes and self-assessments, together with levels of doing proof, opinions and perceptions about the proof of the teachers and pre-service teachers who will train the students increasingly gain importance in order to avoid the possible problems to occur in the classroom. Accordingly, this study aims to determine the proof-related opinions of pre-service elementary mathematics teachers who still study at different grade levels.

Method

This research is a developmental research conducted through longitudinal design. Developmental researches have a descriptive feature and inquire questions such as what it was and what happened (Çepni, 2009; Menard, 2008; Miller, 1998)

Sample

The sample group comprises pre-service elementary mathematics teachers studying at 1st, 2nd, 3rd, and 4th grades of Karadeniz Technical University. The

questionnaire prepared to be used in the study was applied to 187 pre-service elementary mathematics teachers. 73 of the said pre-service teachers continued their study at 1st class, 35 at 2nd class, 34 at 3rd class and 45 at 4th class.

Data Collecting Tools

In this study, the questionnaire devised by Lee (1999) was utilized to determine opinions of pre-service elementary mathematics teachers about presenting proof. At this stage, the questionnaire was initially translated into Turkish and validity and reliability studies were carried out through a pilot study. In the pilot study, the questionnaire was applied to 174 pre-service elementary mathematics teachers. Accordingly, the validity of language, content and structure were ensured within the scope of validation studies of the questionnaire. As a result of structure validation, the questionnaire was separated into 4 factors. These dimensions were arranged as "Confidence", "Belief and Attitude", "Mental Process" and "Self-Assessment". At the end of the reliability studies carried out, the Cronbach's Alpha reliability coefficient of the questionnaire was later calculated as .79. "Questionnaire for Constructing Mathematical Proof" developed to reveal views of the pre-service teachers about proof composes of total 30 items, 27 of which are based on 5-point Likert-type and three of which are open-ended questions.

Data Analysis

Likert-type items of "Questionnaire for Constructing Mathematical Proof" covers five points including "always", "often", "sometimes", "rarely" and "never". Scoring of each Likert-type item was done in the order above as 5, 4, 3, 2, 1. Some items were reversed-scored and analyzed as they consisted of negative opinions. Points of each item were collected and the point average of pre-service teachers was separately determined for all grades according to the each factor included in the questionnaire. In addition, One-Way Analysis of Variance (ANOVA), one of statistical methods, was utilized by means of SPSS to analyze and interpret whether there is a significant difference between different grades. In this study, it was sought to see whether different samples significantly differ for each factor through ANOVA (Balci, 2005; Büyüköztürk, 2004; Kalaycı, 2005). For this reason, the average of all participants for each factor was found one by one and, analyses and interpretations were made

using One-Way Analysis of Variance (ANOVA). Frequency and percentage values of the items contained in each factor were also separately tabled and interpreted.

Results

"Questionnaire for Constructing Mathematical Proof" used for this study covers four factors comprising mental process, confidence, self-assessment and belief and attitude. The general average of the questionnaire was founded as 3,59 on the basis of all classes. The foregoing average shows that mathematical proof-related views of the pre-service elementary mathematics teachers are positive. The first factor of the questionnaire is mental process. General average of mental process is 4,02. This average shows that participants frequently utilize their mental processes about proof. The other factor of the questionnaire is confidentiality. The general average of the participants who responded to the questionnaire was found to be 3,04. It indicates that majority of the participants sometimes relies on themselves on the subject of proof. Another factor of the questionnaire is individuals' self-assessments about proof and review of what they have done. The average of self-assessment of all participants about proof is 3,70. This average suggests that the participants frequently assess themselves when proving and they re-review what they have done. The last factor in the questionnaire is attitude-belief. By means of this factor, it was sought to reveal the participants' attitudes and beliefs. The general average of the participating pre-service teachers from different grade levels is 3,58 for the attitude-belief as regards the proof. The foregoing average shows that proof-related attitudes-beliefs of the pre-service elementary mathematics teachers are positive.

As a result of the ANOVA test conducted, it was seen that there was significant difference between grade levels in respect to mental process. According to the test results, it was revealed that there was a significant difference between first-grade pre-service teachers and fourth-grade ones at the level of $p < .05$ in respect with the mental process (.001); and between second-grade pre-service teachers and fourth-grade ones at the level of $p < .001$ (.009). However, it was seen no significant difference between third and fourth grade levels in respect with mental process.

Discussion, Conclusion and Recommendations

“Questionnaire for Constructing Mathematical Proof” used for this study covers four factors comprising mental process, confidence, self-assessment and belief and attitude. The general average of questionnaire based on all grade-levels shows that mathematical proof-related views of the pre-service elementary mathematics teachers are positive. The results of the study are consistent with the study by Lee (1999). However, the results of the study conducted by Morali et al. (2006) with the elementary and secondary school mathematics teachers are not parallel with this study. The most important reason thereof may be that the questionnaires used in both studies include different factors.

The general average of the first factor and mental process of doing proof show that participants mentally display a positive approach towards proof. In other words, participants often use their mental processes about proof. The results of this study are also parallel with the study conducted by Lee (1999). When the averages are concerned on the basis of separate grade levels, it is seen that mental processes are often used at all levels for proof, and the averages of different grade levels are found to be close to each other. It demonstrates that the participants from different grade levels need definitions and theorems to do mathematical proof. As for the mental process, while the lowest average is scored by fourth grade levels, the highest average was scored by the first grade levels. The main reason thereof may be that the first-grade pre-service teachers know no other way to do proof and consider that it is only required to use definitions and theorems in this regard. Besides, according to the results of the statistical test conducted to reveal which grade levels differs from each other in respect with the factors included in the questionnaire, it is seen that there is a significant difference between first-grade and fourth-grade pre-service teachers and between the second and fourth-grade pre-service teachers.

Another factor of the questionnaire is confidentiality. When the general average of participants who responded to the questionnaire is concerned, it is seen that they rely on themselves on the subject of proof as in the study conducted by Lee (1999). Although the averages of different grade-levels are very close to each other, those who have the highest confidence in proof are the pre-service teachers at fourth-grade while those who have the lowest confidence are the pre-service teachers at second-grade. The reason why the fourth-grade pre-service teachers have higher confidence than those at other grade-levels is may be the courses received at the

level of undergraduate. As a result of the statistical test conducted, there is no significant difference between grade levels.

Another factor of the questionnaire is individuals' self-assessments about proof and review of what they have done. The average of participants' self-assessment on proving suggests that the participants frequently assess themselves when proving and they re-review what they have done. These results are parallel with study performed by Lee (1999). Self-assessment averages of participants at different grade-levels are highly close to each other. As a result of the statistical test conducted, there is no significant difference between grade levels.

The last factor in the questionnaire is attitude-belief. General average of the participating pre-service teachers from different grade levels in respect with the attitude-belief towards the proof shows that proof-related attitudes-beliefs of the pre-service elementary mathematics teachers are positive. When the averages of the grade-levels are concerned, it is seen that participants' of all grade-levels attitudes and beliefs towards are high. Besides, although the averages of grade-levels are highly close to each other, those who have the highest average are the pre-service teachers at first-grade while those who have the lowest average are the pre-service teachers at fourth-grade. These results are consistent with the other studies performed (Harel & Sowder, 1998; Lee, 1999; Senk, 1985; Üzel & Özdemir, 2009). As the grade-level increases, the reason of the decrease in the attitude-belief towards proof may be that the first-grade pre-service teachers who recently learn proof are eager to do proof. As a result of the statistical test conducted, no significant difference was found between different grade levels.

Doing proof is included, albeit very limitedly, in the elementary and secondary school mathematics programs. When the skills provided to the students through proof are taken into consideration, it is necessary to include proof at any level. Therefore, the prospective pre-service teachers should be able to develop mathematical proving activities by training themselves in this field. Because, students will realize the emerging ways of the mathematical information through such kinds of activities and enjoy the mathematics.

The reason why the mental processes of fourth-grade pre-service teachers are low while doing proof may be researched in the future studies. Though the mental processes of those at fourth grade are low in respect of doing proof, those who have the highest confidence are the pre-service teachers at this grade. Causes thereof can be researched.

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