JOURNAL OF THE EXPERIMENTAL ANALYSIS OF BEHAVIOR 2010, **94,** 365–367 NUMBER 3 (NOVEMBER)

RATE, PROBABILITY AND MATCHING: COMMENTS ON "THE IDENTITIES HIDDEN IN THE MATCHING LAWS, AND THEIR USES" BY DAVID THORNE

HOWARD RACHLIN AND MATTHEW L. LOCEY

STONY BROOK UNIVERSITY

David Thorne's (2010) article, ''The Identities Hidden In The Matching Laws, And Their Uses'' performs a valuable service in pointing out alternative expressions of matching. However, some identities tend to obscure rather than illuminate empirical relationships. Three such problematic instances are discussed: interresponse time as a function of interval and ratio schedule parameters; probability equality as implying rate matching; the apparent simplicity of probabilistic functions, as opposed to response rate functions, of reinforcement rate.

Key words: identities in matching, matching law, reinforcement probability, reinforcement rate

David Thorne's (2010) article, ''The Identities Hidden In The Matching Law,'' is a valuable reminder that Herrnstein's (1961) matching relationship, expressed in two ways below:

$$\frac{B_1}{B_1+B_2}=\frac{R_1}{R_1+R_2} \tag{1}$$

$$\frac{B_1}{B_2}=\frac{R_1}{R_2} \tag{1a}$$

is an identity rather than an empirical finding and that the empirical interest in matching lies not in whether organisms match (that they do is our underlying assumption), but in what the parameters of matching are.[1] Thorne points out that matching can be expressed in terms of reinforcement probabilities ($p_i = R_i/B_i$) as well as reinforcement rates ($r_i = R_i/T_i$) and argues that the probabilistic expression may often be simpler than the expression in terms of rates. Thorne shows that in Baum's (1974) generalized matching relationship:

$$\frac{B_1}{B_2}=a\left(\frac{R_1}{R_2}\right)^c \tag{2}$$

exponentiation is not necessary ($c = 1$) provided that the multiplicative constant, a, is replaced by the variable, p_2/p_1. We agree that it is possible to make such a substitution but believe that several steps in the argument leading to the conclusion that the probabilistic form of matching may be better or simpler or more basic than Baum's generalized expression are misleading. Moreover, the apparent simplicity achieved by Thorne's probabilistic expression of matching obscures actual complexities in the data.

Thorne makes what he says is a ''contrived point'' (p. 248): ''In any single experimental session where the experimenter imposes a particular VI (or VR) schedule, there is a post hoc VR (or VI) sequence that would have produced exactly the same recorded result.''[2] The point is not only contrived, it is misleading. A particular VI schedule determines a feedback function (Baum, 1992): reinforcement rate (r) as a function of response rate (b). As response rate increases from zero, VI reinforcement rate rises sharply from the origin, flattens out, and approaches a horizontal asymptote (of a height inversely related to the VI value). The behaving organism may be thought of as selecting a point on the feedback function determined by the reinforcement schedule. As Thorne's statement implies, it will always be possible to draw a straight line from the origin through that point. Such a line would be a VR feedback function. The VR value would be that of Thorne's Equation 7: IRT = VI/VR; VR = VI/IRT. But the IRT obtained with the VR schedule will not generally equal the IRT obtained with the

---

These comments were prepared with the assistance of grant DA02652021 from the National Institute for Drug Abuse.

Address Correspondence to Howard Rachlin, Psychology Department, Stony Brook University, Stony Brook, NY 11794-2500 (e-mail: howard.rachlin@sunysb.edu).

doi: 10.1901/JEAB.2010.94-365

[1] See also Rachlin, 1971, 1973.

[2] It is not clear what is meant by a ''VR sequence.'' A VR is already a sequence—of numbers of responses between reinforcers.

original VI schedule. In no way would the VR schedule determined by that line have ''produced exactly the same recorded result'' as the VI schedule. Various theories of schedule performance may differ on mechanisms, but they all agree that response rate depends strongly on the schedule type (the *shape* of the feedback function). There is typically no VR schedule that would have produced the same result.[3] VI and VR schedules with the same overall response rate (b) will generally produce different reinforcement rates (r's); VI and VR schedules with the same overall reinforcement rate will generally produce different response rates (except near the origin where VI and VR feedback functions approximately overlap). To put it still another way, you may program a VI and observe a response rate (1/IRT). From that, you may use Equation 7 to derive a VR schedule. But the response rate you obtain when you program the derived VR schedule will differ from that of the VI schedule you used originally. Or you could start with a VR schedule and reverse the process. Again, the response rate will differ. Thus, to be meaningful, the IRT term in Equation 7 would need a subscript (IRT_{VI} or IRT_{VR}) indicating which schedule was used to obtain it.

A second misleading statement (p. 250) is that ''…when relative-rate matching [Equation 1 above] does *not* occur…it must necessarily be due to differences in the two probabilities of reinforcement [p_1 and p_2].'' Thorne refers to rate matching and probability equality as ''confounded correlates of equal potential significance.'' But they are more than that; they are one and the same thing. Probability equality *is* rate matching. Substituting measured entities for probabilities, $p_1 = R_1/B_1$ and $p_2 = R_2/B_2$. If the probabilities are equal, then $R_1/B_1 = R_2/B_2$, and $\mathrm{B}_1/\mathrm{B}_2 = \mathrm{R}_1/\mathrm{R}_2$. But this is Equation 1a—rate matching. Equation 1a is just another form of Equation 1.[4] To say that when matching does not occur it is due to differences in the two probabilities is to say that when matching does not occur it is due to the fact that matching does not occur.

A third misleading implication of the article is that, as applied to single-response schedules, data plotted in terms of reinforcement probabilities is simpler and more straightforward than data plotted in terms of response rates. This is the apparent conclusion to be drawn from Figure 2 of the article which plots, in both ways, the data for 3 of the 6 pigeons tested by Catania and Reynolds (1968). Catania and Reynolds studied pigeons' keypecks under a series of VI schedules. On the right, in Thorne's Figure 2, are the data as originally plotted by Catania and Reynolds: response rate (B/T) as a function of reinforcement rate (R/T). On the left are the same data as replotted by Thorne: reinforcement probability (R/B) also as a function of reinforcement rate (R/T). The functions on the left are a lot simpler than those on the right. The data points for the 3 pigeons, as plotted on the left, are remarkably well fitted by straight lines (almost) going through the origin. The points on the right look much messier; they start out low, rise and then level off. The equations used to predict the Catania–Reynolds plots for each pigeon on the right are more complicated than the straight lines that fit the data on the left and, despite greater complexity, fit the data less well than the straight lines on the left. The reader is led to draw the conclusion that the probabilistic plot provides a better picture of these data than the rate plot.

But let us ask what the two plots tell us. Assuming that the lines on the left go close enough to the origin that the y-intercepts are essentially zero, the linear relationship in Thorne's replotting says that $R/B = k(R/T)$ where k is the (constant) slope of the line. The R's cancel out on the two sides and, rearranging terms, $B/T = k'$ where $k' = 1/k$. That is, the straight lines on the left tell us that response rates are constant. But the more messy curves on the right, where response rates are directly plotted against reinforcer rates, tell us that response rates are not constant; they increase from the lowest values to the highest by as much as a factor of 3. How can two plots of the very same data tell us two diametrically opposite things? The answer is that the linear probabilistic plots on the left obscure actual

---

[3] A VT (variable-time) schedule superimposed on a VR schedule, producing a conjoint feedback function tangent to the VI feedback function at the selected point, might result in the same response rate as the VI alone. But this does not seem to be what Thorne had in mind.

[4] Equation 1a is more convenient than Equation 1 for expressing deviations from matching (as in Equation 2) but algebraically Equations 1 and 1a are identical. Equation 1 converts to Equation 1a by cross-multiplying and rearranging terms; Equation 1a converts to Equation 1 by inverting and adding 1 to both sides ($\mathrm{B}_1/\mathrm{B}_1$ on the left and $\mathrm{R}_1/\mathrm{R}_1$ on the right).

variation in the data. As the Catania–Reynolds plots show, over a wide range of obtained reinforcer rates, response rates of these 3 (of the 6 tested) pigeons are nearly constant. But, for the two or three points, at the lowest reinforcer rates, response rates fall off considerably. In Thorne's probabilistic plots, these points are all bunched up around the origin and their deviation from the straight line is obscured. Unless you argue that low reinforcement rates (high VIs) are somehow less important than high reinforcement rates and therefore should be ignored (for which there is no evidence), you must conclude that the probabilistic plots on the left of Figure 2 obscure essential variation in the data.

Despite these issues, we agree with Thorne's most general conclusion (p. 258): ''…we need to consider and measure *both* variables.'' Data should be looked at from all angles. If that is the main point of Thorne's paper it is well taken.

REFERENCES

Baum, W. M. (1974). On two types of deviation from the matching law: Bias and undermatching. *Journal of The Experimental Analysis of Behavior, 22,* 231–242.

Baum, W. M. (1992). In search of the feedback function for variable-interval schedules. *Journal of The Experimental Analysis of Behavior, 57,* 365–375.

Catania, A. C., & Reynolds, G. S. (1968). A quantitative analysis of the responding maintained by interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior, 11,* 327–383.

Herrnstein, R. J. (1961). Relative and absolute strength of response as a function of frequency of reinforcement. *Journal of the Experimental Analysis of Behavior, 4,* 267–273.

Rachlin, H. (1971). On the tautology of the matching law. *Journal of the Experimental Analysis of Behavior, 15,* 249–251.

Rachlin, H. (1973). Contrast and matching. *Psychological Review, 80,* 217–234.

Thorne, D. R. (2010). The identities hidden in the matching laws, and their uses. *Journal of the Experimental Analysis of Behavior, 93,* 247–260.

Received: April 19, 2010
Final Acceptance: June 14, 2010