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Tests of rating models

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The study reports empirical tests of Anderson's, Haubensak's, Helson's, and Parducci's rating models when two end anchors are used for rating. The results show that these models cannot predict the judgment effect called here the Dai Prà effect. It is shown that an extension of Anderson's model is consistent with this effect. The results confirm the main contention of each model that ratings are linear measures of mental magnitude.

In this paper we report the results of tests made to determine which of the current most popular interpretations of the rating operation—Helson's, Parducci's, Anderson's, and Haubensak's models—is best supported by the empirical data. We begin with a description of the rating procedure.

THE RATING PROCEDURE

Typically, in studies using ratings, participants first inspect all stimuli and then rate each stimulus individually, with no contextual stimulus being presented during the rating process. This process imposes an S shape on the psychophysical function, because participants tend to assign the lowest and highest ratings to non-extreme stimuli (Thurstone, 1929). In studies using ratings for metric purposes, this nonlinearity needs to be removed. One way this can be done is by asking participants to rate each stimulus with respect to two extreme standard stimuli, called the end anchors (Anderson, 1982).

In this case, the participant is asked to compare the mental magnitude, ψ, of each stimulus with the magnitudes of the end anchors ψ_L and ψ_U pre-selected such that ψ_L is somewhat smaller then the smallest ψ, and ψ_U is somewhat larger than the largest ψ. The rating of ψ may be an integer, **R**,

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selected by the participant from a predefined set of equidistant integers. The smallest and largest integers of this set, $\mathbf{R_L}$ and $\mathbf{R_U}$, are the ratings for ψ_L and ψ_U, respectively. Participants are asked to select **R** proportionally to ψ considering that $\mathbf{R_L}$ is the rating for ψ_L and that $\mathbf{R_U}$ is the rating for ψ_U.

HELSON'S MODEL

Helson (1947) proposed that

$$\mathbf{R} = u_1 \cdot (\psi - \psi_A) \tag{1}$$

with u_1 constant and ψ_A the ideal mental magnitude corresponding to the so-called adaptation level. The adaptation level has been variously defined as a quadratic, arithmetic, or geometric weighted mean (Adams & Cobb, 1922; Helson, 1938; Judd, 1940). It is most prominently defined as the weighted geometric mean of the stimulus judged, the stimuli and anchors in the background, and the stimuli and anchors from past experience (Helson, 1971).

When ψ_A is constant, Equation 1 states that **R** varies with ψ according to a straight line. With respect to the following discussion, it is relevant to note that changes in ψ_A do not affect the slope of this line.

PARDUCCI'S MODEL

Parducci (1965) proposed that **R** has two components: one, $\mathbf{R_F}$, due to the frequency of the stimulus relative to that of the previous stimuli and anchors, and the other, $\mathbf{R_R}$, due to the apparent magnitude of the stimulus relative to that of the anchors. Parducci (1982) proposed that

$$\mathbf{R_R} = \frac{\psi - \psi_L}{\psi_U - \psi_L} \ . \tag{2}$$

In its most general form, Parducci's model states that

$$\mathbf{R} = a \cdot \mathbf{R_F} + b \cdot \mathbf{R_R} \tag{3}$$

with *a* and *b* constants (Birnbaum, 1974, p. 95). Equations 2 and 3 imply

$$\mathbf{R} = \left(a \cdot \mathbf{R_F} - \frac{b \cdot \psi_L}{\psi_U - \psi_L} \right) + \left(\frac{b}{\psi_U - \psi_L} \right) \cdot \psi \tag{4}$$

which states that **R** varies linearly with ψ when ψ_L, ψ_U, and $\mathbf{R_F}$ are constant.

ANDERSON'S MODEL

Anderson (1982, p. 134) proposed that participants select **R** so that

$$\mathbf{R} = w \cdot \mathbf{R_L} + (1 - w) \cdot \mathbf{R_U} \tag{5}$$

with w the relative similarity of ψ to ψ_L and with $1 - w$ the relative similarity of ψ to ψ_U. These relative similarities may be represented by

$$w = \frac{\psi_U - \psi}{\psi_U - \psi_L} \tag{6}$$

and

$$1 - w = \frac{\psi - \psi_L}{\psi_U - \psi_L} \tag{7}$$

considering that w tends to 1 when ψ tends to ψ_L and that $1 - w$ tends to 1 when ψ tends to ψ_U. Equations 5–7 imply

$$\mathbf{R} = \left(\frac{\mathbf{R_L} \cdot \psi_U - \mathbf{R_U} \cdot \psi_L}{\psi_U - \psi_L} \right) + \left(\frac{\mathbf{R_U} - \mathbf{R_L}}{\psi_U - \psi_L} \right) \cdot \psi \tag{8}$$

which states that **R** varies linearly with ψ when $\mathbf{R_L}$, $\mathbf{R_U}$, ψ_L and ψ_U are constant. Equation 8 is proposed as a tentative model not encoding context effects (Anderson, 1982, p. 155).

HAUBENSAK'S MODEL

Haubensak (2000) proposed that

$$\mathbf{R} = \mathbf{R_l} + \left(\mathbf{R_u} - \mathbf{R_l}\right) \cdot \frac{\psi - \psi_l}{\psi_u - \psi_l} \tag{9}$$

with ψ_l and ψ_u the *remembered* mental magnitudes immediately lower and immediately higher than ψ, and $\mathbf{R_l}$ and $\mathbf{R_u}$ the respective associated ratings.

Elementary algebra shows that Haubensak's (Equation 9) and Anderson's models (Equation 8—see Equation 16 which derives from Equation 8) have identical form with the only difference being that Equation 8 refers to end anchors and Equation 9 refers to remembered stimuli with apparent magnitude immediately lower and immediately higher than that of the stimulus rated.

Haubensak proposed that participants memorize the stimuli together with the associated rating and use these stimulus-rating associations to rate subsequent stimuli. The operations involved in the use of these associations are described in Haubensak (1992b). Haubensak's model does not consider the end anchors. Tests of this model are the tests reported below of the possibility that the end anchors affect ratings.

PREDICTIONS OF MODELS

The predictions made by the above models are in terms of the unobservable magnitude ψ. In the event that the psychophysical function is known, one could derive predictions from these models made in terms of the physical magnitude, ϕ, that determines ψ. Unfortunately, for most sensory attributes the methods of rating, bisection, and magnitude estimation produce different empirical psychophysical functions. However, for the apparent length of lines presented on the frontal plane, all these methods show that the psychophysical function approximates a power function with an exponent of 1 (Masin, 2008; Verrillo, 1983). Although this concordance of methods is not conclusive proof, it gives some confidence to the assumption that the psychophysical function for apparent line length is

$$\psi = k_0 + k_1 \cdot \phi \tag{10}$$

with k_0 and k_1 constants and ϕ the physical length. Equation 10 may be used to derive specific testable predictions in terms of ϕ from the above models.

Helson's model. Michels and Helson (1949) made the unproven assumption that the psychophysical function is

$$\psi = v_0 + v_1 \cdot \log \phi \tag{11}$$

with v_0 and v_1 constants and ϕ the stimulus value. Equations 1 and 11 imply

$$\mathbf{R} = c \cdot (\log \phi - \log \phi_A) \tag{12}$$

with $c = u_1 \cdot v_1$ a constant and ϕ_A the adaptation level (Michels & Helson, 1949). Equation 12 is the classic adaptation-level formula proposed to predict ratings (Helson, 1959).

However, for apparent line length the psychophysical function should be Equation 10. Equations 1 and 10 imply

$$\mathbf{R} = u_1 \cdot k_1 \cdot (\phi - \phi_A) \,. \tag{13}$$

This equation predicts that **R** increases linearly with ϕ when ϕ_A is constant.

Parducci's model. Equations 4 and 10 imply

$$\mathbf{R} = \left(a \cdot \mathbf{R_F} - \frac{b \cdot \phi_L}{\phi_U - \phi_L} \right) + \left(\frac{b}{\phi_U - \phi_L} \right) \cdot \phi \tag{14}$$

with ϕ_L and ϕ_U the physical lengths relative to ψ_L and ψ_U, respectively. This equation predicts that **R** increases linearly with ϕ when $\mathbf{R_F}$, ϕ_L, and ϕ_U are constant.

Note that, when ϕ_L and ϕ_U are fixed, Helson's (Equation 13) and Parducci's models (Equation 14) predict that the slopes of the straight lines relating **R** to ϕ are equal for each different ϕ_A or $\mathbf{R_F}$, respectively.

Anderson's model. Equations 8 and 10 imply

$$\mathbf{R} = \left(\frac{\mathbf{R_L} \cdot \phi_U - \mathbf{R_U} \cdot \phi_L}{\phi_U - \phi_L} \right) + \left(\frac{\mathbf{R_U} - \mathbf{R_L}}{\phi_U - \phi_L} \right) \cdot \phi \,. \tag{15}$$

This equation predicts that **R** increases linearly with ϕ when $\mathbf{R_L}$, $\mathbf{R_U}$, ϕ_L, and ϕ_U are constant.

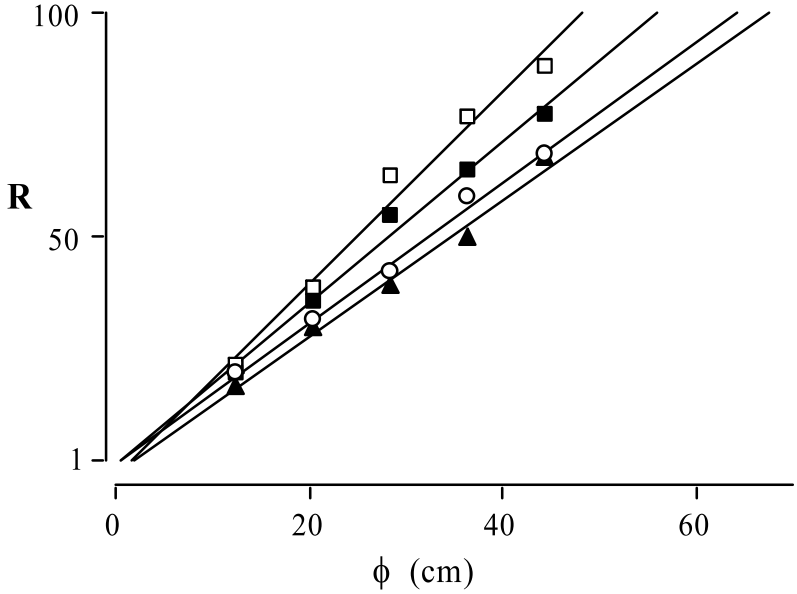

Figure 1. Mean rated length, R**, plotted against physical length, ϕ, for the pairs of anchors with lengths 3.3 and 83.5 (▲), 5.3 and 73.5 (○), 7.3 and 64 (■), and 9.3 and 55 mm (□), respectively. From Dai Prà (2007).**

Haubensak's model. Since Equation 9 does not consider ψ_L and ψ_U (it considers only the remembered magnitudes ψ_l and ψ_u), this equation is mute about whether **R** changes when ϕ_L and ϕ_U change.

THE DAI PRÀ EFFECT

Dai Prà (2007) has recently discovered an important effect of the end anchors on ratings which can be used to assess the validity of rating models. He tested Equation 15 using frontal lines with length varying from 12.5 to 44.5 cm in steps of 8 cm as stimuli. On each trial, two anchor lines preceded the stimulus. For each stimulus, the lengths of these anchors were 9.3 and 55, 7.3 and 64, 5.3 and 73.5, or 3.3 and 83.5 cm. The combinations of stimulus and anchors were presented in random order. The participant rated the apparent length of the stimuli using the integers from 1 to 100 with "1" and "100" being, respectively, the ratings $\mathbf{R_L}$ and $\mathbf{R_U}$ for the anchors immediately preceding the stimulus.

Figure 1 shows mean rated length, **R**, plotted against physical length, ϕ, for each pair of anchors. Triangles and open squares show the data when the anchors were the farthest and closest to the stimuli, respectively. Solid lines depict the least-squares straight lines that fit the data.

These least-squares lines differ in slope due in most part to an effect reported by Rogers (1941; Heintz, 1950; Sherif, Taub, & Hovland, 1958). Some interpret the Rogers effect as a sensory effect (Johnson, 1972; Smithson & Zaidi, 2004) but in reality it is mostly or entirely the consequence of the reformulation of ratings. To make an extreme example, consider three lines *A*, *B*, and *C* with lengths 1, 2, and 3 cm, respectively. If the ratings for the lengths of *A* and *C* are defined to be 1 and 100, respectively, the rater formulates a rating for the length of *B* which should be around 50. Now let the length of *C* be increased to 2 m with its rating still defined to be 100. This increase forces the rater to reformulate the rating for *B*, making it very close to 1 instead of keeping it at about 50. Thus the Rogers effect is mostly or entirely due to rating reformulation.

Anderson's model. To see the effect of the anchors on ratings independently of the Rogers effect one needs to compare the results in Figure 1 with those predicted by Equation 15. This comparison is necessary because Equations 5–7, from which Equation 15 derives, imply

$$\frac{\mathbf{R}-\mathbf{R_L}}{\mathbf{R_U}-\mathbf{R_L}}=\frac{\psi-\psi_L}{\psi_U-\psi_L} \tag{16}$$

which represents the ideal case of ratings not affected by context.

To compare the results in Figure 1 with those predicted by Equation 15 the results in Figure 1 are replotted in Figure 2 using a different diagram for each pair of anchors. The leftmost diagram shows the results when the anchors were the farthest from the stimuli and the rightmost diagram when the anchors were the closest to the stimuli. The filled circles show **R** and the crosses show $\mathbf{R_L}$ and $\mathbf{R_U}$, 1 and 100, respectively. Each dashed line joining two crosses shows the **R**s predicted by Equation 15. The solid lines drawn through filled circles are discussed at the end of this paper.

In each diagram in Figure 2 each obtained **R** is larger than the respective predicted **R**. This divergence—unrelated to the Rogers effect—is called here the Dai Prà effect. It confirms the tentative nature of Anderson's model (dashed lines) by showing that ratings also depend on context factors not considered in this model.

Do the results in Figure 2 agree with any of the other models?

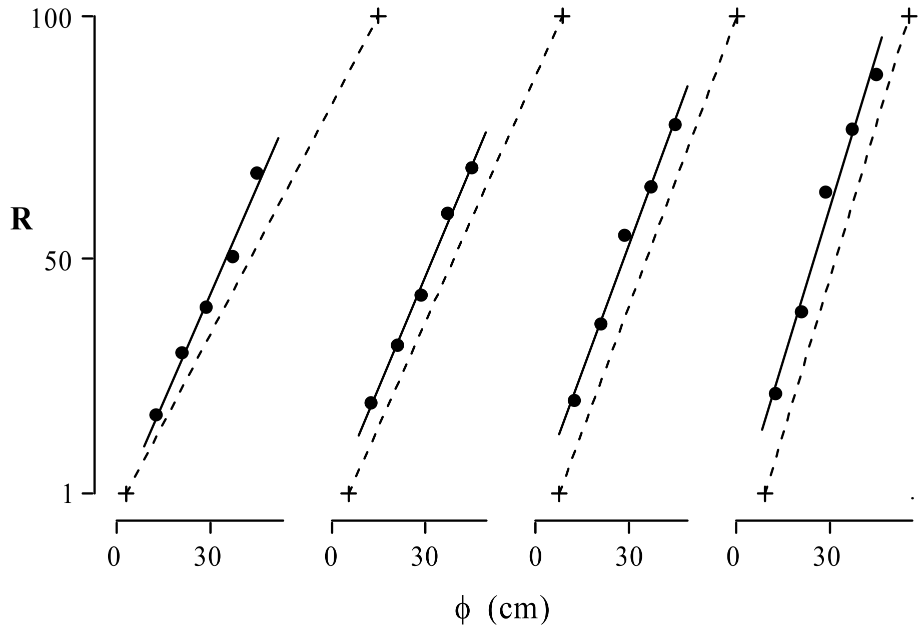

Figure 2. Results of Figure 1 replotted to show the Dai Prà effect. From the left to the right diagram, mean rated length, R, **is plotted against physical length, ϕ, for the pairs of anchors with lengths 3.3 and 83.5, 5.3 and 73.5, 7.3 and 64, and 9.3 and 55 mm, respectively.**

Haubensak's model. Dai Prà's results disagree with Haubensak's model in that they show that the end anchors influence **R**. This conclusion shows that the model should not neglect the effect of end anchors on rating.

Helson's model. Dai Prà's results reject Helson's model since this model implies that the straight lines that fit the data must be parallel to the corresponding dashed line, while instead the difference in slope between these lines increases with the distance of the anchors from the stimuli.

Parducci's model. Since *a* and *b* in Equation 3 are unknown, the results in Figure 2 do not disagree with Parducci's model.

The following experiments were made to test Parducci's model. They provided data which also allowed further testing of Helson's model.

EXPERIMENT 1

Horizontal frontal lines were used as stimuli or anchors. There were three sessions. The anchors were presented only once before the first session and were presented immediately before each stimulus in the second session, or they were presented immediately before each stimulus in the first session and were never presented in the second session. If ϕ_A (Equation 13) or $\mathbf{R_F}$ (Equation 14) cause the Dai Prà effect, the magnitude of this effect should change with sessions since ϕ_A and $\mathbf{R_F}$ change with sessions.

Dai Prà (2007) used anchors which always preceded the stimulus. To explore whether the Dai Prà effect depends on memory, in the third session the stimulus and the anchors were presented simultaneously to each group.

Participants. The participants were 44 undergraduates placed in two groups of 23 and 21, Groups A and B, respectively.

Stimuli. Each stimulus was a 1-mm wide horizontal black line, with length varying from 12.5 to 44.5 cm in steps of 4 cm, presented in the middle of a gray 104 (base) × 53 (height) cm area of the screen of a plasma monitor (NRC PlasmaSync 50MP2) controlled by a computer (Intel Pentium 4). The illumination level was 10 lx. The viewing distance was 2 m.

Two 1-mm wide horizontal red lines with lengths of 5.3 and 64.4 cm were used as anchors. These anchors either preceded the stimulus or were simultaneous with the stimulus.

Anchors preceding the stimulus. Each anchor was concentric with the stimulus. The shorter anchor appeared first. The anchor duration and the interanchor interval were 1 sec. The stimulus appeared 1 sec after the longer anchor and remained visible until the trial was completed.

Anchors simultaneous with the stimulus. The anchors were located at 15.5 cm from the stimulus with the shorter anchor above the stimulus. The centers of stimulus and anchors were aligned vertically. The stimulus and the anchors remained visible until the trial was completed.

Procedure. Participants were asked to rate the length of each stimulus using the integers in the range 10–100 with "10" and "100" defined as the ratings, $\mathbf{R_L}$ and $\mathbf{R_U}$, for the shorter and longer anchors, respectively.

There were three consecutive sessions. In each session the series of nine stimuli was presented twice, each time with stimuli in random order.

For Group A, the anchors were presented once before the first session and never in this session, immediately before each stimulus in the second session, and simultaneously with each stimulus in the third session.

For Group B, the anchors were presented immediately before each stimulus in the first session, were never presented in the second session, and were presented simultaneously with each stimulus in the third session.

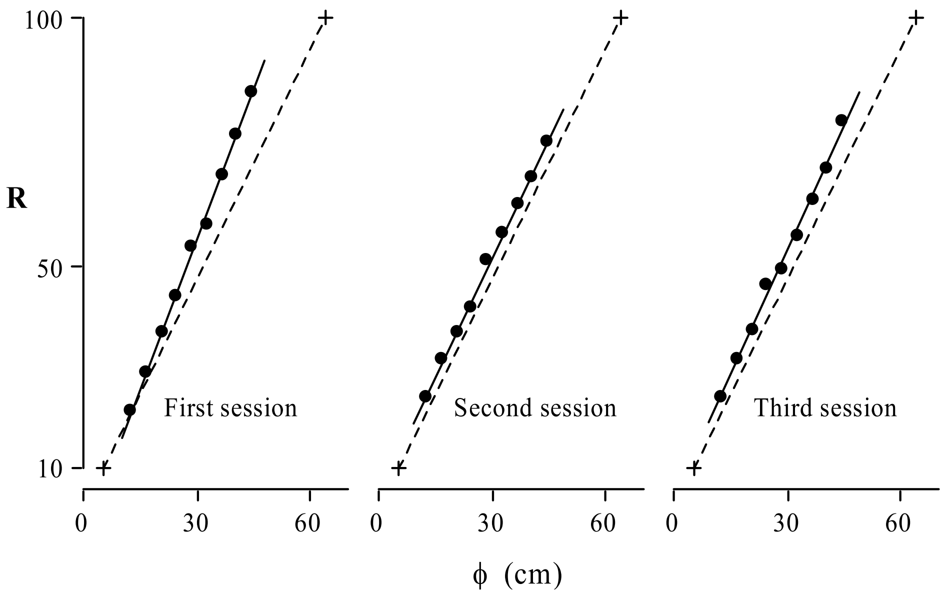

Figure 3. Results for Group A. Mean rated length, R**, plotted against physical length, ϕ, for anchors not presented during the session (first session), presented before each stimulus (second session), and presented simultaneously with each stimulus (third session).**

RESULTS

Figures 3 and 4 show mean rated length, **R**, plotted against physical length, ϕ, for Groups A and B for the first, second, and third sessions. Dashed lines joining two crosses show the **R**s predicted by Equation 15.

First and second sessions. Helson's and Parducci's models predict equal slopes of the straight lines that fit the data from the first and second sessions. For Group A these slopes differed. This result rejects both models.

Parducci's model predicts that ratings change with $\mathbf{R_F}$. Confirming previous findings (Haubensak, 1992a), for Group B the large change in $\mathbf{R_F}$ had no effect in the second session. This result rejects Parducci's model.

A session × length analysis of variance was done on these and subsequent results. Session and the interaction were significant for Group A [$F(1,22) = 5.4$, $p < .05$, and $F(8,176) = 8.3$, $p < .001$, respectively] and not significant for Group B [$F(1,20) = 0.2$ and $F(8,160) = 0.6$, respectively].

Second and third sessions. For Group A Dai Prà's effect was stronger in the third session and for Group B was equally strong in each session.

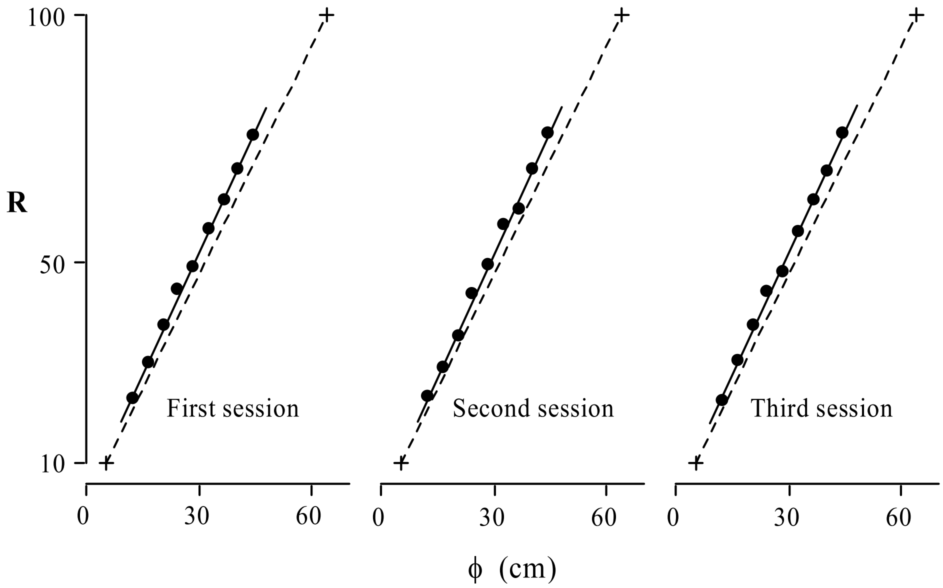

Figure 4. Results for Group B. Mean rated length, R, **plotted against physical length, ϕ, for anchors preceding (first session), never presented (second session), and simultaneous with each stimulus (third session).**

Session was not significant and the interaction significant in Group A [$F(1,22) = 1.6$ and $F(8,176) = 2.7$, $p < .01$, respectively] and both were not significant in Group B [$F(1,20) = 0.2$ and $F(8,160) = 0.9$, respectively].

Thus, the results show that the Dai Prà effect occurred both when the anchors preceded and when they were simultaneous with the stimulus.

The following experiment explored whether perceptual factors altered the Dai Prà effect when the anchors were simultaneous with the stimulus.

EXPERIMENT 2

Participants. The participants were 20 undergraduates.

Stimuli. Stimuli, anchors, and presentation conditions were the same as those used in Experiment 1 for the condition where stimulus and anchors were simultaneous, with the following exceptions.

There were three consecutive sessions. The distance between stimulus and anchors was 15.5 cm in the first and 1.5 cm in the second and third sessions. In the third session, the top anchor and the stimulus were shifted to the left so that the left ends of stimulus and anchors were vertically aligned.

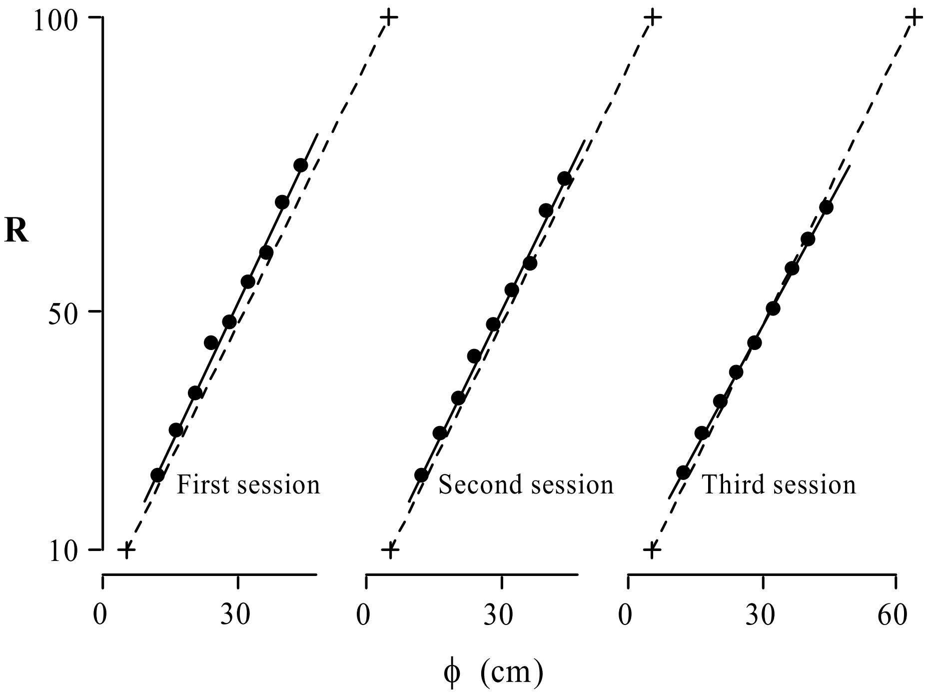

Figure 5. Mean rated length, R, **plotted against physical length, ϕ, when stimulus and anchors were simultaneous. Anchors were far from (first session) or close to stimuli (second and thirds sessions). The left ends of stimuli and anchors were vertically aligned in the third session.**

Procedure. In each session, the participants rated the length of each stimulus by the same procedure as that of Experiment 1 for the condition in which stimulus and anchors were simultaneous.

RESULTS

Figure 5 shows mean rated length **R** plotted against physical length ϕ. From left to right, the diagrams report the results for the first, second, and third sessions. The distance between lines did not alter the Dai Prà effect but, in the third session, the change in alignment of lines did.

For the results of the first and second sessions, session and the interaction were not significant [$F(1,19) = 2.4$ and $F(8,152) = 0.4$, respectively]. For the results from all sessions, session and the interaction were significant [$F(2,38) = 10.1$, $p < .001$, and $F(16,304) = 2.5$, $p < .005$, respectively].

The results show that the spatial arrangement of stimulus and anchors may influence the Dai Prà effect. The finding that distance did not alter this

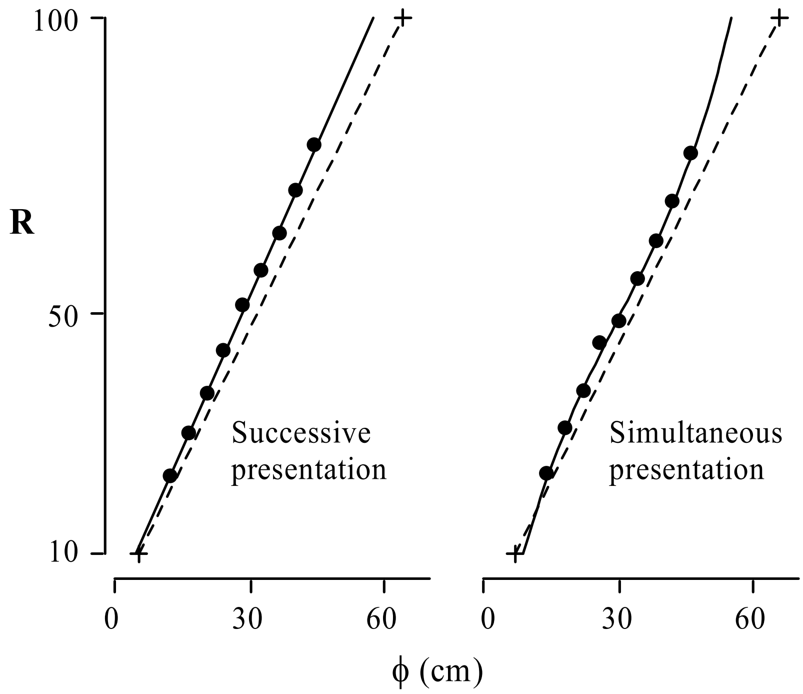

Figure 6. Mean rated length, R**, plotted against physical length, ϕ. Results for successive (left) or simultaneous (right) stimulus presentations.**

effect could suggest that a non-perceptual factor was operative when stimulus and anchors were simultaneous. Experiment 3 explored this possibility.

GENERAL RESULTS

When the anchors precede the stimulus, the results support the predictions of the Helson's, Parducci's, and Anderson's models that **R** is a linear measure of ψ.

Figure 6 shows mean rated length **R** plotted against physical length ϕ for all the conditions where the anchors preceded the stimulus (left diagram, 44 participants) and all the conditions where the stimulus was at 15.5 cm from the anchors (right diagram, 64 participants).

In the left diagram the results show that the relation of mean rated apparent length to apparent length was linear as predicted by the above models. In the right diagram the results show that some perceptual factor imposed a cubic trend on the Dai Prà effect.

Dashed lines depict the straight line predicted by Equation 15. In the left diagram the solid line depicts the least-squares straight line that fit the data. Its slope was 1.71. It differed significantly from the slope of 1.52 predicted by Equation 15 [$t(43) = 4.8$, $p < .001$]. The quadratic and cubic trends were not significant [$F(1,43) = 0.3$ and $F(1,43) = 0.6$, respectively].

In the right diagram the solid curve depicts the least-squares cubic polynomial that fit the data. The quadratic trend was not significant while the cubic trend was significant [$F(1,63) = 0.01$ and $F(1,63) = 21.6$, $p < .001$, respectively]. The slope of the straight line that fit the data was 1.63 and differed significantly from 1.52 [$t(63) = 3.6$, $p < .005$].

EXPERIMENT 3

In Figure 5, the results in the right diagram show that some perceptual factor affected the ratings. The results in the left and central diagrams could suggest that some non-perceptual factor influenced the ratings. The following experiment tested whether non-perceptual factors also alter the Dai Prà effect when stimulus and anchors are simultaneous. The number of stimuli was used as a non-perceptual factor for the test (Parducci, 1982).

Participants. The participants were 20 undergraduates placed in two groups of 10 each, Groups A and B, respectively. There were four sessions.

Stimuli for Group A. Stimuli, anchors, and presentation conditions were the same as those used in Experiment 1 in the condition where the anchors immediately preceded each stimulus, with the following exceptions. Stimulus length varied from 12.5 to 44.5 cm in all sessions. It varied in steps of 16, 8, 4, or 2 cm in the first to the fourth sessions with 3, 5, 9, or 17 stimuli per session, respectively. The series of these 3, 5, 9, or 17 stimuli were consecutively presented 12, 7, 4, or 2 times, each time in random order, with 36, 35, 36, or 34 trials per session, respectively.

Stimuli for Group B. Stimuli, anchors, and presentation conditions were the same as those used for Group A with the exception that the stimulus and the anchors were presented simultaneously as in Experiment 2. The distance between the stimulus and each anchor was 10.5 cm.

Procedure. In each session, Group A rated the length of the stimulus by the same procedure as that of Experiment 1 for the condition in which the anchors immediately preceded the stimulus, and Group B did this for the condition in which stimulus and anchors were simultaneous.

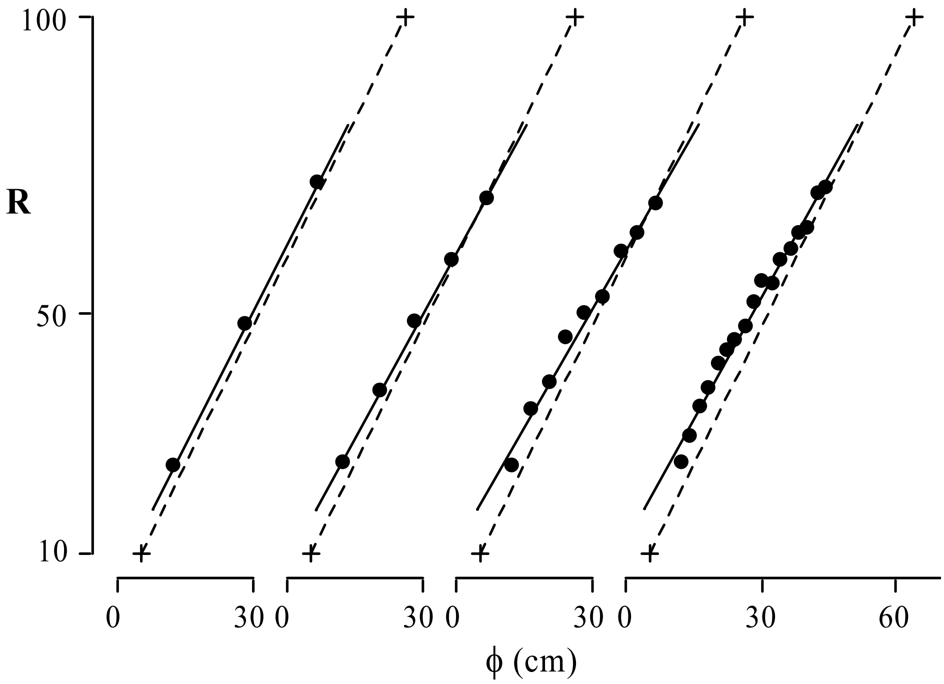

Figure 7. Results for Group A. Mean rated length, R**, plotted against physical length, ϕ, for anchors preceding each stimulus.**

RESULTS

Figures 7 and 8 show mean rated length **R** plotted against physical length ϕ for Groups A and B, respectively. In each figure, from left to right, the diagrams show the results for 3, 5, 9, or 17 stimuli obtained in the first to the fourth sessions, respectively. For each session, a solid line depicts the least-squares straight line that fit the data.

In both Figures 7 and 8, the slope of the solid lines decreases as the number of stimuli increases. These results show that a non-perceptual factor such as the number of stimuli can influence the Dai Prà effect when the stimulus and the anchors are presented successively and when they are presented simultaneously.

These results reject Helson's and Parducci's models since these models predict that the solid lines in Figures 7 or 8 must have equal slopes.

For both Groups A and B, an analysis of variance made on individual slopes showed that mean slope decreased significantly as number of stimuli increased [$F(3,27) = 3.8$, $p < .05$, and $F(3,27) = 9.7$, $p < .001$, respectively].

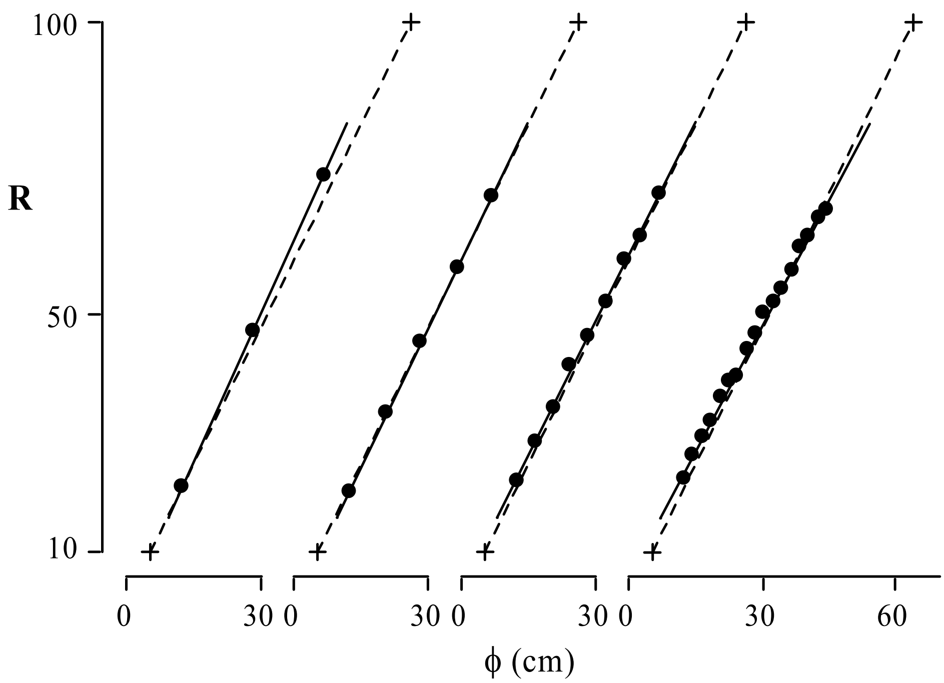

Figure 8. Results for Group B. Mean rated length, R**, plotted against physical length, ϕ, for anchors simultaneous with each stimulus.**

For Group A, the quadratic and cubic trends of **R** were not significant when there were 3 or 5 stimuli, the quadratic trend [$F(1,9) = 7.1$, $p < .05$] was significant and the cubic trend not significant when there were 9 stimuli, and the quadratic and cubic trends were significant [$F(1,9) = 7.1$, $p < .05$, and $F(1,9) = 9.7$, $p < .05$, respectively] when there were 17 stimuli.

For Group B, the quadratic and cubic trends were never significant.

In the last two sessions, the significant quadratic or cubic trends for Group A could have occurred due to the overcrowding of ratings within the range of ratings learned in the first and second sessions.

CONCLUSION

Haubensak's, Helson's, and Parducci's models fail to explain the Dai Prà effect. Haubesak's model (Equation 9) fails because it disregards the anchors although the anchors influence ratings. Helson's (Equation 13) and Parducci's models (Equation 14) fail because they predict parallelism of the straight lines that fit the data obtained in Experiments 1 and 3 whereas the present results show that these lines are not parallel. For an overview of further results rejecting Helson's model see Masin (1999) and for further results rejecting Parducci's model see Haubensak (1992a) and Haubensak & Petzold (2002).

Anderson's model (Equation 8) could account for the Dai Prà effect without referring to context effects. It could do so on the following assumptions. So far we have assumed that participants compare ψ with ψ_L and ψ_U. However, we could alternatively assume that participants first linearly convert ψ_L and ψ_U into the judgment representations

$$\rho_L = a_0 + a_1 \cdot \psi_L \tag{17}$$

and

$$\rho_U = b_0 + b_1 \cdot \psi_U \ , \tag{18}$$

with a_0, a_1, b_0, and b_1 constants, and that they subsequently compare ψ with ρ_L and ρ_U.

Equations 8, 17, and 18 imply

$$\mathbf{R} = m_0 + m_1 \cdot \psi \tag{19}$$

with

$$m_0 = \frac{a_1 \cdot \mathbf{R_L} \cdot (\rho_U - b_0) - b_1 \cdot \mathbf{R_U} \cdot (\rho_L - a_0)}{a_1 \cdot (\rho_U - b_0) - b_1 \cdot (\rho_L - a_0)} \tag{20}$$

and

$$m_1 = \frac{a_1 \cdot b_1 \cdot (\mathbf{R_U} - \mathbf{R_L})}{a_1 \cdot (\rho_U - b_0) - b_1 \cdot (\rho_L - a_0)} \tag{21}$$

and Equations 10 and 19 imply

$$\mathbf{R} = c_0 + c_1 \cdot \phi \tag{22}$$

with $c_0 = m_0 + m_1 \cdot k_0$ and $c_1 = m_1 \cdot k_1$ constants.

That is, Equations 8 and 10 and the assumption that participants convert the perceived anchors into judgment representations (Equations 17 and 18) predict that **R** is linearly related to ϕ. In agreement with this prediction, in Figure 2 the solid lines drawn through filled circles represent Equation 22 with best-fit constants c_0 and c_1.

There is considerable converging empirical evidence that ratings are linear measures of mental magnitude, that is, measures on an interval scale (Anderson, 1996, pp. 94–98). All the models considered here assume that **R** is linearly related to ψ in agreement with this converging evidence. On the reasonable assumption that the psychophysical function for apparent length of frontal lines is a power function with an exponent of 1, the results shown in the left diagram in Figure 6—obtained in conditions that minimized the influence of context factors—nicely confirm that ratings are linear measures of mental magnitude.

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