DEVELOPING COMPUTATIONAL FLUENCY WITH THE HELP OF SCIENCE: A TURKISH MIDDLE AND HIGH SCHOOL GRADES STUDY

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ABSTRACT
Students need to achieve automaticity in learning mathematics without sacrificing conceptual understanding of the algorithms that are essential in being successful in algebra and problem solving, as well as in science. This research investigated the relationship between science-contextualized problems and computational fluency by testing an instructional method that was implemented as a non-traditional drill exercise. The study employed a quantitative analysis of pre- and post-test scores of Turkish middle and high school students after two interventions. The questions addressed were: Do the science-contextualized drill practices (SCP) improve students' computational fluency better than traditional drill practices (TP)? Is there any statistical significance between middle and high school students in terms of their benefit from SCP? This study showed that science-contextualized drill exercises produced higher gains for both middle and high school students' computational fluencies than the traditional context-free drill exercises.
Keywords: computational fluency, integration, mathematics education

INTRODUCTION

Mathematics conceptualization was the primary focus of the *Principles and Standards for School Mathematics* (PSSM) document which was developed and published in 2000 by the National Council of Teachers of Mathematics (NCTM). While it was an improvement over the standards of 1989 and 1995 that may have inadvertently given the impression that computation should be de-emphasized (Hartocollis, 2000), in fact, improving automaticity with computational facts and mathematical precision was presented as an integral part of conceptual mathematics learning in PSSM (Ferrini-Mundy, 2000). In order to strengthen its position on this matter, NCTM issued a document called *Curriculum Focal Points* (CFP) in 2006 that listed the most critical mathematical topics to be taught at schools. As a result, CFP was emphasizing the importance of basic arithmetic skills in lower and middle grades stronger than ever.

However, CFP was perceived by the media as an admission that the PSSM had originally recommended reduced instruction in basic arithmetic facts (Lewin, 2006). NCTM responded to the image displayed by the mainstream media and concluded that CFP fully supported the PSSM and "it is in no way a reversal of the NCTM's long-standing position on teaching students to learn critical foundational topics (e.g. multiplication) with conceptual understanding" (National Council of Teachers of Mathematics [NCTM], 2006, ¶ 9).

Today, NCTM's position on teaching students critical foundational topics is clearer: Teaching mathematics should be based on conceptual understanding with an emphasis on computational fluency, and "computational fluency should develop in tandem with understanding of the role and meaning of arithmetic operations in number systems" (NCTM, 2000, p. 32). Still, by not providing examples of concrete instructional methods, FCP did not diminish the apprehensions of those who has been worried about math wars (Van de Walle, 1999), thus teachers are left to fix on between two sides of the war or to segregate their lessons into conceptual and computational sections (Galley & Manzo, 2004). Even more to worry is the speculation that FCP might be interpreted by educators as if it encourages computational fluency (including algorithmic precision) to be taught only within the domain of arithmetic, and thus students would not need to practice it after elementary grades. A similar topic of discussion was explained in Howe's (1998) critic over *decreased attention* tables in standards document, and how it was often interpreted as *no attention* to certain topics of mathematics.

In correlation with NCTM's position, the question of how to build computational proficiency was paramount and several studies have shown that rote, behaviorist traditional practice was not an effective means for developing computational fluency (Davis, 1984; Ginsburg, 1997; Webb, 1991). Thus, the driving question in this study was to understand the relationship between science-contextualized problems, and computational fluency by testing an instructional method that was implemented as a non-traditional drill exercise — in the sense of a creative and interdisciplinary approach — to foster computational fluency.

The questions addressed in this study were:

1) Do the science-contextualized drill practices (SCP) improve students' computational fluency better than traditional drill practices (TP)?
2) Is there any statistical significance between middle and high school students in terms of their benefit from SCP?

THEORETICAL FRAMEWORK

A Philosophical Debate on Algorithms

The composition of mathematics is a controversial issue; however people from all disciplines agree that arithmetic, algebra, problem solving and geometry are the essentials of mathematical thinking. Almost all advanced applications of mathematics rely on good knowledge of arithmetic that makes the conceptual understanding of the latter topics possible. Despite the consensus between the two sides of the math wars on the importance of arithmetic, there has still been an on-going debate on the role of algorithmic computations in teaching arithmetic. Although some claim this debate is a philosophical one (Davison & Mitchell, 2008), there are philosophers who believe it is a pedagogical issue. For example, the ideas of 20th century mathematician and philosopher A. N. Whitehead who believed education should attain automaticity so that the mind is free to learn higher-level problems (Ocken, 2007) are noteworthy. Whitehead diverged from the notion that students are obliged to always think about what they are doing. He believed *thought* should come with a critical consciousness, which results from attention and elimination of the unrelated (Whitehead, 1938). Whitehead's automaticity is better-responded in second language learning (McLaughlin,1979), and is defined as the point at which a person no longer has to think about the rules of pronunciation, grammar, or the syntax (Brown & Campbell, 2002). Yet, Whitehead considered life as an organic entity and according to him learning should be beauty-centered and holistic (Ernest, 2000), and mathematics is not to train the mind for future challenges but should work for the moment (Whitehead, 1929). Therefore, Whitehead's philosophy suggested that emphasizing algorithms as a way to achieve automaticity in learning mathematics does not sacrifice conceptual understanding (Wu, 1999) or dispute scientific constructivism in the sense that it is self-dependence on individual cognitive abilities to understand, recognize, and create. Because, according to Whitehead, education is "…the acquisition of the art of the utilization of knowledge" (1929, p.4).

Clearly, when the focus is on pedagogy rather than philosophical differences, it is easier to reach a consensus on the role of algorithms in mathematics teaching. Whitehead's views might be helpful in achieving what Dr. Ball tried to do by bringing concrete examples of instruction so that educators may discuss two sides of the story and develop their own styles in achieving computational fluency through effective instructional strategies according to their students' individual needs (Galley & Manzo, 2004).

The consensus on the role of algorithms in mathematics education suffered from extremist approaches in the past. In response to proclamations that technology changed the very nature of mathematics by making calculations easier (NCTM, 1989), and to articles such as Leinwand's *It's time to abandon computational algorithms* (1994), a committee of the American Mathematicians Society (AMS) had to remind that the algorithms of arithmetic form the basis of more advanced mathematics topics such as the concept of real numbers and algebra of polynomials (Howe, 1998). On the other edge, there was the *back to basics* curriculum in the U.S. which "…focused largely on skills and procedures" (Schonfeld, 2004, p. 258) and to which NCTM reacted with its *standards* approach in 1980's.

The Definition of Computational Fluency

Computational fluency has been misunderstood by many as the set of rules of arithmetic; similar to problem solving which was once interpreted as students solving simple word problems so that algorithmic calculations could be avoided (Schonfeld, 2004). The lack of a common definition caused researchers to use concepts such as algorithmic thinking, algorithms, computation, arithmetic, etc. interchangeably, and even sometimes incorrectly (Howe, 1998).

Fuchs et al. (2006) provided 3+2 as an example of arithmetic whereas 35+29 of algorithmic computation due to incorporation of automaticity in the first example. If students were still using a strategy for 3+2, for example, counting up; for determining the answer then it would be algorithmic (Carpenter, Fennema, & Franke, 1996). Thus, algorithmic computation as it is used in this study involves systematic processes comprised of operation(s) and relative symbols to reach to the solution rather than memorized answers to a mathematical problem.

However, efficiency, in the sense of automaticity, should still be an essential target of algorithmic computation teaching (Howe, 1998) as well as flexibility, and accuracy (Russell, 2000).

Lastly, NCTM (2000) brought a general clarity and defined computational fluency as having and using efficient and accurate methods so students can perform computations in a variety of methods such as mental calculations, estimation, and paper-and-pencil calculations by using mathematically sound algorithms.

Computational Fluency and Concepts around It

National Advisory Panel's report (2008) on mathematics education in U.S. pointed out that algorithmic computational skills and conceptual understanding reinforce each other. Similarly Turkish Ministry of National Education's (MONE) report (2005a) also recommended that mathematics education at Turkish schools should focus on conceptual understanding together with the goal of improving computational skills.

However, we know a little about the role of computational skills on student achievement, particularly in middle and high school levels. Some researchers followed cross-national studies and concluded that computational abilities of American students might substantially narrow down the achievement gap between East Asian students (Geary, Liu, Chen, Saults, & Hoard, 1999; Mayer, Tajika, & Stanley, 1991). Tolar, Lederberg, and Fletcher (2009) said that computational fluency was correlated with achievement in higher-level mathematics courses and suggested that classroom practices should include a component that improves adolescents' computational fluency, even during the high school years. Although, the researchers indicated that algorithms as operational mathematical expressions were not enough to be successful in algebra (Tolar, Lederberg, & Fletcher, 2009), automaticity with numerical operations were found to be noticeably influence problem solving skills of pre-adolescent children (Royer, Tronsky, & Chan, 1999).

In Fuchs et al. (2006) research on elementary grades, it was found that arithmetic ($r=0.56$) and attentive behavior ($r=0.60$) were the only two significant predictors of success in algorithmic computation. Russel and Ginsburg (1984) and Ackerman and Dykman (1995) also found that inattentive behavior can cause low achievement in computational fluency. Another research suggested that an empirically-selected intervention over student-selected one produced higher gains in computational fluency (Carson & Eckert, 2003).

Torigoe (2008) suggested that numbers allow students to compare their results with their everyday experiences and numeric problems helped students deal with unit conversions. Moreover, Torigoe recommended that science teachers should present the new material with numeric examples until the idea was understood. The researcher found that most science teachers taught their subject symbolically and this did not help students master the content. In Torigoe's research, when answering questions with contextualized computation, ~95% of the students answered correctly as compared to ~25% when answering parallel questions without computation. It showed that computation was facilitated when students were provided with numbers in the problem context as compared to the abstract algebraic notation.

Turkish Case

Initially, school mathematics and science in Turkey followed a strictly linear hierarchical curriculum model until the 2004 reform movement (Bulut, 2007). It was assumed that students would master computational fluency by the end of elementary school. Besides the fact that reform has not yet had any positive effect on Turkish students' performance in international comparison studies (Zembat, 2010), students are still not allowed to use calculators neither in the classroom (Ozgun-Koca, 2009) nor during central examinations. Baki and Celik's study (2005) found that teachers believed using calculators in class would inhibit the computational fluency of their students and would result in not being able to finish their work in time during multiple-choice tests.

Mathematics in Science and Mathematics Taught by Science Teachers

It was suggested by MONE (2005a) that more emphasis on applications taken from science curricula should be applied in Turkish mathematics classrooms. However, it is documented that science and mathematics teachers in Turkey do not work in collaboration, and most math teachers were not equipped with necessary science knowledge (Turkish Academy of Sciences [TUBA], 2004). The consequence for this lack of coordination and integration between subjects was that Turkish students learned mathematics as an abstract science with minimal real-life connection (TUBA, 2004). Similarly, MONE (2005b) proposed that science teachers should work along with teachers from other disciplines to enrich classroom learning.

There is a need of empirical research on the mathematical content knowledge (CK), and pedagogical content knowledge (PCK) of science teachers because "unlike the mathematics teacher who can choose to avoid science, the science teacher is not able to cover most topics without calling on mathematical concepts and skills"

(Frykholm & Meyer, 2002, p. 504). The assumption that science teachers know mathematics needs to be substantiated, because having mathematical sense means more than just manipulating numbers (Kulm, 2008), and according to the RAND Mathematics Study Panel's report (2003) even math teachers are unable to explain why basic mathematics algorithms worked. Considering the fact that students in Turkey are introduced to scientific constants such as Avogadro's number or Newton's gravitational constant as early as in eight-grade, the importance of a revision of computational fluency in science classes is prominent. High school physics and chemistry classes are also subjects in which computational fluency is heavily used. Torigoe (2008) indicated that algorithmic manipulation was what most physics tests measure.

Webb (1975) delineated some causes of difficulty for science teachers in teaching mathematical concepts as a) unfamiliarity with the symbols, mathematical language and methods, b) the timing and depth of development of certain mathematical topics in science classroom, c) lack of familiarity with the use of mathematics concepts, and d) failure to relate teaching of mathematics to other subjects adequately (in particular science).

METHODOLOGY

Intervention

Exponentials (indices) was chosen as the unit of study because it is one of the two common mathematics-related topics (the other one is dimensional analysis) that is explicitly covered in middle and high school science curricula in Turkey. Secondly, all participating students had seen this topic before. The first intervention was a review of basic concepts of exponentials unit (including scientific notation) which lasted for two class periods and implemented by the science teachers. Lesson plans were designed together by the researchers and the corresponding science teachers. Conceptual understanding of the algorithms in exponentials was the main objective of this intervention.

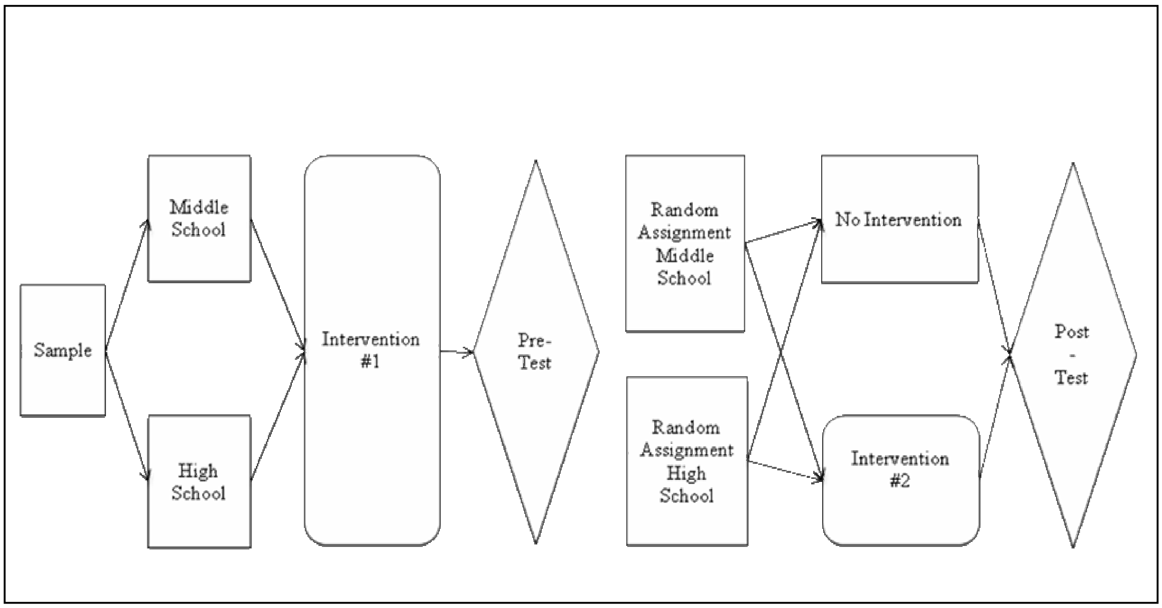

Figure 1. Two Interventions Experiment Model

Instrumentation

This study employed a quantitative analysis of pre- and post-test scores of middle and high school students after two interventions. Both pre-test (Cronbach's $\alpha = 0.60$) and post-test (Cronbach's $\alpha = 0.63$) each comprised of 10 multiple choice questions on exponentials were produced alike to end of middle school exam questions in Turkey by an expert mathematics teacher. Both tests were required to be solved in 20 minutes each and students were not allowed to use calculators.

The pre-test was administered before the second intervention, which was a set of assignments given to the students to be completed individually at home. Homework assignments were handed out to evenly divided control and experiment groups in each category, and students were reminded that they had to show their work and that they were not allowed to use calculators. The correct answers of the assignments together with a complete detailed solution of the problems of that day were given to the students with their new homework. Control group homework included TP type questions whereas the experiment group homework contained the same questions which were presented in contexts chosen from science curriculum (SCP). By this way, problem solving component was controlled so algorithmic computation could be tested for significance between the groups. An example question from the homework intervention was as follows:

> TP type question: Find the solution in scientific notation $(4.2 \times 10^{18}) / (150 \times 10^{14}) = ?$
> SCP type question: The Richter magnitude scale, also known as the local magnitude (M_L) scale, assigns a single number to quantify the amount of seismic energy released by an earthquake. An earthquake that measures 8.0 on the Richter scale releases 4.2×10^{18} Joules of energy, while the same energy for a 5.0 magnitude earthquake is 150×10^{14} Joules. How many times more energy is released at 8 Richter scale than 5 Richter scale? Calculate the answer by dividing the two given amounts and write the answer in scientific notation.

Participants

Participants of this study were students of a public elementary K—8 school, and 9—12 grades public high school in a major metropolitan city in Turkey (N_{total}=150). The public school list was drawn from MONE website (http://istanbul.meb.gov.tr/) and the elementary school was randomly selected with the help of a computer program. The area where the participants were selected is a developing district of the city and it is heavily populated by working class. The schools are located in the district center, and they admit students on the basis of proximity to the schools.

Seventy-five of the participants were middle school grades 7 and 8 students, whereas the rest (n_{high_school} = 72) were high school grades 9 and 10 students. Our sample consisted of 84 male students and 66 female students. The attendance during the course of the study never dropped below 90% in any class.

Data Analysis

The student scores from pre- and post tests were calculated by adding up the exactly correct answers they have given in pre- and post-tests. Their achievement in exponentials unit was calculated by students' total score as a measure of their computational fluency, which was the independent variable in our study. In order to determine the statistical differences in the pre-, post tests, and delta scores (pre-test score subtracted from post-test score) between the control and experiment groups, normality of the data was confirmed by comparing the histograms of pre-, post-tests, and delta scores in the whole sample. Secondly, homogeneity of variance assumption was tested with Levene's test for equality of the variances, and no statistical significance in variances was found ($p<0.05$). These two steps made it possible to conduct an independent samples t-test.

In order to answer the research question about the differences on computational fluency between two grade levels, the data was split into middle and high school student categories. Although the normality was achieved for each category on all variables, homogeneity of variance assumption was found to be violated only for middle school post-test variable according to Levene's test ($p=0.001$). Statistical significance between the means of control and experiment groups was investigated with the consideration of this result.

Effect sizes (ES) were calculated by adjusting Cohen's d for delta scores (Wilson, 2010). Pearson's r was computed for the correlation between the post- and pre-test scores. Thus,

$$ES = (Mean_{post\text{-}test} - Mean_{pre\text{-}test}) / SD_{pooled}, \quad (1)$$
$$SD_{pooled} = SD_{delta} / (2*(1-r))^{1/2} \quad (2)$$

were used to calculate the effect sizes.

RESULTS

Table 1 contains the descriptive analysis of the whole sample in which means and standard deviations of pre-, post-test scores and their difference between —delta scores—are given. The table also presents these statistics as they are calculated separately at middle and high school grade levels.

Table 1: Mean and Standard Deviation Scores of Pre- and Post-tests with Grade Levels

	Control Group (n=75)						Experiment Group (n=75)					
	Pre-Test Mean	SD	Post-Test Mean	SD	Delta Mean	SD	Pre-Test Mean	SD	Post-Test Mean	SD	Delta Mean	SD
Middle School (n=72)	6.26	1.76	6.80	1.92	0.54	1.39	6.61	1.29	8.05	1.30	1.44	1.31
High School (n=78)	8.61	1.02	8.69	0.98	0.08	1.46	8.44	0.88	9.28	0.91	0.83	1.16
Whole Sample (N=150)	7.39	1.87	7.71	1.80	0.32	1.43	7.49	1.44	8.64	1.28	1.15	1.27

Note. Both pre- and post-tests contained 10 questions. SD = Standard deviation.

Independent samples t-test at 95% confidence level provided no statistically significant difference (p=0.70) in pre-test scores between the groups after the first intervention, and it was concluded that the groups were at a close starting point before the second intervention in terms of their achievement on exponentials topic. The analysis procedure was repeated for each sub level data —middle (p=0.31) and high school (p=0.46) — and verified to be the same.

After the second intervention ended, the post-test was administered in the classrooms. The analysis of the data (independent samples t-test) indicated a statistical significance for p<0.05 between the groups in post-test (p=0.000363) and delta scores (p=0.000266). Figure 1 contains confidence intervals around the means of pre- and post-tests for control and experiment groups, while Figure 2 shows the confidence intervals of the mean delta scores.

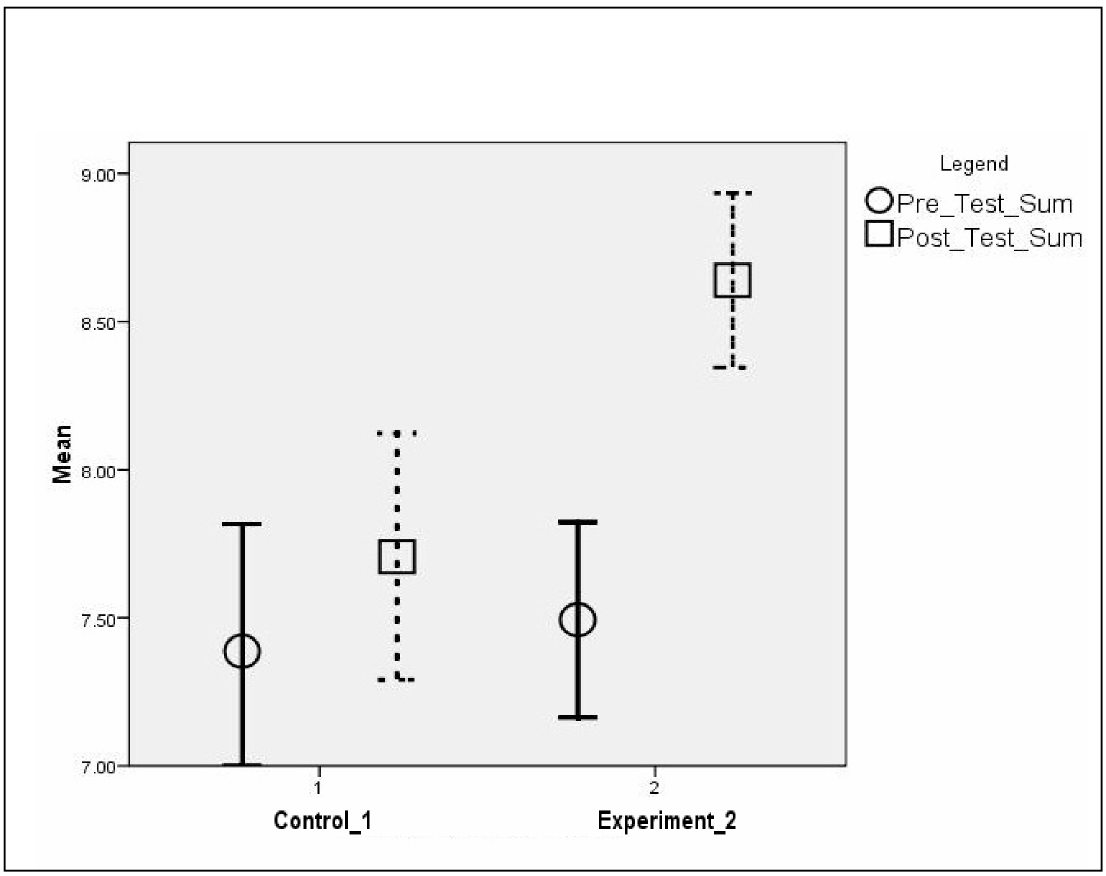

Figure 2. 95% Confidence Intervals around the Mean of Pre- and Post-test scores

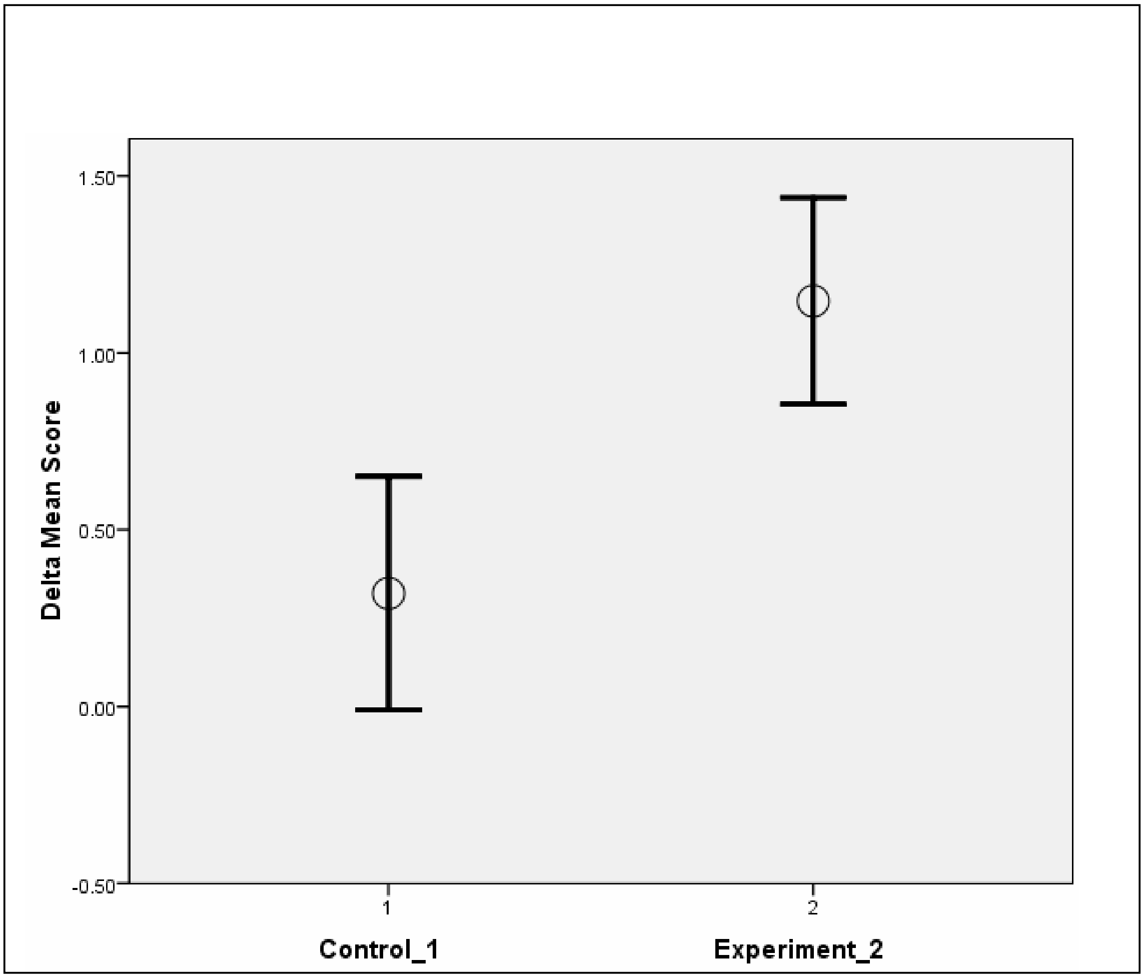

Figure 3. 95% Confidence Intervals around the Mean of Delta Scores

In addition, there was also a statistical significance at post-test and delta scores between the control and experiment groups when middle and high school data were analyzed separately ($p < 0.05$ for each four categories). The study found evidence in both the pre- and post-test scores that middle and high school student scores differed significantly ($p < 0.05$) from one another. However, when the delta scores were investigated, middle school students had a statistically higher result then their peers in high school ($p < 0.05$).

Finally, there was a correlation between pre- and post-test scores for the whole sample ($r=0.63$, $p<0.01$), as well as for middle ($r=0.63$, $p<0.01$) and a very strong relationship for the high school sample ($r=0.95$, $p=0.08$). Thus, according to equation (1), ES was calculated as 0.63 accounted for the variance in the whole sample post-test score. The ES value for middle school post-test was very high (ES=0.82) but relatively low for high school significance (ES=0.17).

CONCLUSIONS

Previous research had already shown that traditional drill exercises that contain non-contextualized repeated algorithmic procedures are not effective ways to improve computational fluency (Davis, 1984; Ginsburg, 1997; Webb, 1991). Thus, mathematics teachers should develop alternative instructional methods to help their students practice algorithms together with their conceptual understanding. This study showed that science-contextualized drill exercises produced higher gains for both middle and high school students' computational fluencies than the traditional drill exercises.

An important conclusion derived from this finding is that science is an effective domain to foster computational fluency. On the other hand, since computational fluency is frequently used in science, science teachers can easily integrate it into their own lessons. In relation to previous research (Torigoe, 2008), two possible benefits of practicing computational fluency in science lessons that are supported by our findings were; 1) there would be more class time to foster conceptual understanding of more complex algorithms in mathematics and focus on problem solving strategies, 2) students would learn science topics better when they deal with numerical examples.

Gobet and Campitelli (2007) suggested practice drill require effort and are not enjoyable to the students. They claimed “Most students are incapable of working on practice activities for long periods of time” (p.160). The intervention in this study was designed in a way to increase student attention while practicing computational fluency. Thus, it was also noteworthy that the delta scores of middle school students were higher than those of the high school students. This can best be explained by the comfort of the high school students with exponentials unit. However, in correlation with Fuchs et al’s findings (2006), it can also be speculated that the intervention of this study increased middle school students’ attentive behavior towards the drill exercises more than it did for high school students.

DISCUSSION

Teachers of mathematics should focus on pedagogy and try to find better instructional methods to reach to their students instead of being a part of philosophical discussions in math wars. Computational fluency is one of the key areas of mathematics in which better instructional methods should be developed at all grade levels. In order to help our students excel and maintain their skills in computational fluency, computational fluency should be the target of each grade level, it should be re-visited and practiced extensively throughout the k-12 curriculum. However, computational fluency should not be the concern of mathematics teachers only. In particular, science and technology, and in general, all subject teachers should help students develop computational fluency by integrating mathematics into their lessons.

Due to the increased pressure of high-stakes testing in the world, and the large volume of participants waiting to get admitted to universities in Turkey, there seems to be no change in the selection-based multiple-choice centralized testing method in the near future. It is also very unlikely that there will be a policy change in the use of calculators during these exams since computational fluency is a skill that causes large variance among students. Students definitely need to excel on computational fluency to save time at a test to spend on more complex dimensions of mathematics questions. The conceptual understanding of the algorithms is also important to be successful in those complex dimensions. Teachers of mathematics need to help their students achieve both aspects of computation. Science emerges as a convenient medium to get this help from, as it naturally provides the context. Having said this, practicing computational fluency in science classes will also ease the load of the mathematics curriculum, and thus teachers of mathematics can have more time to focus on developing their students’ problem solving skills. Students will also have more time to focus on conceptual understanding of science topics if teaching and practice of computational fluency is shared between subjects.

Recommendations for future research include an extension of this study into the qualitative domain and investigate the possible reasons behind the usefulness of the SCP and its relationship with attentive behavior. Secondly, there is a need for further research on the mathematical ability of science teachers, particularly on their mathematics CK and PCK.

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