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Creativity in problem solving: Uncovering the origin of new ideas

Carol R. Aldous
School of Education, Flinders University *carol.aldous@flinders.edu.au*

Innovation and enterprise depend for their success on the development of new ideas. But from where do new ideas come? How do they arise? Finding solutions to such questions is at the heart of creativity research and the solving of novel problems. Reflection, not only in cognitive processes but also in the non-cognitive ones used in solving novel mathematics problems, is uncovering a way in which the origins of new ideas occur. A study involving protocol analysis of five expert problem solvers identifies three critical elements. These elements have been employed to construct a framework of creative problem solving which may be used to foster creativity among young people under instruction and provide a cognitive explanation of the origin of new ideas.

Creativity, problem solving, cognitive, non-cognitive, reflection

INTRODUCTION

Uncovering the origin of new ideas conjures many benefits in the resolution. These benefits include the innovative, the entrepreneurial, the educational, the social, and the global. The source of new ideas also merits a search from different orientations. These orientations may include the historical, the sociological, the philosophical and the theological to mention but a few. While each orientation contributes a different perspective, a comprehensive disclosure as to the origin of new ideas may ultimately depend for its resolution on the synthesis of many such perspectives.

The orientation adopted in this paper is one taken from cognitive psychology and neurobiology. In this orientation the focus is on differentiating the creative processes in human cognition that may be used to solve novel mathematics problems. The purpose of the study is to identify and describe some elements of creativity that may be used to construct a framework of creative problem solving. By so doing a cognitive explanation as to the origin of new ideas may be found.

A Working Definition of Creativity

Many definitions of creativity can be found within the research literature on creativity. However one definition finding increasing acceptance in both education and psychology is that describing creativity as the production of effective novelty (Cropley, 1999; Lubart, 2001; Mumford, 2003a). This definition implies that for something to be creative it must be both original and useful.

The National Advisory Committee on Creativity, Culture and Education in England, for example, advises that creativity is “Imaginative activity fashioned so as to produce outcomes that are both original and of value” (NACCCE, 1999, p.30). In the same vein cognitive psychology, adopting a more processed orientation, defines creativity as “the sequence of thoughts and actions that leads to a novel adaptive production” (Lubart, 2001, p.295).

One definition that makes explicit the nature of thought and action within the creative process is that by Koberg and Bagnall (1976) who describe creativity as:

> both the art and the science of thinking and behaving with both subjectivity and objectivity. It is a combination of feeling and knowing: of alternating back and forth between what we sense and what we already know. (Koberg and Bagnall, 1976, p.8)

This definition implies that not only is cognitive activity involved in the creative act but non-cognitive activity as well. According to Koberg and Bagnall (1976), the act of creation, involves oscillating between what individuals think or know (namely, cognitive activity) and what they sense or feel (i.e. non-cognitive activity). This conceptualisation is significant in light of the protocols that are described below.

Valuing Cognitive and Non-cognitive Elements

That there is a need to value both subjective and objective elements in the creative process, is a view shared by a number of notable proponents in the field. Russ (1993) for example has developed a model of affect and creativity. Cropley (2001) has mapped different emotions to particular phases of the creative process and Shaw (1989) highlights positive and negative poles of emotion arising at different stages of creative production. These poles of emotion are over-layed with a series of feedback loops involving conscious and non-conscious mental activity (Shaw, 1989).

Each of the affective models referred to above employs an adaptation of the classic four-stage model of creativity put forward by Wallas (1926) and others (Hadamard, 1945). The four stages in the classic model are preparation, incubation, illumination and verification. Although the phases of preparation and verification are marked by conscious activity, the phases of incubation and illumination may include non-conscious activity.

Cycles of Conscious and Non-conscious Activity

Shaw (1989) predicts that a series of feedback loops arises between each of the phases of the creative process. The 'Areti loop' for example, predicts conscious and non-conscious oscillatory behaviour occurring between the phases of preparation and incubation, the 'Vinacke loop' to cycles of non-conscious and conscious mental activity arising between incubation and illumination, while the 'Lalas loop' predicts cycles of illumination and verification occurs whenever a given explanation leads to further illumination. The 'Communication loop' predicts feedback between the stage of verification and further validation of the creative product. Finally, multiple feed-back loops, involving conscious and non-conscious mental activity are theorised to exist from the verification and validation stages of creativity to all previous stages in the creative process. These multiple feed back loops are collectively referred to as the 'Rossman loop'.

This interplay of conscious and non-conscious mental activity with respect to each of the four phases of creativity can be explained in neural network theory in terms of a large net of interconnecting neurons (Martindale, 1995) or nodes. A neural network explanation of the stage model of the creative process follows.

Neural Networks and the Creative Process

During the preparation stage, when attention is greatly focused, a small number of highly activated nodes dominate consciousness (Martindale, 1995). These highly activated nodes inhibit other nodes from becoming active in a process of lateral inhibition. However, when attention is gradually defocused, such as occurs during incubation, lateral inhibition is diminished and other nodes primed through remote association with the problem may be activated. Should a partially primed node become fully active, it may enter consciousness in an act of illumination (Martindale, 1995). The verification phase is once again marked by focused attention when an idea or solution is being examined for suitability and a small number of nodes are highly activated.

Thus, in the confines of this study both cognitive and non-cognitive elements of creativity need to be identified and described in order that a comprehensive representation of the creative process may be articulated within the proposed framework of creative problem solving.

Creativity within the Education Context

In the field of creativity research some argument arises as to the degree of effective novelty required before a given production may be classed as creative. Some proponents consider only eminent productions to be creative (Simonton, 1988) while others regard more modest representations of the everyday kind as creative (Cropley, 2001; Richards, 1999a). Creativity in this study has been interpreted relative to the originator of the novel production. If an individual found an effective solution to what was to him a new and different problem, even if others had already solved the problem, then, for the purposes of this study the individual demonstrated creativity. Thus, an interpretation of creativity consistent with the everyday kind has been adopted, a perspective useful in the education context.

METHODS OF ANALYSIS

Protocol Analysis

Verbal data from five expert problem solvers were collected using protocol analysis (Ericsson and Simon, 1993), a technique developed to study cognitive processes in human subjects. Three of the experts were teachers of secondary mathematics, one a university lecturer in applied mathematics and one a Grade 11 student who was a proficient problem solver in mathematics. The verbal data were recorded by audio, video or on-line means.

Two forms of verbal report were gathered. The first of these involved concurrent reporting. The second involved retrospective reporting. Concurrent reporting involves subjects verbalising their thoughts while performing a specified task or problem. Such reports are often disjoint since the subject does not explain what he or she is doing, but verbalises only that to which he or she is attending. Without the encumbrance of explanation, the subject’s sequence of verbalised thought is considered to reflect the sequence of information processing (Ericsson and Simon, 1993). The theory of sequenced thinking reflecting sequenced information processing is significant in light of the patterns of alternating and concurrent thinking reported in this study.

Retrospective reporting requires the subject to recall the sequence of thoughts from start to finish at the completion of the task. Retrieval cues remaining in short term memory allow effective recall of the sequence of thought. Since concurrent reporting does not permit explanation of process, when it is combined with retrospective reporting a more coherent picture of human information processing is achieved (Ericsson and Simon, 1993). It also provides an opportunity for seeking clarity on any point in the protocol.

Indeed, a limiting factor of protocol analysis is that, although the brain may be a parallel processor (Tank and Hopfield, 1987), speech is sequential and thus the sequential nature of thought may be over accentuated (Das, 2003; Khandawalla, 1993). To alleviate this difficulty both kinds of reporting were used in the study.

An Expanded Focus of Information

Unlike traditional studies of protocol analysis where the focus is on functional mechanisms of cognition that can be replicated in a computer program, an expanded focus that included non-functional as well as functional mechanisms was used in this study. Non-functional mechanisms include such processes as sensing, feeling, incubating, reverie, imagination and inspiration. Thus, an expanded notion of information to be heeded in the information-processing context was adopted, that included both non-cognitive and cognitive aspects.

RESULTS AND DISCUSSION

Exploratory Protocols and the Identification of Themes

Content analyses of five exploratory protocols led to the identification of three themes around which cognitive and non-cognitive elements within the creative problem solving process could be described. Consistent with published research findings (Damasio, 1994; Dehaene, Spelke, Pinel, Stanescu, and Tsivkin, 1999; Epstein, 1994; Sloman, 1996) these themes involved:

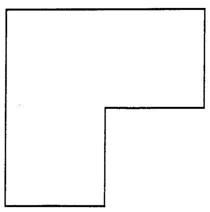
- the interaction between visual-spatial and analytical reasoning
- the role of feeling in listening to the 'self'; and
- the interaction between conscious and non-conscious reasoning.

The interplay between visual-spatial and analytical reasoning was the first theme to be identified. Analyses of protocols within this theme led to the identification of the second theme, namely the role of feeling in listening to the 'self'. One individual, for example, consistently used the expression "I think" when displaying analytical reasoning, but the turn of phrase "I feel" when displaying visual-spatial reasoning. In addition, a second individual articulated the need to obtain a "visual feel for the shape" linking a feeling approach with visual spatial reasoning. Since the concurrent turns of phrases were not, in the main, consciously chosen, (as revealed by the retrospective accounts) the third theme, the interaction between conscious and non-conscious reasoning, was identified.

The following are extracts taken from each protocol broadly grouped on the basis of theme. While the labels of A, B, C, D and E have been applied to each individual, pseudo-names have also been given in order to facilitate semantic memory and preserve participant anonymity.

Theme 1: On the Interaction between Visual-spatial and Analytical Reasoning

The protocol abstracts from individuals A, B and C, referred to as Anne, Barbara and Chelsea, relate to problem solving attempts of a novel geometric problem related to area that may be solved using either analytical or spatial reasoning or a combination of both. Anne is a new graduate teacher in secondary mathematics education. Barbara is an established secondary mathematics teacher with more than ten years experience, while Chelsea is a Grade 11 student of mathematics. A copy of the problem and its rubrics solved by Anne, Barbara and Chelsea is given in Figure 1.



Divide the shape above into four pieces, which are exactly equivalent in shape and area. The shape is a regular shape with each of its short sides being half the length of its long sides. Use any materials or procedures you think will help you complete this task.

Figure 1. The L-shaped area problem (investigated by Anne, Barbara and Chelsea)

The solution to this problem involves identifying an arrangement of 'L' shaped patterns nested along side each other within the diagram.

Individual A: Anne

Individual A: Anne is first observed using an analytical approach to solve the problem but when this approach fails Anne switches to a visual-spatial approach. Anne eventually solves the problem by visual-spatial means.

While using analytical reasoning Anne concurrently reports:

> I am trying to think of irregular shaped patterns, shapes that are more complex that will divide the area …
>
> I think what I am trying to do is keep everything sort of basically symmetrical…
>
> As I say, if I can put a diagonal line from corner to corner. If I can, then just deal with one side of it, then whatever I do to that one side I can do to the other, but I'm not sure if I can do this either. May be it shouldn't be a diagonal line.

While using visual-spatial reasoning Anne verbalises:

> I'm just trying to visualise patterns in my head to see what areas spatially will fulfil the requirements…
>
> I definitely feel that the shapes are going to be geometric and that will most easily allow me to make sure the areas are the same size…
>
> So I feel that somehow it is like that (pointing to a swastika superimposed with L shapes she has drawn upon the page). It will be made of Ls.

Anne then silently draws the solution upon the page and counting four L shapes embedded within the diagram exclaims:

> Where did that come from?

It is interesting to note that when using an analytical approach to solving the problem Anne is recorded using the expression ' I think' but when using a visual-spatial approach to solving the problem she is recorded using the expression 'I feel'. In addition Anne's final exclamation indicates that the solution process involving visual-spatial reasoning was not entirely conscious.

Invited to reflect on her unconscious choice of the word 'feel' when using a visual-spatial approach Anne reports:

> It was a feeling in the respect that it had its own self-similarity. I know I was sort of seeing these sorts of patterns in my head… If I had plasticine I would have started shaping these Ls. In my head I was moulding them… which was like an emotional shaping … the only way I could describe it, is that it was a kind of an instinct, which I would argue, wasn't thinking because it was more primitive than that.

Anne elaborates:

> You've got to sit back and see how you feel … there are no boundaries, you have to open your mind and see what comes in… I was outside of my mathematics domain… I just went global.

Anne concludes:

> It was an awakening of ideas. I'm thinking the images but feeling their correctness… It's taken all of me to solve it… True problem solving is using all of you to solve it.

When invited to reflect on her final comment, "Where did that come from?" Anne explains:

> I must admit, when I was drawing it … I didn't know the solution until I'd finished drawing it. You know what I mean? I must have had a glimmer of it in my head. It was almost like my head wasn't controlling my hand…my subconscious just fully took over.

This protocol indicates that in the case of Anne a total engagement of the 'self' was essential to the formation of a successful solution. Anne opened her mind to conscious and non-conscious aspects of cognition and employed visual, spatial, analytical and feeling elements in the problem solving process. Of note is the observation that the feeling approach used by Anne served an evaluative function, in the navigation of a solution path. Further this feeling aspect was expressed during the employment of visual-spatial reasoning.

Individual B: Barbara

During this interview Individual B: Barbara was observed alternating back and forth between analytical thinking and spatial thinking until the solution was found. While an analytical approach was used to determine the unit shape and its area, a spatial strategy was employed to determine the collective orientation within the figure. Beginning with an analytical approach Barbara reports:

> Well the first thing I think when I look at it, is that it's very easy to divide it into three shapes; because we have three squares… one on each side, placed together to form an L.

Later changing to a visual-spatial strategy Barbara verbalises:

> So I'm trying to visualise how I can put lines in that will allow me to make four pieces.

Returning to an analytical approach Barbara relates:

> Starting to think that straight lines aren't the answer, starting to think that perhaps I need to step my lines.

Reverting to a spatial-verbal approach Barbara reports:

> So I'm putting markers on the shape just so I can get a visual feel for the shape.

As mentioned above it is interesting to note the use of the word 'feel' with a visual representation of the shape. It is also interesting to note the word 'think' being articulated during an analytical approach. Further when invited to comment on her alternating analytical and visual spatial behaviour Barbara retrospectively explains:

> Definitely… I could feel it. I could actually feel it in my brain. The analysis would take over, and then that would reach a dead end and then I would look for some intuition of where to go. I could feel it happening in my head.

Of relevance to this protocol is the suggestion by neuroscientists that mathematical intuition may emerge from the interaction between visual-spatial and linguistic reasoning circuits (Dehaene et al., 1999). Barbara's articulation of the word intuition in the context of her alternating behaviour concurs with this view.

Further when invited to elaborate upon her feeling of what was happening in her head Barbara explains:

> It's pretty hard, but it's something I've always known about myself mathematically that if I can't see the answer straight away; if I just sort of let my head go fuzzy and stare at it, it comes. The answer just comes. And I've always been the same. It's not always the right answer. Sometimes I go off on the wrong path but when that path is exhausted, same procedure. And that's how I get my direction in solving things. So

> here, once I'd eliminated the straight lines and I'd got my 3 and 6 and that wasn't going to work then it was an analytical thing to think OK, go to shapes. And then to work out areas, that was definitely analytical. But then the actual orientation (of shapes) was very much intuition. Was very much just look at it, not even think, just let your brain, work on it.

This verbal account indicates that Barbara utilised analytical, rule based reasoning together with associative, spatial reasoning in an alternating fashion in order to solve the problem. A conscious non-conscious interplay in the problem solving process is also indicated by the statements " If I sort of let my head go fuzzy and stare at it, it comes" and "the actual orientation was very much intuition … not even think, just let your brain work on it." Such statements are also indicative of defocused attention and pre-conscious activity.

It is noteworthy that Barbara reports feeling the reasoning processes alternating in her head. Such feeling reflects a total engagement of the 'self' in the problem solving process and relates to the second theme 'the role of feeling in listening to the self'. The fact that Barbara is definite about knowing she has a feeling and further, that she is able to heed that feeling in the problem solving process, is relevant to work in metacognition.

Barbara's retrospective protocols also reveal that she models the use of this intuitive reasoning to her pupils. Indeed, in the event of becoming stuck in a problem Barbara advises her students to relax their minds while continuing to absorb the problem. This strategy, she advises, is particularly useful in solving deductive geometry problems.

> When I really notice myself doing it is in a class with kids, particularly with deductive geometry… I say to the kids "OK I need to have a think". That's how I do it. I just pull up a chair and sit at the front board and just stare at it and it just happens, … the answer just jumps off the black board at me. But nobody can talk to me while I'm doing it… I actually say, "Have you just sat and looked at it. Just sort of let your mind go blank?" And quite often the kids are then able to go on and solve things … I won't help a student until they've actually sat and looked at it… if they have just sat and looked I'll give them one clue, and then, you know, say "Have another try". And you can see the lights come on. And I'd have to say I've been reasonably successful in teaching the topic.

Although this account of fostering intuitive reasoning among students warrants closer examination, the fact that Barbara takes the trouble to model this approach to students indicates that an intuitive reasoning approach works successfully for her, particularly in the field of deductive geometry. Moreover it is evident that Barbara has confidence that the approach will work for her students. When asked to confirm the success of just sitting back, looking and letting your mind go blank during problem solving, Barbara comments:

> I've got to say, it used to worry me … that I didn't appear to be thinking, like other people think, or… how I thought other people thought… for me to actually think about it… was actually more the emptying of the mind than the filling of it. But I pretty well always got the right answer.

This account points to the benefits of defocused attention in a novel problem solving context and the usefulness of incubation or semi-incubation in reaching a solution. Given a recent experimental finding showing that an instructional strategy used to encourage children between the ages of 8 and 10 years to defocus their thinking before attempting a problem increased their ideational productivity (Howard-Jones and Murray, 2003), the above protocol shows a degree of forward thinking.

Individual C: Chelsea

Unlike Anne and Barbara, who are experienced mathematics problem solvers, it may be recalled that Individual C (Chelsea) is a relatively inexperienced Year 11 school student. In this interview Chelsea relies totally on rule based, analytical reasoning and is concerned with attempting to solve the problem in the so-called 'correct way'. She appears to look for an algorithm to solve the problem, and lacks the confidence needed to step outside the traditional rule-based mathematical approach. Had Chelsea been able to do so it may have been more economical on her effort and time.

> I don't know how I'm going to do this… I'll probably just try a few things… What if I did triangles? Is that all right? …OK I'm thinking how I can make these two sections, cut in half to make four… but it's an odd shape? … Is there like some sort of a rule that I have to use? … No geometric rules? … Just trying to think of that.

However, Chelsea does finally solve the problem analytically using trial and error reasoning. She retrospectively reports:

> Once I had it into 12 squares and tried a few patterns, which didn't work out, then I finally found this (pointing to the solution)… Just trial and error, I just tried a whole lot of different sequences and just came to the right one.

Chelsea's total reliance on rule based reasoning is perhaps not surprising given the emphasis that school based mathematics curricula place on training students to think in this way. By contrast, it may be recalled that Anne, who stepped outside the traditional mathematics domain and in her own words "went global", and Barbara, who defocused attention through an "emptying of the mind", solved the problem relatively quickly. Both of these strategies were employed during visual-spatial reasoning, an approach Chelsea did not report.

Theme 2: On the Role of Feeling and Intuition in Listening to the 'Self'

Individual D: David

In addition to the protocol extracts relating to the theme 'The role of feeling and intuition in listening to the self' obtained from Anne and Barbara, an interview conducted in an on-line mathematics enrichment program also provides material relevant to this theme.

Individual D (David) is a highly experienced mathematics teacher who possesses a talent for divining original ideas for mathematics questions such as those used in the Australian Mathematics Challenge, a national event conducted in mathematics problem solving.

Invited to discuss how he came up with creative ideas for particular problems David reports:

> … I look at life through a mathematical glass – stoplights, rates of cordial concentration, the price of stamps, whatever comes up from the murk in my tortured mind.
>
> I just kind of like numbers and the way they behave. There's order within their disorderliness. I can't help finding it interesting and sometimes the things I'm interested in interest the kids too.
>
> … I noticed that 135 and 136 were a corrine pair[1] one night. $135=3^3x5$ and $136 = 2^3x17$. It was on my clock. I thought, "I bet that doesn't happen too often". Then

[1] A corrine pair is a pair of consecutive integers with a prime factorisation of the same form.

> I started fiddling around with other consecutive integer pairs and prime factorising them.

This protocol indicates that David uses associative forms of reasoning to assist in the development of creative ideas. He employs, in an automatic fashion, the operations of similarity and contiguity in generalising digits appearing on a clock as corrine pairs.

When questioned about finding his way in a problem task, David reports:

> …some numbers feel prime to me. Some answers I get don't feel good and those ones usually aren't
>
> Eventually you do have to sit down and flog out the answer formally though

In acknowledging the role of 'feeling' in the problem-finding context David demonstrates one way he evaluates the correctness of his intuitive insight and the importance of listening to the 'self'. However David finally tidies up the problem activity using a rule based approach, involving the formal documentation of ideas.

Theme Three: On the Interaction between Non-conscious and Conscious Reasoning

Individual E: Eddie

In addition to the protocols from Anne and Barbara revealing the involvement of semi-conscious or pre-conscious reasoning in the problem solving process, a second hand data study involving a professional mathematician was found to reveal relevant information.

Individual E, Eddie, is a university lecturer in mathematics who had been video taped solving six mathematics problems. The retrospective protocol of one of these six problems is reported. A copy of the reported problem is located in Figure 2

Suppose that each of five people know exactly one piece of information and all five pieces of information are different. Every time one person phones another the first person tells the second everything he knows while the second tells the first nothing. What is the minimum number of phone calls between pairs of people needed for everyone to know everything?

Figure 2. The telephone problem (investigated by Eddie)

In this particular interview Eddie had been asked to solve the problem (to which the solution was '8'). Initially Eddie came up with the answer '8' by intuitive means, but then with prompting by the interviewer endeavoured to justify his answer until he was confident '8' was the correct answer. Eddie reports:

> … it was a case of the method that I suggested occurred almost sort of naturally as being the way one would go about the problem in an optimal fashion, and so when I got my initial "8" I was reasonably confident about it on an almost intuitive basis, because it just seemed to me the obvious way to do it. I then had to go back and think, "Right! How can I somehow formalise its optimality." …It was in part a case of trying to determine why my intuitive feeling was my intuitive feeling, so now looking back on the problem, it's because, having thought about it, it was the realisation at some point someone has to first know all the information and so there is really two parts to the problem. There's first of all the collection in one spot of all the information and secondly the dissemination of that information to every one else so you can in fact optimise the two halves of the procedure.

Eddie's response would seem to imply that his initial thinking processes relied heavily on associative reasoning which, according to Sloman (1996), was consistent with the intuitive function. During this period the video showed long pauses of silence interspersed by the comment "Now I'm searching for inspiration". Such behaviour is compatible with thinking occurring in the non-verbal, visual spatial circuit of the brain. However, with the need to verify the solution, Eddie included the use of rule-based reasoning and the explanation was translated into two distinct steps. In the initial stage Eddie is unable to articulate how he obtained the answer of 8 indicative of pre-conscious awareness. This is followed by conscious awareness facilitated by the forced explanation of the solution.

It is interesting to observe that the professional mathematical problem solver (namely, Eddie) drew upon associative forms of reasoning almost immediately in solving a novel problem, before calling upon his rule based analytical reasoning to verify the solution. Each of the other individuals described above (except David), first began to solve the problem provided relying upon rule based rational reasoning before embracing associative, experiential forms. It is worthy of note therefore, that Chelsea the Grade 11 mathematics student appeared to shut out associative reasoning entirely, relying totally on an analytical approach. Such an observation, should it generalise to other students, is cause for concern and has implications for mathematics and educational curricula.

CONCLUSIONS

The protocols reported above indicate that the formation of a new idea has three aspects. First, preverbal and non-verbal processes (including spatial thinking but not exclusively so) are involved. Second, creativity may incorporate pre-conscious or non-conscious activity. Third, creativity gives rise to a feeling or intuition. Indeed the production of a new idea would appear to entail moving between thinking and feeling, and between focused and defocused states of attention.

Translation of Themes into Elements

Thus three elements emerge as critical to the formation of a conceptual framework of creative problem solving. These elements, consistent with documented research involve the:

- visual-spatial and linguistic circuits within the brain;
- conscious and non-conscious mental activity; and the
- generation of feeling in listening to the 'self' including that of intuition.

Brain imaging evidence has been used to locate two circuits involved in mathematical thinking (Dehaene et al., 1999). One circuit, used in processing approximate quantities, employs a region strong in visual-spatial processing. The other circuit, used in processing exact quantities, utilises a region strong in linguistic processing. It is theorised that mathematical intuition emerges from the interaction of these two brain circuits (Dehaene et al., 1999). Thus the interaction between visual-spatial and analytical reasoning observed in the above protocols is consistent with neuro-biological evidence related to visual-spatial and linguistic thinking and is embedded within the first element of the framework.

The second element detailing the interaction between conscious and non-conscious activity is also supported by documented research. In particular, cognitive psychology has identified two systems of reasoning (Epstein, 1994; Sloman, 1996). One of these, the rule based or rational system, is characterised by conscious activity. The other, an associative or experiential system, is characterised by non-conscious activity. Indeed Epstein (1994) proposed that creativity, among other higher order functions, involved the complex processing of both the experiential and rational systems.

The third element on the generation of feeling in listening to the self is also supported by scientific research. In particular, neuro-scientific evidence indicates that certain processes related to emotion and feeling are indispensable to rational thinking (Damasio, 1994). Patients with lesions in a small frontal area of the brain, impairing the connection between reasoning and feeling, were unable to bring reasoning to any practical conclusion. Without feeling, such patients were unable to decide which of two rational alternatives was better (Damasio, 1994). This notion, that feeling is important in the creative problem solving process, as was evident in the protocols above, is embedded within the third element of the framework.

The Formation of a Conceptual Framework

This study proposes that creativity may emerge from the interactions arising between each of the three elements. These elements and their interactions are represented diagrammatically in Figure 3. The lower part of the figure displays the interactions arising among the elements themselves. The upper part of the figure displays the components of the stage model of creative problem solving, together with cycles of feedback theorised to exist between each of the stages within the creative process. These stages and their associated cycles of feedback have been superimposed onto the elements of the conceptual framework. A synergy results from the interactions arising among the critical elements of the framework and the stage processes and feedback cycles implicit in creative problem solving.

Among the elements of the framework, the interactions arising between the visual-spatial circuit and the linguistic circuit are thought to give rise to mathematical intuition (Dehaene et al., 1999). These interactions may be direct or indirect and involve either or both non-conscious and conscious paths of thinking. Similarly the interactions arising between non-conscious and conscious mental activity represented by ‘self state one’ and ‘self state two’ may provide the necessary cognitive space where learning can take place (Davis, 1996).

Central to the framework however, is the intuitive function. The generation of feeling within and between non-conscious and conscious activity (represented by ‘self state one’ and ‘self state two’) and between and within the visual-spatial and linguistic circuits serves to evaluate, monitor and filter a particular solution path. Feeling, as is revealed in the exploratory protocols, (refer Anne, Barbara and David) plays a significant role in crystallising possible solutions generated in the visual spatial circuit of the brain and is likely to be one of the body’s internal communication mechanism particularly in instances of non-verbal reasoning. This suggestion of feeling serving an internal communication function is consistent with the neuroscience findings of Damasio (1994) and the work of Epstein (1998) in constructive thinking.

The Framework and its Implications for Teaching

Much mathematics instruction focuses primarily within the bottom right hand corner of the conceptual framework. Emphasis on linguistic and conscious processes tends to be the norm with students and teachers alike, solving problems that have been practised many times before.

While routine practice of skills may well enhance the student based repertoire of problem solving strategies, there is an increasing need, in an ever-changing technological world, for greater exposure to more novel and real life problems. Continued but balanced experience, in solving more novel problems should help provide the forum by which other forms of reasoning may be fostered, including the visual spatial and non-conscious elements of the self. Such experience encourages students to draw upon all of themselves in solving a problem and lead to greater fulfilment of their developing potential.

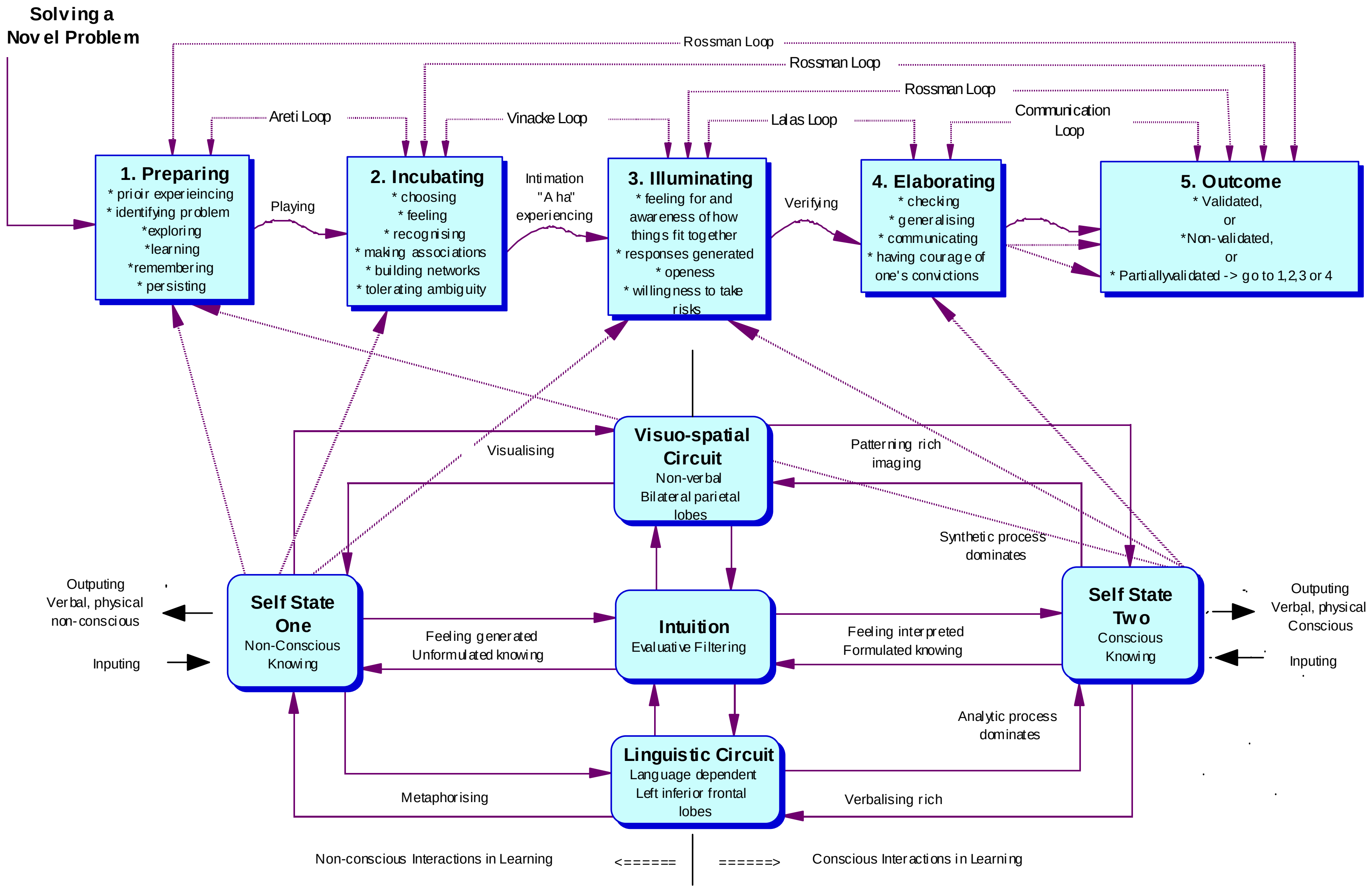


Figure 3. A conceptual framework: Listening to the 'self' in learning, creating dynamic interactions of flow

The exploratory protocols revealed that, for novel problem solving, feeling thinking and intuition play a significant role and are a necessary component of the creative process particularly in instances of visual spatial or non-verbal reasoning. The non-cognitive elements of feeling and intuition may be among the first steps in the path to solving a novel problem. The cognitive elements of rule based analytical thinking may then follow, playing some interactive role. But the complete sequence of reasoning remains to be determined and in all likelihood is far more complex than this preliminary explanation reveals.

CONCLUDING COMMENT

Creativity in this study has been found to revolve around the interactions of three critical elements. These are the visual-spatial and linguistic circuits of the brain, conscious and non-conscious mental activity and the generation of feeling in listening to the self. The vital component linking each element in the framework is the intuitive function made manifest through attention to feeling. Indeed feeling is the communicating link in the interactions arising between any or all of the elements within the framework. While feeling without action does not of itself constitute creativity, reflection as fostered by listening to the self in learning, not merely in cognitive but also in non-cognitive ways (as revealed by the proposed framework of creative problem solving) is more likely to result in a positive outcome.

In writing about the inseparable nature of feeling and reason, neuroscientist Damasio (1994, p.xiii) had this to say. “Feelings point us in the proper direction, take us to the appropriate place in a decision making space, where we can put the instruments of logic to good use”. And furthermore he adds, “Educational systems might benefit from emphasising unequivocal connections between current feelings and predicted future outcomes” (Damasio, 1994, p.247).

In summary then this paper finds that increasing creative productivity will involve attending to feeling and reflecting on the world within. Indeed new ideas are heralded with a wellspring of feeling from within. The added value to be obtained in so doing is that both individual and collective rationality are likely to increase.

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