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# CAS-enabled Technologies as 'Agents Provocateurs' in Teaching and Learning Mathematical Modelling in Secondary School Classrooms

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This paper draws on a one year study of three secondary school classrooms to examine the nature of student-student-technology interaction when working in partnership with computer algebra systems (CAS) on mathematical modelling tasks and the classroom affordances and constraints that influence such interaction. The analysis of these data indicates that CAS enabled technologies have a role to play as provocateurs of productive student-student-teacher interaction in both small group and whole class settings. Our research indicates that technologies that incorporate CAS capabilities have the potential to mediate collaborative approaches to mathematical enquiry within life-related mathematical tasks.

There has been interest by educational researchers and curriculum developers in the use of both mathematical modelling and digital technologies to enhance the learning experiences of students in secondary mathematics classrooms for at least the past two decades. Mathematical modelling – formulating a mathematical representation of a real world situation, using mathematics to derive results, interpreting the results in terms of the given situation and if necessary, revising the model – is a significant element of the senior mathematics syllabuses in Queensland, Australia, and appears, as applications of mathematics, in the curriculum documents of most other Australian states. The need to make use of digital technologies in learning mathematics is now included in most state curriculum documents within Australia and is increasingly apparent in curricula internationally. Recently, Computer Algebra System (CAS) enabled technologies have begun to make an impact on teaching and learning practices, most notably within Australia as part of the Victorian Curriculum and Assessment Authority's (2006) CAS active version of *Mathematical Methods*. CAS-enabled technologies not only have the capability to perform a wide range of mathematical procedures, such as function graphing, matrix manipulation and symbolic operations, but also the capacity to provide users with real time advice about errors as mathematics is done. As CAS-enabled technologies develop increasing acceptance in mainstream mathematics instruction, there is a need to explore and understand the synergies that might be developed between CAS and other areas of foci in mathematics education, such as mathematical modelling, and to identify implications of these synergies for classroom practice. This is particularly important as, to date, research has usually lagged behind implementation of CAS active curricula (Zbiek, 2003).

In addition, the current emphasis on the quality of interactions between students and between students and teachers in school mathematics classrooms (e.g., see Goos, Galbraith, Renshaw, & Geiger, 2003; Manouchehri, 2004) means that any innovation that has the potential to influence these interactions must be of interest to researchers and teachers alike. The synergy that is likely to exist between mathematical modelling and CAS-enabled technologies is one such innovation. While there is significant research related to solving contextualized problems through the use of the multiple representational facilities offered by digital technologies (e.g., Doerr & Zangor, 2000; Huntley, Rasmussen, Villarubi, Santong, & Fey, 2000; Yerushalmy, 2000) and substantial argument to support the use of CAS to enhance the process of mathematical modelling (e.g., Kissane, 1999, 2001; Thomas, 2001), literature that deals with CAS-enabled technology mediated interaction in mathematics classrooms is only just emerging. The aim of the research reported in this article was to develop a greater understanding of how CAS-enabled technologies can support students' learning when they are engaged in mathematical modelling tasks, including ways in which CAS can mediate and support productive social interaction.

## The Role of Technology in the Process of Mathematical Modelling

While there are now significant bodies of literature on the use of CAS-enabled technologies and on mathematical modelling in school contexts, comparatively little has been written on how technology can be used to enhance the processes of mathematical modelling. An examination of the volume produced from the 14th Study of the International Commission for Mathematical Instruction entitled *Modelling and Applications in Mathematics Education* (Blum, Galbraith, Henn, & Niss, 2007) for example, contains only one chapter out of 58 which focuses on technology use in mathematical modelling. This is despite acknowledgment from the editors that:

> Many technological devices are highly relevant for applications and modelling. They include calculators, computers, the Internet, and computational or graphical software as well as all kinds of instruments for measuring, for performing experiments etc. These devices provide not only increased computational power, but broaden the range of possibilities for approaches to teaching, learning and assessment. On the other hand, the use of calculators and computers may also bring associated problems and risks. (Niss, Blum, & Galbraith, 2007, p. 24)

Niss et al. (2007) go on to list nine questions related to the potential benefits of technology to mathematical modelling and to the possible dangers. These questions are not addressed directly by the authors who propose them and are only responded to, in passing, by other authors in the volume. This is an indication that, while there is acknowledgement of the potential for technology to enhance processes associated with mathematical modelling, there is limited literature that addresses this issue directly. Despite this limitation, a number of models for the role of technology in mathematical modelling have been proposed. Two of these models (Galbraith, Renshaw, Goos, & Geiger, 2003 and Confrey & Maloney, 2007) are presented below.

## *Technology for Dealing with Routines and Mathematical Processes*

Based on a three year longitudinal case study of a class of students studying mathematics in technologically rich environments, Galbraith, Renshaw, Goos and Geiger (2003) provide a description of the role of technology in the process of working with applications of mathematics and mathematical modelling. In this description, illustrated in Figure 1, mathematical modelling is presented as a cyclic process that starts with a problem set in a life-related context. The problem is then abstracted into a mathematical representation of the contextualised situation and solved through the application of mathematical routines and processes. The resulting solution is brought into relief against the original problem to consider its fit with the original context. If the fit is not considered sufficient, adjustments are made to the model and the process repeated until a satisfactory fit is achieved. Galbraith et al. (2003) argue mathematical routines and processes, students and technology are engaged in partnership during the *Solve* phase of a problem. This view identifies the conceptualization of a mathematical model as an exclusively human activity while the act of finding a solution to the abstracted model can be enhanced via the incorporation of technology.

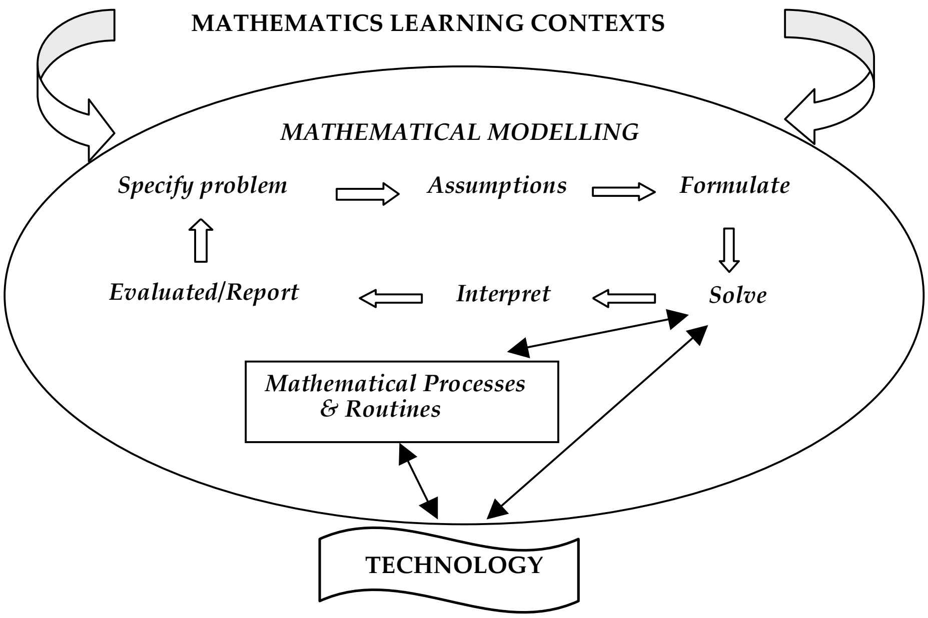


*Figure 1.* Some technological and mathematical interrelationships from Galbraith, Renshaw, Goos, & Geiger (2003, p. 114).

## *Modelling as Transforming Indeterminate Situations into Determinate Outcomes*

Confrey and Maloney (2007) identify four approaches to technology in mathematics instruction:

1. Teach concepts and skills without computers, and provide these technological tools as resources after mastery;
2. Introduce technology to make patterns visible more readily, and to support mathematical concepts;
3. Teach new content necessitated by technologically enhanced environments (estimation, checking, interactive methods);
4. Focus on applications, problem solving, and modelling, and use the technology as a tool for their solution. (p. 57)

While acknowledging that each of these approaches has its place, Confrey and Maloney regard mathematical modelling as a central goal of mathematics instruction. Drawing on a Deweyian definition of inquiry, they argue that the process of modelling is founded on two activities: inquiry and reasoning. They see inquiry as a means of gaining insight into an indeterminate situation – such as a loosely bound problem in the real world. Reasoning is the process which draws on bodies of knowledge to transform the indeterminate situation into a determinant outcome – a model. In their view:

> Mathematical modelling is the process of encountering an indeterminate situation, problematizing it, and bringing inquiry, reasoning, and mathematical structures to bear to transform the situation. The modelling produces an outcome – a model – which is a description or representation of the situation, drawn from the mathematical disciplines, in relation to the person's experience, which itself has changed through the modelling process. (p. 60)

The process of inquiry gives rise to observations, responses, measurements, interactions, indicators, methods of sampling and data collection that are typically mediated by various forms of technology. Confrey and Maloney (2007) claim that it is through the coordination of these artefacts and the processes of inquiry, reasoning and experiment, that an indeterminate situation is transformed into a determinate situation (see Figure 2). The role of technology in this model is many-fold. Technology can generate and incorporate representations which can assist in transforming an indeterminate situation into a determinate one. Technology also plays a central role in coordinating the inquiry, reasoning, and systematising that lead to a determinate situation.

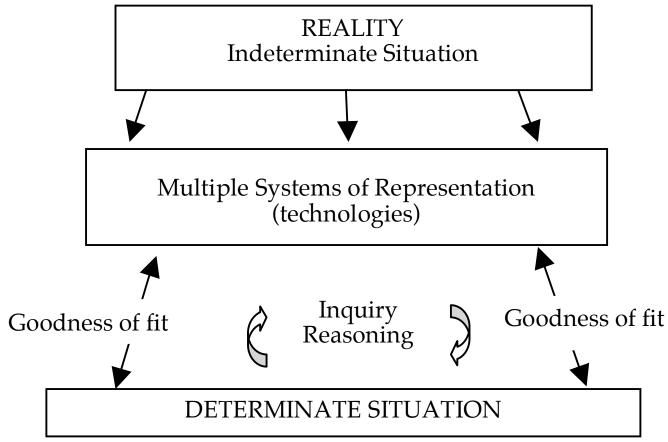


*Figure 2.* An inquiry and reasoning approach to mathematical modelling from Confrey and Maloney (2007, p. 67).

### *Commentary on the Role of Technology in Mathematical Modelling*

Both of these descriptions of the role of technology recognise the cyclic nature of the modelling process and the important interplay between exploration, conjecture, reasoning, the use of mathematical procedures and processes, and the validation of solutions. In both of these models technology can be used to deal with mathematical procedures; however, Confrey and Maloney (2007) suggest technology has a role to play in the coordination of inquiry and reasoning and, by association, interaction. What is not explored in this approach is the nature of the interaction between human participants and technology in the cycle of inquiry and reasoning.

In the Galbraith, Renshaw, Goos, and Geiger (2003) model, technology is seen as a tool used to assist with the solution of a problem after a mathematical model is developed rather than as a tool for the exploration and development of a model or its validation as a reliable representation of a life related situation.

The section that follows provides an outline of research into the role of technology in mediating productive student interaction in mathematics classrooms.

## The Role of Technology in Promoting Social Interaction in Technologically Rich Mathematics Classrooms

While there is now an extensive body of literature related to the use of technology to learn mathematics, this has been principally concerned with the acquisition of mathematical knowledge and the advantages offered by the capacity of technology to present multiple representations of mathematical ideas as an aid to learning. Other authors (e.g., Burrill, 1992; Galbraith, Renshaw, Goos, & Geiger, 1999; Goos, Galbraith, Renshaw, &

Geiger, 2000b), however, have suggested that the most significant changes related to the introduction of technology into mathematics classrooms will be in the ways students and teachers interact. From this perspective, questions such as the role technology can have in mediating social interaction, or how technology is entwined into the fabric of a learning discourse in collaborative learning environments, receive greater primacy. There is now a growing interest in social perspectives of learning with technology and a number of authors have attempted to define the territory. Simonsen and Dick (1997), for example, in a study of teachers' perceptions of students' use of graphics calculators, conclude that this technology has a role to play in shifting the orientation of the classroom towards more student centred, discursive and exploratory approaches. The availability of technology alone, however, will not ensure the development of collaborative practices (Beatty & Geiger, 2009) and so the teacher has a vital role to play in mediating the type of social interaction that is regarded as collaborative within a classroom of learners.

Some time ago Willis and Kissane (1989) introduced the notion of Computer as a Catalyst. In this mode, the computing environment is used as a means of provoking mathematical explorations and discussion or to invoke the use of problem solving skills. This recognises the potential of technology to support learning-focused interaction between students and suggests a mediating role for technology in learning. The metaphor of Computer as a Catalyst is further extended by Goos and Cretchley (2004) in a review of the role of technology in education in the Australasian region. In their view, the computer is a tool that can be used as a catalyst for visualisation, higher order thinking and collaboration.

The use of tools to mediate higher order thinking and collaborative learning is consistent with a socio-cultural perspective on intellectual development where learning takes place via social interaction and is supported by cultural artefacts or physical tools. The inseparability of cognitive activity from both the process of learning within the group and from the tools that help mediate the activity is consistent with a Vygotskian view of the social nature of learning and also with Pea's (1985, 1987) description of the role of cognitive tools in distributed cognition. This view considers humans are elements in a reasoning system that includes human minds, social contexts and tools. Digital tools, such as computers and graphics calculators, can be used to mediate productive collaborative interaction even though these tools have not been specifically designed to support collaboration (Beatty & Geiger, 2009; Geiger, 1998; Geiger & Goos, 1996; Goos, Galbraith, Renshaw, & Geiger, 2000a, 2000b; Trouche, 2005). Studies which exemplify the use of technologies designed primarily as mathematical tools but are used to mediate collaborative practice include those of Geiger and Goos (1996) and Manouchehri (2004).

In a case study designed to investigate the social and material mediation of computer-based learning in an upper secondary mathematics classroom, Geiger and Goos (1996) found that interaction was both tool and task dependent. In that study, the computer was intended to act as both a tool, in enabling students to generate and manipulate data in a spreadsheet, and as a catalyst, in provoking exploration of the patterns that emerged from the data. However, it was found that the extent to which such exploration occurred depended on the type of task the students were given. Differences in the social organisation of students' work, identified in the function of their talk and the structure of their interaction, were associated with differences in task focus. A focus on process, rather than products or means, was found to encourage collaborative discussion. Results implied that computer environments do not automatically facilitate peer interaction and that careful attention needed to be given to the structure of tasks if they were to elicit high-level verbal reasoning. Students were most likely to interact if there was a genuine problem to be solved, consistent with the findings of research on talented students summarised by Diezmann, Faragher, Lowrie, Bicknell and Putt (2004). This finding is of particular relevance to the current study because the nature of mathematical modelling ensures students will be challenged by authentic, loosely bound problems.

In a study involving undergraduate preservice teachers using NuCalc, an interactive algebra application, Manouchehri (2004) observed students' mathematical discussions displayed greater complexity while using NuCalc than when they used no mathematical computer application. Manouchehri identified the following four ways that the software supported discourse:

1. by assisting peers in constructing more sophisticated mathematical explanations;
2. by motivating engagement and increased participation in group inquiry;
3. by mediating discourse, resulting in a significant increase in the number of collaborative explanations constructed;
4. by shifting the pattern of interaction from teacher directed to peer driven.

Further, Manouchehri concluded that because of the immediacy of feedback to students, the software also supported a culture of conjecturing, testing and verifying, formalising mathematics and collaboration and shifted the locus of power from the teacher to the students.

These studies offer support for the premise that technology can play a role in the mediation of collaborative learning processes. The immediacy of the feedback provided by technology can offer enhanced possibilities for classrooms where conjecturing, testing and verifying mathematical argumentation is viewed as important aspects of learning and doing mathematics. These aspects are also consistent with the model for the use of technology in mathematical modelling proposed by Confrey and Maloney (2007) outlined earlier in this paper.

## The Study

The research questions guiding the study reported in this article were:

1. How can CAS-enabled technologies support students' learning when they are engaged in mathematical modelling tasks?
2. How can CAS mediate and support productive social interactions between students?

Different aspects of the models proposed by Galbraith, Renshaw, Goos and Geiger (2003) and Confrey and Maloney (2007) were used as a theoretical framework to investigate the first research question. The cycle which forms the basis of Galbraith, Renshaw, Goos and Geiger's (2003) model outlines phases of activity in which a problem is specified, assumptions are made, a mathematical model is formulated, solved, interpreted and evaluated. These phases were used as a framework to analyse the types of activity impacted upon by CAS-enabled technologies. In particular, activity within and between these phases was examined for evidence of the influence of CAS upon the coordination of inquiry and reasoning that Confrey and Maloney (2007) suggest technology can enhance. The role of CAS in mediating social interaction within the inquiry and reasoning approaches employed by students was also documented as it relates to the cyclic model and is used to address the second research question.

## *Participants*

The research reported here is based on data sourced from a 12 month study of the use of CAS-enabled technologies in senior secondary classroom settings. Participants consisted of three secondary mathematics teachers, one from a government school and two from non-government schools, one class of students for each teacher and a university based researcher (first author). Teachers were selected because of their interest in exploring the use of CAS-enabled technologies in teaching senior mathematics and because of a history of effective use of mathematical modelling tasks in their teaching practice. In addition, all three teachers were known to be supportive of collaborative approaches to learning mathematics. While each teacher had responded to mandatory curriculum requirements by developing considerable experience and expertise in teaching mathematical modelling and the use of non-CAS technologies such as graphing calculators, there were differences in their facility with using CAS-enabled technologies which ranged from that of expert to novice.

The three cohorts of students consisted of one Year 12 class and two Year 11 classes. All classes were studying Mathematics B, a subject that includes substantial elements of calculus and statistics. Each class was equipped with a set of Texas Instruments CAS-enabled Nspire handheld devices (at least one for each student and the teacher) and one licence for software that mirrored the facilities of the Nspire handheld device. These technologies possess all of the features of a typical graphing calculator, such as function and graph plotting modules, but also include a CAS capability that is highly integrated with other calculator facilities. Other features include a fully functional spreadsheet (again with CAS integrated capability) and a sophisticated feedback mechanism for reporting on input errors.

Students' experience in the use of technology to learn mathematics varied across the three classes. While none of the students had used the Nspire handhelds before the beginning of the year in which the study was conducted, two groups experienced substantial previous use by the time they were first observed by the researcher, the other group receiving very limited exposure.

## Methodology and Research Design

The data and analysis presented here concern the interactions between students and teachers in two different classrooms. Research into interactions between multiple participants and between participants and digital tools in an authentic classroom setting must employ a methodology with the capacity to accommodate educational phenomena that are situated, temporal and complex. Further, the nature of the classroom environment brings with it, in the case of this study, the prospect of unanticipated or emergent outcomes (Ramsden, 1997) in terms of both the usage of digital technologies and in the type and quality of the interactions between participants and technologies – thus a naturalistic research design was employed (Lincoln & Guba, 1985).

A case study approach (Burns, 2000) was used to document the actions and interactions of the teachers and students who are the focus of this report. Sampling was purposive and opportune as cases were chosen for the capacity to illuminate and enhance understanding rather than for representativeness (Stake, 2005). In particular, the cases reported in this paper were selected because they represent independent occurrences of a learning/teaching phenomenon which emerged (Ramsden, 1997) during the course of the study.

The researcher was responsible for classroom observations and video recording as well as conducting interviews with both students and teachers. Data collection methods included observational field notes, video and audio recording of small groups of students working on specified tasks, video and audio recording of episodes of whole class activity as well as follow-up individual teacher interviews, and a focus group interview involving the three teacher participants in the project. In the individual interviews, teachers were asked what they had noticed during a lesson in relation to students' use of CAS during the phases of the cyclic model and also technology influenced approaches to inquiry and reasoning. They were also asked to comment on any unexpected events and to speculate on reasons for their occurrence. The purpose of the focus group interview was to further document the participant teachers' perceptions of classroom events and the benefits offered by CAS to mathematical modelling and also to triangulate, through the teachers' responses, the researcher's observations, reflections and initial theory formation. Each class group was observed on three different occasions, each time for periods ranging from 45 minutes through to 90 minutes. On the majority of occasions, students and teachers worked on tasks that incorporated some element of mathematical modelling.

Consistent with a naturalistic methodology, data collection and analysis were conducted simultaneously with the development of theory. In this case, theory is being developed for the role of CAS-enabled technologies in supporting student learning, within the social milieu of secondary school mathematics classrooms, while engaged in mathematical modelling tasks. This theory will expand on the frameworks offered by Galbraith et al. (2003) and Confrey and Maloney (2007) by specifically addressing the role of CAS in the human interaction involved in modelling.

The mode of analysis employed is that of explanation building (Burns, 2000). The explanation building process is iterative as the explanations of initial observations of phenomena are formulated into theoretical propositions which are in turn tested, revised and refined against further data. Instances of emergent behaviour were documented and categorised. Where emergent phenomena were noted and documented, the researcher made use of follow-up interviews with the relevant teacher in order to triangulate the occurrence of the identified phenomena and to discuss possible explanations for what was observed (Lincoln & Guba, 1985). By ascertaining the participant teachers' perspective on the event, occurrence or episode the researcher gathered additional evidence in favour of one explanation over other rival explanations. The researcher then incorporated the observed phenomena and explanation into an initial theoretical proposition. After initial theory was developed the researcher was sensitised to the observation of similar phenomena during further data gathering from participants' classrooms which in turn initiated additional iterations of follow-up interviews with teachers and further revision of theory. The congruence between theory and observed phenomenon received further scrutiny during the final focus group interview which included all participating teachers.

The following analysis is based on data drawn from two classroom episodes, which are reported as vignettes, and the focus group interview conducted towards the conclusion of the project. These episodes were chosen because they represent two instances of similar teaching/learning activity that occurred independently in different classrooms in different schools. The vignettes are developed from observational field notes on whole class activity, audio and video recordings of students and teachers working together in both small group and whole class settings and follow-up interviews of the teachers. The analysis of focus group data is based on video and audio records of the focus group meeting.

## Results and Discussion

### *Two Vignettes where CAS Promotes Contention*

The two vignettes reported below come from classrooms in two different schools – one a government school and the other a non-government school. The teacher in the government school had personal experience with the use of CAS but had not used it previously in his teaching. His students had begun to make use of CAS from the beginning of the year; approximately two months before the first vignette was documented. The second vignette records an episode from the classroom of a teacher from a non-government school. This teacher had extensive experience with the use of CAS and his students had been using CAS, in at least a limited way, for two years of schooling. These students, however, had been using the Nspire handheld for only two months. In both cases, the teachers challenged students to make use of CAS based technology as an aid to working with mathematical modelling problems. The analysis presented here builds on the work previously presented by Geiger, Faragher, Redmond, and Lowe (2008).

## *Vignette 1*

The vignette described below took place in a Year 12 (final year of secondary school) mathematics classroom where students were investigating the nature of population decay towards extinction. The teacher had set students the following question.

> When will a population of 50 000 bacteria become extinct if the decay rate is 4% per day?

One pair of students developed an initial exponential model for the population $y$ at any time $x$, $y = 50000 \times (0.96)^x$ as they believed the decay of the bacteria population would be exponential in nature. They then equated the model to zero in order to represent the point at which the bacteria would be extinct. Their intention was to solve this equation using CAS in order to find the number of periodic cycles and hence the time it would take for the population to become extinct. When students entered this equation into their Nspire handhelds, however, the device unexpectedly responded with a false message, as illustrated in Figure 3.

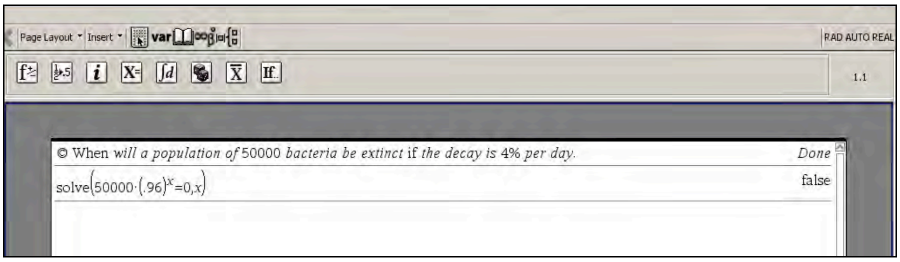


*Figure* 3. Nspire display for the problem $y = 50000 \times (0.96)^x$

The students were initially concerned that this response had been generated because they had made a mistake with the syntax of their command. They re-entered the instruction several times and tried a number of variations to the structure of the command but did not consider that there was anything at fault with the parameters they had entered. When the students asked their teacher for assistance, he looked at the display and stated that there was nothing wrong with the technical side of what they had done but they should think harder about their assumptions.

After further consideration, and no progress, the teacher directed the problem to the whole class. One student indicated that the difficulty being experienced was because "you can't have an exponential equal to zero". This resulted in a whole class discussion of the assumption that extinction meant a population of zero. The discussion identified the reason for the unsatisfactory calculator output as inappropriately equating an exponential model to zero and then considered the possible alternatives. Eventually the class adapted the original assumption to accommodate the limitations of the abstracted model by accepting the position that extinction was "any number less than one". Students then made this adjustment to their entries on the handheld and a satisfactory result was returned.

In a follow-up interview, directly after the lesson, the researcher (R) asked the teacher (T1) about the episode.

R: I saw an element of what we just talked about today when conflict was generated by an interpretation of the question about bacteria. Students developed an equation and then, because no bacteria were left, they equated it to zero. The calculator responded with a false message. In some ways you could think it was a distraction and that the procedure didn't work; some kids might just give up. But on the other hand, what it provoked in your class was an opportunity to discuss. "Did you push the wrong buttons? Oh, you think you did – let's look at the maths. Well your maths is right! Do you understand why it couldn't be? Let's talk about the assumption".

T1: Simon was one of those, he said – "no way you could get that to equal zero", without necessarily understanding why. Not that he couldn't solve it when it equalled zero, it was that concept he couldn't see; that population couldn't become zero.

R: Yes didn't need CAS to understand that, they just understood it because they knew their maths well enough.

T1: Yeah we actually use the CAS to create the confrontation.

In this excerpt, the teacher identifies the message created by the CAS – that the equation was *false* – as a mechanism for confronting a student's lack of understanding of the interplay between the demands of developing an appropriate mathematical model and a valid mathematical expression. The context of the problem indicated to the students that the model should be equal to 0 to represent the extinction of the bacteria although, from a purely mathematical perspective, this was not valid. Interestingly, this conflict or "confrontation" was viewed by the teacher as an opportunity to promote productive interaction among the class, which ultimately led to the resolution of the problem and a broader understanding of the role of assumption in the mathematical modelling process.

## *Vignette 2*

In this second vignette the other teacher was working with a Year 11 class on a unit about a variety of mathematical functions including linear, quadratic, cubic, exponential and power functions. During the observed session they were asked to work on the following task.

> The CSIRO has been monitoring the rate at which Carbon Dioxide is produced in a section of the Darling River. Over a 20 day period they recorded the rate of $CO_2$ production in the river. The averages of these measurements appear in Table 1.
> The $CO_2$ concentration $[CO_2]$ of the water is of concern because an excessive difference between the $[CO_2]$ at night and the $[CO_2]$ used during the day through photosynthesis can result in algal blooms which then results in oxygen deprivation and death of the resulting animal population and sunlight deprivation leading to death of the plant life and the subsequent death of that section of the river.

Table 1
*Rate of $CO_2$ Production Versus Time*

| Time in hours | Rate of $CO_2$ Production |
|---|---|
| 0 | 0 |
| 1 | -0.042 |
| 2 | -0.044 |
| 3 | -0.041 |
| 4 | -0.039 |
| 5 | -0.038 |
| 6 | -0.035 |
| 7 | -0.030 |
| 8 | -0.026 |
| 9 | -0.023 |
| 10 | -0.020 |
| 11 | -0.008 |
| 12 | 0 |
| 13 | 0.054 |
| 14 | 0.045 |
| 15 | 0.040 |
| 16 | 0.035 |
| 17 | 0.030 |
| 18 | 0.027 |
| 19 | 0.023 |
| 20 | 0.020 |
| 21 | 0.015 |
| 22 | 0.012 |
| 23 | 0.005 |
| 24 | 0 |

From experience it is known that a difference of greater than 5% between the [$CO_2$] of a water sample at night and the [$CO_2$] during the day can signal an algal bloom is imminent.
Is there cause for concern by the CSIRO researchers?
Identify any assumptions and the limitations of your mathematical model.

Students were expected to build a mathematical model by inspecting a scatterplot that would then be used to determine the general form of function that would best fit the data. This general form was then to be adapted for the specific data presented in the question and used to address the questions at the end of the task. Students had earlier studied strategies for determining if a particular function type was most suited to a data set. Most recently, students were introduced to a technique where *ln* versus *ln* plots of data sets were used to determine if a power function was an appropriate basis on which to build a mathematical model. This appears to have influenced the actions of two students as the transcript below indicates.

> R: So you are up to building the model are you?
>
> S1: Well we worked out a plan of what we are going to do, we are just putting it on paper.
>
> R: So do you want to tell me what the plan is?
>
> S1: The plan is to do the Log/Log plot of both the data to see if they are modelled by a power function. We have previously seen that the………..
>
> R: So that is something you have learnt to do over time? Whenever you see data look like that, you check if it's a power function by using Log/Log?
>
> S1: Yes.

These students experienced problems with this approach, however, as the technique employed, in this case, meant the students tried to find the natural logarithm of 0.

> S1: 0.44 zero … (entering information into the Nspire device). Don't tell me I have done something wrong. Dammit. Mumbles … Start at zero is it possible to do a power aggression (sic)? I don't think so!

This comment was in response to the display that resulted when the students attempted to find the natural logarithm of both *Time* and $CO_2$ *output data* using the spreadsheet facility of their handheld device (see Figure 4). Students were surprised by the outputs they received for both sets of calculations, that is, the #UNDEF against the 0 entry in the *Time* column and the lack of any entries in the $CO_2$ column. In addition, an error message was produced indicating the results of the students' entries were problematic for the handheld device.

After a little more thought students realised where the problem lay.

> S1: *Ln* time is going to be equal to the *Ln* of actually time. Time…. Oh, is that undefined 'cause it's zero?
>
> S2: Yep.
>
> S1: Right now if I go back to my graph… Enter
>
> S2: If you try zero fit, it will just go crazy.

The students eventually identified the problem with their approach and realised their initial assumption, that is, the best model for the whole data set was a power function, was at fault. Eventually, they realised it was best to model the data with two separate functions.

> S1: So we have fitted a linear model for the top data and then we fitted a power function to the bottom data given we take the absolute value of those the question asks, the difference greater than 5% we need to look at the actual $CO_2$ produced, now what we have got is the rate, to go back to the actual $CO_2$

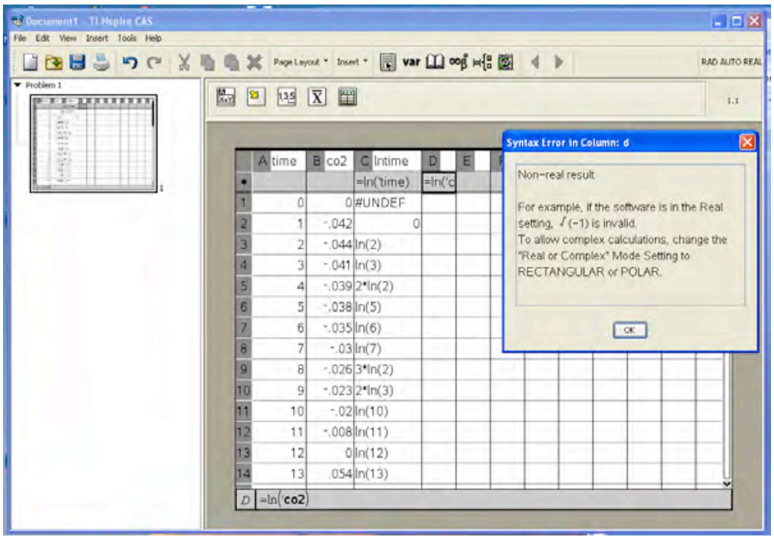


*Figure 4*. Nspire display of spreadsheet for natural logarithm of time and $CO_2$ output data.

absorbed we need to integrate the model or both models and then use the percentage difference formula – predicted minus actual divided by actual or, in this case, night minus day divided by day multiplied [by] 100 to look at whether for any *x* or any *t* there is any percentage difference greater than .05.

The second teacher (T2) was interviewed approximately two weeks after the episode described above.

R: One of the things that's come out quite strongly in your classes, and also I noticed it with John's as well, is there have been quite a few occasions where something has happened with the technology and it's really provoked a discussion.
The last time I was here the students tried to put in a logarithmic model and make it equal to zero and Nspire just said "No - not doing it!", which provoked a discussion about where the problem lay.
Today, there was a problem about the difference between a model developed using integration, either on the calculator or by hand, versus the regression model that was produced by doing the "area-so-far curve" which also provoked a discussion. Do you consciously do that or are these just incidental things that you just run with as they come up?

T2: Just as they come up, yeah.

R: That last one looked to me like you deliberately did it, but you are saying it just happened and that it was a good thing to follow-up.

> T2: Yeah basically, I mean they crop up and we call them teachable moments. There are things that crop up and you work through and you sort of think, yeah that's pretty cool. I mean yeah it worked out all right.

In this excerpt the teacher acknowledges the value of discussion that flows from unexpected outcomes but makes the point that he did not attempt to deliberately catalyse the collaborative discourse that developed by building in a "confrontation" into the lesson design. Despite the benefit that this teacher believes is an outcome of debate around a blockage to students' progress, and the apparent prevalence of this type of discourse in his classroom, he does not believe the type of scenario described above should be contrived, but simply embraced when it occurs. Indeed, it may well be impossible to contrive such provocations for thinking. It would depend on particular students' current understandings and the approach they take to open tasks. None-the-less, to take advantage of such instances a teacher must have a disposition to recognise an opportunity and the confidence and facility with both mathematics and technology to do so.

## *Focus Group Interview*

In the final focus group interview, participant teachers confirmed that the productive discussion arose from instances where technology produced unexpected results. This can be noted in the following transcript where Teacher 1 commented on events during the lesson on the decay of a bacteria population.

> T1: It was pretty obvious to me why it didn't work but I deliberately made a point of that with a student to see what their reaction would be. And it was a case of pretty much what I expected. That they just grasped this new technology Nspire and were so wrapped up in it that they believed it could do everything and they didn't have to think too much. And so suddenly, when it didn't work, it took a fair amount of prompting to get them to actually go back and think about the mathematics that they were trying to do and why it did not give a result ... and so forth.

> R: ...Interestingly you didn't just go over and tell them what to do. You just looked at it and said the syntax is all right – go and have a think about it. And they did for quite a while, and I don't know if anyone sorted it out. They may have but they didn't say. You then brought it back to the whole class and said "what's gone wrong here?" Someone eventually said that you can't have an exponential equal to zero. What happened out of that – you might want to fill in more – is that there was quite a protracted discussion about what happened. Extinction is zero isn't it? So there is a little bit of a conflict between the way students think about it mathematically and the way it works in context. The context implies zero but there are other answers that could still make it work. So, you have to do this bit of a fudge and say the equation has to be equal to anything less than one – if it is a bacteria.

T1: Even if the kids were solving that by traditional methods, they would still need to have that discussion. It was an issue with CAS that they were just expecting an instant answer and they didn't want to go and think about what was really going on.

R: What is it about CAS-enabled technologies that would be different to ordinary technology, in this instance?

T1: I'll just reiterate and say with CAS that kids are looking for the quick solution, the immediately obvious without looking at what is underlying the discussions and the decisions that they are making. And they assume – like I did in the second section that the machine can handle it, sort of thing.

In this discussion, the teacher identifies a "blackbox" use of CAS (Drjivers, 2003) as the source of the impasse the students experience in attempting to determine when the bacteria will become extinct. That is, the students attempt to use the technology to solve the problem without engaging deeply enough with the mathematics inherent in the solution to the problem. While the technology has the capacity to solve complex equations, the display for this indeterminate case is false. The teacher accommodates this surprising result by taking the opportunity to explore both the source of the problem, from a purely mathematical perspective, and what adjustments are needed to find a way forward mathematically while remaining true to the original context. This instance exemplifies the type of unexpected results that can be generated by CAS-enabled technologies. The above discussion demonstrates such instances can be used to the advantage of students' learning. It would appear this depends on the teacher having the disposition, mathematical expertise, technological competence and confidence to explore and promote students' mathematical knowledge and their understanding of mathematics in context.

## Discussion and Conclusions

The research reported in this paper provides insight into how CAS-enabled technologies can support student engagement with and learning through mathematical modelling and the role of CAS in mediating the type of human interaction that emerges when exploring such tasks. The findings expand on previous theory about the role of technology in mathematical modelling.

One set of findings relates to the use of CAS within different phases of the modelling cycle (research question 1). In contrast to the role attributed to technology in mathematical modelling by Galbraith, Renshaw, Goos and Geiger (2003) the electronic output forced students to re-evaluate fundamental assumptions they had made within the context of the described problems and then to reformulate, solve, interpret and evaluate the problem in the light of an adapted assumption set. This means that student-student-technology related activity takes place during all phases of the mathematical modelling cycle, as illustrated in Figure 5, rather than only at the solve juncture outlined in Figure 1. Consequently, this assigns a role to technology in the conceptualisation of the model rather than simply as a tool which is used to solve a mathematical problem after it has been abstracted. This is a position more consistent with that of Confrey and Maloney (2007) who acknowledge a role for technology in the inquiry/reasoning cycle.

A second set of findings is concerned with the role of CAS in mediating productive social interaction within the context of mathematical modelling activity (research question 2). In both of the vignettes described above, teachers found students were experiencing blockages to their progress and used this as a catalyst for whole class discussion in which the problematic issue was explored and then resolved. While these blockages were the result of erroneous input due to students' gaps in mathematical understanding, or perhaps carelessness that stemmed from a false belief that Nspire could solve any equation, the unexpected set-back catalysed discussion in a way consistent with Goos and Cretchley's (2004) commentary. It is important to note, also, that during both small group and whole class discussions, students themselves contributed to the generation of knowledge and understanding. Technology has therefore played a role in catalysing student participation in their own learning through small group and more public interactions with the teacher and their peers.

The unexpected output on the handheld devices in both vignettes influenced the inquiry/reasoning cycle by confronting students with an unanticipated result which, in turn, provoked the rethinking of their original assumptions, sometimes with the guidance of their teacher, and led to an adjustment to their approach to solving these problems. This rethinking was characterised by highly collaborative modes of discourse in which interactions between students and between students and the teacher focused on the processes of conjecture, knowledge testing and validation by a classroom community that included all classroom participants – both students and the teacher. As a result, students were forced to reshape their early thinking to satisfy the demands of both the context and the limitations of their abstracted model. Thus technology, in this case, has fulfilled a more interactive role than simply that of a powerful computational tool – it has mediated interaction by producing an initially unexpected result and also played a part in the final resolution of the conflict through its use to validate alternative solutions. We would argue this is an example of where the CAS - enabled technology provoked learning – the role of provocateur. The way in which technology mediated discourse, facilitated collaborative interaction and shifted the locus of interaction from the teacher to the students is consistent with the findings of Manouchehri (2004).

The provocations also represent opportunities for teachers to gain an awareness of students' misconceptions and then to provide appropriate scaffolding in order to move students forward in their understanding of the issue that was proving problematic. As reported above, both teachers used the consternation generated by the error messages recorded on students' handhelds to structure a forum in which student-student-teacher interaction played an important role in resolving the issue of concern. Despite the value teachers placed on this process, neither teacher believed that this could be implemented in a contrived way – rather they indicated that such opportunities are by nature serendipitous and that it was part of a teacher's repertoire to accommodate and take advantage of such events as they occurred.

The role of CAS based technologies as a provocateur of productive student-student-teacher interaction, in both small group and whole class settings, appears to have potential to mediate collaborative discussion, and within the particular context of this article, provides possibilities for enhancing the teaching and learning of mathematical modelling, and is therefore an area worthy of further research.

## References

Beatty, R., & Geiger, V. (2010). Technology, communication and collaboration: Re-thinking communities of inquiry, learning and practice. In C. Hoyles, & J-B. Lagrange (Eds.), *Digital technologies and mathematics teaching and learning: Rethinking the terrain* (pp. 251-284). New York: Springer.

Blum, W., Galbraith, P., Henn, H. & Niss, M. (Eds.) (2007). *Modelling and applications in mathematics education: The 14th ICMI study*. New York: Springer.

Burns, R. (2000). *Introduction to research methods* (4th ed.). Melbourne: Longman.

Burrill, G. (1992). The graphing calculator: A tool for change. In J. F. C. Hirsch (Ed.), *Calculators in mathematics education* (pp. 14-22). Reston, VA: NCTM

Confrey, J., & Maloney, A. (2007). In W. Blum, P. Galbraith, H. Henn & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 57-68). New York: Springer.

Diezmann, C., Faragher, R., Lowrie, T., Bicknell, B., & Putt, I. (2004). Exceptional students in mathematics. In B. Perry, G. Anthony & C. Diezmann (Eds.), *Research in Mathematics Education in Australasia 2000 - 2003*. Flaxton, QLD: Post Pressed.

Doerr, H. M., & Zangor, R. (2000). Creating meaning for and with the graphing calculator. *Educational Studies in Mathematics, 41*(2), 143-163.

Drijvers, P. (2000). *Learning algebra in a computer algebra environment*. Utrecht, The Netherlands: CD-B Press.

Galbraith, P., Renshaw, P., Goos, M., & Geiger, V. (1999). Technology, mathematics, and people: Interactions in a community of practice. In J. Truran & K. Truran (Eds.), *Making the difference* (Proceedings of 22nd annual conference of the Mathematics Education Research Group of Australasia, Adelaide, pp. 223-230). Sydney: MERGA.

Galbraith, P., Renshaw, P., Goos, M., & Geiger, V. (2003). Technology-enriched classrooms: Some implications for teaching applications and modelling. In Q. Ye, W. Blum, S. K. Houston, & Q. Jiang (Eds.), *Mathematical modelling in education and culture* (pp. 111-125). Chichester, UK: Horwood.

Geiger, V. (1998). Students' perspectives on using computers and graphing calculators during mathematical collaborative practice. In C. Kanes, M. Goos & E. Warren (Eds.), *Teaching mathematics in new times* (Proceedings of the 21st annual conference of the Mathematics Education Research Group of Australasia, pp. 217-224). Gold Coast, QLD: MERGA.

Geiger, V., & Goos, M. (1996). Number plugging or problem solving? Using technology to support collaborative learning. In P. Clarkson (Ed.), *Technology in mathematics education* (Proceedings of the 19th annual conference of the Mathematics Education Research Group of Australasia, pp. 229-236). Melbourne: MERGA.

Geiger, V., Faragher, R., Redmond, T., & Lowe, J. (2008). CAS enabled devices as provocative agents in the process of mathematical modelling. In M. Goos, R. Brown & K Maker (Eds.), *Navigating currents and charting directions* (Proceedings of the 31st annual conference of the Mathematics Education Research Group of Australasia, Brisbane, Vol. 1, pp. 246-253). Adelaide: MERGA.

Goos, M., & Cretchley, P. (2004). Computers, multimedia, and the internet in mathematics education. In B. Perry, C. Diezmann & G. Anthony (Eds.), *Research* *in mathematics education in Australasia 2000-2003* (pp. 151 - 174). Flaxton, QLD: Post Pressed.

Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2000a). *Classroom voices: Technology enriched interactions in a community of mathematical practice.* Paper presented at the Working Group for Action 11 (The Use of Technology in Mathematics Education) at the 9th International Congress on Mathematical Education, Tokyo/Makuhari, 31 July – 6 August 2000.

Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2000b). Reshaping teacher and student roles in technology-enriched classrooms. *Mathematics Education Research Journal*, *12*(3), 303-320.

Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2003). Perspectives on technology mediated learning in secondary school mathematics classrooms. *Journal of Mathematical Behavior, 22*(1), 73-89.

Huntley, M. A., Rasmussen, C. L., Villarubi, R. S., Santong, J., & Fey, J. T. (2000). Effects of standards-based mathematics education: A study of the Core-Plus Mathematics Project algebra and function strand. *Journal for Research in Mathematics Education, 31*(3), 328-361.

Kissane, B. (1999). The algebraic calculator and mathematics education. In W.-C. Yang, D. Wang, S.-C. Chu, & G. Fitz-Gerald (Eds.), *Proceedings of the 4th Asian Technology Conference on Mathematics* (pp. 123-132). Guangzhou, China: Program Committee.

Kissane, B. (2001). The algebra curriculum and personal technology: Exploring the links. In A. Rogerson (Ed.), *Proceedings of the International Conference: New Ideas in Mathematics Education* (pp. 127-132). Palm Cove, QLD: Program Committee.

Lincoln, Y. S., & Guba, E. G. (1985). *Naturalistic inquiry*. Beverly Hills, CA: Sage.

Manouchehri, A. (2004). Using interactive algebra software to support a discourse community. *Journal of Mathematical Behavior, 23*(1), 37-62.

Niss, M., Blum, W., & Galbraith, P. (2007). In W. Blum, P. Galbraith, H. Henn & M. Niss (Eds.), *Modelling and applications in mathematics education: The 14th ICMI study* (pp. 3-32). New York: Springer.

Pea, R. (1985). Beyond amplification: Using the computer to reorganize mental functioning. *Educational Psychologist, 20*(4), 167-182.

Pea, R. (1987). Cognitive technologies for mathematics education. In A. H. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 89 - 122). Hillsdale, NJ: Lawrence Erlbaum Associates.

Ramsden, P. (1997, June). *Mathematica in education: Old wine in new bottles or a whole new vineyard?* Paper presented at the Second International Mathematica Symposium, Rovamiemi, Finland.

Simonsen, L., & Dick, T. (1997). Teachers' perceptions of the impact of graphing calculators in the mathematics classroom. *Journal of Computers in Mathematics and Science Teaching, 16*, 239-268.

Stake, R. (2005). Qualitative case studies. In N. Denzin & Y. Lincoln (Eds.), The Sage handbook of qualitative research (3rd ed.). Thousand Oaks, CA: Sage.

Thomas, M. O. J. (2001). Building a conceptual algebra curriculum: The role of technological tools. In H. Chick, K. Stacey J. Vincent, & J. Vincent (Eds.), *The future of teaching and learning of algebra* (Proceedings of the 12th ICMI Study Conference) (pp. 582-589). Melbourne, Australia: Department of Science and Mathematics Education, The University of Melbourne.

Trouche, L. (2005). Instrumental genesis, individual and social aspects. In D. Guin, K. Ruthven & L. Trouche (Eds.), *The didactical challenge of symbolic calculators: turning a computational device into a mathematical instrument* (pp. 197-230). New York: Springer.

Victorian Curriculum and Assessment Authority (2006). *Mathematics Victorian Certificate of Education study design*. Melbourne: VCAA. Retrieved 20 November 2008 from http://www.vcaa.vic.edu.au/vce/studies/mathematics/mathsstd.pdf

Willis, S., & Kissane, B. (1989). Computer technology and teacher education in mathematics. In Department of Employment, Education and Training (Ed.), *Discipline review of teacher education in mathematics and science* (Vol. 3, pp. 57 - 92). Canberra: Australian Government Publishing Service.

Yerushalmy, M. (2000). Problem solving strategies and mathematics resources: A longitudinal view on problem solving in a functional based approach to algebra. *Educational Studies in Mathematics, 43*(2), 125-147.

Zbiek, R. M. (2003). Using research to influence teaching and learning with Computer Algebra Systems. In J. Fey, A. Cuoco, C. Kieran, L. McMullin, & R. M. Zbieck (Eds.), *Computer Algebra Systems in secondary school mathematics education* (pp. 197-216). Reston, VA: NCTM.

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