

Identifying cognitive processes important to mathematics learning but often overlooked

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Mathematical competencies

Communication — Incoming: reading, decoding, interpreting statements and mathematical information. Outgoing: Explaining, presenting and arguing.

Mathematising — Transform a real world problem into a mathematical problem. Interpret mathematical objects or information in relation to the situation represented.

Representation — Devising or using depictions of mathematical objects or relationships: equations, formulae, graphs, tables, diagrams, textual descriptions.

Reasoning and argument — Logically rooted thought processes that explore and link problem elements to make inferences from them; or to check a given justification; or to provide a justification.

Strategic thinking — Selecting or devising, and implementing, a mathematical strategy to solve problems arising from the task or context.

Using symbolic, formal and technical language and operations — understanding, manipulating, and making use of symbolic expressions; using constructs based on definitions, rules and conventions, formal systems.

In August 2010, ACER held its annual conference in Melbourne. The theme of the 2010 conference—*Teaching Mathematics? Make It Count*—was chosen to highlight that mathematics education is an area of high priority in Australia. Almost 800 delegates met to review research in mathematics education and debate how lessons learned from this research can be put into practice.

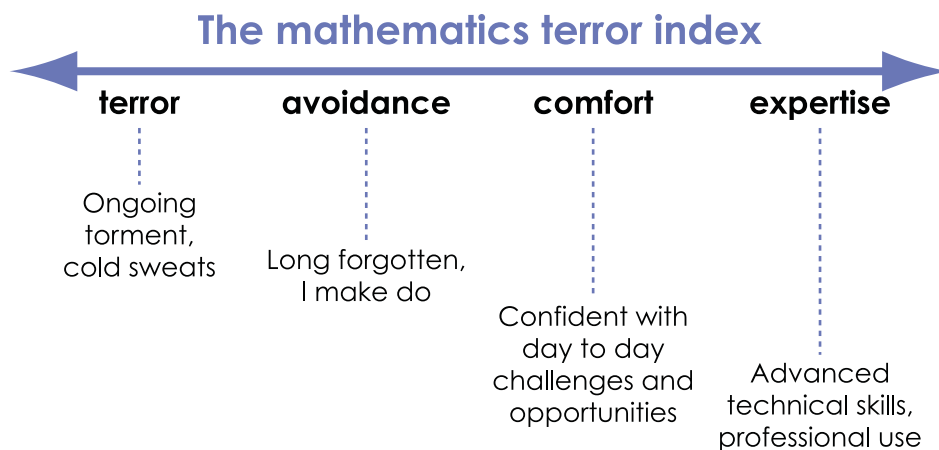
In my own presentation to the conference I outlined research into an area that I believe is very important to mathematics learning but often overlooked. I outlined a set of competencies that are fundamental to the development of ‘mathematical literacy’, or a person’s ability to apply their mathematical knowledge to practical situations.

The competencies are communication, mathematising, representation, reasoning and argument, devising strategies, and using symbolic, formal and technical language and operations (see box for more detail). These competencies can be thought of as a set of individual characteristics or qualities possessed to a greater or lesser extent by each person.

Recent research indicates that the more you possess and can activate these competencies, the better able you will be to make effective use of your mathematical knowledge to solve contextualised problems. In other words the possession of these competencies relates strongly to increased levels of mathematical literacy. In contrast a lack of these competencies contributes to unacceptably large measures on what I like to

call the mathematics terror index, where many people feel unable to deal effectively with mathematical problems in their daily lives.

A strong argument in support of giving mathematical competencies more attention in Australian classrooms comes out of the OECD's Programme for International Student Assessment (PISA). PISA aims to measure how effectively 15-year-olds can use their accumulated mathematical knowledge to handle 'real-world challenges'. The measures we derive from this process are referred to as measures of *mathematical literacy*. The *literacy* idea has really taken hold among those countries that participate in PISA, and is increasingly used in areas including mathematics and science, as well as in its more usual context of reading. It is generally regarded as very important that people can make productive use of their mathematical knowledge in applied and practical situations.

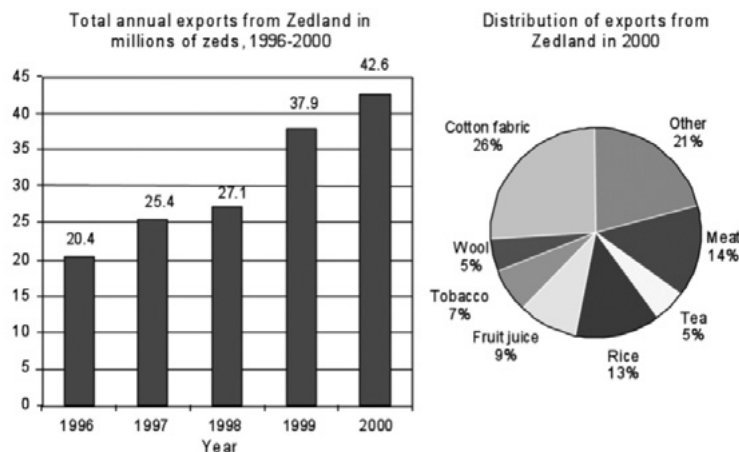


Some examples of questions used in PISA in its 2003 survey can help us to illustrate how different mathematical problems call for the activation of the six competencies to a differing extent and help explain why some problems appear to be harder than others.

Two survey questions from the unit titled *Exports* involve interpreting data presented in a bar graph and a pie chart. The first question calls for the direct interpretation of a familiar graph form: identifying that the bar graph contains the required information, locating the bar for 1998 and reading the required number printed above the bar.

EXPORTS

The graphics below show information about exports from Zedland, a country that uses zeds as its currency.



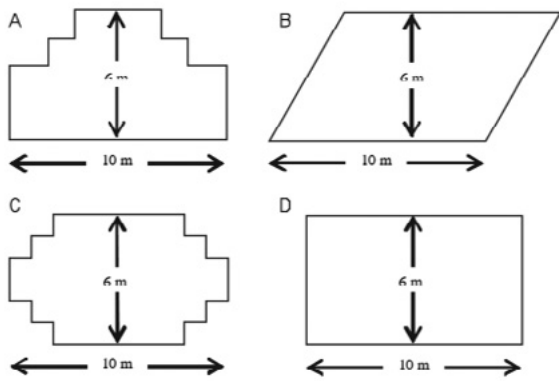
Q 1: What was the total value (in millions of zeds) of exports from Zedland in 1998?

Q 2: What was the value of fruit juice exported from Zedland in 2000?

A: 1.8 million zeds. B: 2.3 million zeds. C: 2.4 million zeds. D: 3.4 million zeds. E: 3.8 million zeds.

CARPENTER

A carpenter has 32 metres of timber and wants to make a border around a garden bed. He is considering the following designs for the garden bed.



Circle either "Yes" or "No" for each design to indicate whether the garden bed can be made with 32 metres of timber.

Garden bed design	Using this design, can the garden bed be made with 32 metres of timber?
Design A	Yes / No
Design B	Yes / No
Design C	Yes / No
Design D	Yes / No

The second question is more involved, since it requires linking information from the two graphs presented: applying the same kind of reasoning required in the first question to each of the two graphs to locate the required data, then performing a calculation using the two figures found from the graphs (find 9% of 42.6 million). A further question *Carpenter* is presented, which requires some geometrical knowledge or reasoning. Familiarity with the properties of basic geometric shapes should be sufficient to establish that while the 'horizontal' components of the four shapes are equivalent, the oblique sides of Design B are longer than the sum of the 'vertical' components of each of the other shapes.

What do we find when problems such as these are given to random samples of 15-year-olds across over 60 countries around the world? Table 1 shows that Australian students surveyed in PISA have answered these questions more effectively than the international average. The information in the table also shows that more students were able to correctly answer some questions than others; indicating that some questions were more difficult. Fewer than 20 per cent of all students and 23 per cent of Australian students could answer the 'Carpenter' question correctly.

Table 1. Item facilities for three PISA questions.

Question	Facility (all students)	Facility(AUS students)
Exports Q1	67.2%	85.8%
Exports Q2	45.6%	46.3%
Carpenter	19.4%	23.3%

As a mathematics teacher, I would have hoped that most 15-year-olds could answer questions like these correctly. This also has implications for what happens to these students when they leave school, since the mathematical capabilities students demonstrate by the time they are nearing school leaving age foreshadow the approach those individuals will take to using mathematics later in life and where they might place on the mathematics terror index.

I do not believe it is good enough to see such a high proportion of students unable to answer these mathematics problems correctly. Is the problem that many students do not know the required mathematical concepts; that they have not learned the required mathematical skills? Or could it be that too many 15-year-olds are simply unable to activate the required knowledge when it could be useful; that there is a disconnection between the way in which many of us have been taught, and the opportunities to use mathematics in life outside school? To attempt to answer these

questions we need to look in more depth at the mathematical competencies alluded to earlier.

The frameworks that governed the mathematics part of the PISA surveys conducted in 2000, 2003, 2006 and 2009 describe a set of eight mathematical competencies. For the purposes of a research activity we have carried out, these have been configured as the set of six competencies described in this article.

Our research has shown that these competencies can be used to explain a very large proportion of the variability in the difficulty of PISA mathematics test items, possibly as much as 70 per cent of that variability. A group of experts assigned ratings to PISA mathematics items according to the level of each competency demanded for successful completion of each item. The researchers found that questions requiring students to activate more of these six competencies proved more difficult for students to answer correctly. To have thus identified factors that explain so much of what makes mathematics items difficult is an important finding.

Table 2 summarises outcomes when the experts assigned competencies to the three example questions. Items with a higher facility (they are easier) have lower totals for the 'competency demand', whereas more difficult items demand activation of the competencies at a higher level. For Exports Q1, a relatively easy item, the communication and representation competencies are the most strongly demanded, with the others demanded little or not at all. The communication demand lies in the need to interpret reasonably familiar nevertheless slightly complex stimulus material, and the representation demand lies in the need to work with two graphical representations of the data. For Q2, the representation demand is even higher because of the need to process the two graphs in relation to one another. Each of the other competencies is also called on to some degree, with the need for reasoning, some strategic thinking, and calling on some low-level procedural knowledge to perform the required calculation. For Carpenter, the reasoning required comprises the most significant demand, but each of the other competencies is demanded to some degree.

Table 2. Competency demand for three PISA questions

Question	Facility (all students)	Competency demand
Exports Q1	67.2%	3.2
Exports Q2	45.6%	6.7
Carpenter	19.4%	8.9

Returning to the questions posed earlier on why so few students were able to answer the more difficult PISA items, I would argue that the problem is the opportunities to use mathematics that we come across in life are not packaged in the same way they were in school. At school you knew when you were going to mathematics class and you knew the mathematics teacher would show you new mathematical ideas or skills, give you some examples and then point you to a set of exercises more or less like those used to demonstrate the idea or skill you were learning. In the real world, that's not normally how opportunities to use mathematics come to us. We have to make the judgments and decisions about what mathematical knowledge might be relevant, and how to apply that knowledge.

While this research into the role of mathematical competencies has further to go, the results of this work indicate that we must not underesti-

mate the importance of this set of competencies to developing students' mathematical literacy. There are three conclusions that I believe can be drawn from the research so far.

1. Possession of these six competencies is crucial to the activation of mathematical knowledge.
2. The more an individual possesses these competencies, the more able he or she will be to make effective use of his or her mathematical knowledge to solve contextualised problems.
3. These competencies should be directly targeted and advanced in our mathematics classes.

In general, not enough time and effort is devoted in the mathematics classroom to fostering the development in our students of these fundamental mathematical competencies. Moreover, the curriculum structures under which mathematics teachers operate do not provide a sufficient impetus and incentive for them to focus on these competencies as crucial outcomes, alongside the development of the mathematical concepts and skills that typically take centre stage.

These competencies must be given a conscious focus in our mathematics classes, through teaching and learning activities, and through assessment. A key place to start is with the nature of discussion that is facilitated in mathematics classrooms. Students need to be given opportunities to articulate their thinking about mathematics tasks and about mathematical concepts. Obviously teachers play a central role in orchestrating that kind of discussion in class and this provides the basis for encouraging students to take the next key step, writing down their mathematical arguments. Giving emphasis to the communication of mathematical ideas and thinking, both in oral and written forms, is essential both to improving communication skills, but also to developing the mathematical ideas communicated and the capacities to use them.

Acknowledgement

This article, reproduced from ACER's "Research Developments" for summer 2010–2011, is based on a presentation to ACER's Research Conference 2010: Teaching Mathematics? Make it count. The full paper and presentation slides are available from www.acer.edu.au/conference where you will also find the presentations of four keynote addresses and 15 concurrent sessions.

From Helen Prochazka's

SCRAPbook

Six is a number perfect in itself, and not because God created the world in six days; rather the contrary is true. God created the world in six days because this number is perfect, and it would remain perfect, even if the work of the six days did not exist

St Augustine (AD 354–430), theologian and philosopher