

## USING THE INTERACTIVE LEARNING ENVIRONMENT APLUSIX FOR TEACHING AND LEARNING SCHOOL ALGEBRA: A RESEARCH EXPERIMENT IN A MIDDLE SCHOOL

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### ABSTRACT

Most software tools that have been developed with the aim of helping students to learn school algebra have not yet achieved successful results in classroom. Almost all of them are menu-based systems that provide transformation rules in menus and buttons. Aplusix is a new interactive software tool for learning school algebra. In contrast to existing software tools in mathematics education, Aplusix has been developed to allow students to freely build and transform algebraic expressions as they can do on paper. In addition, Aplusix provides appropriate feedback and interactivity, and as such, it becomes a source of learning. This work reports on a research project on the investigation of students' learning of elementary algebra with Aplusix. The work uses the Theory of Didactical Situations (TDS) to analyze and evaluate the learning potentialities of Aplusix. The paper also reports on implications for the learning of school algebra and the integration of Aplusix into mathematics classroom.

**Keywords:** Aplusix, learning, mathematics education, school algebra, Theory of Didactical Situations, utility value

### INTRODUCTION

The difficulties of learning school algebra in a paper-pencil environment relate to different analyses: The evolution from procedural to structural conception (Sfard, 1991), semantic/syntactic difficulties (Drouhard, 1992), the transition from arithmetic to algebra (Vergnaud, et al, 1988), the status of letters and the notion of variable in algebraic expressions (Kieran, 1992). Another difficulty is that algebra is often seen as a formal, isolated system where manipulations of symbols are often dominating (Kieran, 1991). There has been a great deal of research in teaching and learning school algebra the last 15 years (Kieran, 2007).

However, despite the fact that researchers know relatively more about the learning of algebra, a great deal remains to be researched, in particular the integration of ICT into classroom. Several software tools aiming at helping students to learn algebra, have been developed, but few of them have achieved successful results in classroom yet (Kieran, 2007; Zbiek, Heid, & Blume, 2007). A promising interactive software tool in school algebra is Aplusix (Chaachoua, Nicaud, Bronner, & Bouhineau, 2004; Nicaud, Bouhineau & Chaachoua, 2004). The aim of this research work is to gain theoretical and practical insights into the learning and teaching of school algebra using the interactive learning environment Aplusix.

The work is structured as follows. First, the theoretical framework is outlined. Second, the literature review is reported. Third, the research experiment in a middle school is described. This is followed by the data collection methods. The evaluation results are then outlined. Some implications on the use of Aplusix for the learning and teaching of school algebra conclude the article.

### Conceptual Framework

The modes of use of Aplusix depend on the chosen conceptual framework in terms of interactions with the students in classroom. The Theory of Didactical Situations (Brousseau, 1997) is one of the most appropriate frameworks for investigating interactions with Aplusix. According to the Theory of Didactical Situations (TDS), learning occurs by means of interaction between learner and a "milieu". Each action of the student in the "milieu", which consists of a material and non material "milieu", is followed by a feedback of the "milieu" itself, which generates difficulties and contradictions. Learning happens through adaptation of the student to the "milieu" (Figure 1).

Aplusix can be considered as an element of the "milieu", and as such, its feedback becomes a source of learning. Aplusix provides three categories of feedback (Nicaud, Bouhineau & Huget, 2004):

- Feedback about the equivalence of expressions,
- Feedback on the state of the current step provided by indicators in the software, and
- Feedback provided by textual messages.

The role of the teacher is that of setting a situation or experiment in which students interact with Aplusix to achieve a given educational goal (Cerulli, Pedemonte & Robotti, 2005).

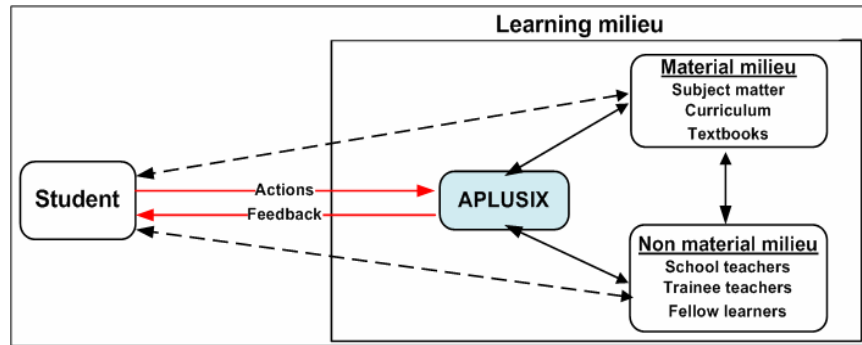


Figure 1: Interactions of the student with the “milieu” according to TDS

## LITERATURE REVIEW

Several experiments with Aplusix have been carried out in mathematics education in different countries (Bouhineau et al, 2005; Nicaud et al, 2006; Nicaud, Bouhineau & Huget, 2004): remediation piloted by researchers in Italy; remediation integrated into the regular functioning of classes in Brazil; collaborative learning in India, and regular use during an entire year in France. On the basis of these experiments, researchers (Bouhineau & al, 2005) concluded that Aplusix has been shown to be a usable computer system, favoring the students’ learning of school algebra. In addition, the cost of integrating Aplusix into the teaching of algebra is low. Furthermore, the students gained autonomy and improved their knowledge. Finally, Aplusix facilitated the teachers’ work because of the students’ autonomy and of already-made lists of exercises.

However, despite the promising benefits of Aplusix, research work still remains, among other things develop adapted experiments for many features of the software, collect a large number of protocols and study transformation rules by-hand, use an algorithm to diagnose the transformation made by the students, narrow the domain (in order to get more actions for each student on one domain), determine typical conceptions of students in this domain, use of Aplusix for a very large number of students, exploiting the feedback of Aplusix to mediate the equivalence between algebraic expressions (Maffei, Sabena, & Mariotti, 2009), and understanding the relation between design and usage of Aplusix (Trgalova & Chaachoua, 2009). Finally, there is a need to perform comparative studies between different countries.

## Research Experiment in Classes of Grade 10

The research experiment with Aplusix described in this paper took place in two classes of grade 10 having 30 students each and 2 teachers in a middle school. The major goal was to evaluate the student progress in learning school algebra after the training phase with Aplusix. The experiment consisted of a teaching sequence with four major phases. It is similar to the one described in (Nicaud, Bouhineau, & Chaachoua, 2004):

- First, a pre-test of 30 minutes, using paper-pencil techniques, was given to the students.
- Second, before any teaching of algebra, the students were introduced to the functionalities of Aplusix.
- Then, the learning of equation solving with Aplusix began with the training phase, using the feedback of the system. This phase had a two weeks’ duration.
- Finally, a post-test of 30 minutes using paper-pencil techniques was organized in order to measure the students’ progress in comparison to the pre-test.

## Data Collection and Analysis Methods

Both qualitative and quantitative methods were used to collect data. Quantitative data collection consisted of three methods. The first one used pre- and post-test with paper-pencil techniques to measure the students’ progress in learning school algebra between the pre- and the post test. The second method relied on statistics that is automatically produced by Aplusix. The data collected statistically can be analyzed and displayed on the screen (Nicaud, 2006). These consisted of students’ past activities such as attempted exercises, well-solved exercises, calculation errors, scores and time. The teacher can select the students individually in order to analyze their performances. Finally, a survey questionnaire was used to collect data about the utility value of Aplusix. The questionnaire used a five-point Likert scale from 1 to 5, where 1 is coded as the lowest and 5 as the highest.

Three methods were used to collect qualitative data. The first one used the students’ protocols that are produced by Aplusix. Protocols permit the analysis of difficulties encountered by the students, their strategies used in the resolution of problems, the students’ acquisition of relational understanding of the equal sign, and the analysis of exercises with bad percentage of success to identify possible didactical variables. The second method used semi-structured interviews with teachers ( $N=2$ ) and students ( $N=6$ ). The third method used observations of

students' interactions with Aplusix. There are three types of interactions that can be observed with this method: Student-teacher, student-student, and student-textbooks.

## RESULTS

The results are described with regard to the following issues:

- Students' understanding of the equal sign
- Students' performances in solving equations
- Students' algebraic strategies and resolutions, and
- Utility value of Aplusix.

### Students' Understanding of the Equal Sign

The exercises in the pre- and posttest contained similar tasks. The goal was to identify students who had a relational understanding of the equal sign both in the pre-and post-test. The results indicate a progression of 23% from the pre-test (11%) to the post-test (34%) with regard to the percentage of students who had a relational understanding of the equal sign (Table 1).

Table 1: Students' progression in their understanding of the equal sign from pre-test to post-test

Pre-test (in %)	Post-test (in %)	Progression (in %)
11	34	23

### Students' Performances in Solving Equations

Regarding the students' performances in solving equations the following results were achieved (Table 2):

Table 2: Students' performances in solving equations

	Task	Pre-test (in %)	Post-test (in %)	Progression / regression (in %)
1	$ax = b$	68	90	22 %
2	$-a = -bx$	18	42	24 %
3	$a/b \cdot x = -c/d$	2	0	-2 %
4	$-ax+b = 0$	21	24	13 %
5	$ax+b = -cx-d$	11	22	11 %

The results show that tasks using fractions (exercise 3) were very difficult for all students. Exercise 1 and 2, which were more familiar to the students, achieved a progression of 22% and 24% respectively. Most students (90%) solved exercise 1 in the post-test. Exercise 2 was interesting from a pedagogical point of view because it contains a didactical variable (the minus sign). Students had difficulties with this type of exercise in the pre-test (18%). It seems that Aplusix helped some students to solve this exercise (42% achievement in the post-test), a progression of 24%. Exercise 4 and 5 achieved lower results (13% and 11% respectively). This is not surprising since these exercises require the understanding of two didactical variables (the minus sign and the number zero).

Summarizing, the students' performances seem to be dependent on the task type (increase from 0 to 24%). In particular, fraction tasks were a very difficult problem for all students (-2% regression). In addition, problems with the equivalence principle play a role, and it may be well suited to train students with Aplusix.

### Students' Algebraic Strategies and Resolutions

The following analysis is about the students who acquired a relational understanding of the equal sign in the post-test, that is to say an understanding of the equivalence principle. The analysis concerned their algebraic strategies they used to acquire such an understanding. The analysis was based on the students' protocols produced by Aplusix. To be able to reason algebraically and develop strategies students need to understand the equivalence principle, manipulate didactical variables, and solve complex equations. The following experiments show the steps the students went through to acquire an appropriate understanding of elementary algebra.

#### a) Understanding of the Equivalence Principle

The following examples (Figure 3 and 4) show a gradual learning process in three steps. First, the student was not able to use the equivalence principle correctly. In the second step, the student seemed to be able to make progress, before he/she managed in the third step to solve the exercise using the equivalence principle.

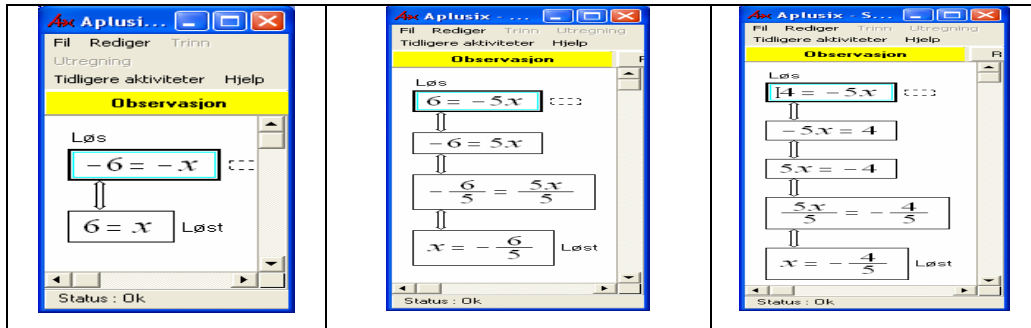


Figure 3: Example showing a gradual understanding of the equivalence principle

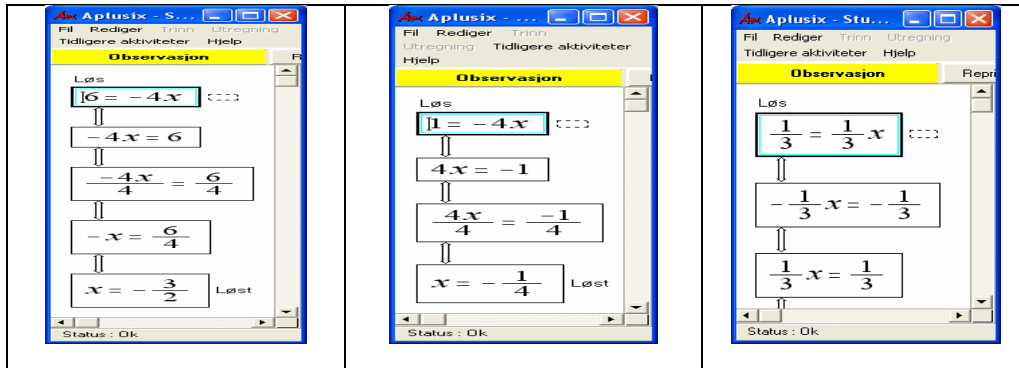


Figure 4: Example showing a gradual understanding of the equivalence principle

**b) Use of the Didactical Variable Minus and Arithmetic Knowledge about the Number Zero**

This example shows that the student was first not able to understand the role of zero and minus sign in algebraic equations. In the second step, the student improved her/his understanding of the didactical variables considered (Figure 5).

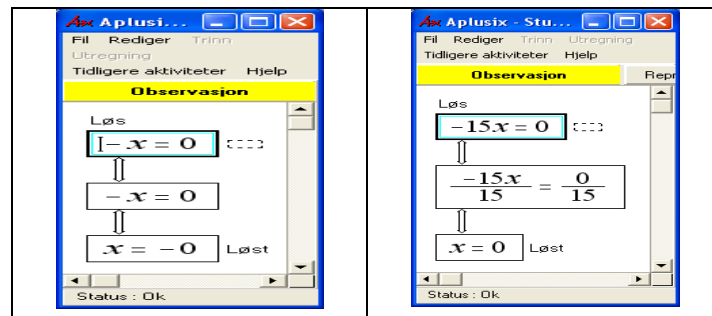


Figure 5: Example showing a gradual understanding of didactical variables

**c) Solving Complex Equations**

This example shows the students' ability to solve complex equations (Figure 6). It appears that the student made progress in solving complex equations in comparison to his/her performance in the pre-test.

Summarizing, the following conclusions can be drawn:

- The lack of arithmetic skills may have been an obstacle to learning algebra. In particular, problems with the didactic variables fraction, minus, and zero, but also the lack of awareness of arithmetic conventions, such as omitted count characters and calculation priorities, leading to problems when solving equations.
- The performance of many tasks seems to have helped the students' learning.
- One way to help students to acquire an understanding of algebraic equations is to increase the number of equivalent steps in the solution process when using Aplusix.
- Students, who had a relational understanding before they began to use Aplusix, could quickly improve their understanding and performance.

- Several students used algebraic strategies in the post-test than in the pre-test.

Figure 6: Example showing students' ability to solve complex equations.

### Utility Value of Aplusix

The utility value of Aplusix was evaluated on the basis on 5 criteria: Technical usability, pedagogical usability, feedback, differentiation, motivation, and interaction. The evaluation was carried out by means of survey questionnaires. The results achieved are as follows:

- *Technical usability:* There is a relatively large consensus that the Aplusix is easy to learn and to use for most students.
- *Pedagogical usability:* Aplusix seems to be pedagogically suited for both average and strong students, but students at a low level do not appear to have benefited from the software. The reason may be the students' limited prior knowledge, unfamiliar task types, and the mathematical language used in Aplusix. The students' different ways of working (Procedure-oriented, solution-oriented or reflected) seem to affect the pedagogical usefulness of Aplusix. The integration of Aplusix into classroom may have a positive impact on learning, even for students at a lower level.
- *Feedback:* Students from the analysis group ( $N=13$ ), that is to say the group of students who changed their understanding of the equal sign from operational to relational, responded differently to feedback, either unstructured or structured trial and error or with a targeted improvement of the error. All ways of working with Aplusix lead to learning. However, it is difficult to conclude that this applies to all students.
- *Differentiation:* Teachers believed that Aplusix takes into consideration some students' needs. In contrast, students' responses to the questionnaire indicate uncertainty in relation to differentiation opportunities among students.
- *Motivation:* At the beginning of the experiment with Aplusix all students were motivated and task-focused, and some of them were highly motivated. However, the motivation decreased over time, especially for weak students. Lack of prior knowledge may be a plausible reason for decreased motivation.
- *Interaction:* Most students used Aplusix interactively. They needed little teacher help than in normal hours, and there were few interactions between with the "milieu" and the students. However, the interactions with the "milieu" (fellow students, teacher) were difficult to measure due to a number of contextual factors.

### CONCLUSIONS AND RECOMMENDATIONS

As with all educational research of this nature, it is difficult to conclude direct causality between the characteristics of the experiment and the learning effect of Aplusix since a number of contextual factors may implicitly affect the learning process. However, by considering the various parameters, both technical and pedagogical, that have been taken into account in this work, it has been possible to make some reasonable interpretations of the results. These indicate that Aplusix shows potential for learning school algebra, although not all types of students benefited equally well. Aplusix may have a positive impact on the students' learning if some conditions are met. Firstly, students need to have a basic knowledge of the relational understanding of the equal sign. Secondly, students should have prerequisites, especially a good arithmetic basis. Third, the design of didactical situations is important, such as choice of task types that are adapted to the students. Then, teachers' ICT expertise, both technical and didactical, is important. Furthermore, the time aspect needs to be considered, both for the learning and use of Aplusix. Finally, the integration of Aplusix into classroom could increase the value and benefit of the program for learning elementary algebra. This may be an important condition for improving student achievement and performance.

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