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Assessing Teachers' Developing Interpretive Power: Analysing Student Thinking

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A cohort of middle school mathematics teachers in the U.S. participated in a two-year professional development program that focused on developing a deeper conceptual understanding of the mathematics of middle school with connections to instructional practice. We assessed the teachers' developing interpretive power, specifically developing interpretations of student work, which Ball & Cohen (1999) call one of the core activities of teaching. In interviews before and after the first year of the program, we found significant shifts in their capability in anticipating student responses to a task and in the range of pedagogical moves. On further analysis, these shifts were related to positioning – position of a teacher relative to the students (role of the teacher), perception of the position with regards to mathematics (primacy of formal solutions in mathematical understanding) and positioning of students in relation to each other (as pedagogical resources). Here we draw upon four teachers' responses to a situation-specific task in interviews prior to and following a year of professional development and teaching that typify the shifts teachers made in the dimensions of interpretive power.

Traditional teacher education programs in North America treat the domains of mathematics content, mathematics pedagogy, and student thinking separately and, typically, the mathematics content has been provided without deep consideration of its use in teaching. There is, however, growing international interest by mathematics teacher educators in teachers learning in, from, and for practice, where the different domains are intertwined and tied to settings in teaching (Knight, 2002; Nickerson, 2008; Silver, Clark, Ghousseini, Charalambous, & Sealy, 2007; Visnovská, 2007). This is spurred, at least in part, by the challenges teachers face in linking different aspects of this knowledge and in using this knowledge to manage student learning (Putnam & Borko, 2000; Perrin-Glorian, Deblois, & Robert, 2008).

Activities in practice-based teacher education are often focused around collectively analysing artefacts of practice, such as samples of student work and videos or narratives of classroom teaching episodes, thus making the work of teaching available for investigation and inquiry. The belief is that these highly contextualized learning opportunities can support teachers in developing the ability to make the complex judgments required for teaching. If we are interested in teachers' development in the complex judgments required for teaching, it seems important then that an investigation of teachers' learning is explored in situation-specific contexts (Biza, Nardi, & Zachariades, 2007; Hill, Rowan, & Ball 2005; Watson, Beswick, & Brown, 2006; Watson, Beswick, Caney, & Skalicky, 2006/2007).

We engaged middle school mathematics teachers in a professional development program with a central focus on learning mathematics, by necessity, in the context of university mathematics courses. Aware, however, of the challenges teachers face in using this knowledge to support student learning, we constructed the professional development with a focus on making connections to practice. We then assessed the teachers' developing interpretations of student work. We illustrate the dimensions of change in what we are calling their interpretive power by excerpts from interviews with teachers, prior to and after one year of the professional development.

Settings and Goals of Professional Development

The professional development program was designed for teachers who were not considered 'highly-qualified' middle school (lower secondary school) mathematics teachers according to the 2001 U.S. Federal No Child Left Behind (NCLB) legislation. The NCLB legislation requires mathematics teachers demonstrate subject matter competency either by completion of a required number of upper-division mathematics courses or by passing a subject matter exam. Therefore, the professional development work in mathematics and the teaching of mathematics was offered in the form of 12 semester units of mathematics content courses and 3 semester units of mathematics pedagogy.

This was a cohort of practicing middle school mathematics teachers with credentials to teach primary school (multiple subjects) who had taken fewer than six additional junior or senior level mathematics courses. Roughly 40% of the teachers in these schools were second-year teachers. Most participants were first career teachers; a few were undertaking teaching as their second career.

The program encompassed a number of aspects meant to help teachers make connections to practice:

1. mathematics teacher educators from the university [MTEs] modelled instructional practice focused on students' making meaning;
2. teachers routinely reflected on their experience as students in the university mathematics classes;
3. teachers and MTEs engaged in discussions of middle school students' learning in the content area; and
4. teachers had site-based support for trying new pedagogical strategies.

The MTEs from the mathematics department who were teaching the weekly mathematics courses implicitly modelled teaching for understanding and explicitly reflected on aspects of their pedagogical decisions. The pedagogy classes that met once a month were focused around explicit discussion of teachers' experiences as learners in the mathematics classes. Furthermore, MTEs had a focus on how to use knowledge of student thinking to guide instruction.

As recommended by researchers describing effective professional development, teachers at each school site also had opportunities to collectively engage in focused inquiry and refinement of practice in communal settings (Darling-Hammond, 1998; Sowder, 2007). School-site administrators or university MTEs and, occasionally both, led the site-based collaboration. For example, the school-site administrators led content-focused coaching with teachers at their site and led lesson study sessions with teachers across school sites. The university MTEs facilitated lesson-planning sessions of grade-level teachers across sites and teachers' observations of each other's instructional practice. Thus, there were a number of avenues for teachers to make connections between the coursework and teaching.

In sum, the focus of our efforts was to work with middle school mathematics teachers for the purpose of increasing their mathematical content knowledge, and supporting improved practice. Teachers enrolled in university classes spanning two years in which the emphasis was on mathematics and the pedagogy of mathematics lessons. The MTEs modeled and explicitly discussed pedagogical decisions, particular instructional strategies, and orienting instruction around teaching for meaning. Due to the importance of learning connected to practice, the school-site administrators and the university MTEs worked with teachers at their sites to "try on" new instructional practice.

Investigation into Developing Interpretive Power

One common element of all activities was that teachers' instructional practice would be guided by their students' mathematical understandings. Understanding students' mathematical thinking is an important task regularly required of teachers seeking classroom practices responsive to the needs of students (Franke, Carpenter, Levi, & Fennema, 2001). We were interested in how to assess what teachers had learned about interpreting and responding to student understandings.

Biza et al. (2007) give us some insight into what might be learned by engaging in-service teachers in situation-specific tasks. They asked Greek in-service teachers to solve and reflect upon the mathematical objectives of a mathematics problem, examine a flawed fictional student solution, and describe (in writing) feedback to the student. Teachers' responses were analysed along the mathematical, didactical, and pedagogical dimensions of knowledge revealed by engagement with the tasks. The teachers' responses provided insights into teachers' pedagogical aims, their ability to identify the student's error and the cause of the error, and how a teacher might respond to the student.

Watson and her colleagues designed a profiling instrument for chance and data using Shulman's (1987) seven types of teacher knowledge (Watson et al., 2006; Watson et al., 2006/2007). Watson et al. (2005/2006) explored using a variation of the teacher profiling instrument in a middle school numeracy professional development program to measure teacher change prior to and following six calendar days of professional development distributed over 12 weeks. Researchers described three items for which teachers were asked what responses to a posed mathematics task they would expect from their students. They were asked to select an inappropriate response and describe how they would respond pedagogically. The second set of profiles showed increased diversity and detail in teachers' suggested possible student solutions and possible pedagogical responses.

These two studies give us insight into the realm of identifying student errors, valuing student solutions, and the range of pedagogical responses. The studies also looked at shifts as a result of the instructional intervention. We conducted multiple interviews to examine whether a teacher's response to a classroom-specific scenario would help us construct an image of teachers' interpretation of student thinking at the beginning of our work with them, as well as provide evidence of change in one of the core activities of their practice.

In our study, we interviewed 12 teachers with the intention of examining the dimensions along which teachers' interpretations of student work were affected by their participation. We conducted semi-structured interviews in which we asked teachers to interpret student work in order to provide insight into what they noticed and the instructional strategies teachers thought would be appropriate. The student work on which they reflected was embedded in classroom scenarios that were part of their everyday classroom routines. Significantly, we asked them to look across multiple student solutions—either by examining several students' solutions to one problem or examining several solutions of one student.

Description of the Interview

An analysis of student learning for the purpose of making instructional decisions requires an understanding of the landscape of mathematical content domain, an understanding of the pedagogical stages (including a recognition of student difficulties), and the ability to identify instructional tools and strategies to extend students' thinking. During the interview, the teachers engaged with four situation-specific tasks. Three mathematical tasks were drawn from 6th, 7th, and 8th grade textbooks that the teachers were using. The teachers were told that the problems originated in the classroom setting, such as in homework (in the case of multiple solutions of one student) or in the course of launching class work (in the case of multiple students' solutions).

The strategy, *Launch, Explore, and Summarize* describes the instructional segments in mathematics classes used throughout the teachers' school district. The *Launch* refers to the beginning of the class in which the teacher creates an *anticipatory set* (Hunter, 1982) by gathering important information about knowledge and skills that are prerequisite to the new concept. The model of this usually involved the teacher posing a problem then circulating among groups of students, asking questions, and preparing to start the *Explore* portion of the lesson. The *Explore* portion is an opportunity to develop the concept through discussion of the *Launch* problem or to solve a new task. Finally, the teacher is encouraged to use the *Summary* portion of the lesson to bring together connected ideas that have been explored, and summarize the concepts.

The tasks situated in the whole class scenario asked teachers to consider a range of student examples as if they were observing students' work in the *Launch* portion of the lesson and to consider how their observations might guide the *Explore* portion of the lesson. For both the homework scenario and the whole class launch scenario, the interview questions had the general structure shown in Table 1.

We chose interviews because, although not a completely authentic recreation of assessment practice, an interview provides insights into the criteria teachers bring to bear and the range of possible responses (Morgan & Watson, 2002). We recognize interviews provide a more reflective environment for responding than a classroom situation. Though the teachers were teaching different grade levels, for lower secondary we chose pre-algebra topics.

Table 1
The structure of the situation-specific interview tasks

I. Knowledge and view of the mathematics 1. The teacher solves the problem. 2. The teacher explains the mathematical big ideas.
II. Anticipating and interpreting children's thinking 1. The teacher anticipates students' responses. 2. After being shown multiple student responses, the teacher explains what each student's work tells them about the student's understanding of the mathematics.
III. Proposing pedagogical moves 1. The teacher articulates questions they would ask to help the student reflect on the strategy. 2. The teacher suggests how this helps them make pedagogical decisions.

To illustrate our findings, we have drawn from interviews with four teachers whose change typified what we saw as the dimensions of change. The participants of focus include two females and two males. They are of diverse ethnicities: Hispanic, African-American, and Caucasian. Three of the teachers were in their second year of teaching; the fourth teacher had four years of teaching experience. Two of the participants were beginning teaching as a second career. Their backgrounds, as well as their responses, were representative of the 12 interviewed teachers.

Data and Methodology

The teachers were interviewed in a semi-structured format (Goldin, 2000) before and after the first year of the professional development program. The interviews were transcribed and coded by two researchers with the assistance of a graduate student. There were two phases of our analysis. In the first phase of the analysis, we began with open coding (Strauss & Corbin, 1990) of individual teachers' interviews on a case-by-case basis, based on categories structured by the format of the interview. The interviews were coded according to the levels of proportional reasoning (Rubenstein, Beckmann & Thompson, 2004) as they applied to the teacher's understanding of the mathematics, the teacher's description of a student's thinking, and the methods/questions the teacher would use to extend a student's thinking. In a second phase of analysis, we looked for patterns in the interviews to see if there were consistencies across pre-interviews or post-interviews and sought trends across interviews. Using the results of the first two phases of analysis we generated a list of hypothesized strands of change. We returned to the interviews to test this analysis. *Positioning* emerged from the analysis of teachers' responses to situation-specific tasks in interviews.

Results

We found the situation-specific tasks in the interview were a powerful mechanism for revealing teachers' developing interpretive power. As articulated earlier, the goals of the professional development centred on supporting the development of instructional practices that drew on their knowledge of mathematics, how they used this knowledge to guide instructional decisions, and included a focus on organizing instruction around teaching for meaning. We identified three dimensions of change in a teacher's position:

1. view of the teachers' role in supporting student learning;
2. perception of what it means for a student to understand mathematics; and
3. differentiation of instructional strategies for moving students along the trajectory.

One task, *Tom's Bike* problem, is used as an illustrative example. (See Figure 1.) The student responses were chosen to represent a range from informal to formal and encompassed algebraic and arithmetical structures (Van Dooren, Verschaffel & Onghena, 2002).

View of the Teacher's Role

In the interviews, teachers were asked to anticipate student responses to the mathematical problem. In the first interview, many teachers seemed to struggle with answering the question from the perspective of what the student might know. There were two types of responses to the questions in the initial interview. One type of response was a description of where students might have difficulty. For *Tom's Bike* problem, teachers suggested that students would have difficulty translating the story problem to mathematical symbols and many cited students' difficulties with fractions.

The second type of response reflected how the teacher would direct student responses, rather than anticipating them. To a question about how a student might solve the problem, even after having been pressed, some teachers spoke only of what a teacher should do. For example, in the first interview, Charles repeatedly used the phrase "I would have them… ", for example:

> I would have them draw it out. I would have them look at the words. Underline what they are asking for… I would have them look at the things like Mike travels 1/2 mile to the lake and then 3 times around the lake.

Another teacher, Lexi, in the first interview also answered from the perspective of the teacher's actions. She talked about herself as a student and what scaffolding she would need in order to solve the problem.

> Because I've sat there as a student and said "I know they want me to figure this out, but I've never even thought about this before. I have no idea at all. Can't they give me something?" And so, that's why I give 'em a picture, and maybe it might be too much scaffolding that I do, but I've just been in that position and I don't like it at all.

Tom bikes $1\frac{1}{2}$ miles to the lake. He bikes 3 times around the lake and then bikes back home. He travelled a total of 21 miles. How far is it around the lake?

Marissa

$$1\tfrac{1}{2} + 3x + 1\tfrac{1}{2} = 21$$
$$3 + 3x = 21$$
$$-3 \qquad\quad -3$$
$$\frac{3x}{3} = \frac{18}{3}$$
$$x = 6$$

Mike

$$1\tfrac{1}{2} + 2 + 2 + 2 + 1\tfrac{1}{2} = 9$$
$$1\tfrac{1}{2} + 4 + 4 + 4 + 1\tfrac{1}{2} = 15$$
$$1\tfrac{1}{2} + 6 + 6 + 6 + 1\tfrac{1}{2} = 21$$

Margot $1\frac{1}{2}m + 3m + 1\frac{1}{2}m = 6m$ ☆ 6 miles

Marcus

Maria

$$21 - 3 = 18$$
$$18 \div 3 = 6$$

Figure 1. Students' solutions to one situation-specific task, Tom's Bike problem

In the second interview, all teachers were able to articulate possible student responses, sometimes only one and sometimes many. The striking aspect from our perspective was the shift in position of the teacher's role from what the teacher should or could do to how the students might respond. Note the language in an excerpt from the second interview with Charles:

> Um, I think they might draw a picture and label the parts of it. (pause) Label the trip there, the trip back, [and] draw circles around…[to represent going around the lake]. They might miss that they are going back home that is included in there.

We cast this as a shift in positioning–position of a teacher with respect to students. The teachers had shifted from what the teacher would do to focusing on student understanding.

Perception of What it Means to Understand

In the first interview, teachers valued algebraic solutions but more importantly thought this was an indication of student understanding of the problem. They requested very little additional justification if the solution was formal. Teachers signalled they would press students with informal solutions to make connections to formal solutions. In the second interview, teachers were more inclined to accept informal solutions and to press for explanations from those that produced formal or algebraic solutions.

Sarah's response in the first and second interview typifies teachers' changes with regards to the question about what the solutions show about students' understanding. In the first interview, she simply said that Marissa understood and "knew the answer." When asked about questions to extend student thinking, she posed questions for every student except Marissa. Furthermore, she indicated she would press Mike to move away from informal solution strategies. Sarah said, "Maybe ask him [Mike] why the 2, 2, 2? Was he guessing? Is there an easier way to write 2… You know, like repeat it … instead of repeated addition, writing that as a multiplication?"

By the end of the academic year, Sarah has a focus on what Mike does understand. She said,

> He [Mike] understands that there has to be one-and-a-half on both sides and that the three times around the lake has to be the same term. It's a guess-and-check strategy and he's successful, so I think it's a good one. … it works for him and I think it's a good strategy because he understands all the numbers that play into the problem.

Like other interviewees, Sarah shifted in the second interview to questioning Marissa:

> This is the traditional way that most books want to see it. Maybe just to explain their thinking and to label some of the things. Maybe this one [Marissa] I would ask to draw a diagram, maybe just to show visually what they did.

This is indicative of what we cast as a shift in position with respect to mathematics. The interviews provided evidence of a shift away from the view that the formal algebraic solution is a transparent indication of understanding.

Differentiating and Choosing Instructional Strategies

After the teacher gave an assessment of what students understood, the question that was asked was, "What would be your next steps?" During the first interview, when they could, teachers formulated questions for individual students; these suggested one-on-one dialogue. During the second interview, many teachers suggested next steps that involved pairing up students to share ideas, how they could learn from one another, and then suggested having a final class discussion tying student work into the algebraic equation that was provided by the student. Alex in the first interview suggested enacting the word problem, then proceeded to describe each student and the support required. Here is one example:

> Well, I guess for Marcus, I would ask him questions like how far did it take for you to get to the lake from the house. Just by looking at his diagram I knew that he did not label the length of the trip there and on the way back. I would ask him, "If you were Tom here, and you traveled your bike around this diagram that you drew and you went 3 times around what was the total number of miles traveled?" I would also ask him, "Does the trip there and back have anything to do with the trip around the lake?"

In the second interview, Alex described the students as resources for each other.

> I would think I would pair with some of these kids together and share their ideas. Margot with Marissa. Well, I think they were both kind of on the same line as far as coming up with that. To put it in an algebraic equation. And Mike and Marcus were more of the visual kind of a thing. And also because these two got it right. Then they could share ideas to it. And, of course, we would look at that as a whole and have Marissa or Mike, or Margaret, after they discussed it with their partners, share it with the class. How to come up with…try to connect this with the algebraic equation.

We characterize this as a shift in positioning of students with regards to other students. Teachers' interpretation of the students and their thinking has expanded to include students as pedagogical resources. The above examples are glimpses from interviews that provide evidence of the changes in the teachers over the first year of the professional development program.

Discussion

The assessment of teachers' interpretive power gave us insights into the teachers when our work with them began. The interviews also provided a means of documenting changes in individual teacher's knowledge of mathematics, student thinking in pre-algebra topics, and the increasing range of pedagogical moves they see as appropriate. Furthermore, the interview gave us an opportunity to view what the teachers valued. We found that the situation-specific tasks elicited numerous aspects of teacher's changing instructional practice. The use of scenarios that involved several students' work situated in a context that was part of their daily lives gave us a window into the shift toward more whole-class pedagogical moves that would have been absent in an analysis of individual student solutions.

Our analysis of the interviews suggests that there were changes along three dimensions: 1) view of the teacher's role in supporting student learning 2) perception of what it means for a student to understand the mathematics, and 3) differentiation of instructional strategies for moving students along the trajectory. We cast these shifts as shifts in positioning: position of a teacher relative to the students (role of the teacher), perception of the positioning of the student with regards to mathematics (primacy of formal solutions in mathematical understanding), and positioning of students with regards to each other (as pedagogical resources). Our next analysis will include the integration of observation, reflection, and class field notes to provide a more robust picture of change.

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