TEMPLATES IN ACTION

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Circle Geometry, a senior mathematics topic, is often regarded as time-consuming and associated relational concepts difficult for students to grasp. This article explores teacher-designed dynamic geometry software templates to maintain student ownership of circle geometry theorems. The teaching sequence is imbedded within the van Hiele Teaching Phases and includes sample materials for teachers.

Introduction

Units of work that introduce students to circle geometry theorems are frequently described as a string of tedious constructions. Whilst construc-tivist approaches are preferred, we are sometimes forced to take shortcuts in an attempt to meet program deadlines. To add to the problem, when these shortcuts are taken, and a student begins following geometrical procedures without conceptual understanding of the process, he/she is unable to think deductively in this context.

This article presents a strategy for using Dynamic Geometry Software (DGS) templates within a theoretical framework. Firstly, the templates, or pre-constructed geometry sketches, enable a pre-designed sketch to be added to, dragged to investigate properties, and facilitate reasoning through evidence. Templates are often designed with the support of action buttons that allow further additional information or instructions to be viewed by the student.

Whilst many pre-constructed objects are freely available, teacher-designed or student-designed templates better meet students' individual needs. When designed by the teacher for a particular class, careful atten-tion can be given to the appropriate time to introduce formal language and emphasis placed on particular mathematical concepts which target the students' level of understanding. Sinclair (2003) found that "the task ques-tion and sketch provision must work together to create an environment for exploration" (p. 289) and "that explicit attention to visual interpretation and

exploration using change is required in order for the students to benefit fully from their experiences with pre-constructed dynamic geometry sketches" (p. 313).

Secondary students of today are equipped with the skill of using Information and Communication Technology (ICT) as a sophisticated means of communication and a display tool. Often little flexibility is available in various forms of mathematics education software. We generally need to adapt our own teaching to fit the software, rather than using ICT materials in the light of our own experiences and the particular needs of the group of students. In the case of DGS, Jones (2000, p. 55) identified that "students' explanations can evolve from imprecise, 'everyday' expressions, through reasoning that is overtly mediated by the software environment, to mathematical explanations of the geometric situation that transcend the particular tool being used".

Whilst it is hoped that each of the students in the class will have passed through van Hiele's Level 2 (van Hiele, 1986), where figures are identified in terms of properties which are seen to be independent of one another before commencing the circle geometry unit, this is often not the case. Some students will still be operating at this level and the templates need to be open-ended in nature to cater for these students. Other students will be operating at van Hiele's Level 3 where they focus on the relationships among the figures and properties. Remaining students will be operating at van Hiele Level 4, where they are able to focus on the complex interrelationships among properties and figures, and hence, the place of deduction is understood. The challenge is catering for all these students within the structure of a two week unit of work targeting circle geometry theorems.

Van Hiele teaching phases: A teaching framework

A teaching framework that suitably addressed the need to assist teachers in using technology to develop mathematical concepts is the basis of the work of Dina van Hiele-Geldof. The five teaching phases represent a framework to facilitate the cognitive development of a student through the transition between one geometrical level of understanding and the next. The van Hiele model acknowledges that progress is easier for students with careful teacher guidance, the opportunity to discuss relevant issues, and the gradual development of more technical language.

The five phases of teaching assist in maintaining student ownership of ideas throughout the learning process. During this process, students can seek clarification from each other and from the teacher concerning the language used. In particular, language plays a central role. It is only after students have identified and described concepts, using their own language that the more technical language is introduced. A description of the phases is provided in Table 1 with an emphasis on the changing role of language as the student progresses through the phases (Serow, 2007, p. 384). The phase names have been adapted.

The five-phase teaching approach provides a structure on which to base a program of instruction. As can be seen, the phase approach begins with clear teacher direction involving exploration through simple tasks, and moves to activities that require student initiative in the form of problem solving.

Table 1. Descriptions of the van Hiele teaching phases.

Phase	Description of Phase Focus
1. Information	For students to become familiar with the working domain through discussion and exploration. Discussions take place between teacher and students that stress the content to be used.
2. Direction	For students to identify the focus of the topic through a series of teacher-guided tasks. At this stage, students are given the opportunity to exchange views. Through this discussion there is a gradual implicit introduction of more formal language.
3. New ideas	For students to become conscious of the new ideas and express these in accepted mathematical language. The concepts now need to be made explicit using accepted language. Care is taken to develop the technical language with understanding through the exchange of ideas.
4. Ownership	For students to complete activities in which they are required to find their own way in the network of relations. The students are now familiar with the domain and are ready to explore it. Through their problem solving, the students' language develops further as they begin to identify cues to assist them.
5. Integration	For the students to build an overview of the material investigated. Summaries concern the new understandings of the concepts involved and incorporate language of the new level. While the purpose of the instruction is now clear to the students, it is still necessary for the teacher to assist during this phase.

Teaching sequence

The target outcome of this two-week unit of work is: "SG5.3.4 At the end of this unit, the students should be able to apply deductive reasoning to prove circle theorems and to solve problems" (Board of Studies, NSW, 2002).

The key ideas are: deduce chord, angle, tangent and secant properties of circles.

There are various forms of DGS available to schools. The type modeled in this article is Geometer's Sketchpad, known as GSP (Jackiv, 2000). There is a range of DGS available to schools, readers may like to explore similar activities using Geogebra and/or Cabri.

Activity 1: Information

Students work through simple constructions using Geometer's Sketchpad (GSP) through the drawing of a circle (using GSP), labelling and naming known features and properties, using a textbook to record known formulae concerning the circle. Class sharing of information is recorded.

Activity 2: Direction

Students investigate circle property relationships through the medium of pre-constructed circle geometry templates. A series of templates are created which when opened by the student appear similar to Figure 1

On the revealing of information through the action buttons, and consequent exploration and recording, the template will appear similar to Figure 2.

Each student's constructions, measurements, explanations and justifications are emailed to the teacher for feedback and preparation for Activity 3 discussion. Instructions for creating templates using GSP are contained in Appendix A.

Activity 3: New idea

Whole class discussion to formalise language conveyed in current responses and emailed material. Class theory for each theorem is produced based on the explorations from the previous lesson.

Activity 4: Ownership

Figure 1. Sample template.
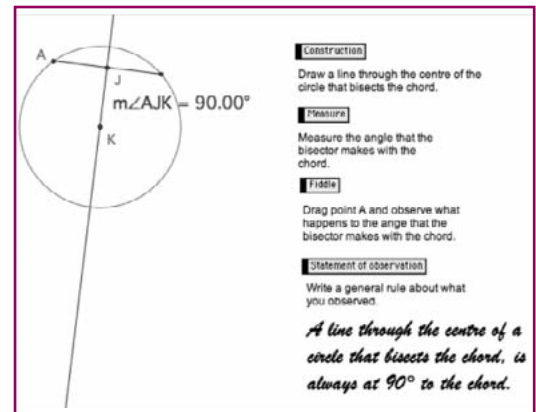
Figure 2. Student completion of sample template.

In small groups, students sort circle geometry question cards into theorem categories. These diagrams may be derived from typical circle geometry textbook and exam style questions. After the students have sorted the cards into theorem categories and have identified a 'plan' to find the unknown values, the values are identified with justification. Class discussion follows and clarifications may be sort using DGS. Figure 3 shows a sample of question cards.
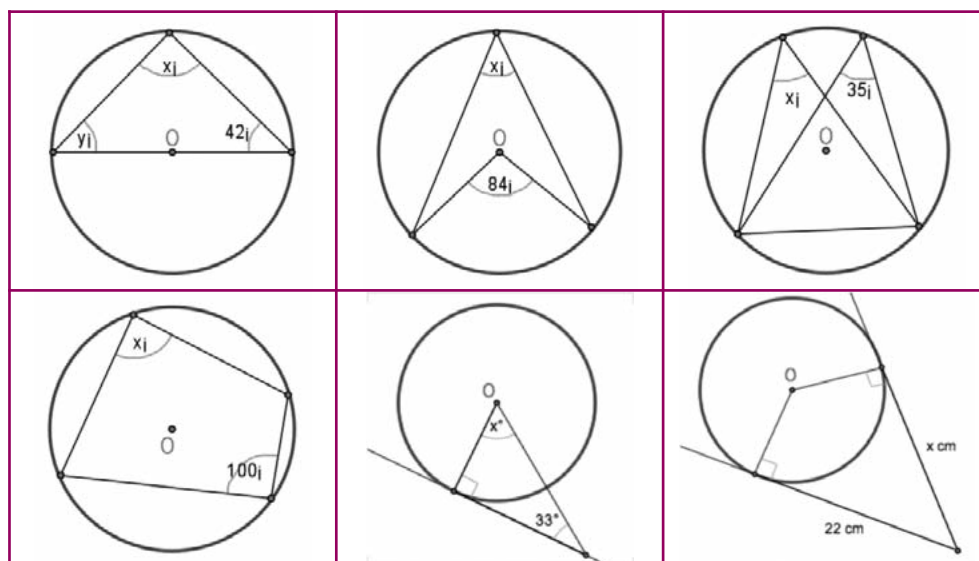

Figure 3. Sample sorting cards.

Activity 5: Ownership

Students complete routine and non-routine circle geometry questions from a range of sources that are not in identified categories.

Activity 6: Integration

Use GSP as a medium for the creation of individual student-designed study guides of circle geometry theorems.

Conclusion

A main feature of the teaching sequence presented is the integration of dynamic geometry software using the van Hiele teaching phases as a framework (van Hiele, 1986) for maintaining student ownership of circle geometry concepts. This is facilitated via student-centred pre-constructed templates that acknowledge students' individual experiences and the progression from informal to formal language use. The teaching sequence combines a range of effective teaching practices involving technological tools, cooperative group tasks, whole class discussion, and a range of question types.

References

Board of Studies, NSW. (2002). *Mathematics: Years 7–10 syllabus.* Sydney: Board of Studies, NSW.

Jackiw, N. (2001). *The geometer's sketchpad: Dynamic Geometry Software for exploring mathematics, version 4.0.* Emeryville, CA: Key Curriculum Press.

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Serow, P. (2007). Incorporating dynamic geometry software within a teaching framework. In H. Reeves, K. Milton & T. Spencer (Eds), *Mathematics: Essential for learning, essential for life* (Proceedings of the 21st Biennial Conference of the Australian Association of Mathematics Teachers Inc., pp. 382–397). Adelaide: The Australian Association of Mathematics Teachers Inc.

Sinclair, M. P. (2003). Some implications of the results of a case study for the design of pre-constructed, dynamic geometry sketches and accompanying materials. *Educational Studies in Mathematics , 52*(3), 289–317.

van Hiele, P. (1986). *Structure and insight: A theory of mathematics education.* New York: Academic Press.

Appendix A: Creating templates using GSP

Making templates for individual guided learning.

Multiple pages in a document

1. Select FILE, then DOCUMENT OPTIONS.
2. You can add a BLANK PAGE, add a DUPLICATE of another page and give the new page any name you desire.
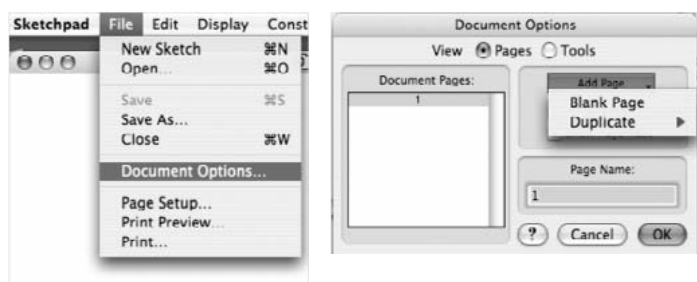
Hiding objects

It is a good idea never to delete parts of your diagram that are undesirable, as many have integral functions for the diagram. Use the HIDE function from the DISPLAY menu to hide objects that complicate the diagram and may cause confusion for the students—such as the "draw" point on a circle.
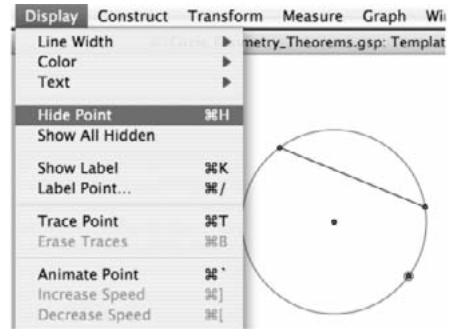
Labelling aspects of diagrams

You can label aspects of diagrams such as points, lines and angles. Use the LABEL option from the DISPLAY menu.
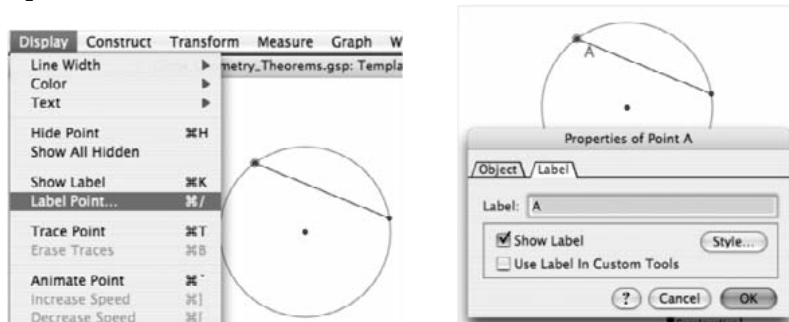
Creating a template for student use, using the HIDE/SHOW animation tool

The following instructions are to create a Circle Geometry template as shown in the Workshop.

1. Draw the base diagram, which the students will see on opening the page.
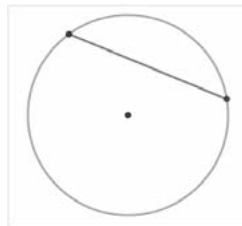
2. Type the first instruction.

    1. Draw a line through the centre of the circle that bisects the chord.
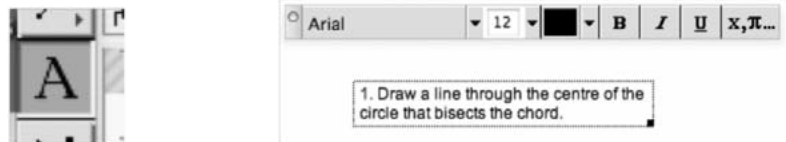
3. Select the text box using the ARROW tool, and select the HIDE/SHOW option from the ACTION BUTTON selection on the EDIT menu.
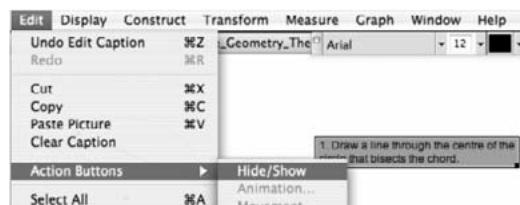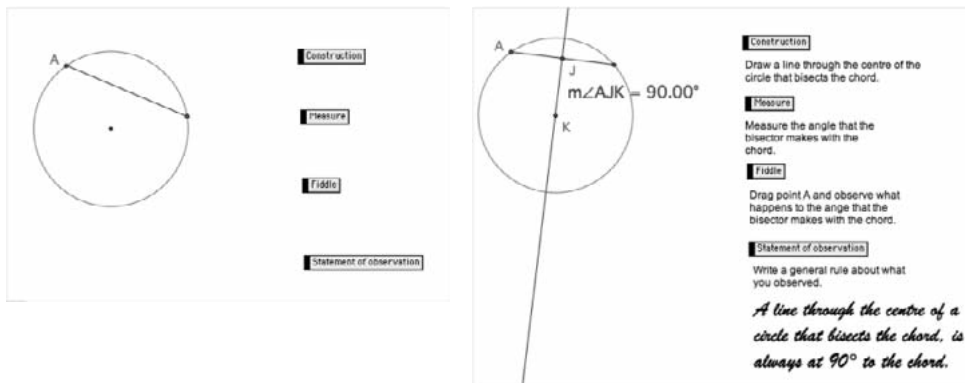
This gives you an ACTION icon that is preset as Hide caption.

| Hide Caption |
| 1. Draw a line through the centre of the circle that bisects the chord. |
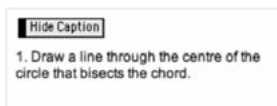
4. You can change the label on the action button by selecting the button using the ARROW TOOL, and going to the DISPLAY menu and selecting the LABEL option.
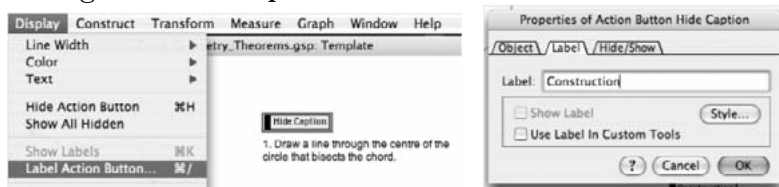
5. To hide the text instruction you typed in point 2, just click the action button.

6. The template can be completed by adding similar instructions and action buttons. This is what the student will see on opening the document.