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Multiple Solutions to Problems in Mathematics Teaching: Do Teachers Really Value Them?

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Abstract: Solving problems in different ways is strongly advised for mathematics learning and teaching. There is, however, little data available on the examination of teachers' openness to and evaluation of different solutions to the problems. In this paper, the author examines classroom teachers' openness to different solutions (or to what extent they value different solutions) to problems and how they evaluate (grade) these solutions. For this purpose, two questionnaires including items on students' different solutions are applied to about 500 classroom teachers. In this paper, only two items related to the focus of the study are analysed. The findings show that teachers do not value different solutions and have difficulties in grading students' different solutions. The issues that these findings raise are discussed in detail.

Introduction and Theoretical Background

Particularly due to the influence of constructivist ideas that value students' individual knowledge construction and emphasise their autonomous development, enabling students to solve mathematical problems and questions that have one outcome in different ways has been strongly advised for the learning and teaching of mathematics (Schoenfeld, 1983; Yackel & Cobb, 1996; NCTM, 2000; Leikin, Levav-Waynberg, Gurevich & Mednikov, 2006). Similar views with even more emphasises have been expressed with regard to the use of mathematical problems and questions that have more than one outcome[1] (Tsamir, Tirosh, Tabach & Levenson, 2010).

With particular regard to the importance of solving mathematical problems and questions in different ways, Leikin (2007, p. 2330) argued that "solving problems in different ways as a (meta-mathematical) habit of mind both require and foster advanced mathematical thinking (AMT)". More to this point, Leikin & Levav-Waynberg (2008, p.234) emphasised that "solving problems in multiple ways contributes to the development of students' creativity and critical thinking". Schoenfeld (1983) also drew attention to the importance of solving problems in different ways and noted that when students perceive that the problem at hand can be solved or is allowed to be solved in different ways, then this situation increases students' engagement and helps them not to give up working on the problem. Similar comments were made by Polya (1973, 6.61) and he emphasised that encouraging students to "derive the results differently" can enable them to obtain even more elegant solutions (see, Leikin & Levav-Waynberg, 2008, for more details).

[1] In this paper, the terms different and multiple solutions are used interchangeably. These two terms are also used to refer to different solutions to the questions and problems with not only one outcome but also multiple outcomes.

One can infer from all these studies that enabling students to solve problems and questions in different ways has important implications for students' mathematical learning. Essentially, solving mathematical problems and questions in different ways are considered to not only help students construct mathematical connections but also show their mathematical thinking styles (Krutetskii, 1976; Schoenfeld, 1983, 1988; NCTM, 2000). It is perhaps this particular benefit of solving mathematical problems in different ways that led Silver, Ghousseini, Gosen, Charalambous & Strawhun, (2005, p.228) to claim that, "different solutions can facilitate connection of a problem at hand to different elements of knowledge with which a student may be familiar, thereby strengthening networks of related ideas". Given that constructing mathematical connections amongst and between mathematical concepts are prerequisite for conceptual (Hiebert & Lefevre, 1986) or relational understanding (Skemp, 1976) in mathematics learning, one can argue that enabling students to use different ways to solve mathematical problems and questions in mathematics teaching is of crucial importance for such understandings to take place as well.

Similar perspectives have been expressed regarding problems and questions with different outcomes as well. In this respect, the use of non-routine problems and open-ended questions in mathematics teaching has been particularly stressed as they are considered to offer opportunities for different outcomes (or more than one single correct solution) and place little restrictions on students' ways of solution (Hancock, 1995). Unlike routine textbook questions, non-routine problems and open-ended questions are deemed to offer opportunities to students to engage in situations that may require them formulating hypotheses, explaining mathematical situation, creating new related problems, and making generalisations (Stenmark, 1989; Lajoie, 1995; Silver & Kenney, 1995). These types of questions are, therefore, considered to be "educative for the child, as well as informative for the teacher" (Sullivan & Clarke, 1991 as cited in Clarke, 1992, p.164).

Research has also shown that different solutions to problems and questions has the potential to change the discourse of the classrooms, enhance the quality of mathematics lessons and contribute to students' conceptual learning (Stenmark, 1989; Lajoie, 1995; Silver & Kenney, 1995; Yackel & Cobb, 1996; Cobb, Gravemeijer, Yackel, McClain & Whitenack, 1997; Boaler, 1998; Stigler & Hiebert, 1999). Boaler (1998), for instance, showed that students who learned mathematics through open-ended activities developed conceptual understanding whilst students who followed a traditional approach developed procedural understanding. Cobb et. al. (1997, p.166-168) presented some dialogues from Ms. Smith's classroom of first grade students in their study. One of the dialogues that they presented is related to the addition of 8 and 9 cookies. The researchers showed that Ms. Smith allowed her students to solve this question in different ways and this process generated 7 different solutions to this 'apparently' simple addition question. Other similar dialogues were also presented in the paper. All the dialogues showed that different solutions to the questions, in fact, changed the classroom atmosphere and such a classroom environment, in turn, enabled students to freely express their own solutions. Yackel & Cobb (1996) regarded such classroom cultures as enabling the development of students' autonomy. Giving the students the opportunity to solve the questions in different ways can, therefore, contribute to the development of their autonomy.

All these studies and their findings clearly point to the pivotal role of the teacher in that it is the teacher who actually can probe for different ways of finding solutions to a problem or question in mathematics teaching. It is again the teacher who can choose and use problems and questions with different outcomes for mathematics teaching. Although teachers have such a pivotal role in making use of problems and questions with different solutions in mathematics teaching, little research appears to have been done to explore the extent to which they are open to different solutions to mathematical problems and how they evaluate these solutions. With this in mind, in this paper it is aimed to look at this issue through the following two research questions:

- How open are the primary classroom teachers to different solutions to mathematical problems and questions?
- How do the primary teachers evaluate students' different solutions to the open-ended questions?

These two research questions are addressed through the teachers' degree of openness to different solutions and grading of solutions to mathematical problems and questions. The issue of how this is done is detailed in the following section.

The Background of the Study

The Turkish education system has been undergoing a massive curricular reform since 2004 and the new primary mathematics education curriculum has also been part of this reform movement (Minister of National Education, 2004 (hereafter, abbreviated as MONE)). The constructivist ideas that have guided the curriculum reform of many developed countries in the last two decades are obviously made use of in the process of the development of the new mathematics education curriculum in Turkey, as well. These ideas particularly have brought about fundamental changes with regard to how learning, teaching and assessment are viewed and should be conducted. Learning, for instance, is interpreted as an active process and students are viewed to have autonomy over this process. Teaching, in a general sense, is considered to enable such learning to take place. Teaching, more specifically, is interpreted as putting the students at the centre and to be conductive to their autonomous development. As far as assessment is concerned, more emphasis is placed upon performance-based evaluation rather than product evaluation. Assessment is considered to contribute to the students' learning and development rather than to merely evaluate what they know.

Shaped around these perspectives on learning, teaching and assessment, the new mathematics curriculum requires new roles for teachers and students. Teachers are expected to take the role of a facilitator rather than that of a transmitter in teaching. With this new role, they are wanted to create a classroom culture that enables students to acquire such basic skills as critical and creative thinking, communication, questioning, problem solving, use of information technologies, initiation and use of Turkish language eloquently. In consistence with teachers' roles, students' roles have also been redefined. Students, in a general sense, are deemed to be the 'subject' of and have control over their learning processes. From this perspective, they are expected to be active participants rather than passive receivers in the classroom discourse. Critical thinking, creative thinking, problem solving, communication and similar skills are underlined on the part of the students and that give insight into the roles which they are expected to adapt in the business of learning.

With particular regard to the purpose of this study, the new primary mathematics curriculum pays serious attention to the issue of multiple solutions and asks teachers to create "classroom environments in which students can bring different solutions to the posed problems so that they learn to value different solution strategies in the process of problem solving" (MONE, 2004, p. 11). The task of creating such a classroom environments is, basically, purely laid upon the teachers and they are strongly encouraged to create such a classroom environment conducive to students' meaningful learning. Bearing this in mind, in this study it is aimed to explore to what extent classroom teachers[2] are open to different

[2] Primary (elementary) education lasts eight years and is compulsory for all in Turkey. Students in the first five years are taught by one classroom teacher and different teachers who are specialists in their subject areas in the last three years.

solutions and probe their grading of different solutions to mathematical problems and questions.

Methodology of the Study

In order to examine the extent to which classroom teachers are open to different solutions and probe their grading of different solutions to mathematical problems and questions, two questionnaires with different questions were developed. Both questionnaires included items concerning mathematical concepts covered in the primary school mathematics curriculum.

In this paper, I focus only on the one item from each questionnaire which is particularly related to different solutions. The first item is concerned with a multiplication (item-1) and the second one is related to the calculation of the area of a rectangle (item-2). In both items, teachers are supplied with students' different solutions to the problems and requested to evaluate these solutions. Due to the "ill-structured" nature of open ended-questions that do not require fixed procedures guaranteeing a correct solution (Foong, 2002), in item-2 erroneous student answers are intentionally used to see how teachers would evaluate them as it is natural that some students would provide wrong answers to the posed problems and questions.

Item-1:

$$\begin{array}{r} 32 \\ \times\ 25 \\ \hline \end{array}$$

Below students' three different responses to this multiplication are presented. All three students have reached the same result. Please evaluate each response and explain which one or ones you would accept as an answer and why? (adopted from Ball & Bass, 2003).

A.

$$\begin{array}{r} 32 \\ \times\ 25 \\ \hline 160 \\ +\ 64 \\ \hline 800 \end{array}$$

B.

$$\begin{array}{r} 32 \\ \times\ 25 \\ \hline 10 \\ 150 \\ 40 \\ +\ 600 \\ \hline 800 \end{array}$$

C.

$$\begin{array}{r} 32 \\ \times\ 25 \\ \hline 50 \\ 150 \\ +\ 600 \\ \hline 800 \end{array}$$

In item-1, a traditional method of multiplication is followed in solution A. This method of multiplication is the one that is often preferred by teachers in the classrooms. In solution B, the distribute principle is used, that is, 32 x 25 = (30 + 2) x (20 + 5) = 600 + 150 + 40 + 10. Solution C is based on the area model of multiplication, where (2x5) and (20 x 2) are first calculated and then (30 x 20) and (30 x 5) are calculated. Common to all these solutions is that they are all correct but only presented in different formats. The aim was to see how

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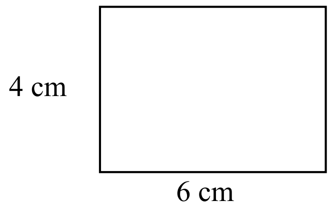
Classroom teachers are generally held responsible to teach five main subjects (Turkish language, mathematics, science and technology, life sciences and social sciences) to the same group of students during these first five years.

teachers would interpret different solution methods to the question with only one correct outcome[3].

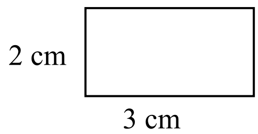
Item 2, on the other hand, is related to the area of a rectangle. In order to find out the dimensions, student K adds up the two dimensions and then divides it by 2. Student L's solution is related to a common misconception among students, that if the dimensions of a rectangle are doubled, then the area will be doubled. The aim was to examine how the teachers would grade these erroneous responses and reasons behind their grading.

Item-2: Fourth and fifth grade students are presented with the following problem:



What can be the dimensions of a rectangle with exactly half the area of this rectangle? Please explain your answer.

The responses of two students (K and L) to this problem are presented below. How would you grade these responses over a range from 0 to 10 and please explain why? (adopted from Hansen et al., 2005).

	Student's Answer and Explanation	**Score (Out of 10)**	**Reason**
Student K	"To find out the half area of the rectangle, I do this: $\frac{6+4}{2}=5$. Then each dimension can be 5 cm."		
Student L	"I would have the half of each dimension: 6 √ 2 = 3 and 4 √ 2 = 2. Then I would come up with a rectangle with a one side being 3 cm and the other 2cm". And draws the following figure: 		

The first questionnaire that included item-1 was administered to approximately 300 classroom teachers and there were 216 returns. The first questionnaire included 5 items in total and it took participants teachers approximately 25 minutes to complete. The questionnaire was applied to these teachers when they took part in an in-service training course. Those teachers taking the first questionnaires were working in 104 different schools in a large province in Turkey. The participant teachers were hence working in almost all different parts of that particular province.

The second questionnaire that included item-2 was administrated to approximately 200 teachers and there were 177 returns. Of them, 148 teachers responded to item-2. The questionnaire was applied to the teachers individually and they were requested to fill it in while they were in their classrooms. It took the participants approximately 20 minutes to complete. Those taking the second questionnaires were working in 10 different schools in three different provinces in Turkey.

[3] All solutions are presented in vertical format. However, it would be interesting to investigate how results are affected when solutions are presented in different formats. The distributive principle format in the following way is only one particular way that can be used for any further research: 32 x 25 = (30 + 2) x (20 + 5) = 600 + 150 + 40 + 10.

Data Analysis and Results

This section contains both data analysis and results. The analysis and related results of each item are presented respectively.

With regard to the item-1, a frequency analysis was first carried out to establish the number of teachers accepting (i.) any one of the solutions of A, B, C; (ii.) any two of A, B, C; or (iii.) all of them together. Note that all the solutions are correct but different. The data analysis presented in Table 1 shows that the majority of teachers (67%) states that they would accept only solution A. Only 15% of them point out that they would accept both A and B, and almost the same percentage of them (17%) state that they would accept all A, B, and C as an answer to the multiplication question.

	A	A&B	A&B&C	No answer	Total
Number	145	33	36	2	216
Percentage	67%	15%	17%	1%	100%

Table 1. Teachers' responses (frequencies–percentages) to item-1

A further analysis is carried out on those teachers who cited accepting only solution A. The aim here was to find out why they would accept only solution A. It was important to look at their reasons as this was considered to give insight into why the teachers did not value multiple solutions. This analysis consisted of repeated re-readings of participants' reasons for accepting solution A. The aim of this process was to determine the categories and their definitions for the data analysis (Patton, 2002, pp. 452-54). The analysis ultimately generated five categories which cover the teachers' reasoning for their choices (Table 2). The categories are as follows: i.) Rule, ii.) Practical, iii.) B and C being difficult (for teachers to make sense)[4], iv.) Accept A but listen to B and C and v.) Not categorised. Table 2 below presents what these categories stand for and provides some exemplary quotations from teachers' reasoning for each category.

Categories	**Definitions of categories**	**Examples from teachers' responses**
Rule	Responses that cite algorithmic rule of multiplication	T1: I would accept only solution A as I also teach the rule of multiplication in this way. T2: I would not accept solutions B and C and if my students attempt to do the multiplication like in B and C, I would interfere at the very beginning not to do so. T3: because there is only one way to the truth (*right conclusion*), I would accept only A.
Practical	Responses that cite the practicality	T4: I would accept only solution A as it is short, practical or maybe this is because we were taught this method.
B and C being difficult	Responses that cite the difficulty of the answer	T5: Student who provides solution A has understood the multiplication and did it

[4] 'B and C being difficult' category stands for two types of teachers' responses. First, some teachers stated that those students who provided solution B and C actually did not understand the multiplication. These kinds of responses were considered as indicating teachers' difficulties in making sense of students' solutions. Second, some teachers directly quoted that solutions B and C were very complex to understand. These kinds of responses were also taken as indicating teachers' difficulty in understanding students' solutions.

	B & C for understanding or teaching	according to its rule. Students who provided solutions B and C have not understood the multiplication. T6: *I would accept only solution A* as the other two are complex.
Accept A but listen to B and C	Reponses that cite teachers being open to both B and C solutions but accepting only A as an answer	T7: I would prefer only solution A as it is short. But I would also talk to the students about B and C.
Not categorised	No reasoning or only stating the preference	T8: I would accept only solution A.

Table 2. Analysis of teachers' reasoning for accepting only solution A in item-1

In analysing the data, the responses of the participants were separately examined and then allocated to each of these categories based on their descriptions. The allocation of responses to the categories was carried out by the author and another mathematics education researcher simultaneously and 100% agreement was reached for the assignment of each participant's response to a category. Frequencies of these categories are presented in Table 3 below. Note that some responses fall under more than one category and that is why the total percentage exceeds 100%.

	Those teachers who accept only solution A (145)				
	Rule	Practical	Accept A but listen to B and C	B and C being difficult	Not categorised
Number	79	39	16	17	20
Percentage	54%	27%	11%	12%	14%

Table 3. Responses of teachers who only chose solution A in item-1

Of the 145 teachers who stated they would accept only solution A, just over half (54%) gave the reason that it fitted with the 'rule' (Table 3). About one quarter (27%) cited 'practicality' to explain their reason. Only 11% of the teachers indicated that they would listen to the B and C solutions too and 12% indicated they found solutions B and C difficult. The findings overall show that teachers have the tendency to accept the standard solution and are not open to different solutions. The notion of a rule and practicality are the two main factors to explain the teachers' reasoning.

With regard to item-2, it was administrated to 144 teachers to examine how teachers would evaluate different solutions to the open-ended question and more specifically to examine how they evaluate erroneous student answers. A frequency analysis is first conducted to ascertain how teachers would grade student K and L's responses over a range from 0 to 10.

Scores	0	1	2	3	4	5	6	7	8	9	10
Student K	76 51%	13 9%	7 5%	7 5%	5 3%	13 9%	4 3%	3 2%	4 3%	1 1%	15 10%
Student L	35 24%	4 3%	6 4%	6 4%	6 4%	18 12%	3 2%	3 2%	1 1%	0	65 44%

Table 4. Teachers' responses (frequencies–percentages) to item-2

The data shows that teachers graded students' wrong responses for different reasons over a range from 0 to 10 (Table 4). For solution K, for instance, 51% of the teachers gave a score of 0 as it was a wrong answer. Interestingly, 10% of the teachers gave a score of 10 to the same wrong answer. Also note that the scores of almost 40% of the teachers ranged from 1 to 9 points. These findings clearly point to the variation of grading given to the same answer and that shows the discrepancy amongst the teachers.

For solution L, the analysis shows even more interesting findings (Table 4). The analysis shows that 44% of the teachers unexpectedly gave a score of 10 (full marks) to the wrong response of the student L while only 24% gave a grade of 0. The table also illustrates that 32% of the teachers allocated scores ranged from 1 to 9 to the same answer. Similar to the findings related to solution K, the findings related to solution L also demonstrates the diversity of grading for the same solution.

A further analysis on teachers' reasons for their allocation of grades reveals some other interesting issues. Those teachers who allocated the grades from 1 to 9 to the wrong answer were actually aware of the erroneousness of the answer. Although this was the case, their grading still ranged from 1 to 9. One of the teachers (T9), for instance, allocated a score of 3 to the solution L and noted that the reason that he allocated this score was because the student "did not completely know the concepts of area and circumference". Another teacher (T10), however, allocated a score of 7 to the same solution L and noted that the student "did not adequately comprehend the concept of area". These two reasons clearly reveal that although the two teachers provided similar reasons, their grading markedly differed.

In addition, those teachers who allocated a grade 0 to the answers often attributed their reasons to the student's lack of knowledge. For instance, one teacher cited that the student "did not know A=axb (T11)" and another one stated that as the student "did not find the correct answer (T12)". Further to these results, those teachers who knew that the responses were wrong but gave grades from 1 to 5 also provided various reasons including "because at least students attempted to solve the problem (3 points, T13)", "as an encouragement or award (4 points, T14)", and "the student tries to find the result, although it is not complete but reasonable (T15)". All these citations show that some teachers valued the student' endeavours and hence gave scores more than 0 to the answer whilst some others valued the correctness of the answer and did not give a grade at all.

Discussion

Although this study mainly aimed to explore classroom teachers' openness to multiple solutions and their grading of different solutions to mathematical problems and questions, the findings raised some other important issues as well. I focus my discussion of these issues around the following four themes: i.) Teachers' openness to multiple solutions, ii.) Teachers' beliefs and orientations about mathematics, iii.) Teachers' difficulties about the assessment of open-ended questions, and iv.) Teachers' mathematical difficulties. I discuss all these themes in relation to the new curriculum implementation as well.

Teachers' openness to multiple solutions

Although the new Turkish primary mathematics curriculum strongly encourages teachers to value different solutions to the problems and questions in mathematics teaching, the results clearly show that classroom teachers are not open to different solutions to mathematical problems. Table 1, for instance, shows that the majority of teachers (67%) state that they would accept only solution A in item-1. Although all the answers are correct, only 15% of the teachers state that they would accept both A and B, and almost the same percentage of them (17%) point out that they would accept all A, B, and C as answers. The results obtained from item-1, therefore, reveal that the majority of the teachers prefer the standard solution to the multiplication question.

Studies from extant literature show that this situation might not only be peculiar to Turkish teachers. Stigler and Hiebert (1999), for instance, show that in the USA teachers of middle grade students hardly ever engage their students in solving problems with multiple solutions. Similar findings are put forward by some other studies as well. The works of Leikin (2007) and Ma (1999) have shown that teachers themselves do not often solve problems in different ways let alone encouraging their students to do so.

With regard to the curriculum implementation, these findings can be interpreted as alarming. Considering that the new Turkish mathematics education curriculum expects teachers to create a classroom culture in which students can produce different solutions, the findings show that the teachers themselves do not appreciate different solutions to the mathematical questions. If there is a consistency between teachers' views and practices, -as often it is considered to be the case (Kagan, 1992)- then one can conclude that the teachers are having difficulty in implementing the curriculum as intended.

Teachers' beliefs and orientations about mathematics

Although initially not intended to be explored, the results also reveal some interesting data regarding teachers' beliefs and orientations about mathematics. An examination of teachers' reasons for their choices in item-1 shows that they value 'routine', 'rule' and 'practical' aspects of mathematical solutions. In this respect, Table 3 shows that 54% of those teachers who point out to accept only solution A cite 'rule' and 27% cite 'practicality' in explaining their reasons. These findings clearly point to the fact that the rule and practicality were important for the teachers in assessing different solutions to the mathematical question.

Teachers' citations of rule and practicality can be related to their already formed personal theories, views, orientations and beliefs with regard to mathematics, its teaching and learning. Some teachers appear to hold the view that solutions to mathematics problems should take little time, be practical and employ procedural rules. Of these teachers, for example, one teacher (Table 2, T4) states that he would accept only solution A in item-1 "as it is short, practical or maybe this is because we were taught this method". Another teacher (T3) notes that he would also accept only solution A "because there is only one way to the truth (right conclusion)". One teacher (T2) even goes further and claims that "I would not accept solutions B and C and if my students attempt to do the multiplication like in B and C, I would interfere at the very beginning not to do so".

These citations are clear indications of the teachers' underlying beliefs about not only their choices but also mathematics, its teaching and learning. The participant teachers appear to hold the beliefs that following some rules and being able to get the right answer quickly is essential in solving mathematical questions. These beliefs, interestingly, have similarities with what Schoenfeld (1992) has reported regarding students' beliefs. Based on the findings of several studies (National Assessment of Educational Progress, 1983; Schoenfeld, 1988, 1989), Schoenfeld (1992) notes that "students come to believe that in mathematics, (a) one should have a ready method for the solution of a given problem, and (b) the method should produce an answer to the problem in short order". Schoenfeld (1992) further cites some findings from the study of National Assessment of Educational Progress and provides that nine students out of ten agreed with the statement "there is always a rule to follow in solving mathematics problems" and 75% of students agreed with the statement "doing mathematics requires lots of practice in following rules". Alongside many other factors, Schoenfeld (1992) refers to the role of the methods imposed on students by teachers and texts in the emergence of such beliefs amongst students about mathematics.

Returning to the findings of this study, the data has revealed that the beliefs that the participant teachers hold are sharply in conflict with what the new curriculum sets out to achieve, which strongly encourages teachers to be open to the different solution strategies. Given that teachers are "the key to change" (Kilpatrick, 2009, p.107) and at the same time their beliefs are extremely resistance to change (Kagan, 1992), this casts a serious shadow on the implementation of the curriculum as intended.

Teachers' difficulties about the assessment of open-ended questions

The new curriculum puts a heavy emphasis on the use of open-ended questions with multiple solutions for both formative and summative assessments (MONE, 2004). Open-ended questions, as one can expect, mean variations and unexpected responses in students' solution strategies to the questions that can raise challenges for teachers to evaluate. Responses to the item-2 in Table 4, in fact, demonstrate a great variation in the teachers' evaluation of one particular answer in that the teachers allocate scores ranging from 0 to 10 to the same answer. Responses also reveal that there is a great variation even in the grading of those teachers who found the solutions wrong. For example, 51% of teachers graded solution of student K in item-2 with 0 and articulated that because it was incorrect. Interestingly, student K's solution also received the grades ranging from 1 to 5 from those teachers who, whilst pointing out the erroneousness of the solution, noted that "because at least students attempted to solve the problem (3 points, T13)" or "as an encouragement or award (4 points, T14). Student L's incorrect answer received even a score of 7 from one teacher (T10) although his reasoning was that "the student did not adequately comprehend the concept of area".

The findings noticeably reveal that there is a great variation in the teachers' grading of different solutions to the open-ended questions. This variation is of great importance as the grades send signals to the students about what is valued by the teachers; mathematical accuracy or making effort? One can expect that a certain level of variation is understandable for individual judgements yet this obviously indicates the lack of assessment criterion which teachers can make use of to evaluate students' work. These findings further, therefore, suggest that the teachers are not open to multiple solutions and also that when they are presented with different solutions, they have difficulty evaluating them.

Teachers' mathematical difficulties

The data, unexpectedly, reveal yet another important discussion issue, that is, the classroom teachers' difficulties in mathematics. This is particularly evident in the teachers' evaluation of student L's erroneous response to item-2 to which 44% of teachers allocated a grade of exact 10 (Table 4). Such a high percentage was unexpected. These findings, of course, cannot be over-generalised to the whole primary teacher population in Turkey. But nonetheless this is an important proportion and draws attention to a possible source of challenge, that is lack of teachers' content knowledge, in teaching mathematics and implementing the curriculum. These findings, therefore, before anything else, raise concern about the mathematical competency of the teachers to teach mathematics to the students.

There is evidence showing that teacher difficulties in mathematics at primary level are not only peculiar to Turkey and it is, in fact, a reality all around the world (e.g. see Ma, 1999; Manouchehri, 1998). This might be understandable as these teachers are not specialised in mathematics and responsible for teaching different subjects. Yet we, whether policy makers or mathematics educators, need to take these difficulties seriously and to seek out ways of improving especially practicing primary teachers' mathematical content knowledge through in-service courses. Neglect in this respect, otherwise, would render any curriculum implementation as a failure.

Conclusions and Implications

The findings of this study have shown that classroom teachers are not open to multiple solutions, have difficulties in evaluating students' responses to the open-ended questions, and experience mathematical difficulties in evaluating whether student solutions to open-ended problems are correct or not. Given that solving mathematical problems in different ways is of great importance for students' conceptual and meaningful learning, these findings can be interpreted as signalling potential difficulties for the implementation of the new curriculum as intended. Further research is, however, needed to reveal more about the teachers' material experience in the actual classroom settings. This strand of research will provide closer insights into the teachers' implementation (or lack of implementation) of the new curriculum.

Given that the new curriculum has been in use in the last five years, concerns have been continuously raised in many circuits with regard to its implementation (Zembat, 2010). These concerns particularly manifest themselves with respect to the issue of teachers' competency and readiness to implement the curriculum as planned by the MONE (Bulut, 2007). The results from this study suggest that teachers need support in many areas to implement the curriculum as intended. There is clearly a need for professional development programs specifically designed to help in-service teachers to implement the curriculum as intended.

References

Ball, D.L., & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. In E. Simmt & B. Davis (eds.), *Proceedings of the 2002 Annual Meeting of the Canadian Mathematics Education Study Group CMESG* (pp. 3–14.). Edmonton, AB.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal for Research in Mathematics Education*, *29* (1), 41-62.

Bulut, M. (2007). Curriculum reform in Turkey: A case of primary school mathematics curriculum. *Eurasia Journal of Mathematics, Science & Technology Education*, *3*(3), 203-212.

Clarke, D. (1992). The role of assessment in determining mathematics performances. In G.C. Leder (Ed.) *Assessment and Learning of Mathematics* (pp. 145-168). Australian Council for Educational Research, Victoria, Australia.

Cobb, P., Gravemeijer, K., Yackel, E., McClain, K., & Whitenack, J. (1997). Mathematizing and symbolizing: The emergence of chains of signification on one first-grade classroom. In D. Kirshner & J. A. Whitson (Eds.), *Situated cognition: Social, semiotic, and psychological perspectives* (pp. 151-233). Mahwah, NJ: Lawrence Erlbaum.

Foong, P.Y. (2002). The role of problems to enhance pedagogical practice. *The Mathematics Educator*, *6*(2), 15-31.

Hancock, C.L. (1995). Enhancing mathematics learning with open-ended questions. *The Mathematics Teacher*, *88*(6), 496-499.

Hansen, A., Drews, D., Dudgeon, J., Lawton, F. & Surtees, L. (2005). *Children's errors in mathematics: understanding common misconceptions in primary scho*ols, Exeter: Learning Matters.

Hiebert, J., & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), *Conceptual and procedural knowledge: The case of mathematics* (pp. 1-27). Hillsdale, NJ: Lawrence Erlbaum.

Kagan, D. (1992). Implications of research on teacher belief. *Educational Psychologist*, *27*(1), 65-90.

Kilpatrick, J. (2009). The mathematics teacher and curriculum change. *PNA, 3*(3), 107-121.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in schoolchildren*. In J. Teller (Trans), J. Kilpatrick & I. Wirszup (Eds.). Chicago: The University of Chicago Press.

Lajoie, S.P.(1995). A Framework for Authentic Assessment in Mathematics. In T.A. Romberg (Ed.) *Reform in School Mathematics and Authentic Assessment* (pp.19-37.). Albany, State University of New York Press.

Leikin, R., Levav-Waynberg, A., Gurevich, I, & Mednikov, L. (2006). Implementation of multiple solution connecting tasks: do students' attitudes support teachers' reluctance?. *Focus on Learning Problems in Mathematics*. FindArticles.com. 11 May, 2010. http://findarticles.com/p/articles/mi_m0NVC/is_1_28/ai_n26986048/

Leikin, R. (2007). Habits of mind associated with advanced mathematical thinking and solution spaces of mathematical tasks. *Paper presented in the Working Group on Advanced Mathematical Thinking*—CERME-5, Cyprus.

Leikin, R. & Levav-Waynberg, A. (2008). Solution Spaces of Multiple-Solution Connecting Tasks as a Mirror of the Development of Mathematics Teachers' Knowledge. *Canadian Journal of Science, Mathematics and Technology Education*, *8*(3), 233 - 251.

Ma, L. (1999). *Knowing and teaching elementary mathematics: teachers' understanding of fundamental mathematics in China and the United States*. N.J.: Lawrence Erlbaum Asc.

Manouchehri, A. (1998). Mathematics curriculum reform and teachers: what are the dilemmas? *Journal of Teacher Education, 49*(4), 276-286.
Ministry of National Education (MONE). (2004). *İlköğretim matematik dersi öğretim programı ve kılavuzu: 1–5. sınıflar [Trans. Elementary mathematics curriculum and manual: Grades 1–5*]. Ankara: Devlet Kitapları Müdürlüğü.
National Assessment of Educational Progress. (1983). *The third national mathematics assessment: Results, trends, and issues* (Report No. 13-MA-01). Denver, CO: Educational Commission of the States.
National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
Patton, M. Q. (2002). Variety in qualitative inquiry: theoretical orientations. In C. D. Laughton, V. Novak, D. E. Axelsen, K. Journey, & K. Peterson (Eds.), *Qualitative research & evaluation methods*. Thousands Oaks, London:Sage Publications.
Polya, G. (1973). *How to solve it. A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.
Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334-370). New York: MacMillan.
Schoenfeld, A. (1989). Explorations of students' mathematical beliefs and behavior. *Journal for research in mathematics education*, *20*(4)**,** 338-355.
Schoenfeld, A.H. (1988). When good teaching leads to bad results: The disasters of "well-taught" mathematics courses. *Educational Psychologist*, 23, 145-166.
Schoenfeld, A.H. (1983). *Problem solving in the mathematics curriculum: A report, recommendations, and an annotated bibliography.* The Mathematical Association of America.
Silver, E. A., Ghousseini, H., Gosen, D., Charalambous, C., & Font Strawhun, B. T. (2005). Moving from rhetoric to praxis: Issues faced by teachers in having students consider multiple solutions for problems in the mathematics classroom. *Journal of Mathematical Behavior*, 24, 287–301.
Silver, E.A. & Kenney, P.A. (1995). Sources of Assessment Information for Instructional Guidance in Mathematics. In T.A. Romberg (Ed.) *Reform in School Mathematics and Authentic Assessment* (pp. 38-68), Albany, State University of New York Press.
Skemp, R. (1976). Relational Understanding and Instrumental Understanding. *Mathematics Teachin*g, 77, 20-26.
Stenmark, J.K. (1989). *Assessment Alternatives in Mathematics*, EQUALS publications, University of Californioa.
Stigler, J., & Hiebert, J. (1999). *The teaching gap*. New York: The Free Press.
Sullivan, P., & Clarke, D. (1991). The Assessment Implications of Open-ended Tasks in Mathematics. In M. Stephens & J. Izard (Ed.) *Reshaping Assessment Practices: Assessment in the Mathematical Sciences Under Challenges*, Victoria, National Library of Australia.
Tsamir, P., Tirosh, D., Tabach, M. & Levenson, E. (2010). Multiple solution methods and multiple outcomes—is it a task for kindergarten children? *Educational Studies in Mathematics.* DOI 10.1007/s10649-009-9215-z.
Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, *27*(4), 458–477.
Zembat, İ. Ö (2010). A micro-curricular analysis of unified mathematics curricula in Turkey. *ZDM—The International Journal on Mathematics Education. 42(5),* 443-455.

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