2009, Vol. 11, 22–35 *Mathematics Teacher Education and Development*

Teachers' Reflections on their Mathematical Learning Experiences in a Professional Development Course

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This study examined the impact on 16 middle-school teachers' conceptions of mathematics learning and teaching of reflections on their learning experiences within mathematics professional development. The research questions were: In their reflections, what do teachers express about how they best learned mathematics through these learning experiences? How do teachers extend these ideas to possible modifications of their teaching practice? Data included the teachers' written reflections and field notes, which were analysed using grounded theory. In their reflections, the teachers noted several supportive processes that aligned with recommendations of teaching for mathematical understanding and that they planned to implement in their classrooms. Teachers' reflections were more aligned with teaching for understanding when they thought about their mathematical learning experiences in the role of a learner before considering implications for their teaching.

Many teacher educators are offering content-based professional development that involves teachers in doing mathematics as a central focus. The facilitators implement the mathematics in a student-centred fashion, and a prominent aspect of most programs is reflecting on mathematical content. Various programs have reported on the impact of reflections on such things as increasing mathematical knowledge (Burk & Littleton, 1995). However, it has not been reported what happens when teachers are asked to revisit their mathematical *learning experiences* within the professional development and consider implications of those experiences for their classroom practice. In the professional development course discussed here, this component was incorporated. The intent of this study was to investigate what teachers noticed in their reflections on their mathematical learning experiences. The research questions were: What do teachers express about how they best learned mathematics through these learning experiences within the professional development? How do teachers extend these ideas to possible modifications of their teaching practice?

Background

Professional developers may utilise various entry points when working with teachers, including examinations of students' work, lesson study, or investigations of video cases (Zaslavsky, Chapman, & Leikin, 2003). In professional development that is classified as content-based, the entry point is to involve teachers in doing significant mathematics (Lappan, 2000; Loucks-Horsley, Love, Stiles, Mundry, & Hewson, 2003). Such approaches are driven by the need for teachers to have strong content knowledge in order to effectively teach mathematics (Ball, 1991; Garet, Porter, Desimone, Birman, & Yoon, 2001; Hill & Ball, 2004; Loucks-Horsley & Matsumoto, 1999; Ma, 1999). With content-based professional development, the mathematics is relevant to the K-12 classroom but examined at a level appropriate for teachers as adult learners (Campbell & White, 1997; Hill, Rowan, & Ball, 2005; Saxe, Gearhart, & Suad Nasir, 2001; Schifter, 1998; Swafford, Jones, & Thornton, 1997). The facilitators of the professional development implement the mathematics in a student-centred fashion, providing a model of how such instruction might look in a classroom. The content focus is then purposely supplemented with pedagogical topics such as investigations of students' mathematical thinking or examining constructivism as a theory of learning.

Research conducted on such programs has shown increases in teachers' content knowledge, teachers recognising mathematics as a sense-making domain, teachers viewing themselves as initiators of mathematical thought, and changes in classroom practice (Campbell & White, 1997; Schifter, 1998; Swafford et al., 1997). In the classroom, teachers have been observed making students' thinking more central in their teaching, using less drill and practice, actively engaging students in the mathematics, and demonstrating confidence and beliefs aligned with teaching in a student-centred fashion. In addition, Hill et al. (2005) and Saxe et al. (2001) have found such programs support student achievement. However, only one of these studies reports asking teachers to reflect on their mathematical learning experiences (Schifter, 1998), and none of them report how such reflections impacted the teachers' thinking about their practice. It is expected that asking teachers to reflect on their own experiences of learning mathematics in a student-centred fashion will influence the perceptions of their own teaching. As such, this reflection component was incorporated in the content-based professional development course described here.

In contrast to content-based programs, White, Mitchelmore, Branca, and Maxon (2004) reported that pedagogically oriented professional development programs are often preferred in Australia. However, they, like others including Loucks-Horsley et al. (2003), recommended that professional development for teachers include content as well as pedagogical emphases. As a way to incorporate content for Australasian teachers, White et al. (2004) suggested emphasising a problem-solving approach. By asking teachers to reflect on their mathematical learning experiences for possible implications to their teaching practice, this professional development course purposely incorporated pedagogical as well as content topics. Furthermore, the examples used in this article will illustrate how this course approached the content in a problem-solving fashion that is relevant and motivating within an Australasian context.

Method

Research Context

The course was similar to the aforementioned content-based professional development approach. The entry point involved focusing on teachers' own mathematical learning and then asking teachers to reflect on their learning experiences to inform their practice. The course consisted of a 2-week summer institute meeting for 6.5 hours per day in 2005 with a content focus on number and operations (National Council of Teachers of Mathematics (NCTM), 2000). The overarching goal was to help teachers understand and be able to illustrate operating on whole, integer, and rational numbers. Two formally-trained mathematicians with over 15 years of collective experience in offering professional development facilitated the course, while the author served as the researcher. Sixteen teachers enrolled in the course. They taught students of ages 10-14 years in grades 5-9. All of the teachers taught at least one class of mathematics, while some of the teachers taught additional subjects such as science. Nine of the teachers were highly qualified to teach middle school mathematics, having at least 24 university credit hours of mathematics (the equivalent of eight university mathematics courses or one year of full-time study).

The course included three elements: deepening teachers' mathematical understanding through problem solving; using mathematical learning experiences to examine practice; and providing support for teachers to modify instruction. First, the course placed teachers in the role of mathematics learners to deepen their mathematical understandings. This was done through careful facilitation of group problem-solving sessions with a strong emphasis on teachers providing coherent explanations of mathematical ideas. Although most of the activities could be adapted to a middle school classroom, the mathematics was extended to levels that were appropriate for the teachers as adult learners. Lessons associated with operating on integers included exploring the area model for multiplication, investigating the partitive and quotitive models for division, and developing number line and counter models for operations with positive and negative numbers. For operations with rational numbers, lessons focused on various models including pattern blocks, fraction squares, and drawings.

Second, the teachers were asked to reflect on those mathematical learning experiences and on possible future applications to practice. The teachers were encouraged to examine "what just happened" in the mathematics component and to consider in essence, "Does my practice provide these kinds of learning experiences for my students?" Examples of these reflection prompts included:

- What have you learned about yourself as a learner from this mathematical experience? What was it like to be in the student role?
- What was hard about this activity? What was easy? What supported your learning? What hindered your learning?
- How might your learning experience influence your future teaching? What are some implications of your learning experience for your teaching?

Third, the teachers were supported to integrate their new understandings of both mathematics and the teaching and learning of mathematics into their practice. This involved completing two classroom implementation projects. For the first project, teachers were required to conduct an in-depth exploration of a topic covered in the course and develop an associated curricular unit. For the second project, the teachers were expected to write a plan for implementing the use of representations to communicate mathematical ideas and to model mathematical phenomena. An example of this three-pronged approach is described in the remainder of this section.

On the second morning of the course, one of the facilitators introduced the teachers to the "Do Math Bear", a teddy bear manufactured by NCTM. The bear was attached to a metre stick, and the facilitator drew a large number line on the board. The facilitator explained that the teachers were to create a model that would describe how to move the Do Math Bear on the number line to demonstrate adding and subtracting integers. The teachers proceeded through several cycles, each associated with an expression, as they developed their models. The initial expressions consisted of computations such as 3 + 5 and 5—2; later expressions included more challenging computations such as -1 + -5, 5 + -3, and -5—3. The teachers began each cycle by individually writing models for the Do Math Bear. Then they talked in their groups, revising their models as needed. Finally, the facilitator led a class discussion. For this discussion, the facilitator asked a teacher to share his or her model, in which the facilitator moved the Do Math Bear along the number line according to the model. The facilitator revealed any ambiguity in the model by moving the bear in ways consistent with the model but different from the teacher's intentions. The facilitator then led a discussion of how to revise the model to address such shortcomings.

The facilitator captured the teachers' models on poster sheets, which the facilitator asked the teachers to compare and contrast. At the conclusion of the lesson, the teachers had created three models for adding and subtracting integers. For example, one of the teacher's models was the following:

> Start by placing the bear at the first number. If the second number is negative, face the negative direction. If the second number is positive, face the positive direction. When adding, walk forward the direction currently facing. When subtracting, walk forward the opposite direction currently facing.

The process of creating these models led the teachers to deeper understandings of operating on integers. The teachers recognised that their models needed to reflect the conceptual interpretations of addition as combining two quantities and subtraction as removing a quantity from some starting amount. Furthermore, the teachers realised that their models had to distinguish between the operation and the sign of the numbers. Finally, the teachers also gained a new appreciation for why adding a negative amount has the same result as subtracting the absolute value of that amount and why subtracting a negative amount has the same result as adding the absolute value of that amount.

Once the teachers had clarified and understood the three models, the facilitator asked the teachers to write a reflection “from the perspective of a learner. Think about the learning processes. What helped you as a learner? What helped you sort out the mathematics?” After the teachers wrote individually on this reflection, the facilitator led a class discussion. This reflection and the associated discussion were intended to encourage the teachers to consider how their experience might eventually affect their practice.

Data Collection and Analysis

The main data source consisted of the teachers’ written reflections completed during the second step of the three-pronged approach. The teachers completed 16 reflections throughout the course. The second data source included field notes so that the teachers’ reflections could be analysed within their respective occurrences in the professional development.

To analyse the data, grounded theory was used (Strauss & Corbin, 1998). First, the teachers’ reflections for each prompt were read and open coding was used to identify how the mathematical experiences contributed to the teachers’ learning and their ideas about teaching. For example, codes were used to identify comments made by the teachers with respect to collaboration, use of visuals, or modelling good instruction. Next, axial coding was used to identify how the codes were related to each other by considering aspects such as who wrote such comments, when did the comments arise, why were the comments expressed, and what consequences came about as a result. As an example, axial coding identified participants who mentioned collaboration, their feelings about collaboration, ways collaboration was found to be helpful, and consequent results. Two written memos, one for each research question, were then prepared to synthesise the teachers’ comments across all of the reflection prompts. One memo summarised the teachers’ comments about how they best learned mathematics in the professional development, and the other memo summarised the teachers’ comments about possible modifications of their teaching practice. Finally, selective coding, a process of integrating and refining theory, was used to explicate the impact of the mathematical learning experiences on the teachers’ conceptions of learning and teaching mathematics. In the results section, the number of teachers expressing each particular type of comment is provided, rather than the number of comments of that type. Providing the number of teachers imparts an indication of the frequency of the comment amongst the teachers.

Near the conclusion of the analysis process, it was also noted that some of the teachers’ reflections were more aligned than their other reflections with processes that engender learning mathematics with understanding (Hiebert et al., 1997; NCTM, 2000; 2007; National Research Council (NRC), 2000; Smith, 2000). As summarised by Hiebert et al., learning with understanding is characterised by seeing connections and relationships with other knowledge and is an ever changing and growing process. A secondary analysis was therefore conducted to classify the teachers’ reflections as strong or weak for alignment with learning mathematics with understanding.

For the secondary analysis, the teachers' comments were separated into two categories: *learner comments* when the teachers wrote about their own mathematical learning experiences within the professional development and teacher comments when they wrote about possible modifications to their practice. *Learner comments* were classified as weak if they expressed a view of mathematics as a collection of distinct rules and procedures (Stigler & Hiebert, 1999) or if they expressed a reliance on others or external authorities to make sense of the mathematics (Amit & Fried, 2005; Hofer & Pintrich, 1997). Learner comments were classified as strong if they expressed learning mathematics as a sense-making process (Hiebert et al., 1997), if they acknowledged affective factors of learning, or if by taking a metacognitive view they provided specific descriptions of teaching or learning processes that enabled their learning with understanding. Teacher comments were classified as weak if they expressed a transmission model of teaching with an emphasis on rote procedures (Stigler & Hiebert, 1999), if they failed to recognise the amount of time and effort it takes students to learn mathematics with understanding (NRC, 2000), or if they discussed removing the cognitive demand on students (Hiebert, et al., 2003; Stein, Grover, & Henningsen, 1996; Stein, Smith, Henningsen, & Silver, 2000). Teacher comments were classified as strong if they described pedagogical strategies that supported students' making sense of the material, (i.e., student-centred instruction) (Smith, 2000). While teaching for mathematical understanding includes numerous pedagogical approaches, some of the strategies regarded as strong in this study included the need for classroom discourse, varied mathematical representations, worthwhile mathematical tasks, attention to students' cognitive difficulties and affective experiences, and the time to explore sound mathematics (Hiebert et al., 1997; NCTM, 2007).

Findings

Impact of the Mathematical Learning Experiences

The teachers' responses from the 16 reflections were categorised into three aspects that supported their mathematical learning: processes and actions; instructional factors; and characteristics of the mathematical tasks. With regard to processes and actions, all of the teachers reported that collaborating with their peers on mathematical tasks supported their learning the most. As Sally commented, "this activity really got our group to work *together*—everyone helped each other see what we were doing, and this really helped me!" The teachers reported that working in groups was helpful because it enabled them to make sense of the mathematics, to hear different ways to think about a task, and to draw on their peers' motivation and enthusiasm. Another process mentioned by 14 of the teachers was the use of visuals such as drawings, physical manipulatives, and written representations. These visualisations helped the teachers represent the mathematics in explicit and meaningful ways. As Sally commented, "I learn better with pictures, diagrams, manipulatives, and any tools that help me 'see' what I'm learning. I also have to use these tools to understand the *WHY* and not just the *HOW* of math." The teachers reported three other processes that supported their learning: the test and revise nature of many of the mathematical activities (described by seven teachers); the process of writing their mathematical ideas (three teachers); and the overall instructional process (two teachers). By overall instructional process, the teachers were referring to working on an activity individually to get started, then working in groups, and concluding with a class discussion and facilitation by the instructor.

The teachers also described instructional factors that supported their learning. Seven teachers commented on how the instructors captured the teachers' mathematical ideas by writing them on the chalkboard or on poster sheets. The teachers reported that this was very helpful as it allowed them to see everyone's ideas, to examine the similarities and differences in those ideas, and to deal with the confusion of multiple ideas. In a similar fashion, seven other teachers commented on how helpful it was that the facilitator used the Do Math Bear to demonstrate their directions for adding and subtracting integers. Five of the teachers further reported that the instructors modelled "good" teaching and the process of asking "good" questions. As Cathy explained, "Active instruction has been excellent—the instructors actually model instruction they are teaching us—asking good questions, presenting information concisely, checking for understanding." Finally, some of the other facilitator actions that were mentioned by two or three of the teachers included providing written feedback on assignments, being well-prepared for class, using a variety of instructors and instructional styles, providing adequate time for the teachers to work on the mathematical tasks, and creating a safe environment for asking questions and sharing ideas.

With regard to characteristics of the mathematical tasks that supported their learning, six of the teachers commented on the challenging nature of the tasks. Six of the teachers also commented on how the tasks required them to look deeply at an elementary idea. As Lisa explained, "It is not like we were learning entirely new concepts, but instead taking a fairly simple mathematical idea (that I thought I knew inside and out) and dissecting it and gaining a much deeper understanding of the material." Five of the teachers commented that they appreciated tasks that made connections to real-life contexts or other cultures. Finally, three of the teachers commented that the tasks were at the right level, not too hard or too easy.

Intended Changes to Classroom Practice

The teachers' written comments about their teaching fell into three categories: comments about: using specific mathematical tasks; teaching mathematics in general; and feeling better prepared to teach mathematics. The facilitators often asked the teachers whether and how they might use in their classrooms modified versions of the mathematical activities that they completed in the professional development. Such questions elicited four types of comments from the teachers about using specific mathematical tasks. First, the most common response from all of the teachers tended to focus on the mathematical ideas addressed through the task. These comments usually identified the overall mathematical concept(s) or recognised the connections that could be made between various mathematical topics. Second, 12 of the 16 teachers commented on aspects of the tasks that their students might find cognitively challenging. For example, with regard to a task that required finding the percentage of the number of shaded squares in a grid with 40 squares, Lisa wrote, "Working with a grid that is not out of 100, such as a 4 x 10 rectangle, could be challenging for students because 40 does not evenly fit into 100." Third, 10 of the teachers considered what level of students the activity was suitable for (i.e., what grade level or what ability level) and how to adapt or extend the activity. Fourth, nine of the teachers often had specific ideas on how they would implement the task. For one task, Cathy wrote, "Students could follow same instructions as we did in class—look for patterns, mark two placements of one rectangle, colour groups of a rectangle, make connections with area."

With respect to general pedagogical strategies for teaching mathematics, 14 of the teachers mentioned that they were going to use and provide students with visuals, manipulatives, written representations, and models. Another common strategy mentioned by eight of the teachers was to slow down their instruction, revisit mathematical topics as needed, and provide multiple opportunities and examples to ensure that all of their students master the mathematical material. Jane, after experiencing the need to revisit a concept multiple times, wrote, "It made me realise that I don't go over 'things' or concepts enough times in different ways or models to help the students in my class who are stuck." Six of the teachers mentioned that they hoped to ask "better questions" in their classrooms, meaning questions that required the students to engage in higher level thinking about mathematical ideas. Other strategies mentioned by two or three of the teachers included having students work in groups, having students explore and discover mathematics themselves rather than telling the students how to complete mathematical tasks, asking students to write about their mathematical ideas, and facilitating more class discussions.

Finally, many of the teachers commented on feeling better prepared to teach mathematics. Nine of the teachers commented that due to learning struggles of their own, they would be able to relate to their students' struggles. Seven of the teachers felt that because they understood the mathematics better and from multiple perspectives, they would be better able to meet their students' varying needs, to see where students might get stuck, and to see their students' different perspectives. Three of the teachers gained confidence in their abilities to teach mathematics. Lauren wrote, "I feel like I am so much better able to teach a topic after I have been to one of these classes and really studied the topic to a deeper level than we teach. ... I know I'll be more confident in the content we covered."

Alignment with Teaching for Understanding

Of 278 comments by the teachers, 131 responses consisted of learner comments and 147 responses consisted of teacher comments. Each type of comment was classified as strong or weak for alignment with processes that engender learning mathematics with understanding. Table 1 provides information for the learner comments, while Table 2 provides information for the teacher comments. Each table includes the number and percentage of strong and weak comments along with two example quotes accompanied by rationales for their classifications as strong or weak.

Table 1
Learner comments: Frequency information, example quotes, and rationales

Alignment	**Number**	**Percentage**	**Example Quote**	**Rationale for Classification**
Weak	20	15%	I wasn't sure how to set up the groupings of the negative and positive numbers. Samantha showed our group how to set up a multiplication problem using the counters. Once I understood how to use her strategy—the process became clear to me.	Reliance on others to make sense of the mathematics.
Strong	111	85%	The writing of all the processes helped me to remember and think through ideas. If they are not written, they flee away. It made it easier to see how the concepts fit together and to take parts of one idea and combine it with another to make the concept more clear. ... It helped to treat confusion and then sort through to organize a conceptual fact.	Identifies a supportive learning process, describes the process in detail, and explains how the process is helpful.

Table 2
Teacher comments: Frequency information, example quotes, and rationales

Alignment	Number	Percentage	Example Quote	Rationale for Classification
Weak	49	33%	It is important to establish terms and concepts before working on the actual problem to be sure all students are on the same page.	Reduces the cognitive demand on students.
Strong	98	67%	I want to get to the point where I can lead better math discussions and stop myself from doing the solving! I like it when different ideas were presented [in the class discussions] and then we were sent back to our groups to discuss. I am thinking that my pedagogy plan might entail actively planning group discussions on topics into my lesson plan.	Emphasises the students exploring the mathematics and describes a specific teaching practice that allows students to do so.

From these tables, 85% of the learner comments revealed that the teachers were experiencing the mathematics as intended by the facilitators. The teachers explored the mathematics and made sense of it based on extensions of their own prior knowledge and through interactions with their peers. In addition, 67% of the teacher comments indicated that the teachers were considering changes to their practice that aligned with teaching for understanding.

In addition to noting the frequency of each type of comment, connections between the learner comments and the teacher comments were examined. It appears important for teachers to consider their own experiences while learning mathematics in order to recognise changes they may implement in their classrooms to enhance learning with understanding. For example, when the teachers were asked to reflect on both their mathematical learning experiences in the professional development and how those experiences may affect their future teaching, their teacher comments were classified as strong 80% of the time. In contrast, when the teachers were asked to reflect only on what they may take from a lesson in the professional development to inform their teaching, their teacher comments were classified as strong only 59% of the time. Consider the two teacher comments provided in Table 2. In the first comment, the teacher only comments on what she may incorporate into her classroom; this teacher did not offer any corresponding thoughts on her own mathematical learning experience. In the second comment, the teacher describes an aspect of the mathematical experience that was helpful for her learning. As a result, her teacher comment is more strongly aligned with teaching mathematics for understanding, and that of allowing students to discuss and explore the mathematical ideas that arise in small groups and in class discussions.

Discussion

One goal of this professional development course was for teachers to experience making sense of and exploring mathematics, so that they may subsequently incorporate similar aspects into their classrooms. When the teachers mentioned their future teaching plans, they included many of the student-centred aspects modelled during the professional development that they found to be supportive of their own mathematical learning. These results align with those of other content-based professional development programs (Campbell & White, 1997; Hill et al., 2005; Saxe et al., 2001; Schifter, 1998; Swafford et al., 1997). Granted, this study only examined what teachers mentioned they intended to incorporate in their teaching practice. A possible limitation is that teachers may have been expressing what they felt the professional developers hoped to hear. However, getting teachers to note student-centred processes that supported their learning and that they intend to incorporate in their own classrooms is an important first step. Furthermore, the contribution from this study is the evidence that reflecting on personal learning experiences may enhance the opportunity for teachers to value and incorporate teaching and learning mathematics with understanding into their classrooms. Specifically, this study shares two lessons learned about engaging teachers in substantive reflections about learning and teaching mathematics.

First, several content-based professional development programs place teachers in student-centred settings in hopes of modelling how teachers may incorporate such instructional strategies in their own classrooms. This study suggests that to enhance the chance that teachers will internalise and consider incorporating similar changes in their instruction, teachers should reflect on their own personal learning experiences within the professional development. When teachers do so, their thoughts about their teaching practice are more often aligned with teaching and learning for understanding. An explanation for this may relate to the influence of teachers' prior learning experiences. Without other intervening factors, teachers tend to teach as they were taught and through which they were successful in learning (Brown & Borko, 1992). These prior experiences often consist of teacher transmission models and an emphasis on rote procedures. To help teachers develop different conceptualizations of mathematics teaching, they were placed in a student-centred instructional setting where they learn mathematics with understanding. These experiences are then paired with deliberate reflection in order to provide teachers with a personal view of such instruction. It is this personal experience that is believed to enhance the likelihood that teachers will incorporate similar pedagogical strategies in their own practice.

A second discussion point emerged after comparing the teachers' learning reflections with their teaching reflections. Some aspects that the teachers mentioned as supportive of their learning were not described when the teachers considered their teaching. When the teachers were engaged in the mathematical learning experiences, they were likely focused on learning the mathematics, as was expected by the facilitators. Since their attention was devoted to learning mathematics, they were likely not able to also pay close attention to the instruction and pedagogy that was occurring during the lesson. Thus, after a mathematical discussion has been concluded and the facilitators ask the teachers to consider associated pedagogical aspects, teachers may need to be provided with tangible reminders of the lesson they just experienced. Tangible reminders could consist of brief transcripts, video clips, or a list of facilitator questions. Such an approach would parallel the efforts of practice-based professional development in which teachers' learning is centred on artefacts from their practice (Ball & Cohen, 1999; Kazemi & Franke, 2004; Scherer & Steinbring, 2006; Steele, 2005; Sykes & Bird, 1992). By examining practice-based artefacts, teachers' interest is often increased, the artefacts capture the many particulars of teaching and learning, and analysis can be more grounded in a realistic context and thereby be more critical. Examining artefacts from mathematical learning experiences during professional development may engender similar benefits as those found in practice-based professional development. Furthermore, it may allow teachers to recognise pedagogical subtleties of their own mathematical learning experiences that they may want to incorporate into their own teaching practice.

The research reported here provides evidence of the value of a professional development model that starts with the mathematics itself and then uses the teachers' experiences with the mathematics to prompt changes in their instructional practice. Future research is needed to examine how enhancing the reflection prompts as described above may further influence the teachers' instructional practices and to investigate whether and how such reflections lead to actual changes in the teachers' classrooms.

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