Why are MATHEMATICAL INVESTIGATIONS important?

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"Exploration is really the essence of the human spirit."
Frank Borman

"Research studies show that when students discover mathematical ideas and invent mathematical procedures, they have a stronger conceptual understanding of connections between mathematical ideas" (Grouws & Cebulla, 2000, p. 17).

Flewelling and Higginson (2005) state that inquiry, investigations and problem solving "give students the opportunity to use their imagination and to get into the habit of doing so. In contrast, traditional text-based tasks provide the student with little or no such opportunity" (Flewelling & Higginson, 2005, p. 19).

"Rich learning opportunities stem from students' engagement in challenging tasks… Classroom tasks can be broadly categorised into those that have the potential for learning and those that provide opportunities for students to practise applying their previously learnt knowledge or skills" (Diezmann, 2005, p. 2). Investigations are a good way of supplying the first need.

Much research points to the need for mathematical inquiry, investigation and problem solving in mathematics lessons. Thus, "Investigations are central to the reforms advocated internationally to improve mathematical learning and develop children's mathematical power" (Baroody & Coslick, 1998 cited in Diezmann, 2005, p. 4).

The investigative approach

The investigative approach is broken down into steps in Maths Charter Working Mathematically (n.d.) and Kanasa (n.d.) has illustrated different methods of investigating and problem solving in a visual way which will add meaning for many students. The ability to pose questions, collect data, investigate and discover answers, solve problems, describe, share and elab-

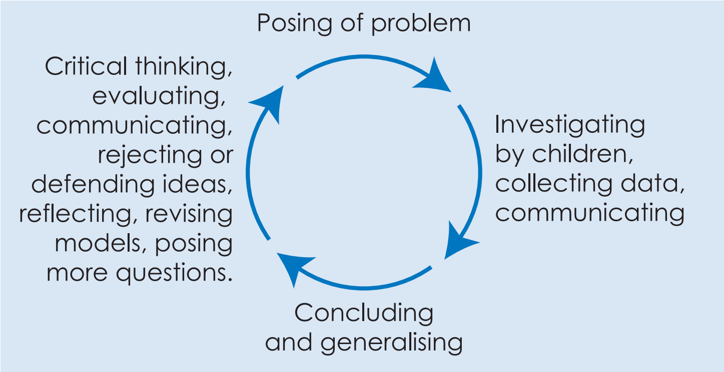


Figure 1. Diagram of the investigative approach.

orate on findings, evaluate, reflect on and judge the accuracy of answers, draw conclusions, revise and test models of investigating and reject or defend ideas are skills that should be stressed in the modern mathematics classroom. These processes need to take place in small groups and in the classroom as a whole.

Examples of mathematical investigations

There are many examples of mathematical investigations in literature. Children who are as young as 7 or 8 can be asked to explore the distribution of colours of smarties in small boxes of smarties (Diezmann, Watters & English, 2001). Later on in primary years children may be asked to investigate whether there is a relationship between the perimeter and area of shapes by investigating whether all pentominoes have the same areas and perimeters (See appendix which follows). Many students find the outcome of this investigation surprising. "It is interesting to note that in some ancient civilizations land was priced by counting the number of paces around the boundary. A shrewd operator in such an arrangement could make a good profit by buying square pieces of land and selling them off in long thin strips!" (Haylock, 2006, pp. 295-296). In later years children may be asked to make as many different cuboids out of twelve or twenty four 1cm^3 cubes as possible and write down the volume and surface area of each to see if there is a relationship (Haylock, 2006). Other examples include investigating the relationship between the circumference and the diameter of circles or investigating which shapes tessellate leading on to conversations about tiling patterns. The foundations of Pythagoras' theorem and Trigonometry can also be laid by getting students to investigate these topics in order to discover the underlying concepts. At higher levels, investigations can be used in Algebra when studying patterns. Lovitt and Lowe (1994), give many examples of Chance and Data Investigations that can be carried out in higher levels. NRich Specialists in Rich Mathematics (http://nrich.maths.org/public/) is a website developed by the University of Cambridge. It includes many examples of rich mathematics activities which can be adapted for use in investigations. These activities can be found by searching "mathematics investigations" in the given website.

Mathematics investigations and the proposed national Australian mathematics curriculum

Details of the proposed three strands for teaching mathematics in Australia can be found in the National mathematics curriculum: Framing paper (2008). Mathematical investigations would generally focus on one or more of the three content strands as well as developing elements of one or more of the four proficiency strands, understanding, fluency, problem solving, reasoning, which are necessary for 'working mathematically'.

Benefits of using investigations in mathematics classrooms.

Investigations utilise a wide range of mathematical concepts and skills, as well as reinforcing the 'language of mathematics'. They often include collecting, organising and analysing information and planning and organising steps to follow. All these skills help to integrate different areas of mathematics. Lovitt and Lowe (1994) suggest that many textbooks isolate and drill particular mathematical algorithms, and divide content into small sections. However, investigations often have a more holistic approach where many mathematical concepts and skills are treated concurrently and links are made to other topics and social contexts. Examples of this can be seen in the books by Lovitt and Lowe (1994).

Another benefit of the investigative process in mathematics is that it can be used to foster a relatively informal atmosphere in a mathematics classroom in which communication and debate are encouraged. It has been stated "mathematics classrooms were once envisaged as silent places" and communication between children was discouraged. "Now talking about the ideas that arise is encouraged and cooperation and communication are desired behaviours rather than forbidden practices" (Booker, Bond, Sparrow & Swan, 2004, p. 20). Investigations can introduce valuable teamwork and debate to a mathematics classroom which is useful as the children will feed off each others' ideas. Different values are attached to mathematics investigations being conducted individually, in small groups and in a class as a whole and teachers should make use of all of these options.

With the help of the teacher or their peers, students can overcome misunderstandings when doing investigations and both students and teachers may get to understand how differently others think and reason.

Investigations allow students to apply mathematical knowledge to solve problems while maintaining some ownership of the steps they take in the process. This 'child centred approach' in which students take an active role in their own learning is a very common element of modern mathematics curricula and is very different from the style in which mathematics was previously taught. Investigations can be used to foster curiosity and exploration in children. "Research has demonstrated that when students have the opportunities to develop their own solution methods, they are better able to apply mathematical knowledge in new problem situations" (Grouws & Cebulla, 2000, p. 19). During investigations students are required to do original thinking and not just follow a set of steps, thus hopefully leading to a deeper understanding of concepts.

Mathematical investigations can focus on real life problems or community issues such as safety, conservation, economics or medical problems. They have become popular with the introduction of computers and many interesting mathematical investigations can be done with the use of data accessed on the internet.

Investigations encourage mathematical inquiry as they are often rich and open-ended tasks which allow students to set their own ceiling in terms of how far they go to solve a problem. They can be used to provide mathematically gifted children with a challenging environment in which to work in order to prevent them from becoming demotivated and frustrated. More able students, in particular, seemingly need more time to learn new skills and less time to practise skills which they already know. Less able children require more scaffolding/guidance during investigations. Because of the open-ended nature of investigations the same investigations can often be used at a number of levels or easily adapted to suit different levels.

It has been stated that "Australia is currently producing inadequate numbers of students who have high level capability in the mathematical sciences" (Thomas 2000 cited in Diezmann, 2005, p. 1). It is hoped that mathematical inquiry inspired partly by mathematical investigation will help to nurture gifted mathematics students.

Culture changes needed in mathematics classrooms to allow successful use of mathematical investigations

There is frequently too much time spent in mathematics classrooms practising skills and too little on discovery-orientated activities. The modern inquiry/investigation approach to teaching mathematics calls for a change in the culture of a mathematics classroom. "There is a need to establish classroom norms in which students develop a commitment to tackling challenging tasks and teachers provide judicious support to students" (Diezmann, 2005, p. 3).

When using investigations students need to be taught to conceptualise and conduct investigations and need to be helped with their observation skills. Also, a non-critical culture where cooperation and communication are encouraged needs to be established in a mathematics classroom to encourage children to discuss their ideas freely. "Effective Mathematics teaching necessitates encouraging talk for learning" (Leach & Bowling, 2000, p. 26), where problems are discussed in order to clarify thinking.

"Learning through problem solving places the responsibility for the learning squarely on the student, and the responsibility for providing the environment and the challenges on the teacher" (Lovitt & Lowe, 1994, p. viii).

Diezmann et al. (2001) state that the main problems with young children learning by investigation are a lack of understanding of the problem, failure to link findings with the answer to the problem, difficulty in posing a problem, lack of knowledge to complete the investigation, failure to explain the unexpected, difficulty with or incorrect use of mathematical ideas, lack of ability with using investigative tools, and difficulty with communicating findings. If teachers are aware of possible problems, they are able to work towards minimising them.

Teachers need to provide very individualised support for students during investigations. However, scaffolding should be given with care as it reduces the value of the task and encourages the students to become too reliant on the teacher for all learning (Henningsen & Stein 1997 cited in Diezmann, et al. 2001).

Investigations need to be prepared carefully and they can be difficult or require new style of rubric to monitor and mark. Flewelling and Higginson (2005) suggest a rubric for identifying student levels or performance. Examples of such marking rubrics can be found in Downton, Knight, Clarke, and Lewis (2006). Teachers also need to be aware that in this modern age when students are asked to investigate a topic, their immediate thought is to look up answers often by a google search or Wikipedia search on the internet (as they would go about researching a topic in another subject). This is not what is required in a mathematical investigation where the students learn from discovering things for themselves.

Summary

The ”investigative approach is in direct contrast to the simple application of rehearsed algorithms” (Lovitt & Lowe, 1994, p. viii). Investigations require teachers to establish a new culture in the mathematics classroom and require a high level thinking and commitment from the teacher. Children need to be helped to see challenges not problems and teachers need to adapt to a different style of teaching.

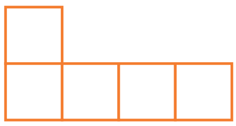
“Engaging young students with investigations requires that teachers reconsider their understanding of the nature of mathematics and how mathematics is learnt” (Diezmann, et al. 2001, p. 4).

Mathematical investigations can be a useful tool when teaching mathematics, including teaching mathematics to very young students (Diezmann, et al., 2001). They can be used to encourage curiosity, debate and communication and often address a range of outcomes and link different subject areas.

With the use of the investigation approach of teaching mathematics, students are provided “with the satisfaction of successfully completing a challenging task and being able to identify and investigate their own problems” (Diezmann, et al. 2001, p. 4).

Appendix: An investigation of perimeters and areas

Pentominoes are shapes consisting of 5 squares. See example below. Each square must share at least one edge with another square, not just a vertex.



1. Using 1 cm^2 grid paper, draw as many different pentominoes as possible. (If a pentanimo can be flipped or turned to give another pentomino, they are considered to be the same pentamino; e.g., the pentominoes below only count as one pentomino).

2. Now next to each pentomino write the perimeter and area of the pentomino. (Assume that the length of the side of a square is 1 cm; unfortunately when reproduced this measurement is often reduced or enlarged by copying).
3. Investigate: Do all pentominoes have the same perimeter?
4. Can it be assumed that all shapes with the same area have the same perimeter?
5. If one pentomino was left out in the above investigation, it would be easy to draw the incorrect conclusion. Which pentomino is this and what conclusion may be drawn if it was omitted? This is the reason why we need to be very careful when making generalisations in mathematics.
6. In some ancient civilizations land was priced by counting the number of paces around the boundary. Would this be a fair measure for pricing land and how could a clever businessman profit from this arrangement?

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From Helen Prochazka's

Scrapbook

The mathematician's artist

The mathematical discipline of tessellations has been evolving since the 17th century. The master of tessellations is undoubtedly the Dutch artist Maurits Escher (1898 – 1972), famous for his interlocking images of animals and intricate drawings that challenge our perception of space. He worked as an artist in relative obscurity until recognised by the scientific world.

Escher was first inspired by the artistic possibilities of mathematics after seeing the intricate Islamic mosaics at The Alhambra in Spain. In 1937 he began to create detailed and complex images saying that he 'felt irresistible joy' when making naturalistic shapes fit together in a structure.

Escher developed a system of categorizing combinations of shape, color and symmetrical properties. In 1958 Escher published Regular Division of the Plane, a paper in which he described the mathematical designs in his art. Escher also studied topology and learned additional concepts from the British mathematician Roger Penrose.

At the age of 68 Escher remarked that 'filling the plane has become a real mania to which I have become addicted and from which I sometimes find it hard to tear myself away.' He numbered his tessellations and produced 137 in his lifetime.