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Young Children's Early Modelling with Data

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An educational priority of many nations is to enhance mathematical learning in early childhood. One area in need of special attention is that of statistics. This paper argues for a renewed focus on statistical reasoning in the beginning school years, with opportunities for children to engage in data modelling activities. Such modelling involves investigations of meaningful phenomena, deciding what is worthy of attention (identifying complex attributes), and then progressing to organising, structuring, visualising, and representing data. Results reported here are derived from the first year of a three-year longitudinal study in which three classes of first-grade children and their teachers engaged in activities requiring the creation of data models. The theme of "Looking after our Environment," a component of the children's science curriculum at the time, provided the context for the activities. Findings include children's abilities to focus their attention on qualities of items rather than the items themselves in identifying attributes, switch their attention from one item feature to another, and create a broad range of models in organising, structuring, and representing their data. Children's development of meta-representational knowledge facilitated their choice and nature of data representations.

One of Australia's national priorities is enhancing learning in early childhood. The Government recognises the beginning school years as critical for the development of key foundational skills, with early learning experiences heavily influencing the opportunities and life chances of young Australians (*An Education Revolution for Australia's Future*, 2009). A significant component of this national priority is promoting young children's mathematical learning. To this end, the National Assessment Program—Literacy and Numeracy (NAPLAN) (ACARA, 2010) has been recently implemented in all Australian schools, beginning with the third grade. The program aims to provide feedback to parents, teachers, and schools on students' learning outcomes and to identify areas in need of improvement. While such assessment programs give an indication of children's basic school achievements, they do not capture the extent of young children's capabilities in creating their own mathematical constructs. We need to expose young children to future-oriented mathematical activities that challenge and extend their thinking, not limit their potential (cf. the Australian Association of Mathematics Teachers & Early Childhood Australia, 2006). One increasingly important area in need of attention here is statistical reasoning (Langrall, Mooney, Nisbet, & Jones, 2008; Watson, 2006).

Recent research has indicated that young children do possess many conceptual resources which, with appropriately designed and implemented learning experiences, can be bootstrapped toward sophisticated forms of reasoning not typically seen in the early grades (e.g., Clarke, Clarke, & Cheesman, 2006; English & Watters, 2005; Lesh & English, 2005; Perry & Dockett, 2008). Most research on early mathematics and science learning has been restricted to an analysis of children's actual developmental level, which has failed to illuminate children's potential for learning under stimulating conditions that challenge their thinking. As Perry and Docket (2008) emphasised:

> …young children have access to powerful mathematical ideas and can use these to solve many of the real world and mathematical problems they meet. These children are capable of much more than they are often given credit for by their families and teachers…The biggest challenge … is to find ways to utilize the powerful mathematical ideas developed in early childhood as a springboard to even greater mathematical power for these children as they grow older … (p. 99)

In this article, I first argue for the need to rethink young children's statistical experiences and propose a models and modelling approach, with a particular focus on data modelling. Such an approach receives limited or no attention in the early mathematics curriculum. Next, I describe the first year of a three-year longitudinal study in which three classes of first-grade children and their teachers engaged in data modelling activities. The theme of "Looking after our Environment," a component of the children's science curriculum at the time, provided the context for the activities. Findings from the first set of multi-component activities are then addressed, with a focus on how students dealt with given complex attributes and how they generated their own attributes in classifying broad data sets, and the nature of the models the children created through organising, structuring, and representing their data.

Statistical Reasoning and Young Learners

Across all walks of life, the need to understand and apply statistical reasoning is paramount. Statistics underlie not only every economic report and census, but also every clinical trial and opinion poll in modern society. One only has to peruse the daily newspapers to see the variety of graphs, tables, diagrams, and other data representations that need to be interpreted. Young children are very much a part of our data-driven society, with early access to computer technology and daily exposure to the mass media where various displays of data and related reports can easily mystify or misinform, rather than inform, their inquisitive minds. Rethinking the nature of children's statistical experiences in the early years of school is imperative—we need to consider how we can best develop the important mathematical and scientific ideas and processes that underlie statistical reasoning (Langrall et al., 2008; Watson, 2006).

Several recent articles (e.g., Franklin & Garfield, 2006; Langrall et al., 2008; Leavy, 2007) and policy documents have highlighted the need for a renewed focus on working with data in early mathematics learning. For example, the USA National Council of Teachers of Mathematics (NCTM) identified data analysis and probability as its "Focus of the Year" for 2007-2008 (September, 2007), while the recent *Statements of Learning for Mathematics* and *Statements of Learning for Science* in Australia (Curriculum Corporation, 2009) as well as the *NCTM Curriculum Focal Points* in the US (2006), highlight the importance of children working mathematically and scientifically in dealing with real-world data in the early school years. One approach to promoting such statistical reasoning in the beginning school years is through data modelling. Past research has indicated that reasoning about data is promoted when children are asked to create and revise models (Lehrer & Romberg, 1996; Lehrer & Schauble, 2000, 2007).

Models and Modelling

The terms, *models* and *modelling*, do not have unique meanings with respect to curriculum issues (Stillman, Brown, & Galbraith, 2008). These terms have been used variously in the literature, including with reference to solving word problems, conducting mathematical simulations, creating representations of problem situations (including constructing explanations of natural phenomena), generating a mathematical model in response to the specifications of a client's need (model-eliciting activities), and creating internal, psychological representations while solving a particular problem (e.g., Doerr & Tripp, 1999; English & Halford, 1995; Gravemeijer, 1999; Greer, 1997; Lesh & Doerr, 2003; Romberg, Carpenter, & Kwako, 2005; Van den Heuvel-Panhuizen, 2003; Zawojewski, 2010).

The overall notion of modelling, as used in the present study, reflects the views of Lesh and Zawojewski (2007) and Zawojewski (2010) on problem solving and modelling; that is, modelling involves a process of interpreting a problem-solving situation where "a system of interest needs to be represented by a mathematical system—which will simplify some things, delete others, maintain some features, and distort other aspects" (Zawojewski, 2010, p. 238). The defining feature of models and modelling lies more in the design of the task than in the problem solver. The nature of task design thus becomes an important feature of research addressing modelling. Stillman et al. (2008) expressed similar sentiments in their discussion on teaching through modelling, in particular, in their reference to "modelling as vehicle" (p. 143).

Modelling as vehicle, which is the form of modelling addressed here, involves choosing contexts in which stimuli for the desired mathematics learning are embedded. In such modelling, genuine problem situations are used as vehicles for students to construct significant mathematical ideas and processes rather than simply apply previously taught procedures. Furthermore, as indicated next, the mathematics that students engage with in solving such modelling problems usually differs from what they are taught traditionally in the curriculum for their grade level (English, 2003a; English, 2008; Lesh & Zawojewski, 2007). This form of modelling differs from the standard textbook-type word problems that constrain problem-solving contexts to those that often artificially house and highlight the relevant concepts (Hamilton, 2007). These textbook-type problems thus preclude students from creating their own mathematical constructs out of necessity.

Data Modelling

Data modelling is a developmental process that begins with young children's inquiries and investigations of meaningful phenomena, progressing to deciding what is worthy of attention (i.e., identifying attributes of the phenomena), and then moving towards organising, structuring, visualising, and representing data (Lehrer & Lesh, 2003). As one of the major thematic "big ideas" in mathematics and science (Lehrer & Schauble, 2000, 2005), data modelling should be a fundamental component of early childhood curricula. There is limited research, however, on such modelling and how it can be nurtured in the early school years. Indeed, the bulk of the research on mathematical modelling has been confined to the secondary and tertiary levels, with the assumption that primary school children are incapable of developing their own models and sense-making systems for dealing with complex situations (Greer, Verschaffel, & Mukhopadhyay, 2007).

Generating and Selecting Attributes

Early experiences with data modelling include the creation, analysis, and revision of data classification models (Lehrer & Schauble, 2000). A fundamental component in creating these models is selecting attributes and classifying items according to these attributes. As Lehrer and Schauble (2007) noted, it is not a simple matter to identify key attributes for addressing a question of interest—the selection of attributes necessitates "seeing things in a particular way, as a collection of qualities, rather than intact objects" (p. 154). Moreover, children have to decide what is worthy of attention (Hanner, James, & Rohlfing, 2002). Some aspects need to be selected and others ignored, the latter of which could be salient perceptually or in some other way. Frequently, however, young children are not given experiences in which they need to consider attributes in this way.

The typical classification activities presented in the early school years involve items with clearly defined and discernable features, such as red square shapes and blue triangular shapes (Hanner et al., 2002). It is thus a rather simple process for children to classify items of this nature. In contrast, problems involving the consideration of more complex attributes, which are not readily discerned or could define more than one classification group present a much greater challenge to young children. In one such study, Lehrer and Schauble (2000) worked with teachers at the first-, second-, fourth-, and fifth-grade levels in creating tasks in which children designed data classification models for determining the grade level of the artists of given drawings. The children were given 12 illustrations of self-portraits and 12 illustrations of two houses (near and far), which had been created by children across these grade levels. The illustrations did not contain the age of the artist or any other identifying information. Over five days, the children were required to explore the features of the drawings and then develop and revise a classification model that would enable them to identify the grade levels of the various artists. Children in the first and second grades, who were given only the self-portraits, found the tasks difficult; their progress mostly involved developing systems of attributes that described their categorised drawings in a post hoc fashion. They did not regard the attributes as rules to guide classification. In contrast, the older children considered their attribute systems as models that logically constrained members that could be included within the categories.

In addressing the difficulties of the younger children, Hanner et al. (2002) offered a number of recommendations for laying the groundwork for a data classification task of this nature. These included allowing students time to sort the drawings by one attribute then mixing the drawings and asking them to re-sort by another attribute, which would likely serve to turn children's attention towards the attributes and away from correct grade-level placement. However, while recommending that we begin data modelling in a more basic way, Hanner et al. cautioned against over-proceduralisation:

> Very often, teachers solve all the interesting issues for kids and present them already resolved to children, without giving children the opportunity to grapple with such questions as, "What attributes should we include?"How many attributes should we consider?" and "How should they be represented?" When teachers take over these decisions, all that's left is a cut-and-dried graphing or sorting activity, in which teachers have done all the intriguing and motivating thinking ahead of time. Our challenge as teachers is to avoid overpreparing tasks as much as it is to avoid overcomplicating them. (p. 106)

Structuring and Displaying Data

Models are typically conveyed as systems of representation (Lehrer & Schauble, 2006). Structuring and displaying data are fundamental here, where "structure is constructed, not inherent" (Lehrer & Schauble, 2007, p. 157). However, as Lehrer and Schauble pointed out, children frequently have difficulties in imposing structure consistently, and often overlook important information that needs to be included in their displays or alternatively, they include redundant information. Providing opportunities for young children to structure and display data in ways *they* choose, and to analyse and revise their creations, is important in addressing these early difficulties.

The need for classroom experiences that provide such opportunities has been emphasised over the years (e.g., Cengiz & Grant, 2009; Curcio & Folkson, 1996; Hutchison, Ellsworth, & Yovich, 2000; Lehrer & Schauble, 2007; Russell, 1991; Watson, 2006), yet young children's typical exposure to data structure and displays has been through conventional instruction on standard forms of data representation. The words of Russell (1991) are still timely today:

> We have two choices in undertaking data analysis work with students: we can lead them to organising and representing their data in a way that makes sense to us, or we can support them as they organise and represent their data in a way which makes sense to them. In the first case, they learn some rules—and they learn to second-guess what they are supposed to do. In the second case, they learn to think about their data. Students need to construct their own representations and their own ways of understanding, even when their decisions do not seem correct to adults. (p. 160)

Constructing and displaying their data models involves children in creating their own forms of inscription. By the first grade, children already have developed a wide repertoire of inscriptions, including common drawings, letters, numerical symbols, and other referents. As children invent and use their own inscriptions they also develop an "emerging metaknowledge about inscriptions" (Lehrer & Lesh, 2003), which diSessa, Hammer, Sherin, and Kolpakowski (1991) termed, *meta-representational knowledge*. Children's developing inscriptional capacities provide a basis for their mathematical activity. Indeed, inscriptions are mediators of mathematical learning and reasoning; they not only communicate children's mathematical thinking but also they shape it (Lehrer & Lesh, 2003; Olson, 1994). As Lehrer and Schauble (2006) highlighted, developing a repertoire of inscriptions, appreciating their qualities and use, revising and manipulating invented inscriptions and representations, and using these to explain or persuade others, are essential for data modelling. Yet, students are often taught traditional representational systems as isolated topics at a specified point in the curriculum, without really understanding when and why these systems are used.

Methodology

Participants

Three classes of first-grade children and their teachers in an inner Brisbane, Australia, school participated in the first year of the study. The school is situated in a middle socio-economic area and has an approximate enrolment of 500 students from the preparatory year through to seventh grade. Each of the first-grade classes comprised 25 or 26 students, with a mean age of 6 years 8 months. The children's previous experiences in working with data were limited to sorting items (e.g., coloured bears) and completing picture graphs (e.g., of favourite pets, hair colour).

Design

A teaching experiment involving multilevel collaboration (English, 2003b; Lesh & Kelly, 2000) was adopted in this study. Such collaboration focuses on the developing knowledge of participants at different levels of learning (student, teacher, researcher) and is concerned with the design and implementation of experiences that maximise learning at each level. Given that the teachers' involvement in the study was vital, regular half-day professional development meetings were conducted with the first-grade teachers. These meetings introduced the teachers to the study, explored their current mathematics and science curricula, developed and refined activities, reviewed children's developments, and reflected on their professional development. The focus in this article, however, is on the students' developments.

Activities and Procedures

A series of three, multi-component problem activities was implemented within small groups in each class. The teachers and researchers acted as facilitators while the children worked the activities but they did not give direct instruction to the children. The activities centred around four picture story books addressing the overall theme of "Looking after our

Environment," a key theme in the teachers' curriculum at the time. The story book contexts were considered a rich basis for the design and implementation of the activities; indeed, it is well documented that storytelling provides an effective context for mathematical learning, with children being more motivated to engage in mathematical activities and displaying gains in achievement (e.g., Casey, Andrews, & Copley, 2008; Casey, Kersh, & Mercer Young, 2004). This article focuses on children's responses to the first of the three sets of activities, based on the purposefully created story book, *Baxter Brown's Messy Room*.

This first set of activities comprised an initial lesson period (activity #1; approx. 35 minutes) in which children read and discussed the picture book of Baxter Brown, the messy dog. The story tells how Baxter Brown collects all sorts of miscellaneous items and hoards them in his bedroom, with the result that he becomes lost among the piles of items. The children are invited to help Baxter Brown work out how to clean his room by considering which items might be recycled, which might be reused, and which should be thrown away.

In the second lesson period (activity #2; approx. 45 minutes), the children were given a packet of small, illustrated black-and-white cut-outs of the various items Baxter Brown had collected. The packets contained multiple items of each type, namely, 8 dog bones, 7 apple cores, 5 plastic bags, 6 old toys, 9 empty drink cans, 7 cereal packets, 8 dog biscuits, 4 old shoes, 5 newspapers, and 6 milk cartons. The children were to explore the different items, count the number of each item type, and then help Baxter Brown determine which items were recyclable, which needed to be thrown away, and which could be reused. The children were to then represent their data classifications on an A3 sheet of paper, in whatever ways they liked. Following this, the children were encouraged to display their data classifications in a different way, however, time limitation prevented this for most student groups. At the conclusion of the activity, the student groups reported back to their class explaining and justifying their classifications and how they represented these.

In the next class period (activity #3; approx. 45 minutes), the children were given a new set of items from Baxter Brown's room, with the set containing only 12 illustrated items (apple core, fish bone, newspaper, toy teddy, shoe, ribbon, sandwich, egg carton, fruit juice container, roll of paper, dog biscuit, dog bone). The children were reminded of the Baxter Brown story and were given the challenge to identify their own attributes ("think differently") for classifying these new items and to represent their classifications however they liked. On completion, the children were given a second set of the same items and were asked to classify and represent them in yet a different way (i.e., the activity required at least two attempts at different classifications and data displays). As before, the groups reported back to their peers. This third class period concluded with a whole-class activity exploring an inappropriate vertical picture graph of how Baxter Brown had classified and represented his own data, which the children were to critically analyse; it is beyond the scope of this paper, however, to address the children's responses to this activity.

Data Collection and Analysis

In each classroom, two focus groups of students were videotaped and audiotaped. The focus groups were of mixed achievement levels and were selected by the teachers, who aimed to place a competent reader in each group. The artefacts of all student groups were collected and scanned and all whole-class discussions and group presentations were videotaped and audiotaped. Digital photographs were taken of the children's artefacts as well as their group and whole-class interactions. Students whose parents did not give them permission to be audiotaped or videotaped were excluded from any analysis, leaving a total of 16 student groups including the six focus groups.

Using iterative refinement cycles for analyses of children's learning (Lesh & Lehrer, 2000), the transcripts of the focus groups were reviewed many times in conjunction with their artefacts and class presentations, as were all group artefacts and whole-class presentations and discussions. The data were coded and examined for patterns and trends using constant comparative strategies (Strauss & Corbin, 1990). From a data modelling perspective, of particular interest in analysing the children's responses to activities #2 and #3 were the following:

1. How did the children deal with the given, complex attributes in classifying the extensive data set involving duplicate items (activity #2)?
2. In what ways did the children generate their own attributes in classifying the reduced data set with non-duplicate items (activity #3)?
3. What models did the children create in organising, structuring, and representing their data in each of the above?
4. What was the nature of the inscriptions the children used in displaying these models?

Results

In this section, consideration is given to the above issues with respect to the model developments of two of the focus student groups in undertaking activities #2 and #3, followed by the range of data models created by all of the student groups for these activities.

Model Developments of Focus Groups in Activity #2

In this activity, the children were asked to help Baxter Brown clean up his room by exploring multiple cut-outs of the various items he had collected and helping him decide which items were recyclable, which needed to be thrown away, and which could be reused. The children were then required to represent their classifications in whichever way they chose.

Thea's Group

Dealing with the given, complex attributes. Thea's group comprised one female and three males. The children began by identifying the items in terms of the given attributes (*reuse*, *throwaway*, *recycle*), for example:

> These are the throw away ones, the apple cores.
> The shoes are for keeping.
> The newspapers you can recycle.

The children continued to identify items in this manner until Thea reminded them that "We need to sort them out." The children then switched their attention from the given attributes to the items themselves, placing them in distinct piles on their desks (i.e., all the bones together, all the apple cores together etc.). After they counted the number of items in their piles, the children redirected their attention to the given attributes, with Harry suggesting, "How about we put the recycling in a pile?" Thea proceeded to remix all the items and the children began to sort them according to the given attributes (e.g., "The cans, we can recycle those."). A good deal of time was spent questioning which attribute the plastic rubbish bags displayed. Thea claimed her mother reuses plastic bags while Alex and Ben indicted that they contain rubbish and thus should be thrown away. Harry suggested that the plastic bag could be emptied and then cleaned. After considerable discussion, Thea concluded that the plastic bags displayed more than one attribute: "Maybe we can put half of those (plastic bags) in the rubbish pile and half of those in the recycling pile." The children did not follow through with this suggestion, however, and moved onto identifying which attribute the dog bones displayed, with Alex commenting that "We could throw the bones away if they're dirty."

Organising, structuring, and representing their data. The children next turned to organising and representing their data, discussing that they could glue the items onto the sheet of paper: "We could put them in a line of which ones we could reuse, recycle." But then Harry queried, "What if there's too much?" and suggested, "Maybe we need to turn it [sheet of paper] landscape." He then suggested, "We could put one on [of each item type]," to which Ben replied, "Need more than one." Harry was persistent, however: "One on recycle, one, one, one of each on each line [referring to horizontal placement of just one of each item that displayed the given attributes]." The other children agreed to use "just one of everything."

On Harry's comment, "Shouldn't we write a word or something?" the children proceeded to record the name of the first attribute, namely, recycle, and then pasted a can, a cereal packet, a newspaper, a milk carton, and a bone in two rows above the label. An argument followed, however, as to whether the bone was recyclable, with Thea arguing that it is reusable not recyclable; the item remained as recyclable. As the children worked on representing their remaining data (one shoe and one toy for the reuse attribute and one biscuit, one apple core, and one plastic bag for the throwaway attribute), Alex reminded the others that "We're doing one of each...we only need one of everything." He reinforced this feature in responding to another child's comment,"[Do we use] different shoes?" by stating, "No we've already got one." On reviewing their completed model, Alex and Harry were concerned that the attribute of *keeping* was absent; Thea and Ben argued that this attribute was incorporated within the attributes of *recycle* and *reuse*:

Harry:	So where's the next one, keeping?
Ben:	We've already done it, look [pointing to recycle].
Thea:	We've finished it all.
Harry:	No we haven't, [there's no] keeping.
Alex:	Yeah, we haven't done keeping.
Ben:	Yeah, but you said that pile was recycling.
Alex:	There's no keeping.
Harry:	Keeping and reusing.

Maria's Group

Dealing with the given, complex attributes. Maria's group, from a different class, comprised four children, two females and two males. Her group also began the activity by identifying the items in terms of the given attributes (e.g., "Reuse, reuse, you could use a shoe again"). Considerable discussion took place on defining the attributes, with Kieran insisting that "Reuse is stuff we can melt down." Like Thea in the previous group, Erin suggested that the group change their approach and just focus on classifying the items according to type (apple cores, cartons etc.): "Let's just sort them here and see what they are. Let's just sort them into what they are. Put them in lines." The group members did not follow her direction, however, and chose to each take one of the given attributes and classify the items accordingly. In doing so, the children changed and added to the attributes, creating those of *reuse*, *give away*, *compost*, and *put in rubbish bin*. Each group member selected one attribute to work with, for example, Maria focused on *reuse* and Sam concentrated on *compost*. On completion, Maria suggested, "Well, how about we count them," to which Erin replied, "Now let's put them in lines like this [in columns]." Erin explained that "By putting them in lines it will be easier [to count]." Sam asked, "Are you sure?" Each child lined up their items and counted aloud.

Organising, structuring, and representing their data. The children then discussed how they should display their data, with Sam recommending they glue the items on the paper, but "not all of them." Discussion followed on how best to display and represent the items, with Erin noting, "Um, don't we do it upside down or something; maybe along this way (indicating landscape position and horizontal lines of items)." Sam proceeded to write the attribute of giveaway across the top of the sheet, to which Erin replied: "You've put it in the wrong place. That's not how a graph goes." The children talked about how to rectify this and decided to write the labels, *giveaway*, *reuse*, *waste*, and *compost* across the top of their sheet in landscape position.

As the children proceeded to paste items in columns on their sheet, they struggled with how to fit all the items on their sheet: "Oh, we can't fit all of this stuff in." "We have so many waste and recycle things." Nevertheless, the group decided to go ahead and paste the items in columns, with Erin reminding Sam to position the items in columns: "That's just putting them on everywhere. We need a pattern of lines, like that." On placing the items in their respective columns in preparation for pasting, Kieran expressed concern: "You said you won't put everything on. Sam said we won't put everything on," to which Sam replied, "We can't put everything on." In their struggle to fit all items in columns, Maria commented to a research assistant that it is "so hard...too much [reuse items]." The research assistant asked if they might move the giveaway items further to the left-hand side of the sheet; the children subsequently decided to reposition their attribute labels at the top of the page and continue to paste their items, with Erin again reminding the group that the items must be positioned in columns.

Data Models Created for Activity #2

Six models were identified from the data analyses according to how the children dealt with the given attributes and how they organised, structured, and represented their data for the second activity.

Model 1. The given attributes of *recycle*, *throwaway*, and *reuse* were ignored and the children just placed like items into sets (i.e, sets of dog bones, sets of apple cores etc.). Three of the 16 student groups across the three classes created a model of this nature, with two student groups displaying their model in rows of like items pasted across their sheet. One of these groups did not include any inscriptions, while the other recorded the number of items in each row. One student group, however, placed their items in several rows and columns around the perimeter of their sheet and recorded the number of items in each set (using arrows with numerals to identify the set).

Model 2. In this model, at least two of the given attributes were addressed in the classification process but like items (e.g., all the shoes) were not placed together in the display of data; that is, the various items displaying a given attribute (e.g., *reuse*) were dispersed within the attribute cluster. Four student groups created such a model, with three student groups displaying their models as columns of pasted items, one under the other; attribute labels were not added to the columns. The remaining student group drew three labelled circles to denote their attributes and pasted the items within the respective spaces.

Model 3. Students classified their items according to the three given attributes and placed like items together within each attribute cluster. However, non-like items were not included in an attribute cluster, for example, the cans and newspapers were not included within the one attribute cluster of recycle. Instead, several clusters displaying the same attribute were created, such as five separate sets of recyclable items (e.g., set of four shoes, set of seven cereal packets, set of six milk cartons etc). Two student groups generated a model of this nature and displayed their models in rows of items pasted across their sheet. The rows were labelled according to the attribute being featured.

Model 4. In this model, children addressed at least two of the given attributes or replaced one of them with a new attribute, such as *compost*, *give* *them to kids*, or *give them to dogs*. Like items were placed together within each attribute cluster. Five of the student groups produced a model of this nature, with two groups displaying their models in rows of items pasted across their sheet (rows not labelled) and two groups pasting their items, one under the other, in columns (one group labelled their columns). The remaining student group drew three labelled circles to denote their attributes and pasted the items within the respective spaces.

Model 5. In addressing the given attributes, one student group came to the conclusion that they could not include all of the items adequately on their sheet and chose to use just four of each item type (with the exception of the apple cores, where they included the entire set); like items were placed together. As indicated in Figure 1, this group displayed their model in rows of items pasted across their sheet; the attribute labels were recorded for each row.

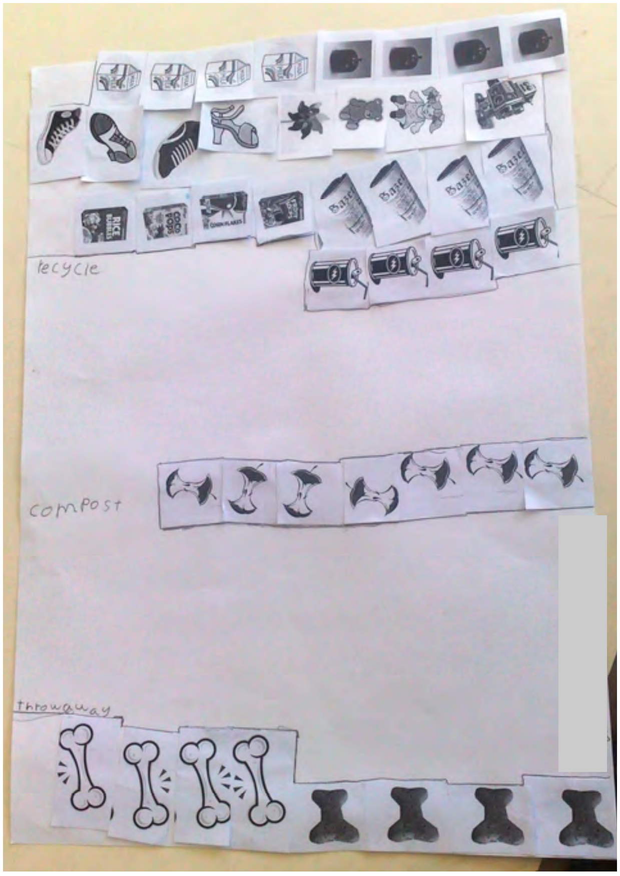

Figure 1. Data model 5 for activity #2.

Model 6. This model was created by Thea's group, who concluded that there were too many items to display effectively on their sheet. In addressing the given attributes, the children decided to use "just one of everything" with their resultant model not comprising any duplicate items.

Model Developments of Focus Groups in Activity # 3

In this activity, the children were given a new set of only 12 items, with no duplicates. The children were to generate their own attributes for classifying the items and were to choose their own way of representing their data classifications. In their second attempt, the children were to identify different attributes for classifying their items and were to find a new way of representing their data. Three student groups made a third attempt at doing this.

Thea's Group

Generation of attributes and organising, structuring, and representing their data. Thea's group commenced the activity by using the attributes of activity #2 to classify the items. Thea began by revisiting Alex's question: "What if it's old and dirty?" However, Alex argued that the attribute of *dirty* was incorrect ("It's [fish bone] all white"). The children subsequently made two sets of items on their desk, which they classified as recycle and waste; a third set, reuse, was subsequently added. In determining how to display and represent their data, the children decided to reduce the size of the given cut-outs (for a better fit across their sheet in portrait position) and proceeded to paste the items in labelled rows according to their chosen attributes: *throwaway* (fish bone, dog biscuit, apple core, sandwich), *recycle* (roll of paper, newspaper, juice carton, egg carton), and *reuse* (shoe, toy teddy, ribbon, dog bone).

The children's next challenge was to classify and represent the items in a different way. The group decided on size as a measure and generated the attributes of *small*, *medium*, *long*, and *large*. Prior to classifying the items according to these attributes, the children recorded the attribute names down the left-hand side of their paper in a portrait layout. On being reminded by the teacher that they were to create a different way of displaying and representing their data, the children decided not to use the actual cut-outs but to initially make templates of the items. Discussion followed as to whether cut-outs or actual illustrations would be more appropriate. The children decided to do a combination of both. In doing so, they debated over which attribute each item displayed, with particular disagreement over the dog bone:

Ben: The bone's not small, Harry.....It's long.

Harry: No, it's not long, it's large.

Thea: No, no, a bone is *that* long.

Ben: No, it's not.

Thea: (making the final decision) It's large, large.

The children subsequently shared the creation of their representation, with some tracing around templates to illustrate the items, while others just drew the items. The children created four rows of items, with the attribute name recorded at the beginning of each row and the name of each item recorded above the item: *small* (biscuit, paper, ribbon, apple core); *medium* (sandwich, shoe, toy, juice carton); *long* (fish bone); *large* (dog bone, egg carton, newspaper) (as shown in Figure 2). On completion, Thea checked that they had the same number of items in their second representation as in their first.

Figure 2. Data model created by Thea's group for activity #3.

Maria's Group

Generation of attributes. Maria's group began the activity with Sam suggesting they could use the attributes of the previous activity, namely, *giveaway*, *recycle*, *compost*, and *waste*. Maria offered another idea: "We could do like food and all the things that aren't food." Other group members suggested different attributes including *empty/not empty*, *old/not old*, and *round/not round*. Sam then commented, "What about another way?" and offered man-made, to which Erin replied, "Man-made and natural." The children proceeded to identify items that were *man-made*, such as the toy teddy, the roll of paper, the juice carton, and the egg carton. In discussing the natural items, the children generated a new attribute namely, *made out of trees/not made out of trees*, and proceeded to identify items that displayed the attribute and those that did not.

Organising, structuring, and representing their data. In representing their classifications, the children divided their sheet in half, in landscape position, with Maria emphasising, "One big straight line and it has to be even sides" and Sam reiterating, "It has to be the same size." The children shared the recording of the attribute labels, with *made out of trees* written in the top left-hand corner and the label, *not made out of trees*, in the right-hand corner. Underneath each label the children recorded, in rows, the names of the respective items: *made out of trees* (ribbon, paper roll, egg carton, newspaper, apple juice carton) and *not made out of trees* (apple core, fish, shoe, sandwich, biscuit, teddy, dog bone). The children had decided just to record the item names "Cause the pictures don't fit" but then Kieran stated that they should also glue the items in the respective columns:

Sam: We don't need glue, we can just write them in.

Erin: Yeah, just write them.

Maria: That was my idea.

The group members informed Kieran that they didn't need glue but then Erin insisted on collecting a glue bottle and the members subsequently pasted the cut-outs in columns in the appropriate half of their sheet. The name of each item was also recorded on the cut-out.

Generation of attributes and organising, structuring, and representing their data. In their second attempt at classifying and representing their data, the group decided on one of the previous attribute suggestions, namely, *eat* and *not eat*. The children chose to represent their data in a similar way to previously, except they did not create the lists of the item names. Their final representation comprised columns of labelled cut-outs for the attribute of *we cannot eat* (egg carton, paper roll, newspaper, teddy, show, apple core, ribbon, dog bone, fish bone) and *we can eat* (sandwich, apple juice, biscuit).

Data Models Created for Activity #3

This activity did not have duplicate items as in the previous activity and engaged the children in at least two attempts at classifying and representing. In contrast to the previous activity, only three main model types could be discerned from the data analyses. However, the children showed greater diversity in the ways they structured and represented their data.

Model 1. Here, children addressed only the attributes of the previous activity or variations of the attributes (*reuse, recycle, throwaway, compost, waste, can keep/can't keep*). Eight student groups created models using these attributes in their first attempt, three groups did so in their second attempt, and one group in their third attempt. The data models the children created were displayed in the following ways:

- Names of items recorded randomly within three circular spaces (labelled *reuse*, *recycle*, *throwaway*);
- Names of items recorded one under the other in labelled columns;
- Cut-outs of items pasted one under the other in labelled columns (with or without the names of the items recorded on the individual cut-outs);
- Cut-outs of items pasted in rows, with the attribute name of each row recorded;
- A vertical picture graph created with attributes labelled across the x axis and cut-outs placed in columns above the respective attribute;
- A vertical bar graph, as above, created but only one cut-out was pasted above each attribute and the remaining items were drawn. The student group who created this model explained that they did not think all of the items would fit onto their sheet (which they had placed in a landscape position) and hence, chose to draw the remaining items.
- One student group attempted a formal bar graph with the attributes labelled across the x axis and a scale of 1-43 created on the y axis (this student group did not complete their graph, however).

Model 2. In this model, children addressed attributes other than those of the previous activity and identified a mix of attributes to classify their items. Three student groups generated attribute clusters as follows: (a) *Food* (sandwich, fish bone, biscuit, apple core, dog bone), *cardboard and paper* (paper roll, newspaper, egg carton, apple juice carton), and *material* (old shoe, ribbon, teddy); (b) *Chew toys* (dog bone, ribbon, fish bone), *cardboard* (juice carton, paper roll, egg carton), *paper* (newspaper), *fabric* (teddy, shoe), and *eat* (sandwich, apple core, biscuit); and (c) *fossils* (fish bone, dog bone), *eat* (sandwich, eggs in egg carton, apple core, juice in juice carton, biscuit), *paper* (newspaper and paper roll), and *fabric* (ribbon, shoe, teddy).

Children displayed their models in one of the following ways:

- Names of items were recorded one under the other in labelled columns;
- Cut-outs of items were pasted in horizontal rows, with the attribute name of reach row recorded.

Model 3. Models of this nature differed from the previous models in one of two ways. The first way involved the use of a single attribute and its complement, for example, *food/not food, things to eat/not eat, made out of trees/not made out of trees, healthy/not healthy, things with white in picture/no white, has writing on it when you buy it* [the item]/*no writing*. Six student groups used such attributes on their first attempt, five groups did so on their second attempt, and one student group on their third attempt.

The second way involved the use of an overarching attribute identified by three student groups on either their second or third attempt: (a) "We sorted in size [of the items]" (*small, medium, long, large*); (b) "What they are made of" (*fabric, bones, seeds, plastic, cardboard, bread, corn*); and (c) "How you use it" (*drink, put stuff in, recycle, make stuff out of*). One student group used the attributes of *soft* and *hard* on their first attempt and another group, the attributes of *short pictures* and *long pictures* on their second attempt, but these two groups did not identify an overarching attribute of texture or length.

It is interesting to note some of the ways in which the children reasoned in identifying and applying their attributes in generating models of this nature. The student group who chose the attribute of "has writing on it when you buy it/has no writing" explained in their class presentation that they were thinking about the items as they would have appeared in a shop display. Hence, they argued that the paper roll would have been sealed with a brand name, the shoe would have come from a labelled shoe box, and the dog biscuit would be from a packet of labelled dog biscuits, whereas the dog bones would have come from the butcher shop in an unmarked packet. The unusual attribute selection of the group who decided on "What they are made of" stated in their class presentation that the apple core was made of seeds, the juice carton was made of plastic, the egg carton of cardboard, the sandwich of bread, and the dog biscuit of corn.

Models of this nature were displayed in one of the following ways:

- Cut-outs of items were pasted one under the other in labelled columns (with or without the names of the items recorded on the individual cut-outs);
- Names of items were recorded one under the other in labelled columns;
- Word lists were created under the names of the identified attributes (fabric, bones, seeds etc.);
- A horizontal picture graph comprising traced outlines of cut-outs and drawings was made;
- Two labelled columns were created with random pasting of items within the columns.

Discussion

This paper has argued for a renewed focus on statistical reasoning in the beginning school years, with opportunities for children to engage in data modelling. As used here, modelling activities are comprehensive learning experiences in which a given problem situation needs to be interpreted and represented by a mathematical system; such a system, as Zawojewski (2010) noted, might simplify some aspects of the problem situation, delete others, maintain some, and distort others. Data modelling engages students in interpreting and investigating meaningful phenomena (in the present case, pertaining to the environment). Students need to decide what aspects are worthy of attention (in the present study, the identification of complex attributes), how these aspects will be dealt with (organising, structuring, visualising), and how the resultant data will be represented to communicate one's findings. A fundamental component of data modelling activities is the selection of attributes and classifying items according to those attributes.

In the first of the two activities addresed here, children were initially given an extensive data set comprising 10 sets of duplicate cut-outs of items that the story character, Baxter Brown, had hoarded in his bedroom. The children were to decide which items were recyclable, which needed to be thrown away, and which could be reused; they were to then organise, structure, and display their classifications in ways of their choice. In the next activity, the children were given a new set of only 12 items, with no duplicates, and were to generate their own attributes for classifying the data and their own ways of representing their classifications. The children were to do so in more than one way. For both activities, the items did not display readily discernable attributes, such as a particular colour or shape.

Deciding on a group definition of an attribute was a fundamental process in the children's engagement in the data modelling activities, especially given that the items in both activities could be classified according to more than one attribute. Considerable debate was generated in defining an attribute. For example, in Thea's group, the attribute of *keeping* (activity #2) was raised initially and then revisited towards the completion of their model, with concerns expressed that the attribute was absent. Thea's group also debated over how to classify the plastic bags in activity #2, with Thea suggesting that they displayed two attributes. Likewise in Maria's group, Kieran emphasised that reusable items were "stuff we can melt down" with the group subsequently refining their attributes.

As the children identified different attributes, they needed to focus their attention on the qualities of the items, rather than the items themselves. For example, the two focus groups commenced activity #2 by switching from item quality to item per se and then back to item quality. Thea's group initially identified items in terms of the attributes of *throwaway*, *recycle*, and *keeping*, but then revisited the task goal of sorting the items and focused instead on like items. The group then reverted to item quality.

The children's talents in identifying a wide range of attributes were clearly evident in activity #3 (e.g., Maria's group offered many suggestions for classifying the items). In particular, children displayed an ability to identify quite obscure attributes, such as *has writing on it when you buy it, made out of trees, fossils*, and *soft/hard* items. This ability may be likened to what Lehrer and Lesh (2003) referred to as "lifting away from the plane of activity" (p. 377), which is a common feature of notational systems. In this case the children were "lifting away" from the actual items in search for features that were not immediately apparent.

In identifying different attributes, children needed to decide on what was worthy of attention and what needed to be placed in the background. Children's skills in switching their attention from one item feature to another in classifying the items was especially evident in activity #3, where the children were required to generate their own attributes, in at least two different ways. For example, one student group classified the egg carton as *cardboard* and then reclassified it as *eat*, switching their focus from its packaging to its contents. Other examples included the student group who classified the items according to the attributes of *keep* and *can't keep*, then reclassified according to *what they are made of*, and in a third attempt, *how you use it*.

In considering the models the children created in working each activity, it appears that the extensive data set of activity #2 provided more scope for data organisation and structure, whereas activity #3 encouraged greater diversity in how the children displayed their models. In activity #2, the children were faced with a number of challenges including whether to include all of the items, how to structure the items including whether to group like items, and how to effectively display all of their data classifications.

In contrast to activity #3, the second activity prompted some children to consider reducing the data sets and using representative items instead. Eliminating features that are not necessarily needed is an important goal of data modelling (Lesh & Schauble, 2003). This ability was observed in the focus groups where Thea's group displayed just one of each item type and Maria's group argued that "not all of them" [items] should be displayed. Another student group decided to use only four items of each type. Other children overcame the problem of displaying all of the data by changing the orientation of their sheet of paper (e.g., landscape to portrait) and making effective use of inscriptions such as recording the item names instead of pasting the items.

Activity #2 also yielded considerable variation in how the children organised the items within their attribute clusters, with some placing like items together and others dispersing the items. A small number of children chose not to include non-like items within a given cluster and created several clusters of the same attribute, that is, their model included redundant information (Lehrer & Schauble, 2007). There was, however, limited variation in how the children displayed their data classifications in this activity, with typical representations being rows or columns of items with half the student groups including inscriptions. The latter included recording the attribute name with the respective column or row. Only a few student groups recorded the number of items for each attribute. On the other hand, there was evidence that children had developed some meta-representational knowledge (diSessa et al., 1991) here, such as in Maria's group, where Erin kept reminding her peers not to place items randomly but to put them "in lines" ("We need a pattern of lines, like that"). An awareness of how this form of representation facilitates data interpretation was evident in her comment that "By putting them in lines it will be easier [to count]."

With the change in task design for activity #3, models of a different nature were created by the children. These models were limited in how the children organised their items (because of the reduced set of non-duplicate items) but more enriched in terms of how their data classifications were represented, including their use of inscriptions. Children broadened their representations by physically reducing the size of the cut-out items, making templates and drawings of the items, recording attribute names either with or without the cut-out items, recording lists of item names, and creating picture and bar graphs.

Again, children displayed meta-representational knowledge in structuring and representing their data classifications; this knowledge guided both their mathematical thinking and how they communicated it (Lehrer & Lesh, 2003; Olson, 1994). For example, in creating their model Maria and Sam insisted on dividing their sheet of paper in half, in landscape position, and emphasised that there must be "One big straight line and it has to be even sides;" "It has to be the same size." This inscription provided the framework for subsequent inscriptions in which the children recorded, in rows, the names of the items displaying their chosen attribute (*made out of trees/not made out of trees*).

As previously noted, task design is a key feature of the models and modelling approach taken in this study. The activities implemented in the first year were designed to challenge and extend the children's mathematical reasoning in working with data, while at the same time be relevant to their learning across the curriculum. By designing the activities so that the children could interpret and work with the problem criteria in ways that they chose, a range of models and inherent processes resulted. Not all components of data modelling were involved in the activities addressed here, specifically, the children did not collect their own data; they did so, however, in the subsequent activities. It was considered important in the present activities to provide opportunities for children to work with complex attributes in organising, structuring, and representing given data. These experiences laid the groundwork for children's subsequent data collection in exploring the environment of their classroom.

Young children also need experiences in which they interpret various data-based contexts, critically analyse data, rank data, search for trends in data representations, use these trends to make predictions, and document and communicate their findings in different formats. For example, in another activity implemented in the first year of this study, the children discussed and answered questions about the book, *Look After Your Planet* (Hurst, 2008). The book involves several story characters discussing which of the items they have collected are recyclable and how these items should be recycled. The children were then given a table of data displaying how many of the various items each of the story characters collected during one week. After answering questions that involved interpreting the given data, the children were invited to help Lola decide who the best recycler was. The children were to determine what "best recycler" might mean and were to rank the characters from best to worst recycler. Finally, they were to document and report on how they decided on their list.

Concluding Points

As communication in contemporary society becomes more data intensive, young children are exposed to increasingly complex statistical displays in myriad ways. Statistical experiences in the beginning school years need to capture some of this complexity and build on young children's capabilities in thinking mathematically (Leavy, 2007; Lehrer & Schauble, 2007; NCTM, 2006). The foundations for statistical reasoning need to begin in these early years not be left for the middle and secondary grades. Data modelling, which engages children in authentic problem solving, provides a rich way of introducing young learners to statistical activity. As indicated here, the starting point for developing statistical reasoning through data modelling is with the world and the problems it presents, rather than with any preconceived formal models (Lehrer & Schauble, 2005). In the present case, this starting point was the problems faced in looking after the environment. The story picture books provided a meaningful and enjoyable context for the children to explore these problems, which were significant and valued ones that they face on a daily basis.

More research is needed on young children's awareness and interpretation of data in their daily lives, both in and out of school, and on how they respond to statistical reasoning activities where they pose their own questions, debate how to address these, source their own data, and decide how to organise, structure, represent, and communicate what they have discovered. In sum, we need to broaden data activities in the early years' curriculum and expose children to experiences that will foster statistical reasoning that extends well beyond the classroom. Young children are indeed capable of undertaking such activities. As Leavy (2007) emphasised, we are under estimating young children's statistical ability if we do not afford them the opportunity to participate in meaningful and authentic statistical experiences. However, despite the increased calls for renewed attention to statistical learning in the early school years, research examining young children's statistical developments remains in its infancy. The modelling perspective adopted here provides a promising avenue for enriching and extending young learners' abilities to work with data and reason statistically.

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