Mathematics Education Research Journal 2010, Vol. 22, No. 1, 81-91

University Students' Difficulties in Solving Application Problems in Calculus: Student Perspectives

Sergiy Klymchuk
Auckland University of Technology

Tatyana Zverkova
Odessa National University

Norbert Gruenwald and Gabriele Sauerbier
Hochschule Wismar University of Technology

This paper reports on the results of an observational parallel study conducted simultaneously at 2 universities – one each in New Zealand and Germany. It deals with university engineering students' difficulties in the formulation step of solving a typical application problem from a first-year calculus course. Two groups of students (54 in New Zealand and 50 in Germany) completed a questionnaire about their difficulties in solving the problem which was set as part of a mid-semester test. The research endeavoured to find reasons most of the students could not use their knowledge to construct a simple function in a familiar context. It was neither lack of mathematics knowledge nor an issue with the context. The students' difficulties are analysed and presented along with their suggestions on how to improve their skills in solving application problems.

There are many papers devoted to investigating undergraduate students' competency in the mathematical modelling process. We will highlight some recent research in this area. A measure of attainment for stages of modelling was developed in Haines and Crouch (2001). The authors expanded their study in Crouch and Haines (2004) where they compared undergraduates (novices) and engineering research students (experts). They suggested a three level classification of the developmental processes which the learner passes through in moving from novice behaviour to that of an expert. One of the conclusions of this research was that "students are weak in linking the mathematical world and the real world, thus supporting a view that students need much stronger experiences in building real world mathematical world connections" (Crouch & Haines, 2004, p. 204). An investigation of undergraduate students' working styles in a mathematical modelling activity has been undertaken by Maull and Berry (2001); whilst a study by Nyman and Berry (2002) investigated the development of transferable skills in undergraduate mathematics students through mathematical modelling. Relationships between students' mathematical competencies and their skills in modelling were considered in Galbraith and Haines (1998) and in Gruenwald and Schott (2000). Obviously, there is a link between mathematical modelling and solving application problems.

We support the view that solving application problems can be considered as a subset of the mathematical modelling process which can be described as "consisting of structuring, generating real world facts and data, mathematising, working mathematically and interpreting/validating (perhaps several times round the loop)" (Niss, Blum, & Galbraith, 2007, pp. 9-10).

Our perception of application problems again is similar to that expressed by Niss, Blum, and Galbraith (2007):

> Standard applications: Typified by problems like finding the largest cylindrical parcel that can be shipped according to certain postal requirements, standard applications are characterised by the fact that the appropriate model is immediately at hand. Such problems can be solved without further regard to the nature of the given real world context. In our example, this context can be stripped away easily to expose a purely mathematical question about maximizing volumes of cylinders under prescribed constraints. So, the translation processes involved in solving standard applications are straightforward, that is, again, only a limited subset of the modelling cycle is needed (p. 12).

Craig (2002) described a number of classifications of word (application) problems. Most common classes are algorithmic and interpretive. Galbraith and Haines (2000) used a similar approach in their study with undergraduate students dividing the problems into mechanical, interpretive and constructive. Another common classification is the division of problems into ill-structured (real-life) and well-structured (school word problems). In our study we use the framework of Galbraith and Haines (2000) and examine the students' difficulties with the constructive type of problem which requires the carrying out of some calculations and drawing a conclusion from some given information and existing knowledge of mathematics.

Common application problems from a typical first-year university calculus course are normally mathematised to a large extent, so there is no need to collect or analyse the data, make assumptions, and so on. Still, in many application problems, students have to go through the formulation step of the mathematical modelling process that often requires choosing/constructing a formula or setting up a function for further investigation. Normally the information given in an application problem includes some numbers, expressions, and stories. This is the stage where many students have difficulties in translating the word problem into the mathematical formula and then deciding which mathematics they should use. As Clement, Lochhead and Monk (1981) pointed out "rather than being an immediate aid in learning mathematics, the process of 'translation' between a practical situation and mathematical notation presents the student with a fresh difficulty that must be overcome if the application (or the mathematics) is to make any sense to the student in the long run" (p. 287). Talking about the nature of that "translation" they continue:

> What makes teaching (and learning) of these translation skills so difficult is that behind them there are many unarticulated mental processes that guide one in construction of a new equation on paper. These processes are not identical with the symbols; in fact, the symbols themselves, as they appear on the blackboard or in the book, communicate to the student very little about the processes used to produce them. There seems to be no way to explain such translation processes to students quickly. (p. 289)

In this paper we report university students' difficulties in solving a common application problem from a typical first-year calculus course. We agree with Galbraith, Stillman, Brown, and Edwards (2007) that for such students

> problems need to be such that the mathematics required for solution is within the range of known and practical knowledge and techniques. It may not be clear however, just which mathematics is appropriate for the job at hand: such decisions are part of the requirements of the modelling process. (p. 131)

Further, we attempt to categorize these students' difficulties in order to try to answer the research question: Why is it most of the participating students could not use their knowledge to construct a simple function in an application problem in a very familiar context? Many researchers and practitioners in different ways ask a similar question (e.g., Crouch & Haines, 2004):

> Why is it that students of engineering science, technology and allied subjects find it difficult to move freely between the real world and the mathematical world, when by their own choice of applied discipline one might have expected strong engagement in modelling or pseudo-modelling tasks? (p. 199)

In their review of the current situation regarding applications and mathematical modelling in education worldwide, Niss, Blum, and Galbraith (2007) identified ten issues that require ongoing research and development. This paper relates to issues 7 and 8 from that list: "teaching/learning practice of applications and modelling; implementation of applications and modelling in practice" (pp. 22-23).

In another review by Stillman, Brown and Galbraith (2008) that focused on the Australasian community, the authors specified eight research questions that related to the most concentrated areas of Australasian research activity. The research question of this paper relates to question 5 from that list: "In what way do mathematical, technological, and modelling knowledge and competencies interact in the solution of contextualized problems" (p. 159)?

The Study

We, the lecturers from a New Zealand university and a German university, were surprised with some results from a routine mid-semester test given to our engineering students (n = 92 in the New Zealand university and n =109 in the German university). Although many students performed well in the procedural test questions that required mathematical techniques and manipulations, the vast majority of the students failed to solve the application problem below:

> *Trucking Cost Problem.* The cost of running a heavy truck at a constant velocity of v km/h is estimated to be $4+\frac{v^2}{200}$ dollars per hour. Show that to minimize the total cost of a journey of 100 km in

the truck at constant velocity the truck should run approximately 28 km/h.

The above is the English version and the students from the German university received it in the German language. From our point of view as their lecturers the students had sufficient knowledge, skills, common sense and practice to solve the problem easily. The vast majority of them studied calculus at school where similar problems would have been common. In our tutorials and lectures about a quarter of the suggested problems were application problems. However, the result was very surprising: within about 6 minutes (an average time a student had for the *Trucking Cost Problem* on the test) only 4 out of 92 students in the New Zealand university and 6 out of 105 students from the German university were able to set up the total cost function correctly recognising that:

TOTAL COST = COST PER HOUR $\times$ TIME IN HOURS

that is

TOTAL COST, $F(v) = (4 + \frac{v^2}{200}) \times \frac{100}{v}$.

As usual we gave our students the model answers to the test questions and discussed them in a class in the following week. Before discussing the solution to the *Trucking Cost Problem* we asked the students the following question: *If someone works at the rate of 10 dollars per hour for 6 hours, what is his/her total earnings?* All students gave the correct answer immediately: 10 dollars per hour $\times$ 6 hours = 60 dollars. Then we asked them the next question: *If someone works at the rate of R dollars per hour for T hours, what is his/her total earnings?* Again all the students replied: $R \times T$, that is, they used the formula:

TOTAL EARNINGS = RATE PER HOUR $\times$ TIME IN HOURS.

This formula for Total Earnings and the formula for Total Cost in the *Trucking Cost Problem* have the same nature and structure. The students had done numerous basic algebra problems in the past at school and university when they used quantities expressed as letters rather than numbers. They had also done applications problems in calculus at school and in our tutorials. The context of the problem did not require any special knowledge – just common sense. The students knew the formula relating the distance, constant velocity and time. The vast majority of the students were well motivated and hard working and they definitely tried to perform well on the test. The students were majoring in engineering! Yet, the vast majority of them failed to set up the function in the *Trucking Cost Problem* correctly. Why?

Questionnaire

To investigate the possible reasons for their difficulties with the *Trucking Cost Problem* we gave our students the following short questionnaire after the discussion of this problem in class in the week following the test:

Question 1. What difficulties did you have while trying to set up the total cost function in the *Trucking Cost Problem*?

Question 2. What can be done to improve your skills in doing this step of an application problem?

Answering the (anonymous) questionnaire was voluntary so it was self-selected sampling. We received 104 responses, 54 from the New Zealand group and 50 from the German group, so the response rate was 57% in the New Zealand group and 48% in the German group. After the questionnaires were completed the lecturers conducted informal interviews with selected students in order to give deeper meaning to the written comments.

Results

Student Difficulties in Formulating Total Cost Function

The main student difficulties while trying to set up the total cost function in the *Trucking Cost Problem* fell into the following two categories:

1. Difficulties related to understanding of the problem (language, use of the given information, identifying the variables); and
2. Difficulties related to the identification and usage of the formula.

Difficulties related to understanding of the problem. Forty eight per cent of the New Zealand group who completed the questionnaire and 36% of the German group responded that they had these difficulties. Some typical comments were as follows:

> "The wording was ambiguous."
>
> "I did not understand the question."
>
> "It was confusing."
>
> "Hard to understand."
>
> "I thought it was too complicated."
>
> "I had trouble deciding how to use the information."
>
> "I did not know how to convert the real life problem into one to solve mathematically."
>
> "I was confused because the result was given."

In the subsequent informal interviews with the selected students it was revealed that the last comment was quite common. The fact that the answer was given hindered many students in their attempt to identify the unknown variable. The students were used to a different way of formulation of the question in min-max problems. They reported that it would be easier for them if the question had been formulated like this: "Find the velocity that minimizes the total cost of a journey of 100 km" instead of "Show that to minimize the total cost of a journey of 100 km in the truck at constant velocity the truck should run approximately 28 km/h". Although they had met mathematics questions like "Show that" before, it was less common for them to see this wording in application problems. The observed sensitivity to the wording of the question in the problem is consistent with anecdotal reports about achievement in final school examinations. In New Zealand every three years there is a notable drop followed by a two year increase in the students' performance at the final school year mathematics examination. It was reported by a chief school mathematics examiner that the reason for the regular drops was the change of a chief school examiner every three years. New chief examiners used their own language style in setting up examination questions which was different from the wording used by their predecessor.

The above difficulties are also consistent with the findings from a research study with upper secondary school students (Years 11 and 12) in Australia by Stillman (2004):

> By far the most frequently reported conditions impeding task access were: language problems related to technical language used in the context, comprehension difficulties, and the unusual wording of tasks. A factor contributing to the high incidence of this condition was the deliberate use of longer verbal tasks in the study than the students were used to in the classroom. Other major conditions (incidence rate ≥ 10) impeding task accessibility were cuing words (of either a mathematical or contextual nature) not being salient (a language-related condition); all the representational conditions, namely, inability to mathematise the context, difficulties with the integration of given or derived contextual information, and difficulties extracting mathematical information from the task context; difficulties recalling a concept, formula, procedure or relevant prior knowledge of the task context (a memory-related condition); and the organisational condition involving difficulties formulating a plan of attack. (p. 53)

Difficulties related to the identification and usage of the formula. Thirty five percent of the New Zealand group and 42% in the German group reported having these difficulties when formulating the *Trucking Cost Problem*. Some typical comments were as follows:

> "Couldn't figure out time as $\frac{100}{v}$."
>
> "I had trouble with the units, because in the given formula I only knew the unit of *v*."
>
> "Could not see the connection between costs per hour and time."
>
> "Did not know where to use $(4+\frac{v^2}{200})$."

These students apparently understood the problem but could not use their existing mathematical knowledge of familiar formulae to set up the required function correctly.

The above two sorts of difficulties are in agreement with the classification from the study by Anaya, Cavallaro, and Domingiez (2007) on novice engineering students' difficulties in mental processes doing a more general modelling task. These researchers found that the students in their study displayed

- Difficulties related to the relational understanding of the situation to be modelled, including difficulties in the identification of variables and unknowns.

- Difficulties related to creativity in establishing associations and relationships between pieces of knowledge that eventually might not have been related up to that moment.
- Difficulties related to the choice of the available knowledge and the use of the given information. (p. 428)

Student Perceptions of How to Improve their Skills in Formulation

In responding to the question about what could be done to improve their skills in carrying out the formulation step of an application problem, the vast majority of the students (87% in the New Zealand group and 78% in the German group) thought that they needed more practice in solving application problems similar to the *Trucking Cost Problem* in class to improve their problem solving skills. Some of them asked to be taught detailed steps in solving such application problems and to be given more "demonstration". Only a small number of the students (6% in the New Zealand group and none in the German group) suggested making the wording of a problem easier to understand despite the fact that almost half of the students in the New Zealand group stated that they did not understand the problem.

Discussion and Conclusions

The two major student difficulties reported in this study dealt with understanding of the wording of the question in the problem and with using their existing knowledge of familiar formulae for setting up the required function in a familiar context. Both those difficulties are typical characteristics of novices that were observed in a number of studies (e.g,. Anaya et al., 2007; Crouch & Haines, 2004; Galbraith & Haines, 2000). According to Crouch and Haines (2004):

> Many students' difficulties may be due to their being comparatively new to mathematical modelling. Such novices, in a variety of fields, tend to perform poorly compared to experts, as may be expected. This seems to be due to novices possessing a much smaller and more poorly structured knowledge base, making it difficult for them to know which information is relevant, what type of problem they are dealing with and to know which techniques and procedures to apply, while experts generally have the experience and knowledge to do this successfully. (p. 199)

Quite surprising for us was the observed sensitivity of many students to the wording of the question where they were asked to show that the required velocity was equal to the given value instead of finding the required velocity.

The vast majority of the students who participated in the study thought that they needed more practice to improve their skills in solving application problems. This is consistent with another study (Klymchuk & Zverkova, 2001) with more than 500 university students from 9 countries where the students also indicated that they felt it difficult to move from the real world to the mathematical world because of lack of practice in application tasks.

Practice is certainly one of the ways that helps students to progress from novices to experts. We can increase time spent on application problems and show their importance by including more such problems in the assessment. At the moment in our (authors') calculus courses about 25% of all test and assignment questions are application problems. This can be increased to 50%. It might encourage students to practise more with application problems preparing for their tests and examinations.

Research on students' difficulties followed up by practical recommendations also can help with a faster transition from novices to experts. As an example, Ubuz (1994) developed a problem-solving method with handout material using Polya's (1945) problem-solving strategies. She tested it on min-max word problems with undergraduate calculus students and suggested a number of potentially successful implications for educational practice. In another study Clement et al. (1981) made several practical recommendations for the improvement of students' translation skills from the real world to the mathematical world. They suggested more practice with emphasis on specific modelling tasks:

> Our own experience suggests that one method for helping students to acquire these (translation) skills is to: (1) allocate time in courses for developing and practising them as separate skills; (2) assign translation problems that cannot be solved by trivial syntactic or other nonoperative approaches; (3) show by many examples the shortcomings of the latter methods; (4) emphasize the operative nature of equations. These techniques have given us encouraging preliminary results. (p. 289)

For a deeper analysis of students' difficulties interviews, interventions and action research might be more effective than an observational study employed in this paper. Those research methods can be applied using the 'Framework for identifying blockages in transitions' developed by Galbraith, Stillman, Brown, and Edwards (2007). As it was described in Stillman, Brown and Galbraith (2008):

> The Framework has direct application arising from the way the research has been conducted - to identify specifically, activities and content with which modellers need competence in order to successfully apply mathematics at their level. This is what the Framework systematically documents. Given that the elements in the framework were identified by observing students working (and, in particular, wrestling with blockages to progress), there are two immediate potential applications. First are the insights obtained into student learning, and how these can inform our understanding of the ways that students act when approaching modelling problems. Second are associated pedagogical insights. By identifying difficulties with generic properties, the possibility arises to anticipate where, in given problems, blockages of different types might be expected. This understanding can then contribute to planning teaching and task design, in particular the identification of prerequisite knowledge and skills, preparation for intervention at key points if required, and scaffolding of significant learning episodes. (pp. 155-156)

The comments of the students who participated in the study make us think that our assumptions about reasonable application problem solving skills of our students were too optimistic. It demonstrates again that assumptions lecturers make about students regarding their knowledge base and successful completion of earlier modules and/or examinations cannot be relied upon as it was shown in Anderson, Austin, Bernard, and Jagger (1998). In our diagnostic test in the beginning of the course we check only students' basic mathematical techniques, not their skills in solving application problems. This study shows that there is a need to teach the students basic skills in solving application problems from the beginning of a calculus course. We should encourage the students to write all steps of the modelling process in detail, even for simple application problems. This can prepare them to deal with real-life problems that require advanced mathematical modelling skills in their other courses and also at work. We should show our students that are not the only ones to experience difficulties in the formulation step of modelling. A survey of non-academic mathematicians undertaken by the Society for Industrial and Applied Mathematics (SIAM) showed that "the hardest task for a mathematician is developing the real problem requirements .... Problems never come in formulated as mathematics problem" (The Society for Industrial and Applied Mathematics, 1998, p. 15). "Some of the most important traits in non-academic mathematicians include skills in formulating, modelling, and solving problems from diverse and changing areas" (The Society for Industrial and Applied Mathematics, 1998, p. 1). Such quotations might encourage students to devote more time and effort to solving application problems. We agree with Kadijevich (1999) who pointed out an important aspect of doing even simple mathematical modelling activity by first-year undergraduate students: "Although through solving such … [simple modelling] … tasks students will not realise the examined nature of modelling, it is certain that mathematical knowledge will become alive for them and that they will begin to perceive mathematics as a human enterprise, which improves our lives" (p. 36).

Acknowledgements

We would like to express our sincere gratitude to Jill Brown from Australian Catholic University and the anonymous reviewers for their constructive comments and thoughtful suggestions which led to many improvements in the paper.

References

Anaya, M., Cavallaro, M. I., & Domingiez, M. C. (2007). Modelling: Difficulties for novice engineering students. In C. Haines, P. Galbraith, W. Blum, & S. Khan (Eds.), *Mathematical Modelling (ICTMA 12): Education, Engineering and Economics* (pp. 324-332). Chichester, UK: Horwood.

Anderson, J., Austin, K., Bernard, T., & Jagger, J. (1998). Do third year mathematics undergraduates know what they are supposed to know? *International Journal of Mathematical Education in Science and Technology, 29*(3), 401-420.

Clement, J., Lochhead, J., & Monk, G. (1981). Translation difficulties in learning mathematics. *The American Mathematical Monthly, 88*(4), 286-290.

Craig, T. (2002). Factors affecting students' perceptions of difficulty in calculus word problems. In I. Vakalis, D. H. Hallet, C. Kourouonoitis, D. Quinney, & C. Tzanakis (Eds.), *Proceedings of the Second International Conference on the Teaching of*

Mathematics at the Undergraduate Level – ICTM-2. Crete, Greece: Wiley, CD format, #411.

Crouch, R., & Haines, C. (2004). Mathematical modelling: Transitions between the real world and the mathematical world. *International Journal on Mathematics Education in Science and Technology, 35(*2), 197-206.

Galbraith, P., & Haines, C. (1997). Some mathematical characteristics of students entering applied mathematics courses. In S. K. Houston, W. Blum, I. D Huntley, N. T. Neill (Eds.), *Teaching and learning mathematical modelling* (pp. 77-92). Chichester, UK: Albion.

Galbraith, P., & Haines, C. (2000) Conceptual mis(understandings) of beginning undergraduates. *International Journal of Mathematical Education in Science and Technology*, *31*(5), 651-678.

Galbraith, P., Stillman, G., Brown, J., & Edwards, I. (2007). Facilitating middle secondary modelling competencies. In C. Haines, P. Galbraith, W. Blum, S. Khan (Eds.), *Mathematical modelling (ICTMA 12): Education, engineering and economics* (pp. 130-140). Chichester, UK: Horwood.

Gruenwald, N., & Schott, D. (2000). World mathematical Year 2000: Ideas to improve and update mathematical teaching in engineering education. *Proceedings of the 4th Baltic region seminar on engineering education* (pp. 42-46). Lyngby, Copenhagen, Denmark: DTU.

Haines, C., & Crouch, R. (2001). Recognising constructs within mathematical modelling. *Teaching Mathematics and its Applications*, *20*(3), 129-138.

Kadijevich, D. (1999). What may be neglected by an application-centred approach to mathematics education? *Nordisk Matematikkdidatikk, 7*(1), 29-39.

Klymchuk, S. S. & Zverkova T. S. (2001). Role of mathematical modelling and applications in university service courses: An across countries study. In J. F. Matos, W. Blum, S. K. Houston, & S. P. Carreira (Eds.) *Modelling, applications and Mathematics Education – Trends and Issues* (pp. 227-235). Chichester, UK: Ellis Horwood.

Maull, W., & Berry, J. (2001). An investigation of students working styles in a mathematical modelling activity. *Teaching Mathematics and its Applications*, *20*(2), 78-88.

Niss, M., Blum, W., & Galbraith, P. (2007). Introduction. In W. Blum, P. Galbraith, M. Niss, & H-W. Henn (Eds.), *Modelling and Applications in Mathematics Education: The 14th ICMI Study (New ICMI Series)* (Vol. 10, pp. 3-33). New York: Springer.

Nyman, A., & Berry, J. (2002). Developing transferable skills in undergraduate mathematics students through mathematical modelling. *Teaching Mathematics and its Applications*, *21*(1), 29-45.

Polya, G. (1945). *How to solve it.* Princeton, NJ: Princeton University Press.

Stillman, G. (2004). Strategies employed by upper secondary students for overcoming or exploiting conditions affecting accessibility of applications tasks. *Mathematics Education Research Journal*, *16*(1), 41-71.

Stillman, G., Brown, J., & Galbraith, P. (2008). Research into the teaching and learning of applications and modelling in Australasia. In H. Forgasz et al. (Eds.), *Research in Mathematics Education in Australasia 2004-2007* (pp. 141-164). Rotterdam, The Netherlands: Sense Publishers.

The Society for Industrial and Applied Mathematics (SIAM). (1998). *Report on Mathematics in Industry*. Retrieved from http://www.siam.org/about/mii /Report.pdf

Ubuz, B. (1994). Problem-solving method with handout material: max-min word problems. *International Journal of Mathematical Education in Science and Technology*, *25*(3), 367-376.

Authors

Sergiy Klymchuk, Auckland University of Technology, Auckland, New Zealand. Email: <sergiy.klymchuk@aut.ac.nz>

Tatyana Zverkova, Odessa National University, Ukraine. Email: <tsklim@rambler.ru>

Norbert Gruenwald, Hochschule Wismar University of Technology, Business and Design, Wismar, Germany. Email: <norbert.gruenwald@hs-wismar.de>

Gabriele Sauerbier, Hochschule Wismar University of Technology, Business and Design, Wismar, Germany. Email: <gabriele.sauerbier@hs-wismar.de>