

How do you measure a slope of 10°?

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Applications for geometry and number skills in day-to-day life can provide a rich context to explore these skills in the classroom and it is also an aim of the *Australian Curriculum: Mathematics*: "to ensure that students... [are] able to investigate, represent and interpret situations in their personal and work lives" (Australian Curriculum, Assessment and Reporting Authority, 2010). This article describes a unit of work that was designed for a Year 9 class of mixed ability and which involved the definition and measurement of angles, as well as the practical application of angle properties—such as complimentary angles and the relationships in a transversal.

The unit considered the numeracy skills required to interpret a safety warning on a ride-on lawnmower—an application which would make even the most cynical and disaffected teenager reluctant to ask, "Why are we doing this?" Such a practical application of mathematical understanding can provide an important illustration for students of how they might use such knowledge in daily life.

There is a real danger of a roll-over when using a ride-on mower across steeply sloping ground. Travelling up or down a slope is much safer but not always practical. Manufacturers are required to publish safety guidelines and make the information readily available to users. One important piece of information is the maximum slope across which the mower can be driven safely. Some manufacturers insist that their machines never be driven across the slope; others vary from 5° to 17° . One popular manufacturer recommends that the mower should not be driven along a slope of greater than 10° (see Figure 1). Here is the problem: how to measure a slope?



Figure 1. A safety label from a ride-on lawnmower.

The first step is to unpack the terminology: what is an angle and how is it measured? An angle is defined as "an opening or amount of turn between two lines that share a common point" (Origo Education, 2008). Ask students to rule a horizontal line on a sheet of paper and then another line representing the slope that rises at an angle of 10°. This represents an 'angle of elevation.' There is value in having students estimate this angle first in order to practice this skill and be prepared to think about what an angle of 10° looks like. In a classroom, one would hope that Year 9 students would take out their protractors and carefully measure the angle on a diagram and easily determine whether it is less than or greater than 10°. A greater challenge for them is how to transfer this understanding to an actual piece of ground.

This type of practical application lends itself well to an investigative approach. It is highly likely that, within any group of secondary students, a number of them will have good ideas about how to approach this problem. These ideas can be collected in various ways but having students write them down or draw a diagram is an effective way of involving the whole group. If the students are grouped in the classroom, a placemat style graphic organiser can aid this process (see Figure 2).

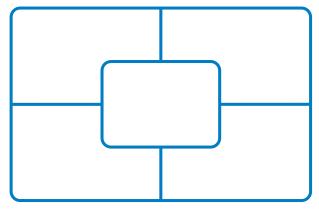


Figure 2. Placemat graphic organiser (Global Education website, 2008).

The placemat in this application has a central section for the group's summary surrounded by a number of sections for each individual to use. Initially, the group's members sit around the placemat, label their section with their names and record their ideas for the solution. They can use words, diagrams or record prompts for a verbal description.

The group then shares ideas in a structured way so that each member is aware of the others' ideas. They discuss the features, strengths and weaknesses of each idea and then determine the best solution, which might be a combination of ideas or the work of one student—so long as the group is agreed that it represents the best solution to the problem.

At this stage, the teacher should determine the effectiveness of each group's idea. If the group are unlikely to be able to solve the problem, another level of sharing can be employed. Each group nominates a spokesperson to explain the group's idea. Spokespersons then visit the other groups in the room and learn about their best ideas. It is a valuable experience for everyone to spend a significant amount of time discussing the different approaches to this problem. Students are often able to better understand concepts and ideas when they are presented by their peers.



Figure 3. A student's solution using a vertical reference.

There are many possible solutions to this problem and the more times this activity is done, more solutions will be found. Each effective solution, however, will involve the creation of some kind of measuring device. This process will probably take up the bulk of a single lesson and it might be appropriate at this stage to instruct each group to determine a list of materials needed for their device as a conclusion to the lesson.

A device for measuring an angle will have at least two components. First, it will need to allow the determination of a 'line of reference' from which the angle can be measured and, second, it must have a scale that allows for the amount of turn to be determined.

The line of reference can be found easily in two general ways: a weight hanging freely, as in a plumb bob, will indicate the vertical, and a body of water at rest will always be horizontal. This notion that a reference, or starting point, is needed from which to measure is fundamental to the conceptual understanding of measurement. By spending some time discussing this, students will have the opportunity to deepen their understanding of the measurement of angles specifically, as well as their understanding of measurement in a general sense.

Using the horizontal as a reference is the most convenient method to use in this situation because the angle of interest is 10° and is measured from the horizontal. To incorporate a 'body of water at rest' in a measuring device can be problematic and it may prove to be more practical to determine the vertical by use of a plumb bob, in which case, an understanding of complimentary angles will be needed. Two angles are complimentary if they sum to 90°. Vertical and horizontal differ by 90° so, if the vertical reference line is used, the device must measure the angle whose compliment is 10°, namely 80°.

If students use the horizontal as their reference, they have an opportunity to learn about the relationships between angles of a transversal. The angle of 10°, as initially quoted, refers to the angle of elevation from the horizontal. To measure this directly, the ground must be dug out to a level platform next to the slope to be measured (see Figure 4). As this is impractical, an accurate alternative must be used. The angle of depression can be measured with reference to a horizontal line that projects out from the slope

rather than in to it. To accept this as an accurate measure of the angle of elevation, it must be understood that, "the alternate angles of a transversal of two parallel lines are equal" (ACARA, 2010).

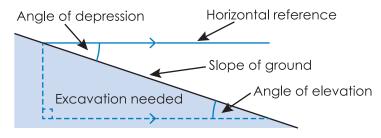


Figure 4. Excavation needed to measure the angle of elevation directly.

What needs to follow in the classroom will be a session or two of construction. The materials list from the previous lesson will vary but should include such items as: string, water, various containers, pieces of timber or cardboard, and nuts (as in nuts and bolts), as well as various tools for marking, measuring and cutting, and the necessary glues, tapes and fasteners to hold it all together. It should be expected that each group will want to use a protractor somewhere in the construction process. It may be used either directly as the scale or in order to determine some reference points that are marked on the device. It is, of course, up to the teacher to allow the use of a protractor and it will depend on the nature of the group and the teacher's intention in using this task.



Figure 5. Some students may prefer the protractor option.



Figure 6. Another student solution. The reference marks on the jar indicate 10 degrees. The arrow shows the user how to orientate this device to the slope.

An important part of the investigation is the testing phase, which should happen as a part of the construction rather than after construction has finished. The opportunity to refine the design is essential to both the success of the device and the students' understanding of invention. A few appropriate locations around the school yard can be set up as testing sites. The slope at these locations should be accurately measured first by the teacher. When each group is ready to test they take their device to the site and use it to determine whether the slope is greater or less than 10°. Their test will determine not only accuracy but also how easy the device is to use.

If the testing has been done largely or solely by the students, it is appropriate to challenge the groups to make a series of measurements of slope in unknown locations. Their results can be compared to accurate measurements as determined by the teacher and will act as a suitable culmination of their learning. A greater challenge will be for each group to create a set of instructions to go with their device and then have a different group carry out a series of measurements with it.

The process of accepting a real-life problem, investigating it and producing a real-life solution will not only consolidate specific mathematical learning for students, it will also consolidate for them the notion that mathematics has important and practical applications in their daily lives.

References

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