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Pre-Service Teachers' Perception about the Concept of Limit

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Abstract

The purpose of this study was to investigate the pre-service teachers' conceptions about the limit of same partial functions given in the form of both graphical and symbolic, the misconception that the pre-service teachers have the concept of limit, and whether there were any differences between pre-service teachers' algebraic performance and their graphical performance. In this study, both qualitative and quantitative methods were used. In the collection of the data, the researcher employed the open-ended question test and interviews. The questionnaire was consisted of five items. Firstly symbolic representation of the questions was administered to 95 pre-service teachers, later graphical representation of the same questions were given to pre-service teachers. Finally, semi- structured interviews were done with eight pre-service teachers. In the data analysis, descriptive analysis method was used and also independent samples t-test was employed with α =0.05 in the analysis of the differences of pre-service teachers' algebraic and graphical performance. The result of current study showed that some students had some misconceptions and misunderstanding related to the concept of limit. It was also observed that there were significant differences found between pre-service teachers' algebraic and graphical performances. The Pre-service teachers had higher scores in graphical representation than symbolic.

Key Words

Limit, Misconception, Concept Image, Multiple Representations, Mathematics Education.

The function is one of the most fundamental and important concepts in all branches of mathematics including calculus, abstract algebra and geometry (Froelich, Bartkovich, & Foerrester, 1991), since the conceptions of modern mathematics were founded on the function such as limit, derivative and integral. Therefore, a lot of research emphasizes the importance of the function (Artigue, 1992; Breidenbach, Dubinsky, Hawks, & Nichols, 1992; Leinhardt, Zaslavsky, & Stein 1990). The function has got multiple representations such as tables consisting of ordered pairs of values, graphs consisting of a pictorial presentation, and equations consisting of algebraic notation (Brenner et al., 1997). Research has shown that students generally prefer the symbolic representation of the function (Leinhardt et al., 1990; Romberg, Carpenter, & Fennema, 1993). However, the graphs are useful and the most economical ways for summarizing large amounts of data (Latour, 1987) and, similarly, the graphs provide the shortest way to obtain information about the function (Kadıoğlu & Kamali, 2009). Therefore, interpreting graphs is important for all the science and mathematics' curriculums and some research emphasizes the importance of interpreting graphs (Fiel, Curcio, & Bright, 2001; Milli Eğitim Bakanlığı [MEB], 2005; National Council for Teachers of Mathematics [NCTM], 2000).

For the last three decades, the mathematics' educators have focused on the students' interpretations of graphs, the students' understanding of different representations of functions and the students' pref-

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erence for multiple representations (Bowen & Roth 1998; Dreyfus & Eisenberg, 1982; Keller & Hirsch, 1998; Knuth 2000; LaLomia, Coovert, & Salas, 1988; Turner & Wheatley, 1980; Vinner, 1988;). Dreyfus and Eisenberg (1982) explored the students' preference for function concepts presented in diagram, graph, and table settings. The results of their study indicated that the students with a high ability preferred the graphical setting throughout all the concepts, while the low ability students preferred the table setting. Keller and Hirsch (1998) conducted a study, whereby they researched into the students' preferences for the representations of the functions. The results indicated that the students had preferences for various representations of the functions and that these preferences varied between the ones, presented in a context and the others, given in a purely mathematical situation. Vinner (1988) examined which style of proof of theorems in calculus, symbolic or graphical solutions students preferred. He gave students two proofs of the mean value theorem. The first proof was the standard algebraic proof. The second was a visual (graphical) proof. Of 74 students, 29 stated that the graphical proof was more useful, 28 stated that the algebraic proof was more useful and 17 considered them both to be of equal value.

The other issue related to the function is the students' understanding of the concept of limit. The concept of limit is prerequisite for the important concepts in calculus such as the continuity, derivative and integral concepts. According to Altun, (2008) if students (male or female) do not understand the concept of prerequisite conditions, these students later have difficulties in understanding things related to the concepts.

Because of the importance of the concept of limit, a lot of research was conducted by mathematics' educators relating to the students' understanding of the concepts of limit (Akbulut & Işık, 2005; Bezuidenhout, 2001; Bukova-Güzel, 2007; Cornu 1991; Cotrill et al., 1996; Çetin 2009; Juter, 2006; Juter, 2009; Hardy, 2009; Szydlik, 2000; Tall & Vinner, 1981). The teaching of the concept of limits has been studied by some researchers from many different theoretical viewpoints, namely: concept image and concept definition (Tall & Vinner, 1981), APOS Theory (Cottril et al., 1996). On the one hand, Tall and Vinner conducted a study (1981), in which they used the term 'concept image' to describe the total cognitive structure that is associated with a specific mathematical concept which includes all the mental pictures and associated properties and processes. On the other hand, Tall and Vinner (1981) used the term 'concept definition' to refer to the words which needed to be spelt out in a mathematical concept. Some researchers investigated the students' performances with the limit concept (Çetin, 2009) and misconceptions relating to the students' understanding of the limit of a function (Bezuidenhout, 2001; Jordan 2005; Özmantar & Yeşildere, 2008). Çetin (2009) explored the first-year university students' skills in using the limit concept. The result of this study indicated that the students were able to compute the limit values by applying the standard procedures but were unable to use the limit concept in solving related problems. Jordaan (2005) investigated the misconceptions that engineering students have in the concept of limit. She found some misconceptions relating to the students' understanding of the limit of a function. Some researchers explored the effect of the learning environment in learning the limit concept (Akbulut & Işık, 2005; Bukova-Güzel, 2007). A study was conducted by Bukova-Güzel (2007). She investigated the effect of a constructivist learning environment regarding the learning of the limit concept. The result of this study showed that the constructivist learning environment provided a positive contribution to the learning of the limit concept.

The Importance and Purpose of Research

The literature has shown that interpreting graphs is important for all people and the limit is an important concept in mathematics. However, pre-service teachers, including participants of the present study, are considered to be the future educational leaders. They will become the new generations' teachers and they will teach the interpretation of graphs to these generations in the future. Therefore, it is important to understand how pre-service teachers' understand the concept of limit as well as the kind of misconceptions pre-service teachers have and whether there are differences between the graphical and symbolic performance of the pre-service teachers. Specifically, in this research the following research questions have been investigated:

1. How do pre-service teachers find the limit of a partial function given in the graphical and symbolic forms?
2. What kind of misconception do pre-service teachers have regarding the concept of the limit?

3. Is it possible to differentiate statically between the pre-service primary teacher's symbolic performance and graphical performance?

Method

Research Design

In the current study, the case study approach was used as a general research strategy (Cohen, Manion, & Morrison, 2000; McMillan & Schumacher, 2006). Two different methods were used to collect data for this study. One was the questionnaire (the open-ended question test) and the other was the use of semi-structured interviews. Firstly, the symbolical representation of the questions was administered. Later the graphical representation of the same questions was given to the pre-service teachers. Finally, the semi-structured interviews were done with eight pre-service teachers.

Participants

The sample of this study consists of 95 pre-service teachers, aged 18–23 years, who attend the Faculty of Education. There were a total of 37 teachers in the department of mathematics teaching and 58 teachers in the department of science teaching. In both groups, the pre-service teachers take the general mathematics course. The researcher followed the convenience of the sampling procedure in which the participants are not randomly selected (Cohen et al., 2000; McMillan & Schumacher, 2006). According to McMillan and Schumacher (2006), a convenience sample is a group of subjects selected, on the basis of being accessible or expedient and it is appropriate to use the group as subjects. The participants in this study were voluntary and they were assured that their answers would be kept confidential.

Instruments and Process

In order to obtain more detailed information on the pre-service teachers' understanding, misconceptions, if there are any, about the limit, and on differences between their symbolic and graphical performance qualitative and quantitative data were collected through the questionnaire, which consisted of three open-ended questions. The questionnaire consisted of two forms of same questions: symbolical representation and graphical representation (see Appendix1-2). In the preparation of questions the related literature was benefitted (Kadıoğlu & Kamali, 2009; Stewart, 2000). In order to establish the reliability and validity of the questionnaire, a number of questions were given to two experts. These experts determined whether the questions in the questionnaire were appropriate for the purpose of obtaining information on the pre-service teachers' understanding, misconceptions and their symbolic and graphical performance on the concept of limit. The questionnaire was administered to 95 pre-service teacher participants. Firstly, the symbolic form of the questions was administered. Fifteen days later the graphical form of the questions was conducted. There was no time limitation for the testing session. However, most pre-service teachers finished the questionnaire within 30-35 minutes. In order to investigate the pre-service teachers' understanding about the limit concept further, eight chosen (n = 8) pre-service teachers were interviewed. These pre-service teachers were asked to "think aloud" and explain their thinking about and reasons on the responses they gave to open-ended questions. Interviews were recorded using the note-taking method.

Data Analysis

In the data analysis, both quantitative and qualitative techniques were employed. Fundamentally, the qualitative descriptive analysis was used to analysis the data of the study (Mcmillan & Schumacher, 2006; Robson, 2002). Firstly, 6 of 95 pre-service teachers' responses and 1 of 8 students' interviews were selected and these were analyzed by the researchers and the researchers' colleagues separately in order to confirm the validity of the analysis. It was clearly seen that the analyses of the researchers and researchers' colleagues were similar. After this process, the open-ended response items were carefully read and examined by the researchers. The pre-service teachers' answers to these items were then coded and categorized and the themes were determined. Percentages of each answer the students gave to each open-ended question were found and were given in tables. The interview data was analyzed with the themes identified in response to the questionnaires. The interview data was only used in order to support the questionnaire data. The researchers also employed the independent sample t-test with $\alpha = 0.05$ in the analysis of the differences of the pre-service teachers' symbolical performance and graphical performance.

Results

In this section, the pre-service teachers' responses to the specific questions were analyzed according to different viewpoints in terms of their understanding of the limit concept. More specifically, the pre-service teachers' ways of solution and their approaches when working on functions presented both graphically and algebraically were investigated in order to understand their misunderstandings and misconceptions in different representational systems better. The pre-service teachers in this study were given several function questions in which the functions are presented symbolically and graphically in order to obtain clear thoughts about their understanding of the concept of limit. In the first open ended question, responses of 95 participants were categorized as shown in Table 1. It was observed that two out of three of them (69.3%) were successful in determining both the points (x=-2 and x=2) where the function had no limit on a symbolic presented function. 5.26% of the pre-service teachers said that the function only had a limit at point x=2, and 3.1% of the pre-service teachers said that the function only had a limit at point x=-2. Alternatively, in the graphical representation of the first question, the majority of the participants (93.7% or 89 out of 95) were successful in determining both of the points (x=-2 and x=2,) as to where the function had a limit or not.

In the second open ended question, participants were asked to determine whether $\lim_{x\to 3} g(x)$ exists for the given .

$$g(x)=\begin{cases}\sqrt{x-3} & x>3 \text{ ise}\\ 3 & x=3 \text{ ise}\\ 6-2x & x<3 \text{ ise}\end{cases}$$

The responses of 95 pre-service teachers were categorized as shown in Table 2. More than half of the pre-service teachers (56.84%) said that the right- and left-hand limits were equal so $\lim_{x\to 3^-} g(x)=\lim_{x\to 3^+} g(x)=0$. Thus, the limit existed and $\lim_{x\to 3} g(x)=0$. 30.52% the pre-service teachers said that $g(x)$ does not have a limit on the point χ=3 since $\lim_{x\to 3^-} g(x)=\lim_{x\to 3^+} g(x)\neq g(3)$. For example, one of these responses was as follows:

"$\lim_{x\to 3^+}\sqrt{x-3}=0$, $\lim_{x\to 3^-} 6-2x=0$ *and* $g(3)=3$ *the limit does not exist because* $\lim_{x\to 3^-} g(x)=\lim_{x\to 3^+} g(x)$ *but when* $g(3)\neq 0$ *the limit does not exist. If* $g(3)$ *is equal to 0 (zero) we will say that the limit exists.*"

It can be seen that the other 28 pre-service teachers have given similar responses. Therefore, it can be said that these pre-service teachers have a misconception. In other words, they think that a function must be defined and continuous at a certain point to have a limit and that the limit is equal to the function value at that point. 7.36% of participants said that $\lim_{x\to 3} g(x)$ does not exist since the right- and left-hand limits are not equal, that is, $\lim_{x\to 3^-} g(x)\neq\lim_{x\to 3^+} g(x)$. One of these responses is as follows:

" $\lim_{x\to 3} g(x)$ *does not exist because of* $\lim_{x\to 3^-} 6-2x=-2$, $\lim_{x\to 3^+}\underbrace{\sqrt{x-3}}_{x=4}=0$ *and* $\lim_{x\to 3^-} g(x)\neq\lim_{x\to 3^+} g(x)$."

It can be seen that these pre-service teachers generally make procedural errors. Then again, in the graphical representation of the second question, as in the symbolical representation approximately half of the pre-service teachers said that the right- and left-hand limits were equal so $\lim_{x\to 3^-} g(x)=\lim_{x\to 3^+} g(x)=0$. Thus, the limit exists and $\lim_{x\to 3} g(x)=0$. However, one out of five of the participants (21.05%) said that the limit exists and $\lim_{x\to 3} g(x)=3$ because of $\lim_{x\to 3^-} g(x)=\lim_{x\to 3^+} g(x)=3$. 8.42% of the participants mentioned that $\lim_{x\to 3} g(x)$ does not exist because of $\lim_{x\to 3^-} g(x)=\lim_{x\to 3^+} g(x)\neq g(3)$. It can be said that these pre-service teachers have some difficulties in interpreting graphs and the students have some misconceptions or misunderstandings about the concept of limit. Finally 12.63% of the pre-service teachers said that $\lim_{x\to 3} g(x)$ does not exist because of $\lim_{x\to 3^-} g(x)\neq\lim_{x\to 3^+} g(x)$.

In the third open-ended question as in the first two questions, it can be seen that the pre-service teachers have some misconceptions or misunderstandings about the concept of limit. However, considering the analysis of the interview conducted with the pre-service teachers, it can be seen that they have some misconceptions and misunderstandings about the concept of limit. The following is a part of the interview conducted with one of the pre-service teachers.

R: *What did you understand from the limit? Could you explain?*

P1: *It is an expression to say that the right- and left-hand limits are equal.*

R: *What does this mean? Could you explain?*

P1: *The right- and left-hand limits must be equal.*

R: *How did you find the right- and left-hand limits?*

P1: *The values were given. I benefitted from these values.*

R: *What can you say about the limit at the point where the function is undefined?*

P1: *If a function is undefined at a certain point the limit does not exist at that point. This shows that the right- and left-hand limits are not equal.*

The following excerpt was taken from the interview with the other student:

R: *What did you understand from the limit? Could you explain?*

P2: *I looked into the right- and left-hand limits. If the right- and left-hand limits are equal, I could say that limit exists*

R: *What are the right- and left-hand limits? Could you explain?*

P2: *The values were given and I put these values into the function.*

Finally, it was investigated as to whether there were differences between the pre-service teachers' symbolical and graphical performances. The responses to the symbolical and graphical representations given by the pre-service teachers were compared. It can be seen in Table 6 that the pre-service teachers had higher scores in the graphical representations than they had in the symbolical representations ($\bar{x}_G = 4.73$ and $\bar{x}_S = 4.03$). These differences were concluded in the independent sample t-test. The t-calculated value of 2.35 was greater than the t-critical value of 1.96 (for the degree of freedom 188). It can be said that pre-service teachers were more successful in the graphical representations than they were in the symbolical representations.

Conclusion and Discussion

The finding of the current study shows that most of the pre-service teachers understand: "when the values are given to the function of the right-hand and left-hand, if the function tends to be the same number, we say that the limit exists." Also, it can be seen that some pre-service teachers have some misconceptions and misunderstandings related to the concept of limit. In the first of these misconceptions, pre-service teachers think that the limit and the function values are the same. The results of this current study are in correlation with some other studies (Bezuidenthout, 2001; Cottrill et al., 1996; Jordaan, 2005). As Bezuidenthout (2001, p. 495) said, "This misconception may be mainly due to the use of a method of substitution to limits algebraically (p. 495)." This method that is in calculus books has been taught to students and they use this method when calculating the limit. Also, the teaching methods in mathematics is a factor to be considered. Teachers generally use traditional teaching methods (Başer & Narlı, 2001). In the traditional teaching methods, most of the teachers and students focus on procedural skills. Thus, most of the students learn the rules, concepts and algorithms without association (Baki & Kartal, 2004). In the second of these misconceptions, it can be seen that the students have thoughts such as "if a function is continuous and this function' limit exists". The findings of this study agree with the findings of the previous research (Bezuidenthout, 2001; Jordaan, 2005). For example, Jordaan (2005) reported, "students think that a function has a limit, and then it has to be continuous at a certain point." The final important result found in this research is that there are significant differences found between pre-service teachers' symbolical and graphical performances. The pre-service teachers had higher scores in the graphical representations than they had in the symbolical representations. This difference may be due to the graphics. The graphics are efficient visualization tools and visualization offers a method of seeing the unseen (Arcavi 2003; McCormick, DeFantim, & Brown, 1987). The result of this study is similar to the findings of previous researchers, who found differences in the students' preferences for multiple representations. For example, Keller and Hirsch (1998) found that students had preferences for various representations of functions and these preferences varied between the concept presented in a context and those given as purely mathematical situations. Similarly, Vinner (1988) found differences in which the students preferred as solutions.

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