My Favourite Ratio — an inquiry about Pi

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The activities suggested in this article are intended for use with lower secondary school students. The *Australian Curriculum: Mathematics* states that students in lower secondary school should "investigate the relationship between features of circles such as circumference, area, radius and diameter" and "use formulas to solve problems involving circumference and area" (Australian Curriculum, Assessment and Reporting Authority (ACARA), 2010, p. 39). It is suggested, however, that teachers may need to work together to plan for the background and level of the students they teach and decide upon intended learning outcomes suited to their own students.

The investigations presented here were used by the authors during a half-day professional learning session with middle school teachers from five rural schools in southern Tasmania (as a part of the ARC-funded research project "Mathematics in an Australian Reform-Based Learning Environment" [MARBLE]). The investigations are likely to be familiar to teachers with a strong Mathematics background but the authors have found them to be unknown to many middle school teachers who are teaching out of area and hence we feel that it is important for these teachers to be provided with the tools for developing their own, and their students' understanding, of Pi as a ratio. In doing so, we recognise that investigation of Pi is not only applicable to study of circles, but the exercises presented here give students the opportunity to develop investigative and problem solving skills in mathematics.

The context chosen for the investigations is Pi, associated with the Greek letter π. Pi is well known in mathematics (Scott, 2008) but is often taught as a number (e.g., 3.14 or even 22/7), formula or rule that must be remembered by rote (Tent, 2001). The problem with this type of rote learning is that some students who rely only on memory may not have basic number sense, so that if they make a mistake along the way to memorising a formula or rule they have difficulty arriving at the correct solution or recognising that their solution is indeed wrong (Munakata, 2006). The power of

Pi, however, comes from understanding its definition as a ratio. The *Australian Curriculum: Mathematics* also makes significant demands for ratio. Ratio is first explicitly introduced in Year 7 including "recognise and solve problems involving simple ratios" (ACARA, 2010, p. 35) and further in Year 8, "solve a range of problems involving rates and ratios" (p. 38). Therefore, because of the wide scope for exploring the mathematics of Pi, when planning a unit of work that aims to develop ideas associated with Pi, the starting point and motivating questions need to be considered carefully. An investigation can start by looking at the questions: "What is Pi?", "How can we work out a value for Pi?", "What can we measure to calculate Pi?", "Why is Pi important?", and "Why is knowing Pi useful?"

This article recommends that students complete a series of investigations in the order presented here, with some optional extension activities, and that students keep notes on the activities for future reference, preferably in a portfolio. In this way, students are guided more gently towards an understanding of Pi and encouraged to formulate their own conclusions about the use and relevance of Pi. Furthermore, these investigations help develop and improve students' ability to solve problems—a learning objective or proficiency strand strongly advocated in the *Australian Curriculum: Mathematics* (ACARA, 2010, pp. 2–3). The other three proficiency strands to be considered in the *Australian Curriculum: Mathematics* are Understanding, Fluency, and Reasoning. The investigations presented here lend themselves to the development and consolidation of each of these four proficiencies, and specific links to these are discussed throughout.

Preliminary discussion 1: Ratio

Depending on students' backgrounds, it may be important to review the term ratio. One way is to brainstorm as a class to fill in the table shown in Table 1, which can then be a class resource for the other investigations. Some possible entries are shown.

The use of a brainstorming table such as this one for ratios helps identify students' prior learning and any gaps in their understanding. As stated in the Understanding proficiency strand of the *Australian Curriculum: Mathematics* it is important that teachers help students to make "connections between related concepts and progressively apply the familiar to develop new ideas" (ACARA, 2010, p. 3). By brainstorming the term ratio before starting the investigations into Pi, ratio ideas studied previously are brought back into focus, encouraging the linking of prior understanding to a new situation.

Table 1

What I know about ratios		
What are mathematical ways of writing ratios?	What are other words related to ratios?	What are some examples of ratios in context?
3:2 3/2 3 to 2 1.5	Fractions Proportions Percent	45 km/h 76c/100 g Foot length to height Girls to boys in the class Cordial to water

Investigation 1: A guided investigation into Pi

Setting the question

What is Pi? How can we work out a value for Pi? Why is Pi useful? Before starting the investigation into Pi it is necessary to ascertain the students' levels of prior knowledge and understanding related to circles, as circles are most commonly associated with Pi (Scott, 2008). Teachers may wish to ask their students the following question: What do you already know about circles?

To help students answer this question, it is suggested:

1. Students brainstorm all the words they know about circles—the students might like to use the "think, pair, share" strategy.
2. As a group, students sort the list into three columns (see Table 2).
3. As individuals, students choose two of these words and write a definition of each.
4. The students' definitions are then collated to make a class mathematics dictionary for future reference.

Table 2

Words I know about circles		
Words I know the meaning of and can explain	Words I think I know the meaning of	Words I have heard of

This task is necessary for teachers to deal with any issues that may hinder the students' completion of the remaining activities in this investigation. Some prerequisite skills, for example, are the knowledge of *circumference* and *diameter* and how to measure these. Other words that might appear in the table are "radius", "centre", "chord", or "segment", depending on students' backgrounds.

Data collection

There are lots of mathematical patterns that can be found in a circle. This activity helps students find one of these patterns, the one that defines Pi. Students are asked to:

- Collect a container of round objects. It is recommended that students collect and measure a sufficiently large number of objects to minimise the potential for measurement error.
- Collect measuring and recording equipment—a ruler, string, some paper, a pencil.
- For each object measure the circumference, and then measure the diameter, remembering to record the units (cm or mm). It is recommended that students use the same unit of measurement each time to allow for ease of comparison between measures.

Data representation

Students are asked to draw up a table (Table 3) to record their results. When they have all their measurements, students are asked to use a calculator to work out the ratio of the circumference to the diameter—teachers may need to remind students that this is done by dividing the circumference by the diameter (Circumference ÷ Diameter), which is represented as

Table 3

Object	Circumference	Diameter	$\frac{\text{Circumference}}{\text{Diameter}}$	= ___ . ___
Dinner plate	79 cm	25 cm	$\frac{79}{25}$	3.16

a fraction and a decimal in the table. Each value obtained is an approximation to the famous ratio, Pi. The use of a fraction to represent a ratio may need further exploration depending on students' prior knowledge and understanding. A ratio is an expression that compares quantities relative to each other, and whilst the most common examples involve two quantities, in theory any number of quantities can be compared. A fraction is an example of a specific type of ratio, in which the two numbers are related in a part-to-whole relationship, rather than as a comparative relation between two separate quantities. A fraction is a quotient of numbers, the quantity obtained when the numerator is divided by the denominator. This quotient then produces a single decimal number. Taking the time to develop students' understanding of ratio and its link to fractions and decimals is an effective way to show students the robust and transferable nature of mathematical concepts and incorporate the "why" as well as the "how" of mathematics, as advocated in the Understanding proficiency strand of the *Australian Curriculum: Mathematics* (ACARA, 2010, p. 3).

Summarising data

Information gathered by students can be recorded in a personal mathematics portfolio along with an introduction to the activity and an explanation of what they found out. Useful questions for the students to consider are:

1. Do you notice a pattern in the values you calculated?
2. Why were all of the values not exactly the same?
3. How could you use this pattern to help you measure circles, say if you could only measure the diameter of a very large circle?
4. Can you think of when you might need to use the pattern from this ratio in everyday life?

It is also recommended that the students' individual information be combined in a class data set, perhaps using a spreadsheet program or a data software package such as TinkerPlots (Konold & Miller, 2005). TinkerPlots and its specific application to this and other investigations are described in Investigation 5. In this way a discussion of accuracy and measurement error can occur, enabling a richer understanding of variance and invariance. This level of discussion links with the proficiency strand, Fluency, of the *Australian Curriculum: Mathematics*, which includes the development of skills in "choosing appropriate procedures, [and] carrying out procedures flexibly, accurately, efficiently and appropriately" (ACARA, 2010, p. 3).

Drawing a conclusion

In this activity students have found a number for the ratio Circumference: Diameter that is about the same for circles of different sizes. The relationship

$$\pi = \frac{\text{Circumference}}{\text{Diameter}}$$

also means that Circumference = $\pi \times$ Diameter

and $$\text{Diameter} = \frac{\text{Circumference}}{\pi}$$

These relationships can be very useful in problem solving and links can be made to the Problem Solving proficiency strand of the *Australian Curriculum: Mathematics* which includes "the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively" (ACARA, 2010, p. 3).

Preliminary discussion 2: Probability

The next two investigations are based on probability and depending on students' backgrounds, it may again be important to review some basic ideas of probability. As with the first preliminary discussion, the activity suggested here links effectively with the Understanding proficiency strand of the *Australian Curriculum: Mathematics* (ACARA, 2010, p. 3). Again a possibility is to brainstorm as a class to fill in the table (Table 4), which can then be used as a reminder for the class as they carry out the next two investigations. Some possibilities for the table are shown.

Table 4

What I know about probability		
What are other words related to probability?	What are some examples of probability?	How does probability link to ratio?
Chance Liklihood Random events	P(head from coin) $= \frac{1}{2}$ P(6 from die toss) $= \frac{1}{6}$ P(rain tomorrow) = 30%	Probability is the ratio of $\frac{\text{favourable outcomes}}{\text{total outcomes}}$

Investigation 2: Count Buffon's estimation of Pi

This investigation provides students with a different (experimental) way of estimating Pi. As well as introducing probability, there is an application of basic algebraic manipulation during the activity. The historical aspect of the investigation is also likely to be of interest to some students. This activity is best carried out in pairs.

Setting the question

What was Count Buffon's estimation of Pi? Who was Count Buffon? Wikipedia and other internet sites give a quick summary of a French naturalist and mathematician of the 18th century who lived to be 80 and published widely in all areas of Science (e.g., http://en.wikipedia.org/wiki/Georges-Louis_Leclerc,_Comte_de_Buffon).

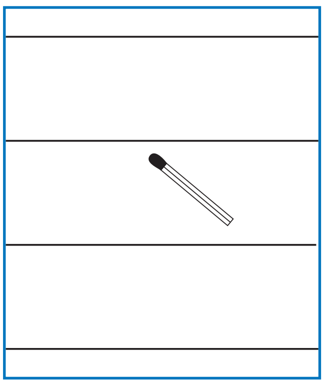

Figure 1. Rule lines an equal distance apart (the length of the match). Drop a match and record if it crosses a line.

Data collection

Student instructions are as follows:

- Collect a piece of paper, a pencil, a ruler and a match.
- Measure the length of the match (see Figure 1).

- Working from top to bottom, rule lines across the paper, so the distance between the lines is the same length as the match.
- Drop the match onto the paper from a sufficient height that it will land "randomly" on the sheet of paper. Ask a partner to note whether or not it falls across a line.

Representing data

The students are asked to record every time the match crosses a line, and to count the total number of times they drop the match (they may like to draw up a table to record this). Each student should drop the match at least 50 times.

Summarising data

Students are asked to use their calculators to calculate the probability of the match landing on a line:

$$\frac{\text{Number of times that the match lands on a line}}{\text{Total number of times the match is dropped}} = \text{Probability}$$

This is the ratio of favourable outcomes to the total number of outcomes attempted.

Count Buffon discovered that if you carry out this activity, and calculate the probability that the match will land on a line, it works out to be the same number as if you do the calculation $\frac{2}{\pi}$. Students might like to work out their own way to get a value of π from their results—but if this is not feasible they can use this method:

1. Calculate the probability of the match landing on the line:

$$\frac{\text{Number of times that the match lands on a line}}{\text{Total number of times the match is dropped}} = \text{Probability}$$

2. Use a calculator to get an answer. Since $\frac{2}{\pi}$ = Probability, this equation can be solved for π.

$$\frac{2}{\text{Probability}} = \pi$$

In this way, students are dividing 2 by the probability they calculated and this gives an approximation of Pi.

Drawing a conclusion

To help students draw a conclusion, they can be asked the following question: How close did you get?

Students should compare their values with other groups in the class, and write a summary of this activity and include it in their portfolios. By answering the question provided, students are using the skills outlined in the Reasoning proficiency strand of the *Australian Curriculum: Mathematics*, including "analysing, proving, evaluating, explaining, inferring, justifying, and generalising" (ACARA, 2010, p. 3).

The explanation of why this works involves calculus. Students who study mathematics at university are likely to work out a proof for this, or it can be found on the Internet (or see Nelson, 1979).

Investigation 3: The Monte Carlo method for the estimation of Pi

This activity is to be carried out in pairs.

Setting the question

What is the Monte Carlo method for the estimation of Pi? A Monte Carlo method is based on the random distribution of points over an area (Hinders, 1981).

Data collection

Instructions to students are as follows:

- Collect a piece of paper, a pencil, a ruler and a round object or compass.
- Draw a circle inside a square so that the circle touches each side of the square.
- Now, randomly place dots onto the square, including the circle, so they fall within the confines of the square (and many, therefore, will be inside the circle as well). This requires considerable discussion on how to ensure a random distribution that is not biased towards the centre or edge (see Figure 2).
- Continue until there are 100 random dots inside the square.

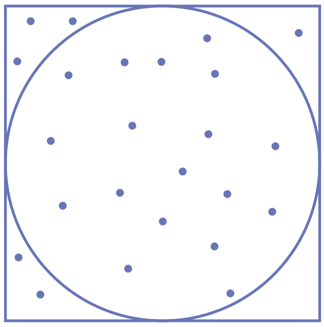

Figure 2

This method uses the idea that random allocation of dots is a way of estimating area. The ratio of dots that fall inside the circle to the dots that fall inside the square (total number of dots) should approximate the ratio of the area of the circle to the area of the square.

To explain how to get Pi, some algebra is needed:
Suppose that the radius of the circle is r.
The area of the circle then is πr^2.
The length of the side of the square is $2r$.
The area of the square is hence $2r \times 2r = 4r^2$.
The ratio of the area of the circle to the area of the square is:

$$\frac{\text{area of circle}}{\text{area of square}} = \frac{\pi r^2}{4r^2} \text{ or } \frac{\pi}{4} \text{ (after simplifying)}$$

The calculation for the ratio of dots inside the circle to dots inside the square (all dots) should equal $\frac{\pi}{4}$.

Representing data

Ask students to calculate:

$$\frac{\text{Number of dots in circle}}{\text{Total number of dots in square (100)}} = \text{Ratio}$$

Summarising data

This investigation should help students appreciate the value of manipulating formulas; that is, multiplying the ratio above by 4, should give an approximation of Pi. Students are asked to compare their values with those of other groups in the class and to write a summary of this activity and include it in their portfolio. The skills required to summarise the data link to the Fluency proficiency strand of the *Australian Curriculum: Mathematics* (ACARA, 2010), whereas the algebraic solution links to Reasoning.

Drawing a conclusion

Some example questions to ask students after the three investigations include:

- How close were you?
- Which of the three methods of obtaining an approximate value for Pi was the most accurate?
- How did you decide this?

Again, these questions link to the Reasoning proficiency strand of the *Australian Curriculum: Mathematics* (ACARA, 2010). To present the information, students are asked to write summaries for their portfolios.

Investigation 4: An application to study elephants

Although it is acknowledged that Investigation 4 can be conducted without any reference to Pi, this investigation can be used to challenge students to use their newly developed understanding of Pi in a creative way. It is also acknowledged that some students will make the link with Pi easier than others. Thus, Investigation 4 can be used as an extension activity for Pi or as a measurement activity that does not involve Pi. This activity has been specifically included to exemplify how different forms of investigation can be included in the middle school mathematics classroom to cater for the diversity of preferred learning styles of students.

This investigation applies the developed understanding of Pi to measuring the physical characteristics of elephants. Here are some facts about elephants (http://elephant.elehost.com/About_Elephants/Anatomy/The_Feet/the_feet.html).

> The elephant's foot size can be used to judge the overall size of a particular animal.
> The forefoot of an elephant has a circular shaped outline and the back foot takes more of an oval shape.
> The circumference of the forefoot is approximately equal to half the shoulder height.

By creating elephants' footprints for students to measure, perhaps on large sheets of paper to imagine a herd passing through the classroom, students can be asked to determine the elephants' heights and how tall they would be relative to the ceiling in the classroom. The following information is useful when drawing the circular front footprints for the elephants.

- A baby elephant's front foot circumference is about 40 cm.
- A juvenile elephant's front foot circumference is less than 1 metre.
- An adult African elephant's front foot circumference is between about 1.8 m and 1.95 m.
- An adult Asian elephant's front foot circumference is between about 1 m and 1.75 m.

An investigation set for students can be used to determine which elephants go with which footprints around the classroom. Creating "herds" of prints for different areas of the classroom should create student interest.

Setting the questions

Is there an unobtrusive way of determining the number of elephants in the herd, and the make-up of the elephant family (that is, how many adults and calves)? What information is needed about an elephant in order to find out its age? How can the shoulder height of an elephant be found if it is not possible to get close enough to measure it?

Data collection

Students have two methods of finding the elephants' heights. Using a ruler and string, students can measure the circumference of each front footprint and use the information provided to calculate the shoulder height of each elephant (height = 2 × circumference). Students can also measure the diameter of the circular footprints and use the results of Investigation 1 to find the circumference, and then apply the height formula.

Data representation

Students can use a table to record their results (Table 5).

Table 5

Footprint diameter	Footprint circumference	Height of elephant	Type and/or age of elephant

Summarising data and drawing a conclusion

Students use the information they obtained to determine how many elephants are in the herd. Given the different heights, students can make reasonable guesses as to the age of each elephant (i.e., calf or adult). The table of results and guesses can be recorded in the students' portfolios, along with an explanation of the procedure followed. How could such information and knowledge about Pi be of use to a park ranger? The underpinning concepts within the Reasoning and Problem Solving proficiency strands of the *Australian Curriculum: Mathematics* are covered in this investigation (ACARA, 2010).

Putting together a portfolio

The following instructions for students may be helpful in assisting them with their portfolios.

Your portfolio needs to show what you have found out about the special ratio, Pi. You should include the following things:

1. **An explanation of Pi**.
   You should write this in your own words and you may like to include some diagrams. You may also like to find out some interesting facts or history about Pi.
2. **Evidence of your investigations to estimate Pi**.
   You should write what you did, record your results and then write a conclusion or summary at the end. Each activity has some questions at the end of the instructions: these should be answered.

3. **A list of examples of where you might use Pi**.
This should include some examples, including those from everyday life. You should also include your answers to the elephant investigation.

It is suggested that teachers use a rubric to assess the students' portfolios (Appendix). The categories for the criteria used in the rubric are based on the work of Boix Mansilla and Gardner (1997). In accordance with the work of Lorna Earl (2003), the rubric can be used for the Assessment of Learning, as it is used at the end of the investigations to assess student outcomes. It may also be used by teachers as Assessment for Learning in terms of planning future learning outcomes. The rubric should be given to the students so that they understand how they will be assessed and what is being assessed. It is a good idea for the students to use the rubric to assess their own work, or they could show their work to peers for their feedback.

Investigation 5: Extension using technology

For teachers and students who are familiar with and have access to the software *TinkerPlots* (Konold & Miller, 2005), it is possible to enter their data from the previous investigations into data cards and use a formula each time to calculate approximate values of Pi. It is then possible to plot values of the data and Pi to observe the variation present. Figure 3 shows the data cards and the formula to approximate Pi for the three investigations, and Figure 4 shows dot plots for each investigation demonstrating the variation in values for Pi when 10 cases are investigated.

Using *TinkerPlots* students can investigate whether adding more cases to their data cards provides greater or less variation in Pi, and can explore the question of how many cases are needed to obtain the most accurate mean value of Pi. For example, if students combined all of the information from each student in the class, would this be enough to obtain a mean value close to 3.14, as an approximation of Pi? Using *TinkerPlots*, it can be seen that for a class of

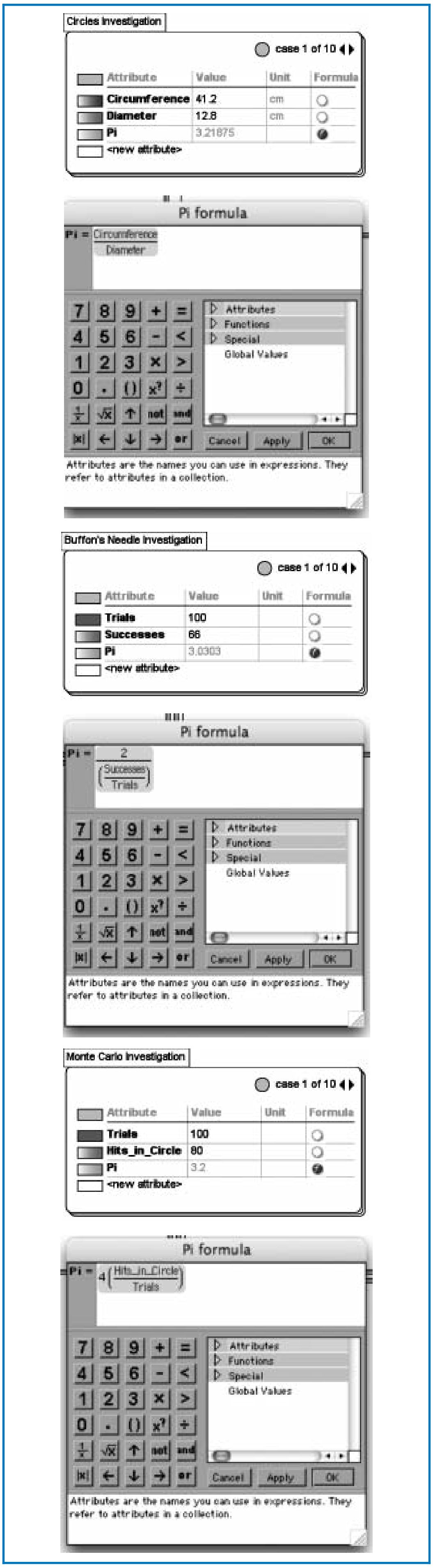

Figure 3. Examples of TinkerPlots cards for Pi investigations 1, 2 and 3, with their corresponding formulas for approximating Pi.

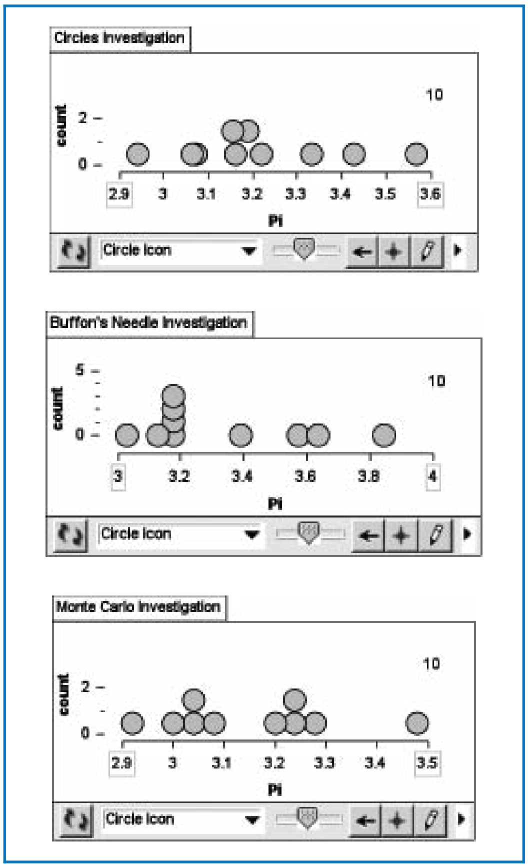

Figure 4. A TinkerPlots stacked dot plot of each investigation showing the variation in Pi calculated for 10 cases in each investigation.

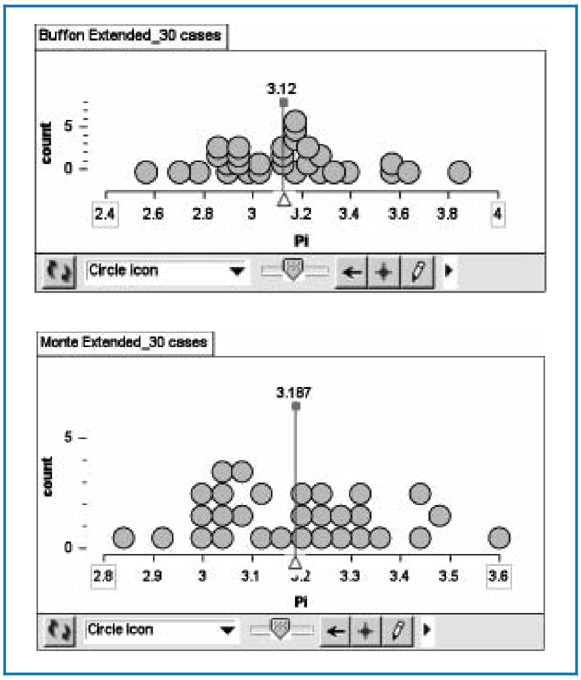

Figure 5. TinkerPlots stacked dot plots showing the mean value for approximating Pi for the Buffon's Needle and Monte Carlo method when using 30 cases.

30 students the distribution of approximate values for Pi could result in a mean value of 3.12 for the Buffon's Needle experiment and 3.19 for the Monte Carlo method, as shown in Figure 5.

Using online simulations for Buffon's Needle and the Monte Carlo method (e.g., http://www.mste.uiuc.edu/reese/buffon/bufjava.html and http://www.dave-reed.com/csc107.F03/Labs/MontePI.html), it is possible for students to run as many trials as they like. The information they obtain from each trial can then be added to their *TinkerPlots* data cards. Students can be asked to consider which method provides a better approximation for Pi. Although the Buffon's Needle method has produced a mean closer to Pi (3.12 – 3.14 = –0.02) than the Monte Carlo method (3.19 – 3.14 = 0.05) from the data in Figure 5, there is more variation in the values produced by the Buffon's Needle method and this might be seen as less desirable. (Using the same scale on the plots is useful to compare the data sets.)

The application of *TinkerPlots* as described in this Investigation challenges students to carry out procedures in a different way and explore the accuracy of their results and interpret the information they obtain, concepts endorsed by the draft *Australian Curriculum: Mathematics* (ACARA, 2010). Furthermore, the *Australian Curriculum: Mathematics* encourages the use of technology in teaching and learning situations as it can aid in developing skills and reduce the tedium of repeated calculations (ACARA, 2010, p. 9). *TinkerPlots*, combined with online simulations, clearly has the potential to enhance several aspects of student learning.

Conclusion

The purpose of this article has been to motivate teachers to present their students with meaningful investigations that lead to an appreciation and understanding of Pi. The preparation of a portfolio is intended to help students consolidate their understanding and enhance their sense of achievement in relation to mathematics. Applying the new-found understanding of Pi to the elephant investigation should be motivating and can lead to further research by students. Extension activities that use online simulation technology, which are readily accessible to most students, and TinkerPlots, where available, can add greatly to the students' understanding of Pi and offer interesting extension opportunities building on the hands-on investigations presented here.

Acknowledgements

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Appendix. Rubric for assessing students' portfolios.

Criterion	On the way	Getting there	Moving well	Really flying
1. What is Pi? (**Knowledge**) *Can explain Pi to an audience.*	You have given a definition but it is not strictly correct, or is a little confusing.	You have given a definition or explanation that is basic, or one that has been directly copied.	You have given an explanation that has evidence of original thought, either in the definition or in diagrams or additional information.	You have given a clear definition that is easy to understand and goes beyond a basic definition. The definition should have evidence of original thought. You have included additional information that clarifies or adds interest to your definition.
2. How can we calculate Pi? (**Methods**) *Can carry out an investigation to approximate the value of Pi.*	You have provided evidence that an investigation has been carried out according to teacher directions and with help from the teacher.	You have provided evidence that an investigation has been carried out, measurements have been taken and recorded and calculations have been completed.	You have provided evidence that an investigation has been carried out, with measurements being taken with appropriate instruments. You have checked your measurements against approximations so there are no way out measures and clearly recorded them.	You have carried out an investigation and clearly explained what you have done. Your results are recorded accurately and presented in an appropriate format. Measurements are accurate, appropriate units used, calculations are correct. A conclusion to the investigation is provided.
3. Why is Pi important? How is knowing Pi useful? (**Purposes**) *Can identify and give examples of how Pi is used; and can apply this to a problem.*	You have given an example of how Pi is used.	You have suggests several different examples where and how Pi is used.	You have suggested different examples where and how Pi is used and you have suggested why knowing Pi is useful.	You have suggested different contexts where and how Pi is used. You have suggested why knowing Pi is useful and important. You have given an example of an investigation that demonstrates Pi is useful.
4. How can I communicate what I know about Pi to others? (**Forms**) *Presentation of the inquiry/inquiries.*	You have handed in your work and it includes a definition and at least one investigation.	You have included the definition, investigations and all other work required by your teacher. It is easy to find each piece of work, and your work is easy to read.	You have included all the work required by your teacher, you have given thought and care to its presentation. You may have made some links between the pieces of work in the portfolio.	Your portfolio is complete, well presented and easy to read. You have included additional components related to extending the investigations or evidence of further research.

Areas of strength:

Areas for further development: