The Mathematics Educator
2011/2012 Vol. 21, No. 2, 34–57

A Singapore Case of Lesson Study

Lu Pien Cheng & Lee Peng Yee

In this article, we present a case study of six Singaporean elementary school teachers working in a Lesson Study team that prepared them for problem solving instruction. The Lesson Study process included preparing, observing, and critiquing mathematics lessons in the context of solving fractions tasks. By conducting Lesson Study, we anticipated that these teachers would develop greater insight into students' mathematics, which would influence their classroom practices. Through the process of planning, observing and critiquing and by purposefully listening to students' explanations, the teachers began to better understand their students' learning, which in turn could help them develop their students' mathematical knowledge.

In Singapore, a range of professional development courses for mathematics teachers are available, from one-session workshops and whole-day conferences to certification programs. Though many commercial providers offer short courses, the main providers of mathematics professional development courses are the National Institute of Education, the Ministry of Education's Curriculum Planning and Development Division, school- or cluster-organized customized sessions, and professional bodies (Lim, 2009).

In addition to the wide range of professional development courses offered to mathematics teachers in Singapore, the concept of learning communities has been encouraged since 1998. Schools in Singapore are grouped into clusters or learning communities according to their geographical locations

Lu Pien Cheng is an assistant professor with the Mathematics and Mathematics Education Academic Group at the National Institute of Education (NIE), Nanyang Technological University, Singapore. Her research interests are in teacher education.

Lee Peng Yee is an Associate Professor with the Mathematics and Mathematics Education Academic Group at NIE, Nanyang Technological University, Singapore. His research area is real analysis, and he teaches mathematics courses for mathematics education students.

to enhance teachers' effectiveness as professionals and this grouping is to encourage teachers to learn and inquire together in order to become more effective in their teaching practices (Chua, 2009). When teachers are engaged in learning communities, they are more likely to innovate their teaching practice as they continually rethink their practice based on how students learn (McLaughlin & Talbert, 2006; Vescio, Ross & Adams, 2008). Lesson Study has traditionally been one of the professional development processes used to encourage teachers to work together in teams to become more effective teachers. In Singapore, mathematics Lesson Study has been adopted by some schools as a school-based professional development program or as a cluster-initiated program. At least 60 schools out of 328 primary and secondary schools in Singapore were attempting Lesson Study in 2009 (Fang & Lee, 2010). Schools reported the use of Lesson Study across various subjects in both the primary and secondary schools. Lesson Study efforts in Singapore have been reported in research briefs, newsletters, school reports, action research projects and book chapters. In this article, we examine teachers' learning and teaching as a result of their experience in one Lesson Study cycle.

Lesson Study

Lesson Study (LS) is a form of teacher professional development that originated in Japan and has been cited as a key factor in the improvement of their mathematics and science education (Stigler & Hiebert, 1999). LS is the primary form of professional development in Japanese elementary schools and its use has been increasing across North American since 1999 (Lewis, Perry, & Hurd, 2009). Through LS, teachers in Japan work together to improve their teaching in the context of a classroom lesson. Perry and Lewis (2009) describe the LS process as follows:

> Lesson Study is a cycle of instruction improvement in which teachers work together to: formulate goals for student learning and long-term development; collaboratively plan a "research lesson" designed to bring to life these goals; conduct the lesson in a classroom, with one team member teaching and others gathering evidence on student learning and

development; reflect on and discuss the evidence gathered during the lesson, using it to improve the lesson, the unit, and instruction more generally. (Perry & Lewis, 2009, p. 366)

Japanese teachers followed eight steps to achieve unified effort in collaborative Lesson Study; (1) define a problem to guide their work, (2) plan the lesson, (3) teach and observe the lesson, (4) evaluate and reflect on the lesson, (5) revise the lesson, (6) teach and observe the revised lesson, (7) evaluate and reflect a second time, and (8) share the results (Stigler & Hiebert, 1999). Rock and Wilson (2005) claimed that completing these steps "requires a group of teachers to collaborate and share their ideas, opinions, and conclusions regarding the research lesson. This process requires substantial time and commitment" (p. 79). They also asserted that the LS process serves as a catalyst to encourage teachers to become more reflective practitioners and to use what they learned to collegially revise and implement future lessons.

Japanese educators have conducted LS at the school, regional, and national level (Stigler & Hiebert, 1999). At the national level, LS may be used to explore new ideas about teaching and curriculum (Murata & Takahashi, 2002). Teachers in the same subject matter or who have common professional interests may form regional or cross-district LS groups (Murata & Takahashi, 2002; Shimizu, 2002). Individual schools may also form their own LS group to serve their school-based professional development needs.

Because LS is a locally designed process, the forms may vary. Across the different variations in LS, four key features can be identified: investigation, planning, research lesson, and reflection (Lewis, Perry, & Hurd, 2009). Another distinctive feature of LS is its constant focus on student learning (Stigler & Hiebert, 1999). In any LS effort, the shared purpose is improved instruction (Fernandez & Yoshida, 2004; Lewis, 2002a, 2002b; Lewis & Tsuchida, 1997, 1998; Yoshida, 1999).

Research on Lesson Study

LS has been implemented widely across Asia, but under several different monikers: in Hong Kong as Learning Study, in China as Action Education, and in many Asia-Pacific

Economic Cooperation (APEC) member countries as LS (Fang & Lee, 2009). Researchers have reported that, in the United States, LS improved teachers' instruction and offered them opportunities to learn (Rock & Wilson, 2005; Lewis, Perry, Hurd, & O'Connell, 2006). Perry, Lewis and Hurd (2009) reported a successful adaptation of mathematics LS in a US school district. They provided an "existence proof" of the potential effectiveness of LS in North America, noting in their case that "teachers used Lesson Study to build their knowledge of mathematics and its teaching, their capacity for joint work, and the quality of the teaching materials" (Lewis, Perry, & Hurd, 2009, p. 302).

Research studies have shown that one way LS improves instruction is through building learning communities. Lieberman (2009) reported a case study of a middle school mathematics department, comprised of seven teachers that had been participating in LS for seven years and found that one "pathway by which LS results in instructional improvement is in increasing teachers' community...Teachers learn that being a teacher means opening their practice to scrutiny, and thinking critically about their lesson plans" (p. 97). Research on mathematics teachers from nine independent school districts in Texas, who participated in three consecutive lesson studies, showed that LS activities "promoted interactions among members within this community of mathematics educators that offered occasions for teachers to explicitly think about their views, influences on instructional choices, and possible changes in practice" (Yarema, 2010, p. 15). In Hong Kong where LS involved secondary English language teachers, Lee (2008) reported that LS "creates a culture of peer learning and learning from actual classroom practice.…[and] provides opportunities for a free discussion of ideas, with participants able to challenge others' and their own way of thinking, and seeing learning from students' perspectives" (p. 1124). In a two-year intervention study for six teachers in one primary Singaporean school, Fang and Lee (2009,) reported that "Lesson Study is a powerful tool to bring together knowledge from diverse communities" (p. 106).

In mathematics LS, the participation of a person more knowledgeable in mathematics teaching and learning has been reported to enhance the pedagogical content knowledge of the learning community. Findings from a case study of two primary school mathematics LS teams highlighted that "the knowledge contribution from the experienced teachers and subject specialists from NIE was significant in developing the pedagogical content knowledge in the community of practice" (Fang & Lee, 2010, p. 3). Lewis, Perry, and Hurd (2009) reported similar findings: "Lesson Study groups may need someone sufficiently experienced in mathematics learning to ensure such [learning] opportunities arise and are used productively" (p. 301).

Research findings also showed that LS affects teachers' instruction in mathematics in particular areas; instructional vocabulary, differentiated instruction, instruction using manipulatives, knowledge of mathematical learning stages, and the establishment of high student expectations (Rock & Wilson, 2005). Similarly, teams in Singapore schools reported that LS "holds tremendous potential in uncovering both students' and teachers' conceptions of and approaches to learning" (Yoong, 2011, p. 4). According to Fang and Lee (2009), participants in their study "developed a well-blended form of pedagogical content knowledge that is directly applicable to improve pupil's understanding of these topics" (p. 126).

The main challenge of implementing LS in Singapore was the time needed for its many iterative cycles (Fang & Lee, 2009). Lee (2008) also reported that the "time constraints and pressure faced by many school teachers" (p. 1124) would diminish interest in LS. He further added that "although Lesson Study is time-consuming, it can be highly rewarding. What is needed is teachers' commitment to the practice, and the support of school administrators and the government" (p. 1123).

Research Questions

The main intent of this study was to gain an in-depth understanding, from the teachers' perspectives, of the LS process used in Singapore. This article examines aspects of teacher professional development through LS and seeks to build upon the previous investigations of LS in Singapore. Several questions regarding the use of LS in Singapore are

important to consider: What are the concerns of teachers in Singapore when implementing LS? What type of support is needed for LS to be effective in Singapore? To what extent might we expect other LS groups in Singapore to conduct LS similar the one discussed here? In particular, we are interested in finding what teachers can learn from the LS experience and if, from the teachers' perspective, LS can be used effectively in Singapore for mathematics lessons. This article presents a school-based professional development initiative using the Japanese lesson-study model described by Stigler and Hiebert (1999) based on a university-school partnership. We report the results of conducting a LS with a group of six elementary school teachers in Singapore. The following research question guides our study: What did the teachers learn as a result of their experience in one LS cycle? In the next section, we outline a theoretical framework of teachers' learning to teach along with our methods of data collection and analysis. Finally we present the teachers' perspectives of their experiences in the LS cycle.

Theoretical Framework of Teachers Learning to Teach

The framework used in this study was described by Lin (2002). According to this framework, teachers learn through reflection, cognitive conflict, and social interaction. Vygotsky's *zone of proximal development* is used to explain the difference between what teachers can do alone and what they can do with assistance from others. Cognitive conflicts caused by observing students, discussing, critiquing, and negotiating during interactions among the teachers, their peers, and professional developers served as catalysts to progress to a higher developmental level. The teachers in the study were involved in a school-based professional development where knowledge is generated from social interaction within a learning community. Similar to Lin's (2002), this study was designed to create opportunities for teachers to develop more specific and deeper mathematical and pedagogical content understanding through observation and discussion.

Research Design and Data Collection

Spring Hill Elementary[1], a neighborhood public school, serves as the setting of this research study. The mathematics department head, who had an interest in using LS as a professional development tool, invited one of the researchers, a university faculty member, to be an LS consultant in 2008. The resulting professional development emphasized deepening the teachers' pedagogical knowledge on mathematics by focusing on students' mathematical thinking. The project started in 2008 and was ongoing during the preparation of this paper. As the LS coordinators and facilitators, the researchers provided strategies to team members to consider before the actual planning of classroom instruction. They listened to the team's input and, if needed, shared insights and posed additional questions to push the team members to think more deeply about what they observed. The team consisted of four teachers (Mabel, Zoe, Jade, and Sarah), the department head of mathematics (Rose) and level head of mathematics (Mary). Rose and Mary were the team leaders for this mathematics LS and they were also considered to be the more knowledgeable in terms of teaching mathematics. They taught upper elementary grade mathematics (which, in Singapore, includes sixth grade) and the rest of the teachers taught first and second grade mathematics during the study. Mabel, Zoe, Jade and Sarah worked very closely together because they shared common interests in enhancing their pedagogies in teaching mathematics. The teachers' role in the LS was to gain better understanding of effective pedagogies through the process of planning, research lesson, and reflection. They volunteered to participate in the LS project as a team when approached by their department head. In this manuscript, we report the results of the first LS cycle conducted by the six teachers.

Lesson Study Procedure

This manuscript focuses on a professional development using the following eight steps for collaborative LS by Stigler and Hiebert (1999). Table 1 describes the schedule for the eight steps.

1. *Define a problem during the first meeting*. The team decided to work on recognizing and naming unit fractions up to $\frac{1}{12}$ in various contexts involving squares, rectangles and triangles because they found that fractions are generally a difficult topic for second grade students.
2. *Plan the mathematics lessons*. Two full days were used to plan a lesson for second grade students on reading fractions. Six elementary school internal faculty members participated in the discussions. By the end of the day, the teachers completed the initial lesson plan (Figure 1) and listed some of the expected student responses.
3. *Teach and observe the lesson in the classroom*. Mabel executed the lesson while the rest of the teachers observed.
4. *Critique and reflectively discuss the lesson after classroom observations*. Following Mabel's lesson, the LS group spent approximately one hour critiquing and reflecting on the lesson. The participants shared and discussed issues of pedagogy and students' learning. Mabel was asked to reflect on her own teaching of the lesson and the rest of the participants were asked to articulate their observations after reflecting generally on their own teaching practices.
5. *Revise the lesson*. Immediately following the critique, the participating teachers spent another hour revising the fraction lesson. The teachers incorporated what was learned from the critique into revised lesson plan.
6. *Teach and observe the revised lesson*. Zoe taught the revised lesson the next day while the rest of the team members observed.
7. *Critique, reflect, and revise*. The team met to critique and reflect on the revised lesson taught by Zoe and the lesson plan was revised again.
8. *Share the results*. The head of department arranged to share results of the LS cycle with the rest of the teachers in the school.

---

[1] Pseudonyms were assigned to the school and the participants to ensure confidentiality.

Table 1

Lesson Study Cycle Schedule at Spring Hill Elementary School

Meeting	Purpose	Data	Duration
1-2	• Discuss the mathematical concept • Discuss how concept is linked to other mathematical topic • Anticipate students' misconceptions on that topic • Identify key factors leading to students' misconceptions or learning difficulties • Plan a mathematics lesson to address the problem	Lesson plan	12 hours (2 full days)
3	Observe lesson (taught by Mabel)	Audio recording and student work	1 hours
4	Critique & revise lesson plan	Audio recording	2 hours
5	Observe lesson (taught by Zoe)	Audio recording and student work	1 hours
6	Critique & revised lesson plan	Audio recording	2 hours
Follow -up	Reflect on LS experience	Recording and questionnaire	

Fractions for Primary Two

Specific Instructional Objectives:
Pupils will be able to recognize and name unit fractions up to $\frac{1}{12}$ in various contexts involving squares, rectangles, and triangles.

Prerequisite Knowledge:
Pupils need to be able to use shapes to represent one whole and fractions with denominators of up to 12 and identify parts and whole of a given situation.

Introduction to the Problem
Using fraction strips (rectangular, triangular, circular), the teacher recapitulates reading unit fractions.

Development

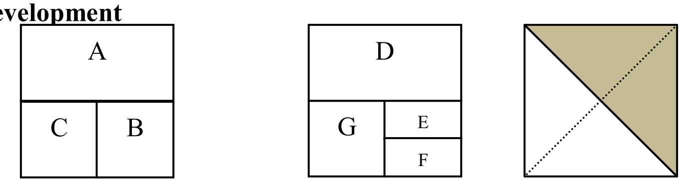


Diagram 1 Diagram 2a Diagram 2b

Key Teacher Questions
T: Look at Diagram 1 and take out the yellow cut outs. Where is the whole? This is the whole of the figure. Let us take a look at A. What shape is A? It is a rectangle. What shape is B? It is a square. What shape is C? It is a square. What shapes make up the figure? 1 rectangle and 2 squares. The teacher points and goes around the respective parts as the teacher introduces shapes A, B, and C to students. T: Now with your partner, discuss what fraction of the whole square is square C.

Expected Student Responses: $\frac{1}{3}$, $\frac{1}{2}$, No answer

The teacher asks the students to explain how they arrive at $\frac{1}{3}$, $\frac{1}{2}$, and why there is no answer to the problem.

Teacher addresses $\frac{1}{3}$ as an incorrect answer: What is the simple rule that you must remember for fractions? They must have equal parts. Does this figure have equal parts? Do you think your answer $\frac{1}{3}$ is correct?

Using the above structure, the teacher continues with the following problems of similar nature as shown in Diagram 2a and 2b.

Figure 1. The problem solving lesson plan.

Data Collection and Analyses

A qualitative design was selected to be the most appropriate research approach for this study because the main intent was to gain an in-depth understanding, from the teachers' perspectives, of the LS process when used in Singapore. Table 1 illustrates the data collection schedule. The data collected in this study consisted of audio recordings of the observed lessons and critiques, questionnaires, a focused group interview, lesson plans, and student work from the observed research lessons. The audiotaped meetings captured the teachers' conversations about their understanding of students' thinking, important suggestions teachers provide to revise the mathematics lesson, and what they learn from the LS cycle. These discussions provided the platform for teachers to constantly reflect on their teaching practices. The researchers administered a questionnaire (Figure 2) at the end of the LS cycle in order to document the teachers' experiences. The focused group interview (Patton, 2002) was conducted with the teachers at the end of the study to consolidate the teachers' reactions from the LS cycle (Figure 3). Interviews were audiotaped and transcribed. The LS team analysed the students' work during the cycle to provide evidence of student learning.

1. What did you learn when you planned the lesson with your colleagues?
2. Did your students respond to the lesson the way you anticipated? (Give specific examples to justify your observations)
3. What did you learn when you observed the mathematics lesson?
4. What did you learn when you critiqued the mathematics lesson with your colleagues?
5. How is the Lesson Study cycle helpful to you as a teacher?
6. How can Lesson Study be best implemented?

Figure 2. Sample of questionnaire conducted at the end of Lesson Study cycle.

1. What did you learn from the Lesson Study cycle?
2. How has participating in the Lesson Study cycle impacted your instructional practice?

Figure 3. Focused group interview questions.

A qualitative approach was used for the data analysis. An explanatory effects matrix (Miles & Huberman, 1994) was used to analyze the data. Data were collected and analyzed mainly to determine what the teachers learned and what they considered to be the effects of the LS. First, we entered quotes from the questionnaire and analyzed the data from Question 1 (see Appendix for a sample of the results). In the last column of the explanatory effects matrix, we added a general explanation of our observations of the data entered (Miles & Huberman, p. 148). During the data entry, we picked out chunks of material and developed codes, such as language, understanding students, teaching style and, manipulatives, by moving across each row of the matrix. We repeated the process for the rest of the questions and once each row was filled in for all the participants, we had an initial sense of emerging themes and patterns. Next, we sought confirming evidence by entering quotes and paraphrases from the interview and analyzing this data for each question. The students' work helped us follow and understand the taped discussions and interviews. Next, we organized and collapsed some of the codes into a theme. For example, *understanding students* and *learning styles* were regrouped and renamed *learning from students*. In the next section, we summarize our findings for each major theme. Our numerous data sources (discussions, focus group interviews, questionnaires, and student work) allowed us to triangulate our findings and provided greater confidence in our interpretations.

Results and Discussion

In the following paragraphs, we present the teachers' reports of what they learned during one LS cycle. In all the meetings, the teachers shared their opinions and observations openly. Our generalizations are not applicable to all the elementary schools in Singapore, but our work can be compared to existing theories of how LS cycles work in neighbourhood public schools in Singapore. We include representative student responses from Mabel's lesson to support this discussion.

Instructional Improvements: Instructional Vocabulary

Instructional vocabulary was one of the key issues brought up for discussion during the critique. Mathematical language was mentioned 11 times in the questionnaire by four of the participants. Jade wrote in her questionnaire that "mathematical language is important and the teacher must be consistent in using the language." Mabel wrote, "I think I have learnt a lot in being more careful in the terms used and more aware of the need in reiterating the terms or concepts that I want the pupils to retain." During the fourth meeting, Rose and Mary pointed out that fractions were read in multiple ways by students and the classroom teacher in Mabel's lesson. The rest of the teachers revealed that they used the fraction language based on their familiarity of it and were unaware of the implications of the differing language for student learning. The team decided to list all the different ways that they posed a fraction question. Table 2 shows the multiple ways that the teachers posed fraction questions, read fractions, and used fraction terminology.

The teachers were all amazed with the repertoire of terminologies they each had for just reading fractions. At this point, Mary commented that if students are unfamiliar with the terms their teachers use in teaching mathematics, they are likely to struggle with their teachers' language. If this occurs, the students become more preoccupied with this struggle than with the thinking processes embedded in the mathematics lessons. Zoe added that, in addition to this problem, students may also encounter challenges when they enter the next grade, in which a new mathematics teacher might use a different term to describe the same idea. At this point, Jade said with excitement:

> I didn't think that saying 3 quarters or 3 out of 4 equal parts matter to the students because I thought they are all common language that second graders should know. Shouldn't we have a vocabulary list clearly listed out for each topic so that students know what the mathematical terms they should know?

Everyone agreed that such a list would be very helpful. Mary suggested that the vocabulary list should be undertaken as a project across all grade levels so that students could focus on learning concepts without being confused by new terminology. can move beyond learning the basic mathematical knowledge.

Table 2

A Summary of Different Ways That the Teachers Posed a Fraction Question

Posing fraction questions	What fraction is shaded? What fraction of the figure is shaded? What is the shaded fraction? Which part is shaded?
Reading fraction	3 out of 4 equal parts 3 fourths 3 over 4 3 quarters
Numerator	The number above The number on top The number above the line The number that represents what the question asks The number that is not downstairs
Denominator	The number below The number below the line The number downstairs The number that represents the total number of parts in one whole

Zoe also brought up the necessity of being precise about the referents of our mathematical terminology. During her lesson, Mabel, referring to the figure in Figure 3, asked the students, "What fraction of the square is shaded?" She asked this without realizing that the square can be the whole figure composed of parts A, B, and C; just part B; or just part C.

Student A treated the smaller square as one whole and was consistent in using the smaller square as one whole throughout the entire worksheet (Figure 4). On the basis of observation and analysis of Student A's work and responses in class, the teachers learned that it is important to be specific when referring to elements of figures, such as the big or the small square. If this is not done, student errors might occur.

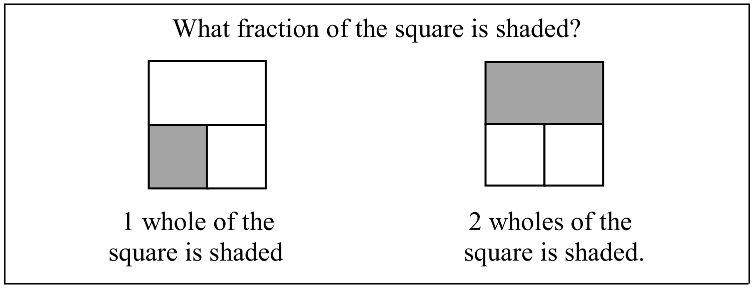


Figure 4. Sample of Student A's written seat work.

The above discussion led the team to realize that having a repertoire of mathematical terms for the same mathematical concept may be counterproductive if students are unfamiliar with some terms. This problem becomes even more significant when the teacher does not help the students relate the terms used in different grades. The LS discussion also challenged teachers to translate their observations into tangible classroom aides—in this case a mathematical language reference sheet integrated with appropriate terminology—which otherwise might not have occurred. The discussion also led the team to be more aware of the role of accurate and precise language as a tool to minimize students' learning difficulties.

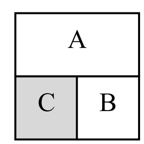
Professional Development Through Lesson Study: Learning From Students

The teachers already knew the importance of listening to students, but, from LS, they gained a deeper and richer perspective of what their students perceive about the classroom instruction. For example, in observing Mabel's lesson, Jade, Mabel, Mary, and Zoe said that they were amazed by some of the interesting, but incorrect, interpretations that students

developed for the concept of one whole. In this discussion, Mary referred to Student B's response and Jade referred to Student C's response (Table 3). The observation and discussions led the team to realize that a focus on student thinking can compel them to listen more closely to their students and that teachers should expect multiple interpretations of mathematical concepts. In the focused group interview, the teachers all claimed that the main benefit of participating in the LS was the opportunity to closely examine and analyze students' learning. They believed that by listening more carefully to their students' responses, they were able to identify factors that might give rise to student learning difficulties. This new understanding led the teachers in the team to recognize the importance of carefully planning every mathematics lesson using the knowledge they built through LS as the basis for making instructional decisions. This result affirms that LS leads to a focus on student learning. The mistakes the students made were directly used to improve classroom instruction (Stigler & Hiebert, 1999) in that the teachers took note of the understanding students demonstrated and the solutions they offered to the fraction problem.

Table 3

Description of Students' Verbal Responses

Student B		Squares B and C have equal parts. Rectangle A does not have equal parts so it cannot be a part of one whole ... one whole makes up of squares B and C. Square B is ½, and Square C is ½.
Student C	The parts are not equal so no fraction of the square is shaded.	

We have already shown that the teachers learned that mathematical language may be a potential barrier to students' learning of mathematics. In addition, the team became increasingly aware of other possible causes of student learning difficulties. During the focused group interview, all of the teachers agreed that their teaching pedagogies grew exponentially at the end of the first LS cycle as a result of their collaborative effort to understand student learning.

Professional Development Through Lesson Study: Learning From Colleagues

During the focused group interview, the teachers expressed appreciation that LS offered a structured system for professional development within the school context. The teachers also shared that their colleagues' observations of the lesson contributed directly to the richness of their critiques because of the variety of student thinking captured. They added that colleagues may also offer new points of view when observing the students. For example, during the sixth meeting, Sarah said "I was hoping Zoe would notice Student D's misconception and ask Student D to explain how they got two sixths during whole class discussion" (Table 4). In another incident, Rose said, "For figure 2(b) Student E and Student F actually wrote one half as an answer, but after Zoe said the correct answer is two fourths, the two students hurriedly changed their answers to two fourths" (Figure 5). Rose felt that Student E and Student Fs' responses provided a great opportunity to connect reading fractions (Grade 2 topic) to equivalent fractions (Grade 3 topic). Such peer observations and critiques offered more feedback to detect and follow up important teachable moments, which would otherwise go unnoticed.

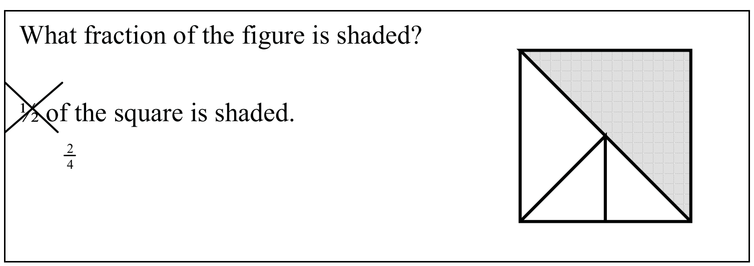
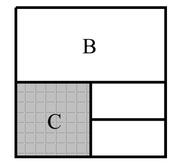
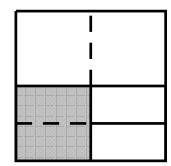
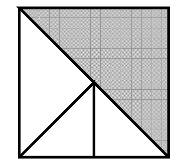
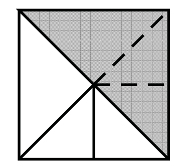


Figure 5. Sample of Student E's and Student's F' written seat work

Table 4

Description of Student D's Verbal Responses and Corresponding Written Seat Work

Verbal Responses	Written Seat Work
 There are 6 parts. A horizontal dotted line should be drawn ... that is how part C was cut ... likewise for part B. A vertical line should be drawn because that was how part B was cut. 	 What fraction of the figure is shaded?  Dotted line drawn by Student D

Teaching can be extremely private because teachers typically work only with their own students and have little collegial interaction (Lortie, 1975). Through their participation in LS, the participants were able to work in teams to challenge their own and their peers' use of instructional vocabulary. We have already discussed how this affirms Rock and Wilson's (2005) findings that LS affects instructional vocabulary. This result also supports Lee's (2008) findings in that the LS created the opportunities for the teachers to freely discuss, as part of a learning community, ideas rooted to classroom practices. In this case study, the teachers organized and built their repertoire of instructional vocabulary in order to attend to student misconceptions. This result affirms that LS offers the teachers a community in which to open the teachers' practice to scrutiny, and together with their community assist one another to think critically about their lessons, resulting in the teachers' instructional improvement (Lieberman, 2009).

During the focused group interview, the teachers said that they were planning and critiquing their daily lessons individually. According to the teachers, observing a live lesson and critiquing the lesson together with their team members gave them opportunities to challenge their hypotheses of students' thinking during lesson planning and test and verify those hypotheses during lesson observation and critique. Furthermore, observing live lessons allowed the teachers to capture more efficiently how students of different ability groups react to different segments of the lesson.

Rich mathematical tasks. Fang and Lee (2009) found that "pedagogical practices in Singapore are dominated by traditional forms of teacher-centred and teacher as authority approaches with little attention to the development of more complex cognitive understanding" (p. 106). Our participants wanted to focus on their pedagogical practices that developed complex cognitive understanding. Hence, the teachers in the team did not want to use the textbook or activities suggested in the teachers' guide. Instead, students explored a task which is usually not found in the Singapore textbook. They did so with the help of teachers who lead the entire class through the exploration by using focused questions. The teachers responded positively to the task on the questionnaire including Mabel who wrote that the task "enables pupils to apply mathematical concepts to solve new problems." Zoe commented that the task "brings about a refreshing way of acquiring the necessary knowledge and concept for the children" and that the unique task required the children to think rather than just be fed information. In addition, all the teachers agreed that the tasks enabled them to study how children learn. For example, Rose said that by analyzing the children's common errors, by utilizing strategies to help those children, and by being able to realize the effectiveness of such strategies, the teachers gained a better understanding of how children learn.

Nonetheless, the teachers had several concerns about implementing the fraction tasks in their own classrooms. The greatest concern the teachers had was the extensive time

required for students to fully explore and investigate the problems. In addition, teachers were not convinced that their students were ready to explore and investigate the problems on their own. Due to the aforementioned situations, the teachers in the team felt that they were likely to have insufficient time to accomplish the designed target stipulated in the syllabus. Given the constraint and tight curriculum, the teachers believed in providing more structure when implementing the fraction task.

Concerns about Lesson Study. Teachers felt similarly constrained by time when implementing the cycles required of LS. Zoe wrote, "Time is the greatest constraint. Even if there is a culture of sharing ... we lack the time to do so." This supports Lee's (2008) finding on time constraints faced by teachers involved in LS. This also affirms Rock and Wilsons' (2005) report that LS process requires substantial time and commitment. Rose suggested that schools could support the LS effort by arranging timetables to include more common time for teachers of the same grade level to meet. Sarah suggested that LS needed to be one of the school's top training plans in order to embed LS as a permanent professional development tool. Although the LS cycle was time-consuming, the results of our case study showed that the teachers found the whole process highly rewarding in terms of enhancing their instructional effectiveness.

Concluding Remarks

In this study, we examined teachers' experiences in one LS cycle. Our findings indicate many positive outcomes: The teachers are more aware of their instructional vocabulary. In particular, they are sensitive to the fact that inconsistencies and inaccurate use of mathematical terms may pose an extra challenge for the students. The LS cycle impacted the teachers' ability to think about the effects on children's learning when mathematical terms are read in multiple ways. Such observations were translated directly into useful resources for the teachers (e.g., a mathematical terminology reference sheet for students across all grades). The LS cycle also motivated the teachers to reconstruct students' thinking and to plan lessons that address students' misconceptions based on their models of student thinking.

During the focused group interview, the teachers in the study said they generally felt that the LS inspired the team to experiment with new tasks and provided them opportunities to evaluate and improvise those tasks. We suggest that LS facilitates the teachers' research on the efficacy of different types of tasks and the teaching approach required by those tasks, and we hypothesize that this enhances the teachers' pedagogical practices. The teachers were able to explicitly think about their views of new tasks, new pedagogies, their influences on instructional choices, and possible changes in practice, similar to the findings reported by Yarema (2010). By providing teachers with such a support system, allowing them to lay the groundwork for rich mathematical learning through reflective and critical thinking, we suggest that LS can serve as a platform to helps teachers cultivate good pedagogical habits.

Because LS requires a significant commitment of teachers' time and energy, the greatest challenge in adopting LS as a school-based professional development approach is time. In order to facilitate teachers' engagement with LS "school administrators can show their support in terms of timetabling… and providing staff development time" (Lee, 2008, p.1123), as suggested by Rose and Sarah.

In this study, when LS was used as a professional development tool, it improved the teachers' reflective thinking about teaching, especially when the teachers worked in a learning community. They were not only there to teach but also to plan, observe, and critique common lessons. Such a platform also provided an avenue of support for teachers to experiment with different teaching approaches. When professional development was embedded in these teachers' practice that included planning, observing, critiquing, and, collaborating, it led to their professional growth. The participants in this study believed that such growth will have lasting impact on their instructional practices.

References

Chua, P. H. (2009). Learning communities: Roles of teachers network and zone activities. In K. Y. Wong, P. Y. Lee, B. Kaur, P. Y. Foong & S. F. Ng (Eds.), *Series on Mathematics Education: Vol. 2. Mathematics education: The Singapore journey* (pp. 85–103). Singapore: World Scientific.

Fang Y., & Lee, C. (2009). Lesson study in Mathematics: Three cases from Singapore. In K. Y. Wong, P. Y. Lee, B. Kaur, P. Y. Foong & S. F. Ng (Eds.), *Series on Mathematics Education: Vol. 2. Mathematics education: The Singapore journey* (pp. 104–129). Singapore: World Scientific.

Fang, Y., & Lee, C. (2010). *Lesson study and instructional improvement in Singapore* (Research Brief No. 10-001). Singapore: National Institution of Singapore.

Fernandez, C., & Yoshida, M. (2004). *Lesson study: A Japanese approach to improving mathematics teaching and learning*. Mahwah, NJ: Erlbaum.

Lee, J. F. K. (2008). A Hong Kong case of lesson study: Benefits and concerns. *Teaching and Teacher Education, 24,* 1115–1124.

Lewis, C. (2002a). Does lesson study have a future in the United States? *Nagoya Journal of Education and Human Development 1*(1), 1–23. Retrieved from http://www.lessonresearch.net//nagoyalsrev.pdf

Lewis, C. (2002b). *Lesson study: A handbook of teacher-led instructional change.* Philadelphia, PA: Research for Better Schools.

Lewis, C., & Tsuchida, I. (1997). Planned educational change in Japan: The shift to student-centered elementary science. *Journal of Educational Policy, 12,* 313–331.

Lewis, C., & Tsuchida, I. (1998). A lesson is like a swiftly flowing river: Research lessons and the improvement of Japanese education. *American Educator, 22*(4),12–52.

Lewis, C., Perry, R., Hurd, J., & O'Connell, M. P. (2006). Lesson study comes of age in North America. *Phi Delta Kappan, 88,* 273–281.

Lewis, C., Perry, R., & Hurd, J. (2009). Improving mathematics instruction through lesson study: A theoretical model and North American case. *Journal of Mathematics Teacher Education, 12,* 285–304.

Lieberman, J. (2009). Reinventing teacher professional norms and identities: The role of lesson study and learning communities. *Professional Development in Education, 35,* 83–99.

Lim, S. K. (2009). Mathematics teacher education: Pre-service and in-service programmes. In K. Y. Wong, P. Y. Lee, B. Kaur, P. Y. Foong & S. F. Ng (Eds.), *Series on Mathematics Education: Vol. 2. Mathematics* *education: The Singapore journey* (pp. 104–129). Singapore: World Scientific.

Lin, P. J. (2002). On enhancing teachers' knowledge by constructing cases in classrooms. *Journal of Mathematics Teacher Education, 4,* 317–349.

Lortie, D. (1975). *School teacher: A sociology study.* Chicago, IL: The University of Chicago Press.

McLaughlin, M., & Talbert, J. (2006). *Building school-based teacher learning communities: Professional strategies to improve student achievement*. New York, NY: Teachers College Press.

Miles, M. B., & Huberman, A. M. (1994) *Qualitative data analysis: An expanded sourcebook (2nd ed.).* Thousand Oaks, CA: Sage.

Murata, A., & Takahashi, A. (2002, October). *Vehicle to connect theory, research, and practice: How teacher thinking changes in district-level lesson study in Japan.* Paper presented at the annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Athens, GA.

Patton, M. Q. (2002). *Qualitative research and evaluation methods (3rd ed.).* Thousand Oaks, CA: Sage.

Perry, R., & Lewis, C. (2009). What is successful adaptation of lesson study in the US? *Journal of Educational Change, 10,* 365–391.

Rock, T., & Wilson, C. (2005). Improving teaching through lesson study. *Teacher Education Quarterly, 32*(1),77–92.

Shimizu, Y. (2002). Lesson study: What, why, and how? In H. Bass, Z. P. Usiskin, & G. Burrill (Eds.), *Studying classroom teaching as a medium for professional development: Proceedings of a U.S.—Japan workshop* (pp. 53–57). Washington, DC: National Academy Press.

Stigler, J., & Hiebert, J. (1999). *The teaching gap*. New York, NY: The Free Press.

Vescio, V., Ross, D., & Adams, A. (2008). A review of the impact of professional learning communities on teaching practice and student learning. *Teaching and Teacher Education, 24,* 80–91.

Yarema, C. H. (2010). Mathematics teachers' views of accountability testing revealed through lesson study. *Mathematics Teacher Education and Development, 12*(1), 3–18.

Yoong, J. I. (2011, September/October). Let the students tell us how they learn. *SingTeach, 32*. Retrieved from http://singteach.nie.edu.sg/files/SingTeach_Issue32.pdf

Yoshida, M. (1999). *Lesson study: A case of a Japanese approach to improving instruction through school-based teacher development* (Doctoral dissertation.) University of Chicago. Available from ProQuest Dissertations and Theses database. (UMI No. 9951855)

APPENDIX

Explanatory Effects Matrix: Lessons learned

Question 1: What did you learn when you observe the mathematics lesson?

		Code	Short-run effects	Longer-run consequences	Researcher explanation
Zoe	Pupils had difficulties in expressing themselves using the appropriate language when asked to explain or justify their answers. Although they know the reason, they need to be taught the proper language so as to be able to support their answers. (questionnaire)	Understanding students Mathematical communication Mathematical Language	Establish the need for mathematical communication	Finding ways to support mathematical communication	The mathematical task chosen allowed teachers to see the need to foster mathematical communication
	There is a need to bring in various strategies in a single lesson to accommodate the various learning styles of the pupils in order to better achieve a higher percentage of pupils grasping the concepts taught. (questionnaire)	Learning styles	More aware of different learning styles	Need to explore differentiated instruction to cater to the different learning styles	Differentiated instruction is a topic of great interest to this group of teachers
Jade	Mathematical language is important and the teacher must be consistent in using the language. (questionnaire)	Mathematical Language	More careful in the use of mathematics vocabulary	Compile a mathematics vocabulary for the school	Teachers have a reservoir of terminologies