Teachers' Influence on Integration of Tools into Mathematics Teaching

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Abstract: This paper examines the process through which three pre-service teachers learn to use mathematical tools; it also looks at pre-service teachers' instrumentation of tools into mathematics teaching. Three pre-service teachers were studying at a primary mathematics teacher training program at Dokuz Eylül University in Turkey. During an eight-week period, workshops were conducted on curriculum tools with pre-service teachers. Subsequently, pre-service teachers' lessons were observed in real school settings. The findings are underpinned by the theoretical framework based on the instrumental approach to tool use. Results indicate that pre-service teachers had difficulty in applying the appropriate use of tools, and teachers' instrumentation schemes influenced students' conceptual understanding. The analysis of data also suggests some implications about the usefulness of the instrumentation framework for effective integration of tools into mathematics teaching.

Many students succeed at lower levels than their expected grade levels in mathematics (Schoenfeld, 2002). The use of tools appears to be one effective way to help students reach higher levels. Research indicates that the use of tools assists students in building connections between mathematical ideas and deeper understanding (e.g. Kober, 1991; Hiebert & Carpenter, 1992; Hawkins, 2007). The need for the use of tools in mathematics teaching appeared in many famous researchers' works, such as that of Piaget (1952), Dienes (1969), and Mueller (1985). These researchers agreed that experiences with a considerable number of physical tools are beneficial for learning mathematics. The underlying idea of the usefulness of tools is a common belief among mathematics educators who think conceptual understanding is a matter of great importance for learning mathematics (Hiebert et al., 1995).

One crucial aspect of tool use to consider is how best to use tools to foster students' learning in mathematics classrooms. This question brings the notion of mediation to the agenda. Bussi and Mariotti (2008) explain mediation as the potentiality of fostering the relation between pupils and mathematical knowledge and define the teacher's role as that of mediator, using the artifact to mediate mathematical content to the student. The mediation of a tool is based on creating a communication channel between the teacher and the pupil (Noss & Hoyles, 1996). As clearly seen, the mediation of tools has strong ties with issues of knowledge for teaching and teachers' pedagogical practices. One of the commonly accepted major domains of knowledge for teaching is pedagogical content knowledge, as described by Shulman (1986), who refers to the mediation of tools. Shulman (1987) explains pedagogical content knowledge as "…the ways of representing and formulating the subject that make it comprehensible to others" (p. 9). Pedagogical content knowledge framework claims that it is not only knowledge of content and knowledge of pedagogy but also a kind of amalgam of knowledge of content and pedagogy that is central to the knowledge needed for teaching (Ball, Thames & Phelps, 2008). Shulman (1987) developed categories to introduce professional knowledge for teaching and evoke the mediation of tools within the major categories of teacher knowledge: "curriculum knowledge with particular grasp of the materials" (p. 8). Teachers should conduct practices which fit with students' ways of learning mathematics (Wood, Cobb & Yackel, 1995); hence, regarding pedagogical content knowledge, when teachers are asked to use tools in their lessons, they are expected to use tools to develop conceptual understanding, which entails a relationship between knowledge and practices.

In 2005, the Turkish Ministry of Education changed the educational plan of action in the schools from a teacher-oriented approach to a student-centered approach (MEB, 2005). The inclusion of tools (e.g., base blocks, geoboard, tangram, etc.) in the elementary mathematics curriculum is a result of this reform movement. However, teachers did not prefer tool use in the development of procedures and skills, especially those who did not know how to make tools a part of mathematics teaching (Kober, 1991; Tooke, Hyatt, Leigh, Snyder & Borda, 1992; Erduran & Yeşildere, 2010). Hoyles (2003) argues that students have to differentiate the tools by interaction and discover whether they have conceptual problems that hinder achievement in mathematical work or if they simply do not appreciate how the tools work. This idea applies to teachers, as well as to students; teachers also need to find out whether they have pedagogical/conceptual problems that may hinder the inclusion of tools in a mathematical activity, or if they simply do not appreciate the pedagogical use of tools.

Some research has been conducted concerning tool use in mathematics teaching (e.g., Thompson, 1992; Clements & McMillen, 1996; Bennett, 2000; Moyer, 2001; Olkun & Toluk, 2004; Hawkins, 2007). Hoyles (2003) points out the importance of tools as part of both the individual and collective experience. Hoyles also takes into account the benefits of the integration of tools that relate mathematical knowledge to mathematical concepts, while both are evolving in the learning process.

In addition to Hoyles's ideas, Trouche (2003) denotes the main ideas shared about the relationships between humans and tools. The first shared idea is the importance of tools in defining humankind; the second is the idea that tools deeply condition human activity; the third is that the use of tools creates automatisms and routine procedures. Although examining the effect of the tools on students' success in light of these shared ideas arouses interest, this paper looks instead at how integration of tools takes place and the influence of teachers on the process. Teachers' difficulties with the effective integration of tools into mathematics teaching will also be discussed. This will be done by relying on the theoretical framework of Artigue (2002)--namely, instrumentation. Among the debates as to what specifically constitutes a tool, a number of French researchers (e.g., Artigue, 1997; Guin & Trouche, 1999; Lagrange, 1999; Trouche, 2003) have presented their own definitions within the instrumentation framework. This framework will help to examine the dynamics of the integration of tools into the mathematics teaching process.

The article is structured around four sections. First, the theory of instrumentation framework is briefly explained. Second, the related literature, in which the researchers used instrumentation framework for analyzing their data, is introduced. Third, research concerning teachers' influence in the instrumentation of tools into mathematics teaching is explored. Finally, an analysis of the data gathered for this study is followed by a discussion of the findings from the case study.

Instrumentation

Instrumentation emanates from Artigue and his team's attempts to put forward a framework to interpret their observations in various studies on using tools, particularly computer algebra systems (CAS). These researchers believe that constructivism is evoked through a few references and some principles without more discussion; therefore, according to the researchers, some other theoretical approaches are needed for research (Artigue, 2002). Consequently, they put Chevallard's (1999) anthropological approach and Vérillon and Rabardel's (1995) ergonomic approach together and define instrumentation framework based on these approaches.

According to the anthropological approach, because mathematics is the product of a human activity, mathematical thinking modes are dependent on the social and cultural contexts in which they develop (Artigue, 2002). Thus, it is a sociocultural approach, and practices are located at its center. Practices are described as having four components: task, techniques, technology, and theory. The first component is a type of task in which the object is embedded; the second is the techniques which are used to solve this type of task (Artigue, 2002). From an anthropological perspective, researchers focus in particular on techniques that students develop while using tools. Technique raises questions about mathematical knowledge. The researchers also extend the definition of techniques with the term "instrumented techniques," which means "techniques involving artifacts." Instrumented techniques are the gestures of instrumented action schemes (Monaghan, 2005, p. 4) and "through mastering instrumented techniques, students develop an operational facility with important components of the conceptual system" (Ruthven, 2002, p. 288). Techniques are accompanied by internalization of associated knowledge (Monaghan, 2003) and, as Artigue (2002) notes, through the naturalization process, techniques lose their mathematical "nobility" and become simple acts. The third component that describes practices is technology: "…the discourse which is used in order to both explain and justify these techniques"; the last component is the "theory," "which provides a structural basis for the technological discourse itself" (Artigue, 2002, p. 248).

Instrumentation framework also rests on Vérillon and Rabardel's (1995) ergonomic approach. This approach, which has a cognitive perspective, introduces the concept of the "instrument." The ergonomic approach is particularly concerned with "how a tool becomes an instrument." A tool is an object that is available for human activity (Trouche, 2003) and an artifact whose purpose is to perform a task (Monaghan, 2003). Vérillon and Rabardel (1995) differentiate an instrument from a tool. They view the tool as a material object and the instrument as a psychological construct: "the instrument does not exist in itself; it becomes an instrument when the subject has been able to appropriate it for himself and has integrated it with his activity" (cited in Monaghan, 2003, p. 3). It is important to stress what "appropriation" means in this case. Monaghan (2003) defines appropriation as "an everyday word associated with making something your own" (p. 6).

As previously stated, French researchers view the instrument as a psychological construct, and they define this component through the Piagetian notion of a scheme. Trouche (2003) distinguishes between gestures as elementary behaviors that may be observed and schemes as the psychological locus of the dialectic relationship between gestures and operative invariants. A scheme, Trouche (2003) argues, has three main functions: a pragmatic function, which allows the agent to do something; a heuristic function, which allows the agent to anticipate and plan actions; and an epistemic function, which allows the agent to understand something.

A tool becomes an instrument through the appropriation of pre-existing social schemes (Artigue, 2002). This process involves the construction of personal schemes. The dialectic between tool and scheme is called "instrumental genesis." Instrumental genesis works in two directions, which are instrumentalisation and instrumentation. The instrumentalisation process is "the component of the instrumental genesis directed towards the tool" (Trouche, 2003, p. 9), and instrumentation is the process "by which the tool prints its mark on the subject, like allowing him/her to develop an activity inside some boundaries" (Trouche, 2003, p. 9).

Trouche (2003) notes that the processes of instrumentalisation and instrumentation are closely interrelated (Fig. 1):

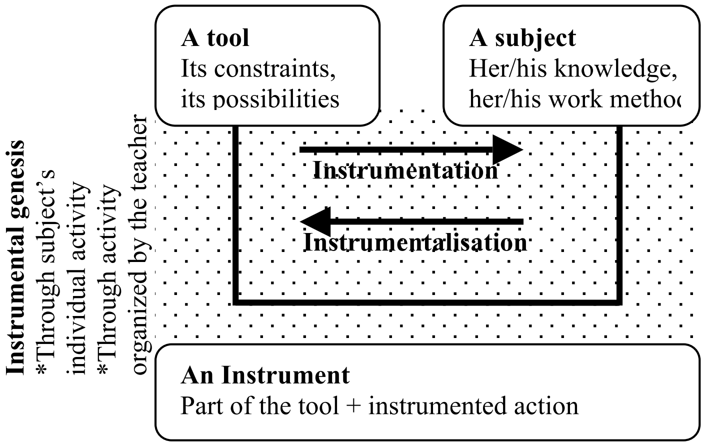

Figure 1. Instrumental genesis as a combination of two processes (Trouche, 2003).

There is a wide range of tools in curricula, and all of them have both benefits and constraints. Deciding which tool to use is sometimes a major problem for teachers. Artigue (2002) states that "…this difficulty poses a didactic obstacle to the progressive building of mathematical activity instrumented in an efficient way" (p. 261). On the other hand, teachers' ways of involving tools in mathematical tasks may also pose a didactic obstacle. At this point, instrumental orchestration comes into question. Trouche (2003) uses this term to point out the necessity of external guidance for students' instrumental genesis. Guin and Trouche (1999) also emphasize teacher involvement in the instrumental process and recognition of the constraints and potential of the artifacts and of student behavior (cited in Monaghan, 2005). "One of the key elements for a successful integration of tools into a learning environment is the institutional and social assistance of this individual command process, and the instrumental orchestrations constitute an answer to this necessity" (Trouche, 2003, p. 19).

In line with the instrumentation framework, the notions of "tool" and "instrument" are discussed, along with following point of view in this paper: Tool is an artifact which is purposed to perform a task (Monaghan, 2005). A tool becomes an instrument when the agent has been able to appropriate it and has integrated it into an activity (Monaghan, 2003).

Related Literature

Some research has been conducted which investigates teachers' ways of using tools in the classroom. Jackson (1999) observed 16 mathematics lessons in terms of tool use in 16 different elementary schools. Ertle (2006) explored teachers' capabilities of making an appropriate assessment of tools. 16 American and 16 Japanese teachers were interviewed to evaluate four tools designed for teaching equivalent fractions. Schorr et al. (2001) examined 58 fourth grade teachers' teaching practices in New Jersey.

Studies have also been done concerning different aspects of mathematical tools, in which an instrumentation framework was used for analyzing the data (e.g., Lumb, Monaghan, & Mulligan, 2000; Monaghan, 2003; Ertle, 2006; Kieran & Damboise, 2007). Drijvers and van Herwaarden (2000) studied 50 ninth-grade students in the Netherlands. They investigated whether performing procedures in the computer algebra environment would contribute to the development of insight in algebraic substitution and in the distinction of the different roles of letters. Kendal (2001) examined how teachers' privileging affected students' learning in CAS lessons (Monaghan, 2003). She investigated how two teachers taught introductory differential calculus to their Year 11 classes, using multiple representations in a CAS-supported curriculum in Australia. Lumb, Monaghan, and Mulligan (2000) examined the practices of two teachers who had previously made no use of Derive and, subsequently, made extensive use of Derive over one school year in England. One of the teachers' schools was for 11- to 18-year-old students, and the other teacher's school was for 13- to18-year-old students. They analyzed the teachers' use of written materials, textbooks, and worksheets and discussed issues concerning the use of materials and pressures on teachers which influence their use of Derive. Kieran and Damboise (2007) carried out a comparative study with two classes of weak Grade-10 algebra students in Canada, who were required by the school to take one month of supplementary algebra. They provided CAS technology to only one of the two classes and implemented two sets of parallel tasks, with the main difference between the two being the use of the CAS tool. They investigated whether students who are weak in algebraic technique and theory benefit more from CAS-based instruction in algebra than from comparable non-CAS-based instruction.

Among the numerous studies that have been conducted on the use of tools, including those cited above, many focus on how use of tools affects students' reasoning. The role of the teacher is occasionally discussed (Monaghan, 2003). There is little research on teachers' thinking about pedagogical strategies and activities to help students construct mathematical ideas or relationships from use of a given tool (Ertle, 2006). There is also little research that has been conducted with primary school students regarding instrumentation framework. In an attempt to contribute to this growing literature, this study examines the role of the teacher in primary school students' instrumental genesis.

Methodology

This study focuses on pre-service elementary mathematics teachers' instrumentation of tools into mathematics teaching in a real context; therefore, a case study with a multiple-case design is the main strategy of the research.

Participants and Context of the Study

Pre-service teachers study at a 4-year primary mathematics teacher training program at Dokuz Eylül University. Candidates enter this 4-year program according to their scores on a university entrance exam. After graduating from the program, they are entitled to a certificate for teaching mathematics in middle schools. During the program, they take courses related to mathematics, pedagogy, and mathematics education. Two courses are offered regarding the pedagogy of mathematics, Instructional Methods in Mathematics I and II; pre-service teachers had these courses through two previous terms. In Instructional Methods in Mathematics I, pre-service teachers learn the nature of mathematical knowledge for teaching and learning methods theoretically; in Instructional Methods in Mathematics II, pre-service teachers are asked to produce classroom activities that can be used in mathematics courses. The Instructional Methods in Mathematics I and II courses are closely related to the school practicum courses in which the data for this study were collected.

There are three school practicum courses in the teacher education program. School practicum courses aim to provide pre-service primary mathematics teachers the opportunity to observe educational phenomena in real school settings and reflect on them. In order to achieve this, classes have, at most, 10 to 12 pre-service teachers. In School Practicum I, pre-service teachers only observe classroom teaching. During School Practicum II, pre-service teachers make observations and do activities in the classrooms, such as: preparing tasks, activities, and worksheets; planning group work; and helping with assessment activities. In School Practicum III, pre-service teachers teach for six hours a week and are observed by mentors and/or university tutors. During the university component of these three school practicum courses, pre-service teachers share their experiences in the schools.

The study took place at Dokuz Eylül University during the School Practicum III course held during the last term of the fourth year in 2009. There were 10 pre-service teachers in the course (7 females and 3 males). All cases are summarized, but three are presented in detail in the paper. In multiple-case designs, the researcher must choose each case carefully, not necessarily following a sampling logic (Yin, 1994). Thus, the selections of cases which are presented analytically are done purposefully with a critical-cases strategy. The fundamental assumption of the critical-cases strategy is “[i]f it is not valid for this case, then it is not valid for any (or only a few) cases” (Flyvbjerg, 2004, p. 426). The first criterion was the pre-service teachers’ success in the Instructional Methods in Mathematics I and II courses. Pre-service teachers who were successful in these courses were preferred. The second criterion was pre-service teachers’ performance in the university component of School Practicum III. Pre-service teachers were preferred who were willing to learn how tools work and who showed high performance in the workshops in understanding how each tool is technically used. Nil, Ela, and Ufuk were three female successful teacher candidates in these courses. They were chosen purposefully, with the assumption that (Flyvbjerg, 2004) if they have problems integrating tools into mathematics teaching, less successful candidates may face similar problems as well. This idea is not intended as a generalization; rather, it was intended to pose some characteristics of the instrumentation process in terms of teachers’ gestures. Other candidates’ names were Ata, Yeliz, Ege, Deniz, Bahar, Ceyda, and Ahmet, whose lessons were explained in a general sense (pseudonyms replace the actual names of all pre-service teachers).

For this study, the focus is on the use of tools in mathematics classrooms in real school environments. Tools were introduced in the national mathematics curriculum of Turkey in 2005 (MEB, 2005). Therefore, current pre-service teachers have had no experience with tools in their educational backgrounds, and tools have recently been introduced into the university component of School Practicum III for the first time. The instructor for this course was the author of this paper. The instructor conducted workshops about curriculum tools during an 8-week period (50 minutes per week), through the university component of School Practicum III. In doing so, the purpose was to develop an understanding of how each tool works mechanically. The curriculum tools included the geoboard, circular geoboard, isometric geoboard, tangram, dot paper, isometric paper, squared paper, pattern blocks, platonic solids, geometry strips, algebra tiles, base blocks, fraction tiles, and fraction cards. The structure of the course was as follows (Table 1):

Week	Course Content
1	**Discuss the national mathematics curriculum’s approach to tool use.**
2	**Interact with all tools and discover their uses in groups.**
3	**Determine which concepts could be constructed with geoboard, circular geoboard, and isometric geoboard.**
4	**Determine which concepts could be constructed with dot paper, isometric paper, and squared paper.**
5	**Determine which concepts could be constructed with tangram, platonic solids, and geometry strips.**
6	**Determine which concepts could be constructed with pattern blocks and algebra tiles.**

7	**Determine which concepts could be constructed with base blocks, fraction tiles, and fraction cards.**
8	**Overview**

Table 1. Structure of School Practicum III course

Pre-service teachers practiced each week's content in classroom in groups of two or three in the university component of School Practicum III. After groups finished practicing, they discussed their experiences with the whole class, with guidance from the instructor. During the first week, they examined and discussed the approach of curriculum to tool use. In the second week, mathematics tools were given to the groups. They were asked to interact with them and figure out their uses in a general sense. Between the third and seventh weeks, groups focused on particular mathematical tools. They were requested to study the use of tools related to mathematical content. Groups examined the outcomes of the mathematics curriculum and related the tools to mathematical concepts. In sum, they discovered which concepts could be constructed with particular tools in groups between weeks three and seven. During the last week, all topics discussed were summarized.

Data Collection Procedure

Data was collected during the School Practicum III course. After workshops were conducted in the university component of School Practicum III, pre-service teachers' lessons were observed in real school settings when they were teaching about quadrilaterals. The pre-service teachers were interviewed after their lessons. Their lessons were videotaped, and the interviews were audio-taped. The following research questions were formulated to be explored:

∞ How do pre-service teachers integrate tools into their teaching?
∞ How does instrumentation of tools take place in pre-service teachers' lessons?
∞ How do pre-service teachers reflect on their teaching concerning the use of tools?

Data Sources and Analyses

The data sources, which permitted qualitative analyses, included (a) pre-service teachers' lesson plans, (b) pre-service teachers' teaching videos, and (c) interviews conducted after their lessons. The audio-taped and video-recorded data were transcribed. The method of analytic generalization was used instead of statistical generalization for data analysis. In analytic generalization, a previously developed theory is used as a template with which to compare the empirical results of the case study, while, in statistical generalization, an inference is made about a population (Yin, 1994). Sampling logic of these two generalization types are quite different; thus, "…one should avoid thinking 'the sample of cases' or the 'small sample size of cases' as if one case study were like a single respondent in a survey or a single subject in an experiment" (Yin, 1994, p. 32). In line with this information, three of the pre-service teachers' transcribed cases are presented analytically, while other cases are presented in a general sense in the paper.

Campbell (1975) asserts that pattern-matching is a useful technique for linking data to the propositions. The data were analyzed to explore the pre-service teachers' influence on the instrumentation of tools, and the patterns noticed in each case were determined. Yin (1994) states that the use of theory in performing case studies is an immense aid in defining the appropriate research design and data collection. Thus, data were interpreted within the instrumentation theoretical framework. Although French researchers suggest using the instrumentation framework with regard to technological tools (e.g., CAS, symbolic calculator), their definition of technology as "the predominant role given to technology as a pedagogical tool" (Artigue, 2002, p. 253) enabled us to use the framework on non-technological tools. Thus, the instrumentation framework served in both gathering and analyzing the resulting data.

The Validity and the Reliability of the Analyses

Yin (1994) proposes that using multiple sources of evidence and establishing a chain of evidence provides construct validity. Pre-service teachers' lessons can be followed through excerpts from their video records and their lesson plans, which constitute a chain of evidence. Triangulation is a strong way to use multiple sources of evidence; therefore, direct observation and focused interview were used in the research. Pattern-matching constructs internal validity (Tellis, 1997); thus, data were analyzed using the pattern-matching approach. In addition, a multiple-case analysis was used, because it increases the external validity of the analysis. The coherence of the case study can be proven with the databases, which are composed of reports written with consideration of the research data (Yin, 1994). Pre-service teachers' cases were analyzed with a cross-case analysis; case study notes were used to compose the database; reports were prepared for each.

Findings

The findings are presented with respect to the research questions previously stated. The analysis of data is carried out considering certain components of the theoretical framework. In what follows, 10 pre-service teachers' experiences and their lesson structures are briefly described. Next, three of these cases are presented in detail, with a common structure considering particular components of instrumentation- tasks and instrumented techniques. As already stated, practices are described by four components, and the first component is the type of task in which the object is embedded. In each case, the pre-service teachers' tasks in teaching quadrilaterals and the interviews about how they prepared them are first reported. Second, the students' instrumented techniques are introduced. Rather than focusing on the techniques that students developed while using tools, the shaping of these techniques by the pre-service teachers' guidance, how the pre-service teachers integrated tools into their teaching, and how this integration affected students' understanding are reported.

Pre-Service Teachers' Experience

Pre-service teachers preferred to use self-produced tools rather than curriculum tools; six of them (Ela, Ufuk, Ata, Yeliz, Deniz, and Ege) produced new tools for their lessons. It was observed that these tools were made in large sizes, with the aim of helping students to see them from anywhere in the classroom; in other words, five pre-service teachers' tools were only for the teacher's use. Therefore, it can be concluded that students were supposed to follow the teacher's use of the tool and try to understand the topic. Only one pre-service teacher, Ela, prepared a tool for use by all students, not merely for the teacher. Techniques are most often perceived by focusing on their productive potential (e.g., efficiency, cost, field of validity) and wideness of application—in other words, their pragmatic value (Artigue, 2002). In the case of these pre-service teachers, they also focused on the pragmatic value of techniques. However, their epistemic value contributes to the understanding of the objects they involve and "… concerns the role of techniques in facilitating mathematical understanding" (Monaghan, 2005, p. 5). Tools for mathematical work have both epistemic and pragmatic sources; they "… must be helpful for producing results, but their use must also support and promote mathematical learning and understanding" (Artigue, 2005, p. 232). The most commonly used curriculum tools among pre-service teachers were dot paper, geoboard, and pattern blocks. Ufuk, Yeliz, and Deniz aimed to teach the area of a trapezoid, while the rest of the pre-service teachers' topic was the area of a rhombus.

Ufuk prepared tools that were different from those included in the curriculum. Her tools were two large, cardboard trapezoids. She explained how to find the area of trapezoid with these tools. She did not interact with all members of the class; rather, she focused on two students. The rest of the students watched the two students' performance during the lesson. Yeliz aimed to teach the area of a trapezoid with the help of a geoboard alongside a self-produced tool. She asked students to construct a parallelogram and a trapezoid in geoboard. Later on, she showed them how to obtain the area of a trapezoid from a parallelogram with the help of a tool that she had created. It can be said that Ufuk and Yeliz produced similar tools and approaches to teach the area of a trapezoid.

Deniz used pattern blocks and large tools that he made. Even though he gave instructions with the use of the pattern blocks for finding the rule of a trapezoid, he did not relate those instructions to the tools, which made their use unreasonable.

Nil's topic was the area of a rhombus. In her lesson, she used dot paper, geoboard, and pattern blocks. She gave students time for the use of dot paper in the task. She used other tools simply to explain the rule. In contrast, Ela preferred to produce a new tool to teach the area of a rhombus. She gave out equal right triangles to each group and asked them to work on the task she provided. However, she did not give the students sufficient time for working with the tools. Ata also aimed to teach the area of a rhombus. He merely used dot paper from the curriculum tools. He asked students to use dot paper and scissors to achieve the task he assigned. However, he did not dwell on the students' performances; rather, he carried on his lesson with the tool he produced, which was the final form of the students' task. He explained the rule rather than let students explore use of the tool by themselves. Bahar and Ceyda both preferred to use dot paper to teach the area of a rhombus. They asked students to construct a rectangle and draw four line segments between the midpoints of each edge. After students constructed this, they explained the rule algebraically without using their construction. Ahmet and Ege followed the same path as Bahar and Ceyda, but with the use of geoboard, not dot paper. Ege used his large tools as well as the dot paper, but it did not make a difference in terms of integrating them effectively into mathematics teaching.

Overall, the pre-service teachers used mathematics tools mostly for visualization of their explanations. They all used at least one tool for teaching the area of quadrilaterals; however, they did not use them to help students explore the rule. As a result, students could not appropriate the tools for themselves, and the naturalization process did not occur. One of three main functions of the scheme observed is pragmatic function, which allows the agent to do something. Students tried to follow teachers' instructions to complete the task. However, the heuristic function of the scheme, which allows the agent to anticipate and plan actions, and the epistemic function of scheme, which allows the agent to understand something, were not put in place. In the end, external guidance for students' instrumental genesis, namely instrumental orchestrations, were weak in the pre-service teachers' lessons. Students were not provided enough time with the tools so as to support and promote mathematical understanding. With respect to the pre-service teachers' views about preparing their tasks, it appears that they preferred to produce tasks themselves instead of using the tasks in the textbooks. All of the tasks required work on only one example to reach a conclusion about the area of quadrilaterals. In terms of the requirement for tools in the tasks, it can be observed that use of tools was not in the foreground in the tasks. They all preferred to produce tasks different from those in the textbooks, and Ela and Ufuk rarely used even the curriculum tools. However, both their tasks and the tools caused the students difficulty in constructing meaning. The pre-service teachers' ways of teaching may compensate for the shortages of tasks. It can be concluded that the pre-service teachers did not use tools for supporting students' mathematical understanding; rather, they used tools as accessory materials. It was observed that the pre-service teachers' instrumentation schemes were not designed to elicit dialogue between the tools and the students' schemes, because students did not interact with the tools. The pre-service teachers' instrumentation schemes influenced the students' conceptual understanding.

Three pre-service teachers' cases are presented in detail, considering the particular components of instrumentation, task, and instrumented techniques. In these three cases, the students' instrumented techniques are reported, along with how this integration affected students' understanding, which was shaped by the pre-service teachers' guidance and the pre-service teachers' ways of integrating tools into their teaching.

Nil's Experience
Nil's Task

Nil aimed to teach how to find the area of a rhombus. Her task consisted of three parts. The first part was as follows: "Ahmet has a rectangular terrain. He wants a building which has a quadrilateral base, but the corners of the quadrilateral must be located in the midpoint of the edges of rectangular terrain. Let's help Ahmet find the needed area for the building." The second part of the task had students draw the figure required by the task and the diagonals of the rhombus; then, students were to find how many triangular regions were inside the rectangle and rhombus. In the last part of the task, students were asked to obtain the area of the rhombus using the area formula of a rectangle.

In the interviews, Nil was asked how she prepared for her lesson. She said that she examined the tasks in the curriculum and in some of the textbooks, but she wanted to create a new activity herself.

Nil's Instrumented Techniques

Nil's task required students to draw a rectangle in which a rhombus was constructed. The rhombus's edges must be located in the midpoint of the edges of the rectangle. For this task, Nil separated students into groups of four. She gave dot papers to each group and asked them to draw the figure requested in the task. After some time, she checked all the groups' drawings. She also drew the figure on the blackboard (Fig. 2).

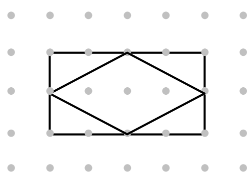

Figure 2. Nil's construction for the task.

She then constructed it on the geoboard and showed it to the students. The students did not work with the geoboard, but simply looked at the shapes that Nil had constructed. Nil asked students to name the quadrilateral they could see. Students gave different answers (e.g., parallelogram, rhombus, square). She then gave pattern blocks to the groups to help them in figuring out these quadrilaterals. Without allowing students any time to examine the properties through the pattern blocks, she gave explanations about them. Students did not interact with the tools in order to relate the quadrilaterals' properties to each other.

Next, Nil asked the students to draw the diagonals of the rhombuses. After each group had done so, she asked how many triangular regions were inside of the rectangle and rhombus that were drawn on the blackboard. She did not take students' drawings into consideration, but, rather, used her own drawing to help them reach a conclusion. Students said that the number of triangular areas amounted to 8 units in the rectangle and 4 units in the rhombus. Nil stressed that the ratio of areas is 2; therefore, the area of the rhombus is half of the rectangle's area. She asked students whether the edges of rectangles and diagonals of rhombuses were related in terms of their length. After students answered that they were equal, she multiplied the lengths of the diagonals and divided by 2. Thereafter, she wrote down a generalization about the area of rhombuses (*e* and *f* are the diagonals of rhombus) as $\frac{e.f}{2}$.

Nil gave the students an opportunity to study with their groups using the dot paper at the beginning. They constructed their own rectangles and rhombuses. However, she did not encourage students either to discover the relationship between the areas of the rectangle and the rhombus or to check other constructions with different sizes. Rather, she asked them to draw a conclusion simply by taking her construction into consideration, which made the use of the tools both meaningless and unnecessary.

After her lesson, Nils was asked about the criteria on which her selection of tools was based. She explained her way of thinking about tools in terms of their technical use:

> Nil: I used dot paper because it facilitates plotting. And I used pattern blocks to remind them of the properties of quadrilaterals. And the geoboard helped me to show the figure I asked them to construct.
>
> Researcher: Why did not you use other curriculum tools?
>
> Nil: I thought about that. Especially, I vacillated on the use of geometry strips. But, later, I decided to use dot paper instead of the strips.
>
> Researcher: Do you think students were able to reach a conclusion by themselves with the tools you chose?
>
> Nil: For some students, yes. But some students were not able to do so. I think my lesson was not fruitful in terms of tool use.

Ela's Experience

Ela's Task

Ela aimed to teach how to find the area of a rhombus in her lesson. Her task consisted of three phases. The first phase was as follows: "You have four pieces of red and four pieces of yellow right triangles. Try to form a rhombus by bringing the right edges of the triangles together. After that, put the rest of the triangle pieces around the rhombus and make a rectangle." The second phase of the task asked for the number of triangles inside the rectangle and rhombus, and students were to show the diagonals of the rhombus. The last phase of the task asked for the relationship between the lengths of the edges of the rectangle and the diagonals of the rhombus and required students to find an algebraic rule for the area of the rhombus.

After her lesson, Ela was asked how she prepared her task. She said, "I examined four textbooks in which numerous tasks were available. I took the important parts of each into consideration and prepared my task… I felt none of the tasks in the textbooks were sufficient because they weren't designed for group work. That is why I prepared the activity myself."

Ela's Instrumented Techniques

In Ela's task, students were supposed to form a rhombus by bringing the right edges of triangles together and placing the rest of the triangles around the rhombus to make a rectangle. Students were separated into groups of four. She gave each student four pieces of red and four pieces of yellow right triangles and asked them to practice the task. After two minutes (between minutes 04:20-06:20), Ela checked the groups' constructions, but students were unable to do it. She then showed the students her construction (Fig. 3) and asked them to do theirs in the same way.

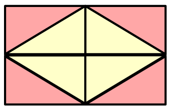

Figure 3. Ela's construction for the task.

Ela asked students the number of triangles inside of the rectangle and the rhombus. After that, she gave an explanation: "Can we say that the area of the rectangle is twice the area of rhombus? Yes, we can. The diagonals of the rhombus are the same length as the length of the edge of the rectangle [*showing her construction*], so the area of the rhombus is half of the multiplication of its edges." She did not make use of students' constructions or allow them to explore through interaction with the tools.

There were many contradictions in Ela's lesson. She separated students into groups, but did not allow them to work on the task together. She gave tools to each group, but did not give them enough time to interact with the tools, which were used simply as an accessory. She requested her students to make a generalization about the area of the rhombus, but she explained everything herself. In her interview, Ela said that she gave priority to the curriculum and to the students' interest when choosing tools:

Ela: First, I took the curriculum into account. And I also tried to choose colorful tools that would be attractive to the students.

Researcher: There were some other tools in the curriculum like the geoboard and geometry strips. Why did you not prefer to use them?

Ela: Well… Actually yes, I could have used these tools. But I did not want to confuse them.

Researcher: So, did you chose randomly? For example, you chose dot paper; is there any reason for that?

Ela: No, not really.

It can be observed that Ela did not consider the tools' epistemic use; rather, she chose them for their effect on motivational issues.

Ufuk's Experience
Ufuk's Task

Ufuk aimed to teach about finding the area of a trapezoid in her lesson. Ufuk preferred to use two separate tasks for the same topic, each of which also included additional steps. She described the first task as follows: "You have two equal trapezoids. How could you put them together to construct a quadrilateral that you are familiar with? And how could you make use of this quadrilateral to find the area of the trapezoid?" The second step of the task was to name the shape and its relationship to a trapezoid. The last step of the task asked students to find a general rule for determining the area of a trapezoid with the help of knowing how to find the area of a parallelogram.

Ufuk described the second task as follows: "You have a trapezoid. How could you cut it up into two triangles to find its area?" The same steps as in the previous task were followed here, as well. In neither task did she ask for the relationship between the height of a trapezoid and a parallelogram or the height of a trapezoid and the triangles. When she was asked how she prepared her tasks, she gave the previously encountered answer: "I looked at the textbooks, but wanted to create the tasks on my own."

Ufuk's Instrumented Techniques

Ufuk used cardboard trapezoids and triangles in her lesson. In her first task, she asked students to construct a quadrilateral with the help of two trapezoids and to use that quadrilateral to find the area of a trapezoid. The construction she asked for is given in Figure 4:

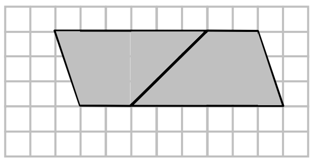

Figure 4. Parallelogram constructed with two equal trapezoids.

Ufuk did not ask students to use any of the tools, nor did she give students time to study by themselves. Rather, she asked one student, Tamer, to come to the blackboard and try it out. She gave Tamer two cardboard trapezoids, and he made some attempts to construct another quadrilateral. However, he was not able to do it, probably because he did not understand what he was supposed to do. Thus, Ufuk constructed it herself and began to talk with Tamer to help him find the formula for the area of a trapezoid.

Ufuk: What was the area formula for the parallelogram? We need to multiply the lengths of its base and height. Is that related to the area of a trapezoid or not?
Tamer: Yes, it is.
Ufuk: What is the length of the base? Count it.
Tamer: 8 units.
Ufuk: And the length of the height?
Tamer: 3 units. [*multiplies 3 and 8, finds 24 unit squares]*
Ufuk: What are we looking for? The area of the trapezoid.
Tamer: *[divides 24 by 2 and finds 12 unit squares]*
Ufuk: What if the bases have 'a' and 'b' unit lengths and the height has 'h' unit length? We are trying to find out a general rule for finding the area of a trapezoid.

Tamer: *[No response]*

Ufuk expected Tamer to be able to transform the numbers to algebra (Fig. 5). Although Ufuk wrote the lengths algebraically on the board, Tamer still was not able to add 'a' and 'b.'

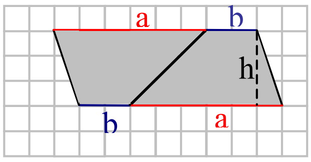

Figure 5. Ufuk's algebraic representation of lengths of edges and height.

Ufuk: Think that 'a' equals 5 and 'b' equals 3. What is the length of the edge?
Tamer: 8.
Ufuk: So what about the lengths of 'a' and 'b'?
Tamer: a + b. It equals …
Ufuk: No, not "equals." We are trying to find a general rule. *[She wrote (a + b).h]* Then what am I supposed to do? *[Asked the whole class]*
Students: Divide by 2.
Tamer: $\frac{(a+b)h}{2}$ *[He writes on the board]*

Ufuk told Tamer what he was supposed to do, step by step, and he did it. In the meantime, he did not communicate with the other students. After that, Ufuk moved to her second task using the same approach. In her second task, she asked the students to separate a trapezoid into triangles to find the area of the trapezoid. The construction she asked for is given in Figure 6:

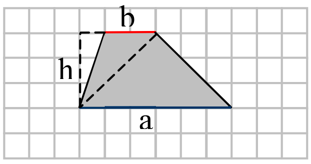

Figure 6. Trapezoid separated into two triangles.

Ufuk again asked one student, Ali, to come and cope with this task. She gave him two cardboard triangles and requested that he form a trapezoid using the triangles. Ali had difficulty in finding the correct directions of the triangles, but he was trying. She allowed him only 15 seconds (between 15:20 and 15:35) before she took the cardboard pieces and formed the trapezoid herself. She talked with Ali to help him find the formula for the area of a trapezoid:

Ufuk: The area formula of triangle is $\frac{a.h}{2}$. What is the length of its height? *[She shows the big triangle]*
Ali: 3 units.
Ufuk: And the length of its base is…
Ali: 6 units.

Ufuk: Then the area of big triangle is $\frac{6.3}{2} = 9$ units. The area formula of triangle is $\frac{b.h}{2}$. What is the length of its height? *[She shows the small triangle]*
Ali: 3 units.
Ufuk: What is the length of the base?
Ali: 2 units.
Ufuk: So the area of the small triangle is $\frac{2.3}{2} = 3$ units. The area of the trapezoid is the sum of 9 and 3. So can you write a general expression using the formulas of these triangles?
Ali: *[No response]*

Ufuk expected Ali to transform the numbers into algebraic expressions, just as she did in the previous task. More explicitly, she expected Ali to write: "The area formula of trapezoid can be found with the sum of $\frac{a.h}{2}$ and $\frac{b.h}{2}$. Therefore, the general expression is $\frac{(a+b).h}{2}$."

Neither teachers nor students were accustomed to tool use in learning mathematics. Therefore, Ufuk had a tendency to perform the entire task herself and to explain every mathematical issue to the students. However, it was observed that both Tamer and Ali needed some time to deal with the task using the tools, to appropriate the tools and to connect the concepts by means of the tools for reaching a conclusion. Rather, they followed Ufuk's practices in completing the tasks. Ufuk used tools to be able to make better explanations, not to help students construct their own understanding. The students just followed the lesson. Unlike the other two pre-service teachers, she did not give any tools to the students; rather, she used the tools herself. When she was asked the reason for not using any of the tools in the curriculum, she said that the curriculum tools were not large enough, so she produced visual ones.

Discussion and Implications

In this section, I will comment on the teachers' influence on the instrumentation of tools into mathematics teaching and the factors that influence teachers' tool use by utilizing the findings that have been noted previously.

These case studies have served to illustrate the teacher misbehaviors that shape students' instrumentation. The pre-service teachers either did not allow students to interact with the tools or did not give them enough time, so students could not transform the tools for specific uses. There were no interactions between the tools and the students' schemes in the lessons, and, for this reason, instrumental genesis did not occur. Therefore, it can be said that the tools did not have an instrumental value in the pre-service teachers' lessons. The pre-service teachers' behaviors obstructed the construction of personal schemes. Nickson (2004) explains this situation in the context of pedagogy:

> If we believe our methodology is of a particular kind that has a set of intended outcomes, but in reality it is not that methodology, then, clearly the intended outcomes will not be achieved. If, for example, being able to reason mathematically is an intended goal, and at the

> same time we do not give pupil the opportunity to explain their thinking, then we are not engaging in the pedagogy appropriate to achieving that goal. (p. 196)

In this case, tools were intended to assist in reaching a conclusion about the area of quadrilaterals; thus, students should have been provided enough time and guidance to interact with the tools for appropriate pedagogy.

The following question is crucial: "What are the factors that make teachers use tools as objects aside from the learning process?" Bishop and Whitfield's (1972) teacher decision-making framework could account for this question. This framework indicates the importance of teachers' individual values in their decision making: "Individual values do play a great part in their decision making, for example, their beliefs about the nature of persons and the nature of their subject material" (p. 6). Borko, Roberts, and Shavelson (2008) explain Bishop and Whitfield's framework, stating that teachers' schemas connect classroom situations to prior experience, values, and teaching goals, which guides decisions and consequent action. They also state that background information, such as psychological theory, as well as general life experiences and, especially, educational experiences, was filtered through the individual's value system and aims for the particular lesson. When the pre-service teachers were asked to design a lesson with the use of tools in this research, they had difficulty in (1) determining the appropriate use of tools, (2) making decisions about which tools to use, and (3) integrating the tools into the task. These difficulties may emerge from their background information, as noted in Bishop and Whitfield's framework. Pre-service teachers met tools for the first time in their school lives, and not having any educational experiences with tools may have caused them to face these difficulties.

Nil, Ela, and Ufuk did not seem to spend time thinking about how tools could aid students in reaching conclusions or how to use tools to support students' mathematical understanding. It looked as though they used tools because they were supposed to. This finding has been demonstrated in other studies. Similar to our findings, many research studies show that teachers do not use tools with the aim of promoting conceptual understanding (Clements & McMillen, 1996; Schorr, Firestone & Monfils, 2001; Stein & Bovalino, 2001). This study adds to the body of literature the influence on students. The pre-service teachers' instrumentation schemes influenced students' conceptual understanding; they did not ask students to make generalizations based on their experiences with particular tools. Although the pre-service teachers knew about all of the curriculum tools, they did not combine them with their pedagogical knowledge. The teachers' lack of previous experience with tools in learning mathematics may be a factor here. Stacey (2001) stresses that teachers design their lessons "from the base of their prior teaching styles and their beliefs about mathematics and how it should be taught" (p. 1). When pre-service teachers were asked to prepare a lesson with the integration of tools, they preferred to use them prescriptively, as they had learned. These findings point out the need for changes to integrate powerful instruments into teaching. Considering how tools are used should become part of a student's conceptualization of mathematical knowledge (Hoyles, 2003), and such consideration may be the first step for teachers to make sense of the nature of tool implementation.

In all three of the cases, the tools served as visual accessories and motivational objects by which students were essentially to construct their own understanding. Moyer (2001) reached similar findings after examining 10 middle-grade teachers' uses of tools for teaching mathematics to explore how and why the teachers used the tools. The results indicated that the teachers used the tools for fun, not necessarily for teaching and learning mathematics. These results may depart from the pre-service teachers' lack of knowledge about the potential of tools because recognition of the constraints and potential of the tools supports instrumental genesis (Trouche, 2003). Artigue (2002) emphasizes the necessity of intertwining "standard mathematics knowledge and knowledge about the artefact and the computational transposition of mathematical knowledge that the use of this artefact involves" (p. 261). Firstly teachers should learn about the constraints and potentialities of tools, and secondly they should initially consider whether selected tools facilitate the construction of mathematical meaning and how they should be integrated into tasks or activities. The former is important, because if tool use does not make learning easy, it may even distract students from conceptual arguments. However, the latter is even more critical, because it supports the emergence of instrumentation.

The pre-service teachers' tasks were a factor that affected the instrumentation of tools. The beginning of Ela's task was quite complicated, unlike the other two tasks. It might be useful for students to deal with challenging tasks, but the complicated part of Ela's task was for the students to understand how to use tools to practice the task. Students had difficulty constructing the figure, and this adversely affected the remainder of the phases. Moreover, the tools became inefficient and pointless, because students could not understand how these tools would help them to practice the task. A study by Vygotsky (1978), which underlines the tools' mediating aspect, may contribute to this discussion.

Vygotsky (1978) asserted that human action is mediated by tools, and he was interested in tools' mediating role. In his research, he asked children some questions about the colors of objects. They were supposed to answer using distributed a set of colored cards. Vygotsky (1978) reported that children could not use the cards in a way that would help them to answer; moreover, he indicated that, in some cases, the cards prevented their performance. According to Vygotsky, it is not meaningful to claim that individuals have mastered a sign without addressing the ways in which they do or do not use it to mediate their own actions or those of others (Wertsch, 1991). Teachers' guidance concerning tool use is crucial.

The pre-service teachers' tasks were alike in many ways, also; they made students examine just one case and asked them to generalize. In the three tasks' final steps, students were supposed to make a generalization from their investigations in the second step. But in the second step, students were required to generalize with merely one example in all cases. It may not be an easy job for students to express the rules of the areas of quadrilaterals algebraically shortly after they have made some arithmetical calculations, and examining only one case might be misleading for them. Hoyles (2005) points out the close relationship of mathematical meanings and the tools used in their construction, as well as the importance of the tools' methods of representing mathematical invariants and expressing relationships. Here, the theory on which the tasks are based comes into question. There were theoretical problems in the pre-service teachers' lessons concerning their techniques to help students solve tasks and the structural basis for these techniques. Monaghan (2005) stresses the relationship of theory, task, and technique, and says that "within a theory, every topic has an accompanying set of tasks and techniques" (p. 6). Tasks should be designed in relation to the theory. Every tool has a social aspect, and, so, this must be taken into consideration while designing tasks in which tools are used. Tasks that are prepared by teachers should have a theoretical framework regarding knowledge construction, especially ones that emphasize tool use in detail.

With regard to the issue of the selection of tools, it was observed that they all preferred to use dot paper, although the geoboard or geometry strips would have been much more convenient for studying quadrilaterals of different sizes. Ela said in the interviews that she did not consider any criteria when choosing tools. The other two pre-service teachers indicated the tools' technical dimensions as a reason for using them. It was seen that the pre-service teachers' ideas about tools focused on their pragmatic function and that they did not consider the epistemic value of practices. The pre-service teachers were not given any rule for the selection of tools, and it was observed that they were not careful enough either in choosing tools or in integrating them into the tasks. The institutional roles of tools should be well defined, as Artigue pointed out, and teachers should be provided with knowledge about both the selection and the integration of tools.

In Nil's case, students were able to use dot paper to facilitate practicing the first part of the task, so that instrumentalisation of the artifact was observed. Nevertheless, the direction of instrumental genesis was only toward the artifacts, which means the instrumentation was not monitored. However, transforming any mathematical tool into an instrument includes a complex instrumentation process for students (Guin & Trouche, 1999). In this study, it was also observed that appropriation of tools requires time. Monaghan (2005) points out the same issue in his paper and emphasizes the need for time to develop rich schemes by using techniques. Therefore, it can be concluded that students should be provided with enough time to allow them to interact with the tools through the task.

Hoyles (2005) discusses tool use on meaning construction and raises two crucial questions. The first is about the design of the tools: how much should be done by the tools, and what should be left for the students to construct? The second crucial question is about the design of the activities in which the tools are embedded: what tasks should be developed that foster students' engagement with mathematical ideas and discourse? The data in this study also imply the importance of the task design and how tools take part in tasks, as Hoyles remarks. It was observed that tasks should be designed in a way that really incorporates tool use, or else tools will be perceived as useful only for demonstration. The findings of this research also emphasize the role of teachers in classroom activities that involve tools. The teachers' competencies in developing a task, the techniques that constitute their instrumentation process, and these instrumentation schemes influenced students' understanding.

The analysis of the data implied the usefulness of an instrumentation framework for effective integration of tools into mathematics teaching. The analysis of data also demonstrated that the role of teachers is crucial through the emergence of instrumental genesis, because students usually practice tasks in the way that teachers show them. In order to clarify teachers' roles in instrumental genesis, it might be useful to define techniques and theory, taking teachers into consideration.

With reference to instrumentation framework, it can be said that techniques involve the teachers' approach to helping students perform tasks, and instrumented techniques are developed by teachers while integrating tools into classroom activities. Relying on these definitions, theory can be defined as a structural basis for the techniques and for technological discourse itself. Monaghan (2005) writes, "Novices progressively become skilled in techniques by doing, talking about, and seeing the limits of techniques. This eventually leads to a theoretical understanding of the topic" (p. 6). This statement can be true for teachers, also: novice (pre-service) teachers become skilled in the techniques they have developed while integrating tools into classroom activities by doing, talking about, and seeing the limits of their techniques.

The analysis of data also implies the existence of some issues in teacher education research. Pre-service teachers spend six hours a week in schools during the last year of their university education in Turkey. The pre-service teachers did not find opportunities to observe the effective use of tools in schools because in-service teachers in schools are also encountering tools for the first time. Thus, they should be observed frequently in their lessons and provided with feedback so that they can transform their theoretical knowledge about tool use into practice.

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