Reducing the Mismatch of Geometry Concept Definitions and Concept Images Held by Pre-Service Teachers

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Abstract

Twenty-three female elementary pre-service teachers were assessed on their ability to answer questions involving geometry concepts. Despite being given the definitions of the altitude of a triangle and the diagonal of a polygon on the pretest, limited understanding of these concepts was evident. A treatment using both graphic organizer and concept attainment teaching strategies improved performance on the posttest but not to the level required for teachers to supplement textbooks that lack adequate definitions or non-prototypical examples. Careful attention to challenging geometry concepts is needed so that pre-service teachers will be able to provide their students with improved conceptual understanding.

Key Words: Concept Image, Geometry, Mathematics Teacher, Pre-service Elementary Teacher

Introduction

Definitions of geometry concepts and examples illustrating them are invariably incorporated as part of instruction by teachers and appear in textbooks designed to enhance student understanding. Yet a limited understanding of geometry concepts exists among students (Fuys, Geddes, & Tischler, 1985; Hershkowitz, 1987; Vinner & Hershkowitz, 1980) and elementary pre-service teachers (Gutierrez & Jaime, 1999; Hershkowitz, 1987; Mason & Schell, 1988; Mayberry, 1981).

This limited understanding of geometry concepts includes what Vinner and Hershkowitz identify as the mismatch between formal concept definitions and students' concept images (1980). Because of how concepts are traditionally taught, a student may be encouraged to memorize a definition, called the concept definition. On the other hand, when in the process of trying to recall a concept, it is not the concept definition that comes to mind. A student typically remembers prior experiences with diagrams, attributes, and examples associated with the concept, instead of the concept definition. All of these experiences embody the concept image (Gutierrez & Jaime, 1999). The concept image held by some students can be limited to a single prototypical image, and an over-reliance on it can impact their understanding (Vinner & Hershkowitz, 1980).

Another impediment to student understanding of geometry concepts may be due to the textbook, which teachers noted as the "most commonly used resource" (Kajander & Lovric, 2009). Some textbooks provide inadequate definitions and a restricted number of examples, making them deficient (Hershkowitz, 1987; Vinner & Hershkowitz, 1987). Although teachers frequently "supplement what they see as inadequacies in the text" (Love & Pimm, 1996), many teachers leave what they believe to be well enough alone since a textbook "organizes the mathematics curriculum… [and] it also organizes the work of the classroom" (Love & Pimm, 1996). Hershkowitz (1987) found that "The textbook and the teacher present mostly the prototypes". Hence, confusions that occur among many students "are not [always] stupidity on the part of the students" (Tall, 1988). Teaching and learning with a deficient textbook is a concern since it may further impede students' conceptual understanding.

Some of the same difficulties experienced by students (Fuys et. al, 1985; Hershkowitz, 1987; Vinner & Hershkowitz, 1980) have also been shown to be a challenge for pre-service teachers (Gutierrez & Jaime, 1999; Hershkowitz, 1987; Mason & Schell, 1988; Mayberry, 1981). Researchers have reported that "pre-service elementary teachers do not possess the level of mathematical understanding necessary to teach elementary school mathematics as recommended in various proclamations from professional organizations such as [National Council of Teachers of Mathematics] NCTM" (Brown, Cooney & Jones, 1990). Mason and Spence attribute some of the difficulties in understanding mathematics with the tendency to compartmentalize rather than to form connections which is what is needed for the highest level of knowledge referred to as *know-to act* (1999). Consequently, elementary pre-service teachers may lack the level of mathematics knowledge (Brown et al., 1990) needed to supplement textbook deficiencies. Because of the primary role played by geometry textbooks in conceptual understanding, further study was warranted.

Textbook Deficiencies

Motivation for this study was partially prompted by a noted inaccurate textbook definition of the concept of the altitude of a triangle. Several studies have pointed to deficient textbook treatment of this concept (Hershkowitz, 1987; Kajander & Lovric, 2009; Vinner & Hershkowitz, 1980). Some textbooks provide an inadequate definition of altitude by excluding the possible need to extend the opposite side in an obtuse triangle. Other textbooks fail to include examples that showcase "special cases" (Kajander & Lovric, 2009), by illustrating for readers only prototypical examples (Hershkowitz, 1987; Vinner & Hershkowitz, 1980) that may hinder conceptual understanding. Since textbooks are such an important resource for the classroom teacher (Kajander & Lovric, 2009; Love & Pimm, 1996), an examination of a current sample of geometry textbook treatments was conducted.

A sample consisting of 12 textbooks used in public schools in central New Jersey were examined for definition accuracy and for the presence of prototypical and non-prototypical examples of altitudes of triangles. To be categorized as adequate, the definition must have included the possible extension of the opposite side. A definition such as "an altitude of a triangle is the perpendicular segment from a vertex to the line that contains the opposite side" (Jurgensen, Brown & Jurgensen, 1994) was regarded as adequate. A textbook's examples were classified as adequate if it included the non-prototypical obtuse and right triangle examples, in addition to the prototypical acute triangle.

Of the 12 textbooks examined, 5 were categorized as not having an adequate definition or not having adequate examples, or both. This result confirms findings in previous studies (Hershkowitz, 1987; Kajander & Lovric, 2009; Vinner & Hershkowitz, 1980). A table containing the list of textbooks and results is contained in Appendix A. These results suggest that classroom teachers must possess a robust concept image of the altitude of a triangle in order to supplement the varying quality of textbook definitions and examples.

Limited Conceptual Images

Regardless of the textbook, even when students are provided with adequate concept definitions during instruction, many exhibit limited understanding and the inability to apply concepts outside of those involving prototypical examples. Hershkowitz notes that "many teachers are themselves not familiar with an 'outside' altitude or an altitude which coincides with one of the sides" (1987). Gutierrez and Jaime discuss altitudes with regard to pre-service teachers (1999) and expand on Vinner's model of concept image and concept definition (Vinner & Hershkowitz, 1980). They point to the mismatch that typically exists between the textbook definitions of the altitude of a triangle and the concept images held by elementary pre-service teachers (Gutierrez & Jaime, 1999). Consequently, the presentation of the concept of altitude to pre-service teachers, some of whom have garnered limited concept images, requires careful attention.

Besides the particular challenge presented when teaching and learning the concept of the altitude of triangle (Gutierrez & Jaime, 1999; Hershkowitz, 1987; Vinner & Hershkowitz, 1980), other concepts such as straight angles (Vinner & Hershkowitz, 1980) and isosceles triangles (Hershkowitz, 1987) have also been reported as problematic. Bassarear (2008) conjectured that the same might be true for the diagonal of a polygon, and pre-service teachers' understanding of the concept of the diagonal of a polygon has not been studied. When one considers concave polygons, the diagonal can fall "outside" much like the altitude of an obtuse triangle. Moreover, the diagonal of a polygon can be presented in the non-prototypical horizontal or vertical positions. Nevertheless, limited concept images held by pre-service teachers of concepts like the altitude of a triangle and other challenging concepts will impact their ability to supplement textbook deficiencies and to successfully teach such concepts to their future students.

Framework for Teaching and Learning

Beyond textbook considerations, teachers are also confronted with a choice of what role they will assume in the classroom, as well as, of what type of instruction they will use, to present concepts. As for roles, teachers can be instructors and transmit information to students; they can be explainers and lay out information for students; or they can be facilitators and teach concepts through problem solving approaches (Ernest, 1989). In terms of types of instruction, a classroom focus can be on "traditional chalk-and-talk learning [or] discovery learning" (Claxton, 1997). An instructional approach that "involves passive learning" is categorized as traditional (Eggen, Kauchak, & Harder, 1979) whereby the teacher provides students with the information. The discovery approach, known as "information processing" (Eggen et. al, 1979) was pioneered by Joyce and Weil in 1972. Its focus is on the invention or construction of new knowledge.

Even though a traditional teaching approach allows for an instructor or explainer to present an accurate concept definition, it is not the definition but rather the student's concept image that is called upon when the student is asked about the concept (Gutierrez & Jaime, 1999; Vinner & Hershkowitz 1980). When figuring out "what kind of instruction could best enable teachers and students to develop a complete concept image" (Gutierrez & Jaime, 1999), "information processing**"** (Eggen et. al, 1979), or "discovery learning" (Claxton, 1997), might outweigh a more traditional approach because of its potential to impact concept image. Therefore, a facilitator using a problem solving approach (Ernest, 1989) might allow for the construction of a complete concept image needed by students.

Sfard suggests that teaching strategies are often most efficient when combined with others in meeting the needs of a variety of learners (1998). "Educational practices have an overpowering propensity for extreme, one-for-all practical recipes…Because no two students have the same needs and no two teachers arrive at their best performance in the same way, theoretical exclusivity and didactic single-mindedness can be trusted to make even the best educational ideas fail" (Sfard 1998). To prevent such failure, Gay (2008) proposes that fusing two nontraditional teaching strategies, graphic organizer and concept attainment, might strengthen the connection between vocabulary, or formal concept definitions, and teachers' concept images. These strategies may improve elementary pre-service teachers' conceptual understanding since they incorporate "discovery learning" (Claxton, 1997) and accommodate the problem solving facilitator described by Ernest (1989).

The goal for using the strategies would be for the pre-service teachers to achieve the highest level of knowledge described by Mason and Spence as being able to "*know-to act* in the moment…in order to engage in problem solving…where context is novel and resolution non-routine" (in Even & Tirosh, 2008). These strategies might engage them in focusing on non-prototypical diagrams and examples to providing experiences needed for robust concept image development. Further, both teaching strategies integrate a problem solving approach that requires students to create their own definition which might reduce the mismatch between concept definitions and their concept images.

Graphic Organizer: The graphic organizer strategy has been influenced by three models. First is the Frayer model which is attributed to Frayer, Frederick, and Klausmeier (in Greenwood, 2002). Its creators incorporate "relevant and irrelevant attributes, examples and non-examples, [and] supraordinate, coordinate, and subordinate aspects of concepts" (in Greenwood, 2002). Greenwood presents an example of the Frayer model by drawing a 2x2 cell grid that includes the term, its attributes, examples, and aspects, each in one cell of the grid (2002).

A second influential model is the "verbal and visual word association" model (Readence, Bean, & Baldwin, 2001). Readence et al. include an example of the model that includes a 2x2 cell grid to teach the word *salubrious*. The four cells of the grid contain the "word…definition…personal association for the word… [and] a word that describes something you do or something you experience that is not [the word]" (2001).

The third model that influences the graphic organizer teaching strategy incorporates Henderson's (1970) three approaches to teaching a concept. Henderson notes that when teachers "talk about the properties or characteristics of objects named by a term" they are showcasing the connotative use of the term (Henderson, 1970). The denotative use of the term is conveyed when teachers present examples and non-examples to students. Lastly, when teachers give a definition, they implicatively introduce the term (Henderson, 1970). A graphic organizer strategy that integrates the three models might be employed to improve pre-service teachers' concept images.

*Concept Attainment**:*** Concept attainment, another teaching strategy, is an "inductive teaching model" (Eggen et al., 1979; Joyce et. al, 2004). Its roots appear in Bruner, Goodnow, and Austin's work from 1956. It was expanded on by Joyce and Weil in 1972 (Eggen et. al, 1979; Joyce et. al, 2004). "Concepts are to be understood as basic units of knowledge that can be accumulated, gradually refined, and combined to form even richer cognitive structures" (Sfard, 1998), which essentially is the concept attainment model. In this model, the teacher "provides examples and non-examples of the concept, and the students determine the concept from the examples" (Eggen et. al, 1979). Students formulate a definition for the concept using only the given examples and non-examples. Additional pairs are shown and students revise and create new definitions after each additional pair is presented (Eggen et. al, 1979; Joyce et. al, 2004). The revisions students make to their own definitions, after each new pair is revealed, is the key component of this model.

Concept attainment requires effective planning to implement effectively. It may be the reason that Henderson notes that, even though there are many "laboratory experiments where the subjects learn to classify objects by practicing with examples and non examples…in the formal education of the secondary school, concepts are infrequently taught in this way" (Henderson, 1970). The order in which pairs of examples and non-examples are presented, the number of pairs presented, the attributes the elementary pre-service teachers are exposed to in each pair, and the implementation of the lesson (Eggen et al.; Joyce et al., 2004) are all components of this strategy that need to be carefully planned.

Furthermore, the examples must contain defining characteristics of the concept (Eggen et al.,1979; Joyce et. al, 2004) while the non-examples should "establish concept boundaries and limits" (Eggen et. al, 1979). The pairs of examples and non-examples need to be developed so that they push students towards confronting the confusions and misconceptions that exist in their conceptual images. This relates to Tall's theory of cognitive conflict "To create actual cognitive conflict, it is necessary for that individual to evoke two mutually conflicting factors simultaneously" (1980). This dissonance within their knowledge might assist in narrowing the mismatch experienced by pre-service teachers.

Combining Strategies: As mentioned, several geometric concepts have been regarded as a challenge in the literature and the graphic organizer could be used to "develop a better conceptual understanding of mathematical words" (Gay, 2008). And, although there is difficulty in devising and implementing the concept attainment model, Eggen et al. note that it is "especially effective" (1979) when introducing challenging concepts. Also, concept attainment contains what Tall deems as necessary for forming strong concept images that teachers should aim to "give the students richer experiences so that they are able to form a more coherent concept [which] is not as easy as it sounds, as it involves a balance between the variety of examples and non-examples necessary to gain a coherent image" (1988).

Employing instruction that uses both the graphic organizer and concept attainment strategies as suggested (Gay, 2008) could bring about enhanced understanding for pre-service teachers. Possibly even the *Know-to act* in the moment level which is most effectively accomplished by "…labelling experiences in which powers have been exhibited, and developing a rich network of connections and triggers so that actions 'come to mind'."(Mason & Spence, 1999). This might allow them to be more effective teachers capable of dealing with the novel and often non-routine responses offered by students. Overall, this might reduce pre-service teachers' mismatch between concept definition and concept image and provide a level of understanding needed to supplement inadequate textbooks and to evaluate and correct student errors involving these concepts.

Research Questions

The purpose of this study was to answer two questions about the conceptual understanding of geometry concepts by elementary pre-service teachers. How many in a sample of pre-service teachers are able to correctly answer questions on geometry concepts, including those commonly identified as challenging in the literature such as altitudes of a triangle, but also the lesser studied diagonals of a polygon when the concept definitions are explicitly provided? How many in a sample of pre-service teachers, who have been exposed to instruction employing the graphic organizer and concept attainment strategies, are able to identify and correct common student errors involving these concepts?

Method

Participants **:** The sample consisted of twenty-three female elementary pre-service teachers enrolled in a highly competitive eastern U.S. state college. Each had volunteered and they were either first or second year students. They were members of an intact mathematics content class that met twice a week for 80 minutes over 14 weeks that is designed specifically for elementary teachers who are not mathematics majors. It is the second in a two-course sequence; the first of which focuses on the development of number systems, algebraic structures, and algorithms, while the second focuses on concepts of geometry, measurement, data analysis, statistics, and probability. The treatment lessons replaced the standard presentation of geometry concepts required in the curriculum. One of the authors was the instructor for the course, and the other author administered the treatment during one regularly scheduled 80-minute classes.

Instruments: A pilot for the pretest instrument was administered to 15 high school students and a revised version to 57 college students from an eastern community college. The resulting instrument consisted of 4 questions concerning parallel and perpendicular lines and non-adjacent vertices, followed by two three-part questions involving the altitude of a triangle and the diagonal of a polygon. For both of these questions the concept definition accompanied the questions. A copy of the pretest instrument is contained in Appendix B.

A posttest was developed which consisted of 3 questions on parallel and perpendicular lines, and non-adjacent vertices. These were followed by two three-part questions similar to the pretest involving the altitude of a triangle and the diagonal of a polygon. However, the definition was removed and context was added to the instructions. The instructions stated that each item showcased a student response and the participant was to evaluate the response as if they were a teacher. The participants were required to determine if the given response was correct and if not, to provide a reason and a correct response. While the student responses were fictitious, the errors resembled those collected from the piloted pretest instrument. A copy of the posttest instrument is contained in Appendix C.

Procedure: A one group pretest-posttest design was utilized in conjunction with one treatment lessons one month after the pretest was administered. During the lesson the participants were introduced to the two teaching strategies, graphic organizers and concept attainment while they sat in groups of 4-5 students per table. A projector and document camera were utilized to showcase their contributions throughout the lesson.

In the first part of the lesson the four parts of the graphic organizer - - the word, characteristics, definition, and example(s) - -, were explained and it was used to introduce the concepts: parallel lines, perpendicular lines and adjacent vertices. These last two topics were considered critical to the concepts of altitude and diagonal. Each topic was presented using the graphic organizer and the form was filled in as a whole class activity. A model form used for a graphic organizer appears in Figure 1 below.

Figure 1: Model form for the graphic organizer

Word	Characteristics
Definition	Examples(s)

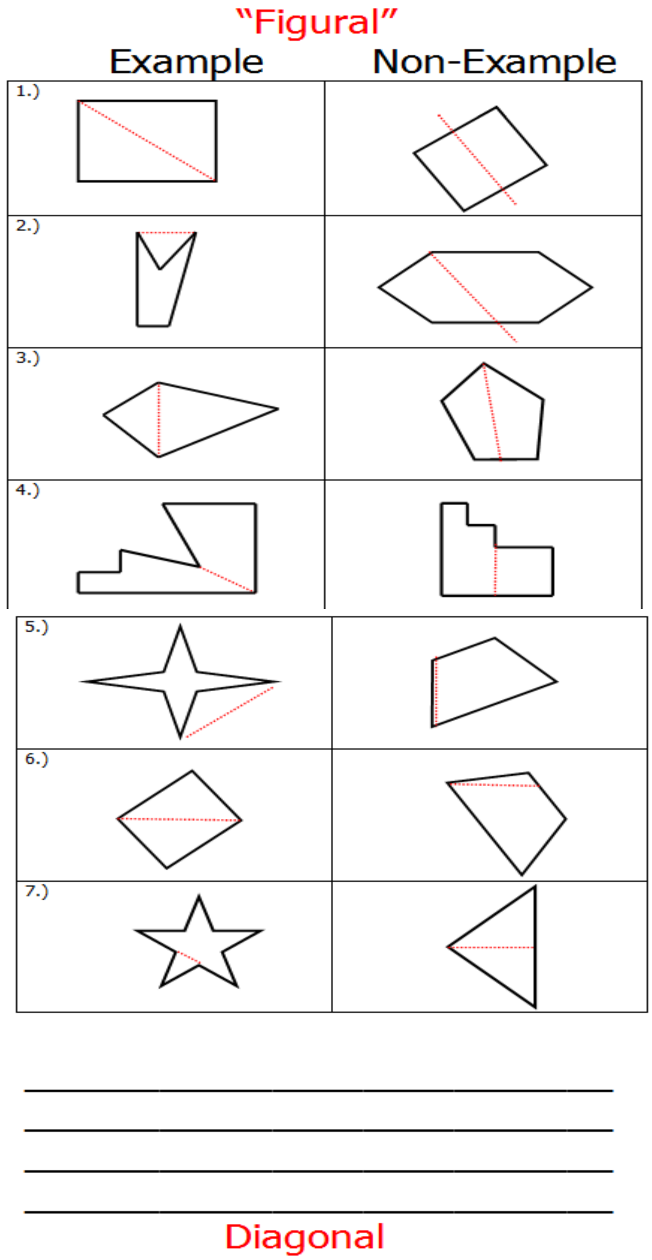
Subsequently, the concept attainment teaching strategy was used to introduce the altitudes of triangle, medians of a triangle, angle bisectors of a triangle and the diagonals of polygons. The participants were shown two diagrams on the document camera (an example and a non-example of a concept) and then given the task of creating a definition that supported the pair. Each group's definition was written down on an index card and given to the facilitator. The facilitator then chose one of the group's definitions to showcase on the document camera. The students were then shown a new set of examples and non-examples and asked to revise their definition accordingly. The facilitator repeated this process until each group generated an adequate definition. Besides creating their own definitions, they verified them as well. Two weeks after the treatment, the posttest was administered. A sample list used for concept attainment for the diagonal of a polygon appears in Figure 2 below.

Results

The pretest results indicate that the majority of these elementary pre-service teachers could correctly draw perpendicular lines and circled non-adjacent vertices. These concepts are critical components of the concept of altitude of a triangle and the diagonal of a polygon, respectively. Yet despite these abilities, and the fact that the definitions of the altitude and diagonal accompanied the items, the non-prototypical altitude of a right triangle (altitude coincides with a leg of the triangle) and the non-prototypical diagonal of a concave polygon (diagonal falls outside of polygon) resulted in 13 and 11 correct responses out of 23, respectively.

On the posttest, the number of students who gave correct responses increased on the non-prototypical item of the altitude of a right triangle which went from 13 to 19 and on the diagonal outside the polygon from 11 to 16 correct responses. However, the item that showed the largest change was the non-prototypical altitude of an obtuse triangle. The number of correct responses decreased from 19 correct to 8. These results appear in Table 1 below.

Figure 2: A sample list of examples and non-examples of the diagonals of polygon.



Discussion

The pretest results confirm that geometry concept of the altitude challenged these non-mathematic elementary pre-service teachers as reported in the literature (Gutierrez & Jaime, 1999; Hershkowitz, 1987; Vinner & Hershkowitz, 1980; Mason & Schell, 1988; Mayberry, 1981), even when the concept definition appeared right next to the item. The results involving altitudes were similar to those reported by Vinner and Hershkowitz's which provided a basis for the lack of conceptual understanding when they found "that less than half of the students succeeded even in the easy items of the [altitude] questions" (1980). The pretest highlighted the challenge of the special case or non-prototypical altitude, that of a right triangle when the altitude coincides with the leg.

Table 1: Number of pre-service teachers giving correct responses (n=23) on each item.

Number of correct responses on:	Pretest	Posttest
Perpendicular line segments (prototypical)	22	22
Perpendicular line segments (non-prototypical)	20	23
Adjacent and non-adjacent vertices	22	21
	Pretest	Posttest requiring corrections
Altitude of an acute triangle (prototypical)	19	22
Altitude of a right triangle (non-prototypical)	13	19
Altitude of an obtuse triangle (non-prototypical)	19	8
Diagonal of a polygon (prototypical)	21	23
Diagonal of a polygon-horizontal (non-prototypical)	19	23
Diagonal of a polygon-outside (non-prototypical)	11	16

Diagonals of a polygon also surfaced as a challenge on the pretest for these pre-service teachers. This was conjectured by Bassarear, who had noted the difficulties when he mentioned a student's apprehension of accepting a horizontal line segment in a hexagon as a diagonal (Bassarear, 2008). It was not so much the non-prototypical horizontal case but rather the non-prototypical outside diagonal that proved problematic. Despite the concept definition being given and the students' demonstrated ability to label non-adjacent vertices, the results pointed to the barrier presented by the non-prototypical outside diagonal of a concave polygon.

While the pretest focused on the conceptual understanding of altitudes and diagonals, the posttest also require respondents to not only recognize but also correct common student errors. A level of conceptual understanding that would be required by these teachers when they have their own classroom. The treatment lesson involving the graphic organizer and concept attainment strategies resulted in some improvement in the number of pre-service teachers making correct responses on the posttest.

Yet, the number of correct responses involving the altitude of an obtuse triangle showed a marked decreased. This decrease is attributed to the fact that the pretest questions on altitude required only the conceptual understanding to recognize and state that the given line segment was an altitude of an obtuse triangle. On the other hand, the posttest question required a higher form of conceptual understanding on a level of the "*Know-to Act* in the Moment" suggested by Mason and Spence (1999) to recognize that a given line segment was not an altitude, give a reason, and then to draw the correct one. Only about one-third of the pre-service teachers demonstrated success in knowing how to correctly draw an outside-altitude of an obtuse triangle or to even recognize that a line segment drawn from a vertex to the opposite side that was not perpendicular to it was, indeed, not an altitude. They verified that their level of conceptual understanding was not sufficient to assess and correct student work on this non-prototypical altitude of a triangle.

Conclusion

Results from this study pointed to the weak conceptual understanding for some pre-service teachers and their need for more than just "passive" or traditional learning (Eggen, Kauchak, & Harder, 1979); learning where the definitions are just presented. This was indicated by the pretest items where the concept definition was explicitly given and yet the response was incorrect. This indicates that memorizing the concept definition does not guarantee success and that student difficulty may not be completely attributed to their not knowing the definition.

In an effort to strengthen their conceptual understanding a treatment lesson was developed where the teacher would not just be an Instrumentalist/Instructor and only transmit the concepts to students but rather a Problem Solving/Facilitator teacher where students form the concepts as the teacher facilitates (Ernest, 1989). Combining the graphic organizer and concept attainment strategies in the treatment lesson conformed with Sfard's recommendations to avoid "theoretical exclusivity" and enact the Participation model (1998). In addition, the treatment lesson was designed and delivered according to Ernest's recommendations and those of Sfard "Participation in certain kinds of activities rather than in accumulating private possessions" (1998).

After this treatment, the posttest results revealed some improvement in the number of pre-service teachers' correct responses but for others it failed to elevate their understanding to the *know-to act* level of understanding needed to recognize and correct student errors, especially with the outside altitude of an obtuse triangle. It remains to be studied why the treatment was not enough to promote a sufficient level of conceptual understanding for these pre-service teachers. But among other factors, it is likely that the limited experience with the two teaching strategies, and the order and number of pairs of examples and non-examples could be improved to provide participants with a more robust and coherent concept understanding (Tall, 1988). Increasing the number of treatment lessons might also yield improved results but such a focus would necessitate the exclusion of other required topics.

Teachers, especially if confronted with unfamiliar material, may lean on textbooks that have inadequate definitions and only present prototypical examples. This study highlighted the challenge facing pre-service teachers who may have limited conceptual understanding as they attempt to teach and promote student understanding. These teachers need to be aware of possible textbook limitations, especially with challenging topics like altitudes of triangles and diagonals of polygons, so that they might be aware of the need to supplement them and provide students with adequate definitions and sufficient examples.

In summary, this study advocates that teaching challenging geometry concepts to pre-service teachers needs careful attention so that the mismatch between concept definitions and their concept images may be minimized. And, only teaching and learning that goes beyond instrumentalism, and promotes participation, will allow them to develop robust concept images. Such concept images are needed to allow them to supplement inadequate textbooks and to provide their students with the support and direction needed so they can develop strong conceptual understanding of geometry concepts.

Appendix A

Textbook definitions of altitudes of a triangle and analysis of examples

Definition of Altitude	Adequate Definition	Adequate Examples
The **altitude** of a triangle is the perpendicular segment from any vertex of a triangle to the opposite side or extension of that side (Stein, 1968, p. 329).	Yes	No
An **altitude** of a triangle is a segment from a vertex of the triangle, perpendicular to the line containing the opposite side of the triangle (Nicholas, Edwards, Garland, Hoffman, Mamary, Palmer, 1986, p. 169).	Yes	Yes
An **altitude** of a triangle is a segment from a vertex perpendicular to the line containing the opposite side (Clemens, O'Daffer, Cooney, Dossey, 1994, p. 189).	yes	Yes
An **altitude** of a triangle is the perpendicular segment from a vertex to the line that contains the opposite side (Jurgensen et. al, 1994, p. 152).	yes	yes
A line segment drawn from one vertex perpendicular to the opposite side is an **altitude** of the triangle (Manfire, Moser, Lobato, Morrow, 1996, p. 314).	No	Yes
Left to the reader; no formal definition ever given (Serra, 1997, p. 105).	No	Yes
An altitude joins a vertex to the line containing the opposite side and is perpendicular to that side (Beech, McClain, 2003, p. 144-145).	yes	Yes
An **altitude** of a triangle is a perpendicular segment from a vertex to the opposite side or to a line containing the opposite side (Serra, 2003, p. 154).	yes	yes
An **altitude** of a triangle is a segment from a vertex to the line containing the opposite side and perpendicular to the line containing that side. Every triangle has three **altitudes** (Boyd, Cummins, Malloy, Carter, Flores, 2005, p. 241)..	yes	No
An **altitude** of a triangle is the perpendicular segment from a vertex of a triangle to the line containing the opposite side (Charles, McNemar, Ramirez, 2007, p. 533).	yes	Yes
An **altitude** of a triangle is the perpendicular segment from a vertex to the line containing the opposite side (Bass, Charles, Hall, Johnson, Kennedy, 2007, 2007, p. 275).	yes	yes
The **altitude** of a triangle is a line segment that connects the base of the triangle with the corner directly opposite from it; the **altitude** needs to be perpendicular to the base (Szcepanski & Kositsky, 2008, p. 254).	No	No

Appendix B

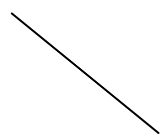
Sample questions from the pre-test given to pre-service teachers.

1. Given the line segment AB below. If possible draw a line segment perpendicular to it, and if not possible circle the words: Not Possible.

A B

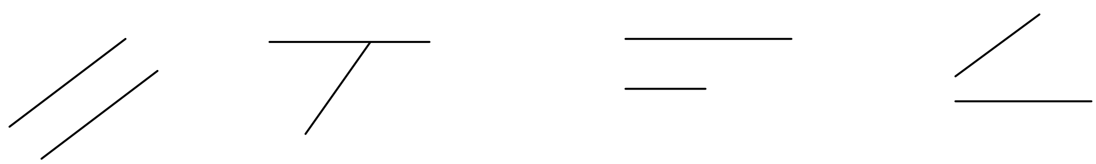
Not Possible

2. Given the line segment CD below. If possible draw a line segment perpendicular to it, and if not possible circle the words: Not Possible.

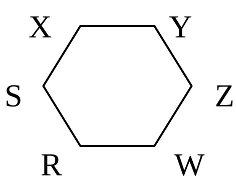


Not Possible

3. Given the pairs of line segments below, circle all segment pairs that are parallel.



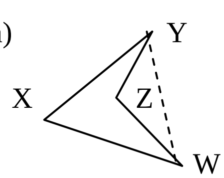
4. In the figure below, draw a circle around a vertex that is non-adjacent to vertex R.



5. A <u>diagonal</u> is a line segment that joins two non-adjacent vertices of a polygon.

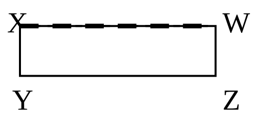
For each of the following figures determine if the dashed line segment is a diagonal and circle your answer. If your answer is no, state a reason.

a)

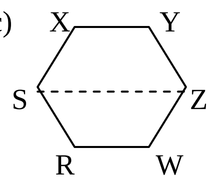


Is the dashed line segment YW a diagonal? Yes No

If no, why not?

b) 

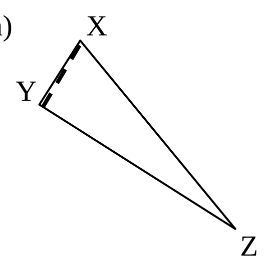
Is the dashed line segment XW a diagonal? Yes No
If no, why not?

c) 

Is the dashed line segment SZ a diagonal? Yes No
If no, why not?

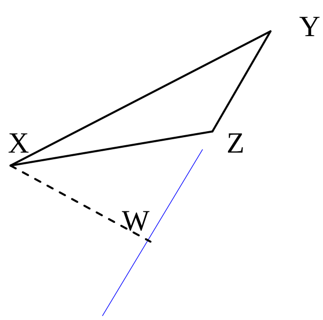
6. An Altitude of a triangle is a line segment drawn from a vertex perpendicular to the opposite side or the line in which it lies.

For each of the following figures determine if the dashed line segment is an altitude and circle your answer. If your answer is no, state a reason.

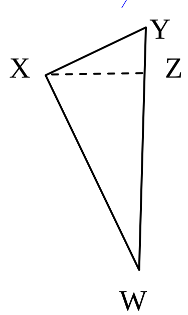
a) 

Is the dashed line segment XY an altitude? Yes No

If no, why not?

b) 

Is the dashed line segment XW an altitude? Yes No
If no, why not?

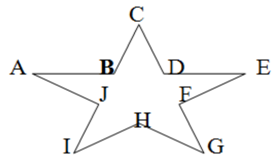
c) 

Is the dashed line segment XZ an altitude? Yes No
If no, why not?

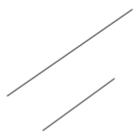
Appendix C

A sample of the post-test questions given after the treatment lesson to non-mathematic elementary pre-service teachers to assess their knowledge of all geometry concepts focused on.

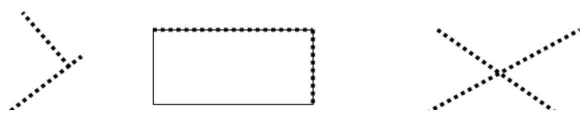
1.) In the figure below, draw a circle around each vertex that is non-adjacent to vertex B.



2.) Is there a line segment that is parallel to **both** of the line segments drawn below? If so, draw it. If not, provide an explanation as to why not?



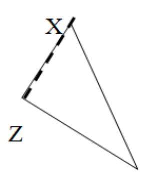
3.) Given the set of line segments below, circle all the dotted line segment pairs that are perpendicular. Provide an explanation as to why the ones you circles are perpendicular and why the ones you did not circle are not perpendicular.



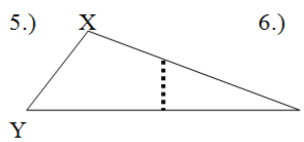
The following questions were given to students on a recent test and their answers are showcased below (as the dotted line segments). Evaluate the student responses to the questions provided as if you were the teacher by examining if the response is correct or wrong. If the student response is correct, defend why their response is correct by using definitions to support your reasoning. If the student response is incorrect, provide the correct answer and explain why the student's response was wrong

For questions 4-6 draw an altitude from vertex *"X."*

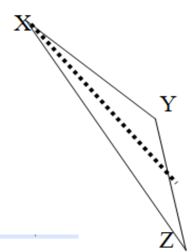
4.)



5.)

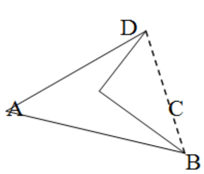


6.)

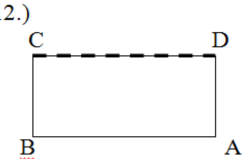


For questions 11-13 draw a diagonal starting at vertex "D."

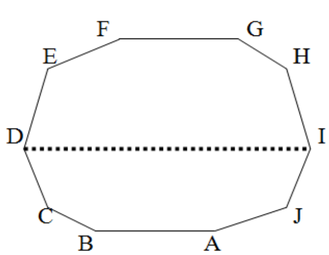
11.)



12.)



13.)



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