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The Academic Merits of Modelling in Higher Mathematics Education: A Case Study

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Modelling is an important subject in the Bachelor curriculum of Applied Mathematics at Eindhoven University of Technology in the Netherlands. Students not only learn how to apply their knowledge to solve mathematical problems posed in non-mathematical language, but also they learn to look actively for, or even construct, mathematical knowledge useful for the problem at hand. A detailed analysis of the academic profile of the curriculum is presented, using a framework of competencies and dimensions, developed at this university by the project, Academic Competencies and Quality Assurance (ACQA). The profile is constructed from the perspective of teachers' ambitions. The research question for the present study is: Are there certain academic characteristics typical for the Modelling Track compared to the characteristics of the other courses in the Eindhoven Bachelor curriculum of Applied Mathematics? The analysis shows that the modelling projects are essential for the development of the designing competencies in the curriculum. Other courses in the curriculum are more intended to develop abstraction capabilities. These results provide supporting arguments for the realistic approach chosen for mathematical modelling education.

In Australasia, much research focuses on mathematical modelling education. The vast majority of this research concerns primary and lower secondary schools, whereas research on higher education is relatively scarce (Stillman, Brown, & Galbraith, 2008). This situation is likely to be similar worldwide. On the other hand, what happens in the first years of university education is indeed relevant for pre-university education in general, as well as for mathematical modelling education in particular.

A case study has been conducted in the context of university undergraduate education: the Bachelor program of Applied Mathematics of the Eindhoven University of Technology (TU/e). The importance of mathematical modelling in the curriculum at this site has increased since a curriculum reform was implemented about ten years ago. The government permitted the curriculum to be extended by an extra year and therefore a series of modelling projects could be introduced: and this sub-program is called *the modelling track*. The main reason for the introduction of the modelling track was employers' dissatisfaction with the level of graduates' skills. According to employers, graduates possessed sufficient technical skills and knowledge of various subjects, but could neither apply nor integrate these skills and knowledge without support. Students should be better prepared to solve practical problems. In general, professional mathematical engineering problems are not pure mathematical problems. Rather, they are of a multi-disciplinary nature and in their first appearance they do not even have a mathematical form. Students should be better able to transform such problems into mathematical problems: the first step of the modelling cycle (Perrenet & Adan, 2002).

Gray (1998) distinguishes three ways of presenting mathematical modelling: 1) various mathematical systems applied in various areas; 2) several mathematical systems, each applied in a single area; and 3) a single area of application. Our approach is closest to the first way, however, sometimes even the mathematics to be applied has to be discovered and studied by the students themselves as part of the project. Students may have to develop, explore and apply 'new' mathematics, taking various levels of problem details into account. For example, it can happen that, by including more details, a (too) complicated differential equation is obtained, which cannot be solved analytically, but for which the students 'discover' a method to produce a numerical solution. Alternatively, if students realise that, for example, `waiting' is an important issue for their problem, they may decide to explore queuing theory, a subject that is taught at a later stage in the curriculum, and apply queuing models that fit their needs. As in practice, the students are brought into a situation in which it is not clear beforehand what (and even if) mathematics is useful to solve the problem. Therefore, the approach enhances three kinds of learning: *learning how to model, learning mathematics through modelling*, and *learning how to apply mathematics*. Within the variety of perspectives on modelling in mathematics education as discussed by Kaiser and Sriraman (2006), our approach is the closest to the so-called pragmatic perspective of Pollak: focussing on the goal of learning to apply mathematics, but doing so in the broad sense as described above.

The approach of not telling the students beforehand what model or even what mathematics to apply differs from current practice in (Dutch) mathematical modelling education at secondary level, where usually the mathematical model is presented along with the problem. In this sense our approach is closer to the original goals of realistic mathematics education (RME, e.g., De Lange, 1987). This approach is not undisputed among staff of our department. We will elaborate on that after describing the program.

Since the Bologna Declaration (European Ministers of Education, 1999), the question whether the academic level of a particular university program is sufficient or not, is no longer an 'academic' question. Compared to the other 44 participating European countries, The Netherlands were relatively quick to implement the Bachelor/Master structure as well as a new accreditation system. Several systems have been developed for quality measurement, see for example, the Dublin Descriptors (Joint Quality Initiative Working Group, 2004) and the TUNING system (Gonzalez & Wagenaar, 2003; 2005). At Eindhoven University of Technology a system of criteria has been developed by the Project Group for Academic Competencies and Quality Assurance (ACQA), which especially aims at higher education curricula in mathematics and science (Meijers, van Overveld, & Perrenet, 2005). The system is in use in the mathematics and science departments of several European universities. At the TU/e all curricula are evaluated by interviewing all teachers about their ambitions in the courses using the detailed ACQA framework. This framework consists of seven competence areas (disciplinary competence, doing research, designing, scientific approach, intellectual skills, co-operate/communicate, and context) and four dimensions of academic activities (analysis, synthesis, abstraction, and concretisation). The competence areas are the same for all disciplines; but the dimensions are discipline specific. In this paper, we will present the general competence areas as well as the specific dimensions for Applied Mathematics. We will use the outcomes of the profile measurement of the Bachelor program of Applied Mathematics to analyse the role of the Modelling Track. The central research question of this study is the following: Are their certain academic characteristics typical for the Modelling Track compared to the characteristics of the other courses in the Eindhoven Bachelor program of Applied Mathematics?

In the following sections, we first give a more detailed description of the Modelling Track as embedded in the Applied Mathematics Bachelor curriculum. Next, we explain details of the method used to measure the academic profile. After presenting the results, we discuss the outcomes. Our modelling education approach will be discussed in comparison with the alternative application approach. This discussion is related to the current debate in the Netherlands about approaches to secondary mathematics education.

Modelling in the Applied Mathematics Curriculum in Eindhoven

The Bachelor Curriculum

In the Bachelor of Applied Mathematics in Eindhoven about 30 students are enrolling each year. In the first year, a series of compulsory courses are taught: analysis, algebra, probabilities and statistics, computer science, computer algebra, mechanics, business administration and modelling. In the second year compulsory courses include abstract analysis, numerical linear algebra, mathematical statistics, optimisation, stochastic processes and modelling, and some elective courses on applied mathematics. In the third year, there are some compulsory courses, but students also choose from three specialisations: statistics, probability and operations research; discrete mathematics and applications; or computational science and engineering, and then follow courses in their specialisation. The regular didactical forms are lectures, practicals, and a mixture of the two. However, in the modelling courses, Modelling 1 – 4, the students undertake a series of projects in pairs.

The Modelling Track

The overall learning goal of the Modelling Track is learning how to use mathematical skills in order to solve problems posed in non-mathematical language. Firstly, it concerns the ability to complete the so-called model-cycle that consists of the following steps:

- Problem analysis: mapping out the problem using common sense;
- Problem translation: constructing a mathematical model for the problem:
- Mathematical analysis: mathematical elaboration of (sub-) problems;
- Implementation: implementation of the solution into a computer program; and
- Retranslation: translation of the solution back to the original problem.

Secondly, it concerns the ability to work along project lines with the following characteristics:

- Working according to a plan,
- Collaborating in a small team,
- Communicating verbally with colleagues and interested layperson (such as the problem owner),
- Giving a verbal presentation to colleagues and interested layperson,
- Reporting on paper to colleagues and interested layperson, and
- Critically following fellow students' mathematical activities.

In the beginning of the track, the students perform small projects with intensive coaching. Gradually, the projects become larger (three projects are done in Modelling 1; two in Modelling 2; one in Modelling 3 and one in Modelling 4). The consecutive projects become more complex and the students work more independently. Each time each pair of students chooses a project assignment out of a set of possibilities. Most of the time two different pairs do not do the same project. Each pair has their own website where they present their plans and products. Integrated with the project work, there are courses in the first and second year on verbal presentation, report writing, website building, LaTeX, project planning, library usage, and interviewing. For each project students have to deliver an intermediate presentation as well as a final presentation and also a final report. The projects are divided according to three application areas: physical-technical applications, business applications, and communication and information applications. Examples of recent project assignments follow.

Physical-technical Applications: Water Clock

> Three thousand years ago, the Egyptians used a simple bin of clay to keep track of time as shown in Figure 1. The clock consists of a bin, which is filled with water every day. The water in the bin seeps out very slowly from a hole in the bottom, and time is measured by means of small dashes inside the bin, where each dash indicates say, an hour. The striking feature is that the dashes inside the bin are distributed *evenly*. This is only possible if the side of the bin has exactly the right angle, and if the ancient Egyptians were precisely aware of this angle. *Develop a model for the water clock and use this model to determine the right angle of the bin.*

Figure 1. Water clock used by the ancient Egyptians.

Example 1 is from the first year of the program and has been created by a staff member. During a period of five weeks, the students worked on it for about one day per week. The students applied the law of Bernouilli to determine the right angle, but then discovered that such a bin is not possible; the shape of the side of the "ideal" bin with evenly distributed dashes is not a straight line with a fixed angle, but a smooth curve (i.e., a polynomial of degree 4). The students used Mathematica to exactly determine this curve and they determined a bin with a straight side. The angle of the side was chosen in such a way that the ideal bin was well approximated (and that the angle was close to the angle of the bin shown in Figure 1). This bin with evenly distributed dashes appeared to produce reliable estimates for time as long as it was filled for at least a quarter of its content.

Communication and Information Applications: Intelligent Vacuum Cleaner

An intelligent vacuum cleaner (see Figure 2) is supposed to automatically clean the whole room at a given frequency, starting from the base station. *Is it possible to develop an algorithm guiding the vacuum cleaner as fast as possible through an arbitrary room (with furniture) so that the whole area is cleaned?*

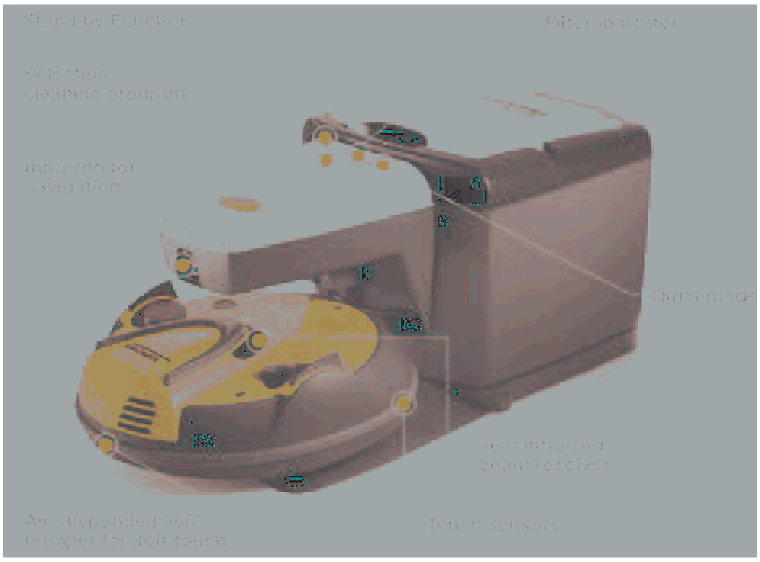


Figure 2. Intelligent vacuum cleaner at base station.

Example 2 is from the second year of the program. The project comes from a mathematics consultancy centre outside the university. During a period of twelve weeks, the students worked on the task for about one day per week. The students developed several heuristic algorithms to guide the vacuum cleaner through a room, of which only its dimensions are known in advance. The obstacles in the room had to be discovered by the vacuum cleaner. Each of these heuristics used an efficient shortest path algorithm as a building block. The students implemented these algorithms in Java and compared their performance for randomly generated rooms with obstacles. In addition, they investigated the effect of the size of the grid representing the room, the optimal location of the base station and the benefits of knowing in advance the exact location and size of all obstacles in the room.

Business Applications: Elevators

> For years, waiting for the elevators in the main building of the Eindhoven University has been a source of irritation. In the near future, the university is planning a big renovation of the main building. *Provide advice to the university with respect to the location, number and control of the elevators in the renovated main building.*

This example is also from the second year of the program. The project is derived from a current issue at the university. During a period of twelve weeks, the students worked on it for about one day per week. They developed a simulation model to evaluate the waiting times for the elevator, paying careful attention to assumptions and limitations of their model. They also developed, at their own initiative, analytical models employing ideas and theory of Markov processes and Markovian queuing models with batch service, which are treated in the following (third) year of the curriculum.

The Modelling Track in the Bachelor curriculum consists of two courses in the first year, taking up a part of 10% of that year's study load, and two courses in the second year, which is about 15% of that year's load. The third year is completed with a small course called Modelling 5 (somewhat more than 3%). The Modelling Track as a whole takes a modest part of nearly 10% of the Bachelor curriculum study load. Modelling 5 consists of a variety of activities of academic reflection on the earlier modelling courses. Examples of themes are the societal relevance of the modelling projects and the possibilities of taking another approach to the projects done before, now that the students have more knowledge and skills. Writing and critically reading essays about proposed new approaches are also part of this course.

The experience of about ten years tells us that most students really appreciate these courses. The majority of the faculty has accepted them as a normal and important part of the curriculum, although there is some criticism about the fact that, contrary to the subject exams, almost every student passes the Modelling Track. Recently, attention has been given to this aspect and, as a result, more detailed rules for assessment have been introduced. A minority of the faculty, however, still has the opinion that mathematics should be taught by means of lectures and practicals only, while project work is 'for kindergarten'. Some accept project work only as a means to apply what has been learned already. We will elaborate on this after presenting the results of our study.

When we look at other curricula with modelling courses, we can conclude that integration with the training of presentation and report writing skills is rather common (e.g., Edwards & Morton, 1987; Usher & Earl, 1987). Contrary to some other programs, our students work in pairs, as the overall staff opinion is that the development of mathematical modelling skills should have priority over the development of skills to work in larger groups. In higher years, the students experience working in larger and more diverse groups. Burton (1997) and Houston (1998) stress the importance of peer assessment and peer tutoring, as occurs in our Modelling 5 course. In the second year of the track, recently some attention has been given to the theory of modelling, but the emphasis is still on creative and empirical model construction coached by a staff member (c.f. Giordano & Weir, 1987).

The Method of Measuring Academic Profiles

In this section, we turn to the measurement of the quality of education in science and mathematics at university level in general. After that we focus again on our Applied Mathematics education. As described earlier, the academic profile is measured by interviewing teachers about their ambitions in the courses. The interviewee for a course is the teacher with final responsibility for that course. The interview is semi-structured. The first main question with respect to a specific course is how much time students should spend – as percentages of total study load – on various areas of competence. Follow-up questions concern which competencies are addressed within each competence area and which are assessed in that course. The second main question asks which of the four dimensions of academic acting and thinking are relevant in that course and for every relevant dimension, how much time students should spend on various levels of that dimension, each time as percentages of the total study load. The profile of the program, or the profile of a subset of courses, mainly consists of the average percentages given as answers in the course interviews concerned. The profile is a global description of how the students are supposed to work - a global description of the realistic teachers' ambitions. To answer the research question, the profile of the Modelling Track will be compared with the profile of the set of other courses. Next we give an overview of the areas of competence and the dimensions with their levels.

Areas of Competence

In the revised edition of the Criteria for Academic Bachelor and Masters Curricula (Meijers et al., 2005), seven competence areas that (should) characterise a university graduate are distinguished. He or she:

1. Is competent in one or more scientific disciplines (*disciplinary competence*). A university graduate is familiar with existing scientific knowledge, and has the competence to increase and develop this through study.
2. Is competent in doing research (*doing research*). A university graduate has the competence to acquire new scientific knowledge through research. For this purpose, research means: the development of new knowledge and new insights in a purposeful and methodical way.
3. Is competent in designing (*designing*). As well as carrying out research, many university graduates will also design. Designing is a synthetic activity aimed at the realisation of new or modified (technological) artefacts or systems with the intention of creating value in accordance with predefined requirements and desires (e.g., mobility, health).
4. Has a scientific approach (*scientific approach*). A university graduate has a systematic approach characterised by the development and use of theories, models and coherent interpretations, has a critical attitude, and has insight into the nature of science and technology.
5. Possesses basic intellectual skills (*intellectual skills*). A university graduate is competent in reasoning, reflecting, and forming a judgment. These are skills which are learned or sharpened in the context of a discipline, and which are generically applicable from then on.
6. Is competent in co-operating and communicating (*co-operate/communicate*). A university graduate has the competence of being able to work with and for others. This requires not only adequate interaction, a sense of responsibility, and leadership, but also good communication with colleagues and non-colleagues. He or she is also able to participate in a scientific or public debate.
7. Takes account of the temporal and the social context (*context*). Science and technology are not isolated, and always have a

temporal and social context. Beliefs and methods have their origins; decisions have their consequences in time. A university graduate is aware of this, and has the competence to integrate these insights into his or her scientific work.

The concept of competence is used as an integration of knowledge, skill, and attitude. A student totally possesses a particular competence if he or she has:

- relevant knowledge of the field of study [k],
- the skill to utilise this knowledge in appropriate contexts [s], and
- the right attitude to utilise the knowledge acquired in these contexts [a].

Clearly, competence areas have definitions in general terms, applicable to all disciplines. Within a discipline, interpretations can be specific. For example in area 4, for the discipline of Applied Mathematics, models should be interpreted as mathematical models. Compared to some other frameworks, our framework is all embracing. Well-known systems in Europe, like the Dublin Descriptors (Joint Quality Initiative Working Group, 2004) and TUNING (Gonzalez & Wagenaar, 2003; 2005) under expose typical engineering activities like designing (area 3). A perspective such as that of Barrie (2006) focuses on generic academic attributes beyond disciplinary expertise and technical knowledge (mainly area 1, partly areas 2 and 3), while the ACQA framework includes both categories.

Within each area, five to eight competencies have been defined. As an example we present the eight competencies of area 3, designing. The graduate:

3.1 Is able to reformulate ill-structured design problems. Also takes account of the system boundaries in this. Is able to defend this new interpretation against the parties involved. [ksa]
3.2 Has creativity and synthetic skills with respect to design problems. [ksa]
3.3 Is able (with supervision) to produce and execute a design plan. [ks]
3.4 Is able to work at different levels of abstraction including the system level. [ks]
3.5 Understands, where necessary, the importance of other disciplines (interdisciplinarity). [ks]
3.6 Is aware of the changeability of the design process through external circumstances or advancing insight. [ka]
3.7 Is able to integrate existing knowledge into a design. [ks]
3.8 Has the skill to take design decisions, and to justify and evaluate these in a systematic manner. [ks]

For each competence, we defined a Bachelor level and a Masters level. The Masters level should be understood as containing the Bachelor level, but also as being more based on attitude, independence, complexity and frontier knowledge.

We illustrate these (design) competencies, by looking back at project examples 1 and 2, described earlier. The activities related to area 3, designing, are clear. In Example 1 the ideal shape of the bin is designed and in Example 2 various algorithms are designed to guide the vacuum cleaner through a room. In Example 1, students discovered the original problem was ill-posed: a straight sided bin satisfying the requirement of evenly distributed dashes does not exist! In Example 2 the students had to explore the boundaries of the system. What information is available in advance and what information is required to efficiently guide the vacuum cleaner through the room? Are the sensors on the vacuum cleaner sufficient to determine the location and size of all obstacles? They used existing, but not yet taught knowledge (i.e., the shortest path algorithm) and also developed new heuristic algorithms. The Example 2 project was more complex than the Example 1 project, in part because of the relations with the external client. The students also had to show more independence in that last project. Therefore, for the project of Example 2, for some competencies in the area of designing even the Masters level is required. Although most other competence areas, *disciplinary competence, scientific approach, intellectual skills, co-operate/communicate*, and *context*, play some role, the students should spend most time *designing*. *Doing research* is not relevant.

Dimensions: Analysis, Synthesis, Abstraction, Concretisation

The framework described so far gives a global indication of two levels in education. What is lacking, is a detailed indication of levels of acting and thinking. In the ACQA project, all program directors of the Eindhoven University of Technology were asked what, in their opinion, were the characteristic ways of academic acting and thinking. Almost all mentioned abstracting, concretising, analysing and synthesising. From their explanations, the following definitions were constructed and agreed upon (Meijers et al., 2005; Meijers, 2006).

- *Abstracting* is to bring a viewpoint (statement, model, theory) to a higher aggregation level through which it can be made applicable to more cases. The higher the aggregation level, the more abstract the viewpoint.
- *Concretising* is the application of a general viewpoint to a case or situation at hand. The more aspects of a situation are involved, the more concrete the viewpoint.
- *Analysing* is the unravelling of phenomena, systems or problems into sub-phenomena, sub-systems or sub-problems with a specific intention. The greater the number of elements involved, or the less clear it is what the elements of the resulting analysis are, the more complex the analysis.
- *Synthesising* is the combining of elements (or components) into a coherent structure that serves a certain purpose. The result can be an artefact, but also a theory, interpretation or model. The greater the number of elements involved, or the more closely knit the resulting structure, the more complex the synthesis.

Looking at mathematics education literature, often abstraction is treated as the most important activity (see, e.g., Gray & Tall, 2008). Sometimes, attention is given to processes of analysing and synthesising to support abstracting (e.g., Ozmantar & Monaghan, 2008). The activity of concretising probably is more characteristic for Applied Mathematics than for pure mathematics. We acknowledge that in a process of acting and thinking all four categories can occur, but we will treat those as four equally important and independent activities. We do not regard analysing - synthesising or even abstracting - concretising as opposites. We will show that for the above definitions engaging in the activity of abstracting does not automatically imply that one is also de-concretising, and vice versa.

The independence of *abstracting* and *concretising* can be illustrated with an example from the classical mechanics of a pendulum. In high school, a pendulum is considered as a point mass, suspended on an infinitely strong (non-elastic) rope, moving at small amplitudes, leading to a simple linear, second order differential equation. In a first-year mechanics course in university, the abstraction is made where the 'point mass' and the 'geometrical motion' are considered irrelevant: the same differential equation is seen to apply to elementary linear electrical networks (e.g., with one capacitor and one coil). In a second-year course, perhaps, a further abstraction is made to arbitrary (linear) networks. Parallel to this, a *concretisation* in a course on mathematical physics could allow for ramifications such as the study of larger deviations, air friction, or the coupling between translational and rotational (torque) modes, and consequently take non-linear differential equations into account. A further concretisation could study the effect of stretching in the rope, and could take the interference between transversal and longitudinal motions into account, etcetera. So the topic of 'pendulum', studied at low abstraction and concretisation levels at high school, will grow both in the direction of more abstraction and concretisation in various courses at university; thus the activity of 'abstracting' is no longer the opposite of 'concretising'. They are both 'academic' activities that can be investigated independently.

Looking for levels for abstracting, concretising, analysing and synthesizing, as a first version, uniform four-point scales were developed, for example, abstraction minimal – low – high – maximal. Abstraction minimal should be typical for the start of the Bachelor program; abstraction maximal should be typical for the end of the Masters program. The program director of a specific program should illustrate the four levels for a specific dimension by constructing examples, which are recognizable and well understood for all the staff of the program. After a trial with two programs, it became clear that firstly, it would be better to look at levels related to a discipline itself than to look at levels directly related to a specific program. Secondly, variation existed between disciplines in the number of levels needed along a specific dimension. Therefore, the choice was made to design a system for level construction that was essentially the same for all disciplines, but different in its disciplinary details. Scales for the four dimensions have been constructed for a number of disciplines (including Applied Mathematics), with the help of experts from these domains. A scale for a dimension in a discipline is obtained by applying a generic construction to a central example from the discipline. We first describe the generic construction principle. Next, we present the four scales used for (applied) mathematics.

Levels in a dimension are constructed in a recursive way, in which the result of a previous step becomes the starting point of the subsequent step, and every step is an instance of the activity of the dimension. Figure 3 depicts this process for the activity of abstracting: starting from an example, A, B, C and D are subsequent reformulations of the example on higher levels of abstraction (elaborated from Meijers, 2006):

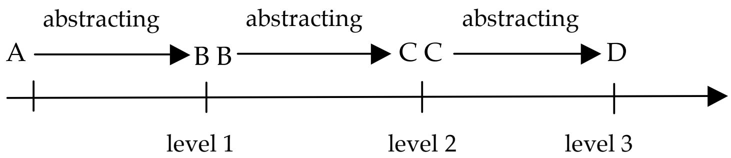


Figure 3. Construction scheme for the scale of abstracting.

Each of the three levels in Figure 3 can be characterised by a combination of a description of the starting point and a description of the activity, for example, for level 1,

1.1 Description of starting point A
1.2 Description of activity (leading to result B)

Using the same principle with the concretising, analysing, and synthesising activities, levels can be constructed for the other three dimensions.

In Table 1 the four scales constructed for Applied Mathematics are given. Experts from the department were closely involved in the process of, firstly, choosing central examples for the dimensions, and, secondly, constructing levels based on these examples. The number of levels of the scale for a specific dimension in a particular discipline is not determined beforehand: the construction is applied until no further steps can be taken inside the discipline (because either the next level does not exist or it is considered to belong to another discipline). The idea is that the resulting scale ranges from the lowest to the highest level in the discipline for the activity concerned. This range is called the scope of the discipline with respect to a given dimension. For a given dimension, scales for different disciplines may have different numbers of levels, and the number of levels across the scales for the four dimensions of a single discipline may vary. In the case of Applied Mathematics four three-point scales resulted. Table 1 shows clarifying examples for each dimension from lowest to highest level.

These examples are not specifically tuned to modelling, however, all four dimensions are relevant in the modelling cycle. As mentioned earlier, the modelling cycle comprises the following steps: problem analysis, problem translation, mathematical analysis; implementation, and retranslation (Perrenet & Adan, 2002). Clearly, *analysis* should be an important modelling activity (it is mentioned twice in the cycle), but so should the other dimensions: *synthesis* comes into play when a model or solution is constructed from sub-solutions; *abstraction* appears when a practical problem is translated into mathematical terms; and *concretisation* applies when theories are specified or models are expanded.

Table 1
Applied Mathematics Scales

Dimension	Scale
Abstract	1.1 A graph of a real-valued function
	1.2 Assessment of the continuity of this function by means of visual inspection of the graph
	2.1 The intuition of continuity of a function based on visual inspection
	2.2 Formalisation of continuity in terms of ε-and δ-arguments
	3.1 A proof of the continuity of a given function using ε-en δ-argumentation
	3.2 A proof of the continuity of a given function by means of the topological definition of continuity
Concrete	1.1 The decomposition of a vector in a generic vector space
	1.2 Expanding a function in a Hilbert space of functions
	2.1 Decomposition of a function in a Hilbert space of functions
	2.2 Developing a periodic function as a Fourier series
	3.1 The development of a periodic function as a Fourier series
	3.2 Specifying the function as a signal and determining the amplitude and phase spectra
Analytic	1.1 Set of measured data of a time sequence
	1.2 Specification of the relevant trends (qualitatively)
	2.1 Specification of the relevant trends from 1.2
	2.2 Quantifying these trends in the form of new data sets (e.g., in terms of period or phase shift)
	3.1 The new data sets from 2.2
	3.2 Observation of some structural behaviour in quantitative trends (e.g., the phase shift increases with the period)
Synthetic	1.1 A list of requirements for simulation of a physical process
	1.2 Specifying methods and techniques within field of Scientific Computing needed to comply with the given requirements
	2.1 Fully specified methods from Scientific Computing as in 1.2
	2.2 Assembling these specifications to obtain a numerical procedure
	3.1 The description of a numerical procedure as in 2.2
	3.2 Construction and implementation of a (prototype) numerical program for a simulation

should be an important modelling activity (it is mentioned twice in the cycle), but so should the other dimensions: *synthesis* comes into play when a model or solution is constructed from sub-solutions; *abstraction* appears when a practical problem is translated into mathematical terms; and *concretisation* applies when theories are specified or models are expanded.

Project examples 1 and 2 also illustrate these dimesnions. As mentioned, Example 1, the Waterclock, is from the first year and Example 2, the Vacuum cleaner, from the second year. We will look at the dimensions *analysis* and *synthesis* first. Example 2 involves many aspects including: What is known about the room? Does the size of the memory of the vacuum cleaner limit the usage of algorithms? Is it possible to determine the dimensions of the room and location of all obstacles with the sensors on the vacuum cleaner? How to compare the various algorithms? In this way, both activities *analysis* and *synthesis* take place on a higher level in Example 2 than in Example 1, because more elements of the situation are involved (see the general definitions given). For the waterclock project some quantification was necessary in *analysis* (level 2, Table 1), for the vacuum cleaner even a quantitative measure had to be developed to compare algorithms (*analysis* level 3, Table 1). Next, we turn to *abstraction* and *concretisation*. Abstraction does not take place on the highest level: visual inspection as well as formularisation do take place (level 1 and 2, Table 1). However, there is no abstraction above that level. On the other hand, *concretisation* was applied while always taking all relevant aspects of the situation into account: level 3 (Table 1) only.

We now proceed to measuring results for the Applied Mathematics program and the Modelling Track as a part of this program.

Results

In total, 46 course interviews were held including all five courses of the Modelling Track. The average length of an interview was one hour. Almost every interviewee was able to give the percentages asked for indicating how students should divide their time for each course using the framework of competencies and dimensions (in an exceptional case, the option to include an uncertainty band in the answer was used). This means that the dimensions and the constructed level examples were generally recognised and could be used well by the staff. This adds to the validity of the method. Another positive aspect of validity followed from the fact that the profile of the Bachelor program differed from that of both Masters programs (not described in this paper; see Projectgroep Academische Vorming, 2007) in a way that could be explained well and used by the program management. In the following sections, we compare the results for the five courses of the Modelling Track with the results for the other courses, beginning with the perspective of competence areas and competencies, and continuing with the perspective of dimensions and levels.

Comparison: Competence Areas

Figure 4 compares the Modelling Track to the rest of the curriculum from the perspective of competence areas. The unit along the radial axes is the percentage of the total study load of the curriculum. The accumulated results of the interviews show that in the Modelling Track the students should spend most of their study time in area 3 (nearly 5% out of 10% of the Modelling Track as a whole), while in the rest of the curriculum relatively most time should be devoted to area 1 (more than 35% out of 90% for the other courses together).

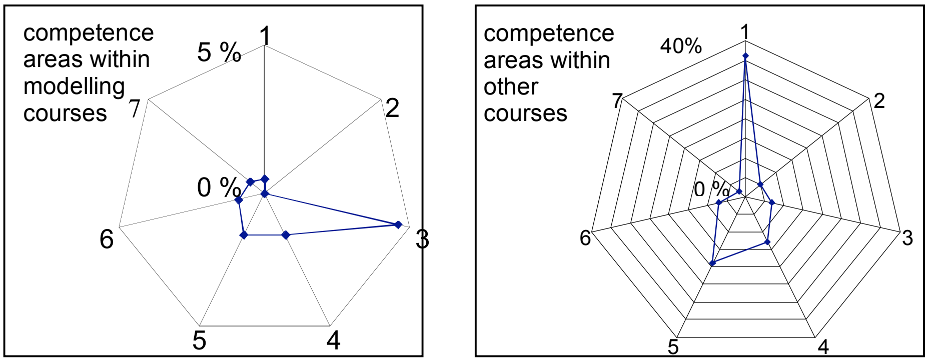


Figure 4. Student study time to be spent on seven competence areas, for the Modelling Track and for the other Applied Mathematics Bachelor courses. 1 = disciplinary competence; 2 = doing research; 3 = designing; 4 = scientific approach; 5 = intellectual skills; 6 = co-operate / communicate; 7 = context.

We conclude that compared to the average of other courses the Modelling Track is very strong on designing competencies. When we analysed the difference at the level of specific designing competencies, it was found that all seven competencies were present in both sub-programs. According to the staff interviewed, only in the Modelling Track were three of those competencies already aimed for at Masters level, (i.e., in the second year projects). These competencies – with the Master level characteristic in italics – are (a) is able to reformulate ill-structured design problems *of complex nature*; (b) has creativity and synthetic skills with respect to c*omplex* design problems; and (c) is able to *independently* produce and execute a design plan. Assessment of these competencies at the last phase of these projects is at Masters level also.

Comparison: Dimensions

In Figure 5, the Modelling Track is compared with the rest of the curriculum from the perspective of the dimension of abstraction. In Figure 5a, the accumulated results of the interviews indicate that abstracting activities are relevant in the Modelling Track and that for about 40% of the time devoted to these activities, students should act and think on level 1 and that for about 60% of the time on level 2. No activities were aimed at level 3 (see Table 1). We conclude that for the Modelling Track, level 3 abstraction is absent; in the rest of the curriculum (Figure 5b) all three abstraction levels are present.

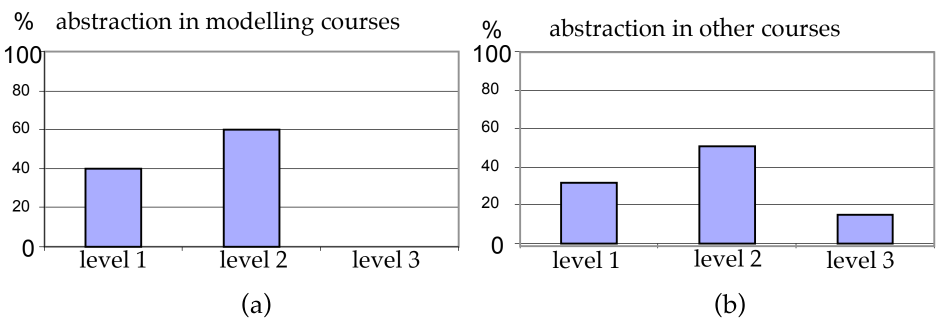


Figure 5. Student study time to be spent on levels of abstraction, for the Modelling Track and for the other Applied Mathematics Bachelor courses.

In Figure 6, the Modelling Track is compared to the rest of the curriculum from the perspective of the concretisation dimension. For the Modelling Track, only level 3 concretisation is present (Figure 6a); in the rest of the curriculum all three concretisation levels are present (Figure 6b). When the Modelling Track (Figure 7a) is compared to the rest of the curriculum (Figure 7b) from the perspective of the analysis dimension, we conclude that for analysing, the two patterns do not differ much; but in the Modelling Track level 2 analysing is somewhat more important. Comparison of the Modelling Track (Figure 8a) to the rest of the curriculum (Figure 8b) from the perspective of the synthesis dimension revealed that, on average, level 3 synthesising is more important for the Modelling Track than for the rest of the curriculum.

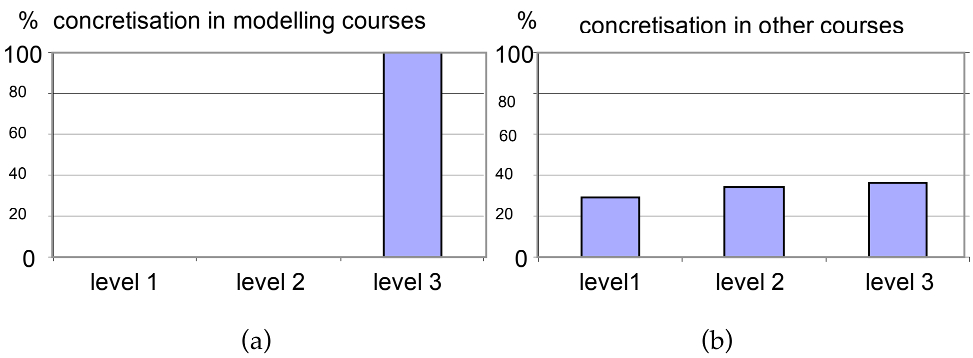
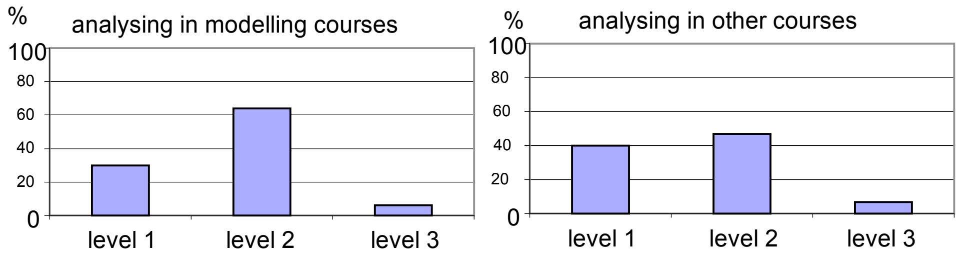
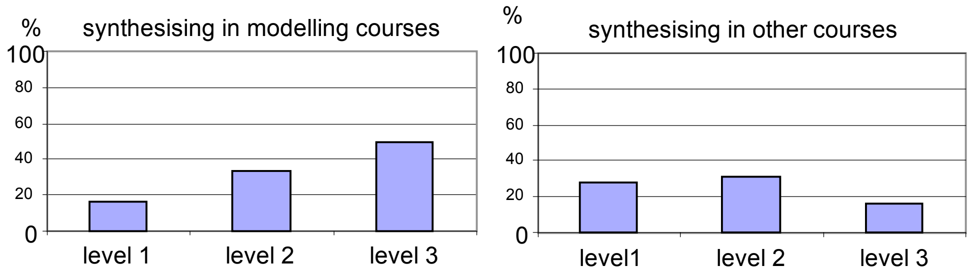


Figure 6. Student study time to be spent on levels of concretisation for the for Modelling Track and the other Applied Mathematics Bachelor courses.



(a) (b)

Figure 7. Student study time to be spent on levels of analysing for the Modelling Track and for the other Applied Mathematics Bachelor courses.



(a) (b)

Figure 8. Student study time to be spent on levels of synthesising for the Modelling Track and for the other Applied Mathematics Bachelor courses.

Summarising, the Modelling Track, on average has, a higher level of concretising and synthesising; the other courses, on average, have a higher level of abstracting. We also analysed the number of levels of a certain dimension, attended to within the same course. This led to the following accumulated results:

- the Modelling Track, on average, has more levels of analysing and synthesising attended to within the same course; whilst
- the other courses, on average, have more levels of abstracting and concretising attended to within the same course.

Conclusions and Discussion

The results show that the Modelling Track is crucial for the curriculum profile of the Bachelor program. It strengthens the competence area of designing in particular. At the level of individual competencies, some competencies are already at Masters level, such as reformulating ill-structured complex design problems, creativity and synthetic skills for complex design problems, independently producing and executing a design plan. Also, in the modelling projects, students have to create and apply mathematics while taking much detail of the problem situation into account (high level of concretising). Finally, within a modelling project, they have to use their analytic and synthetic skills at several levels of complexity, zooming in and zooming out of the problem situation. Nevertheless, the majority of students succeeded in the modelling projects and really appreciate this kind of work. One of the reasons given is the freedom in approaching the problem: the students are not forced to apply prescribed models and techniques. A small minority do not succeed the first time and have to do extra assignments. Few students complain about the situation that only later in the curriculum do they meet the best method to solve a particular modelling assignment. These results effectively counter the criticism of some staff members.

On the other hand, the level of abstraction aimed for in the Modelling Track is somewhat lower, compared to many other courses in the curriculum. Considering ways of raising the level of abstraction, one possibility could be the introduction of lectures on the methodology of mathematical modelling, which recently has occurred. Another possibility is the following: using the activity of generalisation as an aspect of abstraction. After the problem has been solved, and after the mathematical solution has been interpreted in the original context and validated, the following question could be asked systematically at the end of every modelling project: *Is it possible to generalise the applied method to a larger class of problems, and if so, in what ways?* Normally, this question is not asked, as the students, the tutor and the problem owner will be happy when the problem at hand has been solved to everyone's satisfaction. However, in case the students have used self-developed methods or have applied newly studied mathematical theory, this could be too difficult a task. It might be better to leave these kinds of questions to the other subject courses. The argument of some faculty members would be: 'Teach them the models and methods beforehand, so they only have to apply these so that much more abstraction will be possible at the end'. However, the loss would be more severe. It would undermine the training in the important (and realistic) first phase of the modelling cycle: in real life it is not immediately clear which method to use, and ready-made methods do not present themselves. Moreover, having developed new methods or having discovered new mathematical theory (which of course, does not happen in every modelling project) may have the great benefit that these topics will be much better understood by students when these topics are taught later in the program.

The previous discussion is like an echo of a more overt discussion about mathematics education at secondary and primary level in the Netherlands. Louder and louder, criticism is heard about the use of realistic contexts as propagated by the adepts of realistic mathematics education (RME). RME advises the use of contexts in two ways, as a starting point for the development of mathematical concepts, and as a domain for application of mathematical contexts (Doorman et al., 2007). See the website http://www.fi.uu.nl/en/ for the general RME background. Their adversaries, united in the Association for Better Education in the Netherlands (BON - In Dutch: Beter Onderwijs Nederland), plea for much fewer contexts and more training in pure algebra (see http://beteronderwijsnederland.net/ for background information - only in Dutch). The discussion has been so heated now and then, that some even speak of a 'math war' (Feys & Van Biervliet, 2008). Dutch mathematical modelling education at secondary level has already become injured by this war, as it receives less attention in the latest curriculum plans.

It is beyond the scope of this paper to give a substantiated opinion about how modelling should be learnt at secondary education. However, we have made it clear that, in higher education, at a technical university, mathematical modelling, although embedded in a more abstraction oriented curriculum, should be learned in a realistic way. This case study shows that the *realistic* way is also an *academic* way.

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References

Barrie, S. C. (2006). Understanding what we mean by the generic attributes of graduates. *Higher Education 51*(2), 215–241.

Burton, L. (1997). The assessment factor – by whom, for whom, when and why. In S. K. Houston, W. Blum, I. D. Huntley, & N. T. Neill (Eds.), *Teaching and learning mathematical modelling* (pp. 95-107). Chichester, UK: Horwood.

De Lange, J. (1987). *Mathematics, insight, and meaning. Teaching, Learning, and testing of mathematics for the life and social sciences.* Utrecht, The Netherlands: Vakgroep Onderzoek Wiskundeonderwijs en Onderwijscomputercentrum (OW&OC).

Doorman, M., Drijvers, P., Dekker, T., Van den Heuvel-Panhuizen, M., De Lange, J., & Wijers, M. (2007). Problem solving as a challenge for mathematics education in The Netherlands. *ZDM The International Journal on Mathematics Education, 39*(5-6),405–418.

Edwards, D., & Morton, P. C. M. (1987). Formal verbal presentation in project-based mathematical modelling. In J. S. Berry, D. N. Burghes, I. D. Huntley, J. D. G. James, & A. O. Moscarini (Eds.), *Mathematical modelling courses* (pp. 51-57). Chichester, UK: Ellis Horwood.

Feys, R., & Van Biervliet, P. (2008). Wiskunde-oorlog en reddingsoperatie in Nederland [Math war and rescue operation in the Netherlands]. *Onderwijskrant, 146*, 3-15.

Giordano, F. R., & Weir, M. D. (1987). A first course in mathematical modelling. In J. S. Berry, D. N. Burghes, I. D. Huntley, J. D. G. James, & A. O. Moscarini (Eds.), *Mathematical modelling courses* (pp. 70-83). Chichester, UK: Ellis Horwood.

González, J., & Wagenaar, R. (Eds.). (2003). *Tuning educational structures in Europe: Final report phase one.* Bilbao, Spain: University of Deusto.

González, J., & R. Wagenaar (Eds.). (2005). *Tuning educational structures in Europe: Final report phase two.* Bilbao, Spain: University of Deusto.

Gray, E., & Tall, D. (2008). Abstraction as a natural process of mental compression. *Mathematics Education Research Journal, 19*(2), 23–40.

Gray, K. (1998). Perceptions of mathematical modelling in Australia: Is it time to take gender off the agenda? In P. Galbraith, W. Blum, G. Booker, & I. D. Huntley (Eds.), *Mathematical modelling, teaching and assessment in a technology-rich world* (pp. 177-186). Chichester, UK: Horwood.

Houston, S. K. (1998). Get students to do the teaching! In P. Galbraith, W. Blum, G. Booker & I. D. Huntley (Eds.). *Mathematical modelling, teaching and assessment in a technology-rich world* (pp. 45-54). Chichester, UK: Horwood.

Joint Quality Initiative Working Group (2004). *Shared 'Dublin' descriptors for Short Cycle, First Cycle, Second Cycle and Third Cycle Awards*. Retrieved from http://www.jointquality.nl/

Kaiser, G., & Sriraman, B. (2007). A global survey of international perspectives on modelling in mathematics education. *ZDM The International Journal on Mathematics Education, 38*(3), 302-310.

Meijers, A., van Overveld, C., & Perrenet, J. (2005). *Criteria for Academic Bachelor's and Master's Curricula, Eindhoven University of Technology* (Rev. 2nd ed.). Eindhoven, The Netherlands: Eindhoven university of Technology. Retreived from: http://alexandria.tue.nl/repository/books/591930E.pdf

Meijers, A. W. M. (2006). *Academic learning outcomes: A conceptual and empirical approach*. Paper presented at the Accredition Board for Engineering and Technology (ABET) Annual Conference, Florida.

Ozmantar, M. F., & Monaghan, J. (2008). A dialectical approach to the formation of mathematical abstractions. *Mathematics Educational Research Journal, 19*(2), 89-112.

Perrenet, J., & Adan, I. (2002). From mathematical modelling to design-based learning: a bridge too far? *International Journal of Mathematical Education in Science and Technology, 33*(2), 287-297.

Projectgroep Academische Vorming (2007). *Onderzoek Academisch Profiel Opleidingen BSc en MSc Technische Wiskunde; intern rapport* [Study into the Academic Profile of the BSc and MSc programs of Applied Mathematics; internal report]. Eindhoven, The Netherlands: Eindhoven University of Technology.

Stillman, G., Brown, J., & Galbraith, P. (2008). Research into the teaching and learning of applications and modelling in Australisia. In H. Forgasz et al. (Eds.), *Research in mathematics education in Australasia 2004-2007* (pp. 141-164). Rotterdam, The Netherlands: Sense.

Usher, J. R., & Earl, S. E. (1987). Group modelling and communication. In J. S. Berry, D. N. Burghes, I. D. Huntley, J. D. G. James, & A. O. Moscarini (Eds.), *Mathematical modelling courses* (pp. 133-144). Chichester, UK: Ellis Horwood.

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