Modelling the growth of SWINE FLU

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The swine flu pandemic

The spread of swine flu has been a cause of great concern globally. With no vaccine developed as yet, [at time of writing in July 2009] and given the fact that modern-day humans can travel speedily across the world, there are fears that this disease may spread out of control. The worst-case scenario would be one of unfettered exponential growth. Medical scientists and politicians, however, are making every effort to make this exponential growth "change its mind" and come under control. The purpose of this article is not to lay claim to a definitive mathematical model for swine flu, but rather to illustrate some activities that will help students engage with this real world phenomenon in a spirit of curiosity and exploration. [Editor's note: Even with the development of a vaccine, it appears that its supplies are limited. Also, there is now some discussion about the possible over-reaction and vested interests in the proclamation of the pandemic in the first place: see abc.net.au/rn/rearvision, Wednesday 30 June 2010.]

A classroom simulation

The process of encouraging exponential growth to "change its mind" can be simulated in the classroom with the aid of some random numbers and some intrepid students. If we assume that there are 25 students in the class, then each student can be assigned a number from 1 to 25. Some form of random number generator could then be used to select the first student to contract the disease. On a graphics calculator this could be achieved using the command INT(RAND#)*25 +1 or on a device such as the ClassPad by the command RAND(1,25). The first student to be infected can then select a random number to infect someone else in the class. The next "day" these two students can then each select random numbers to infect another two students. The process continues and the whole school would be at risk if not for the courage of the teacher who dashes out the door and locks the students in. This noble act of heroism transforms an uncontrollable pandemic into an epidemic: it persuades exponential growth to "change its mind”. As each “day” passes, the number of students who are infected may increase but the chances of a student being selected who has already been infected also increases. The total number of infected students then levels off to a limiting value (the number of students in the class).

Day	1	2	3	4	5	6	7	8	9	10
Total no. infected	1	2	4	7	11	17	22	23	24	25

Figure 1. The simulation data.

Typical results from such a simulation could be tabulated as follows: A scatterplot of these results is shown in Figure 2.

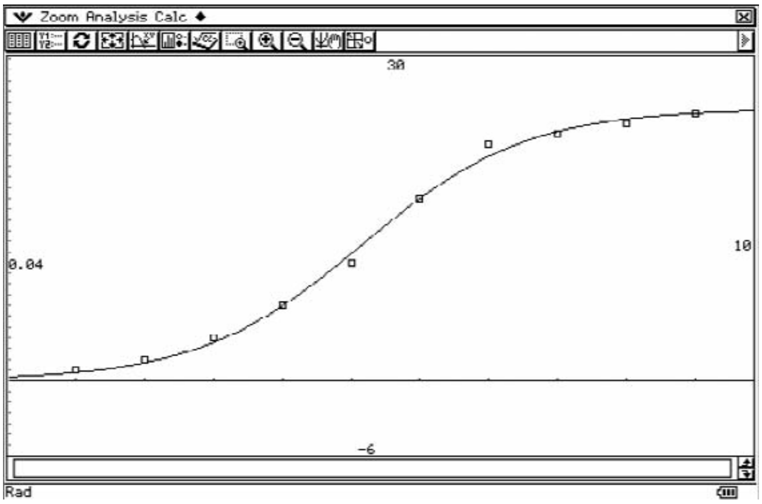

Figure 2. Scatterplot of the simulation data.

Applying the logistic model

The logistic model has been used to describe many phenomena, for example, the growth of a population where constraints such as food supply curtail growth. The general form of the logistic model is

$$y = \frac{C}{1 + ae^{-bx}}$$

Using the regression calculating power of the ClassPad, a logistic model for the spread of the disease through the class is given by

$$y = \frac{25.4}{1 + 75.57e^{-0.838x}}$$

where x is the number of days and y is the total number of people infected.

If we are satisfied that this is a reasonable match for our classroom simulation, can we dare to use our knowledge meaningfully and attempt to model the spread of swine flu throughout the world? The growth in the total number of people throughout the world who have been infected with swine flu since 23 April is illustrated in Figure 3. Is there some hope that the efforts of the world’s scientists and politicians have had some effect? Could the data fit the logistic model?

Using the calculator to find the logistic model for swine flu based on this data, produces

$$y = \frac{195315}{1 + 123.77e^{-0.635x}}$$

where x is the number of days since 23 April and y is the total number of people infected. This would suggest that the total number of people infected would not rise above two hundred thousand world-wide. Whilst this might be far fewer than feared, we must remember that there are assumptions and limitations to any mathematical model. The onset of the flu season, for example, has not been considered. There are lessons from history too. It is sobering to note that the influenza pandemic of 1918 appeared to abate before returning ever more vigorously.

Proceeding with caution, therefore, if we are to assume that the growth rate of the spread of swine flu has peaked, when did this occur? When might we have persuaded the exponential growth of swine flu to “change its mind”? With reference to the graph of the logistic model, this would be the point where the graph changes shape, otherwise known as the point of inflection (no pun intended). At the point of inflection, the growth rate, represented by the first derivative of the logistic model is at a maximum. Hence, at the point of inflection, the second derivative will equal zero.

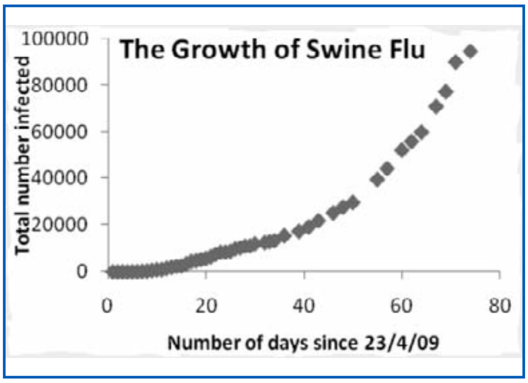

Figure 3. Growth of swine flu (source: WHO archives).

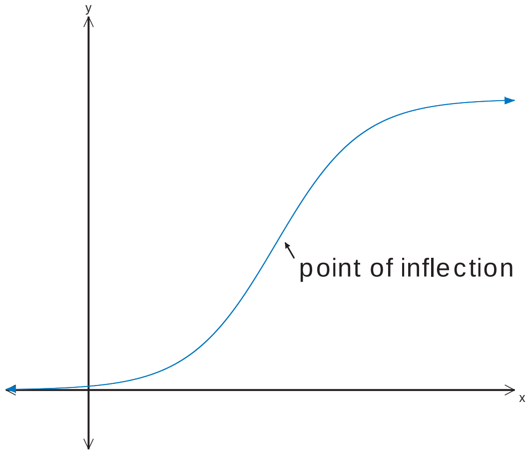

Figure 4. The point of inflection.

A general result using computer algebra

If we take an algebraic approach to finding the point of inflection, we could begin with the general logistic model. Using the computer algebra capabilities of the ClassPad, we could enter the general model

$$y = \frac{C}{1 + ae^{-bx}}$$

into the ClassPad as shown in Figure 5.

Using the ClassPad to find the first derivative we obtain the result shown in Figure 6.

Similarly, the second derivative can be found (see Figure 7; we have to scroll across to see all of it).

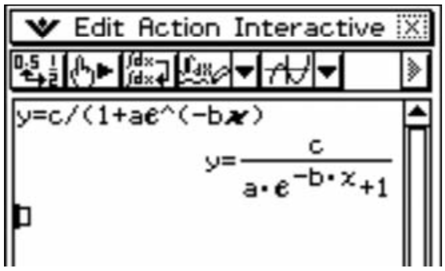

Figure 5

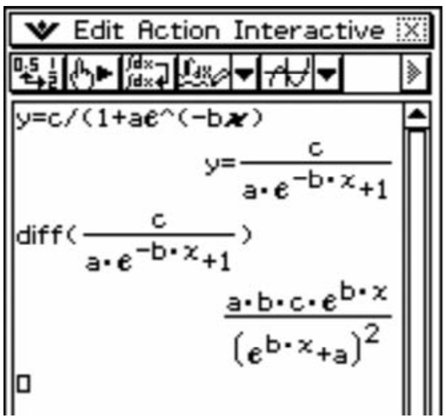

Figure 6

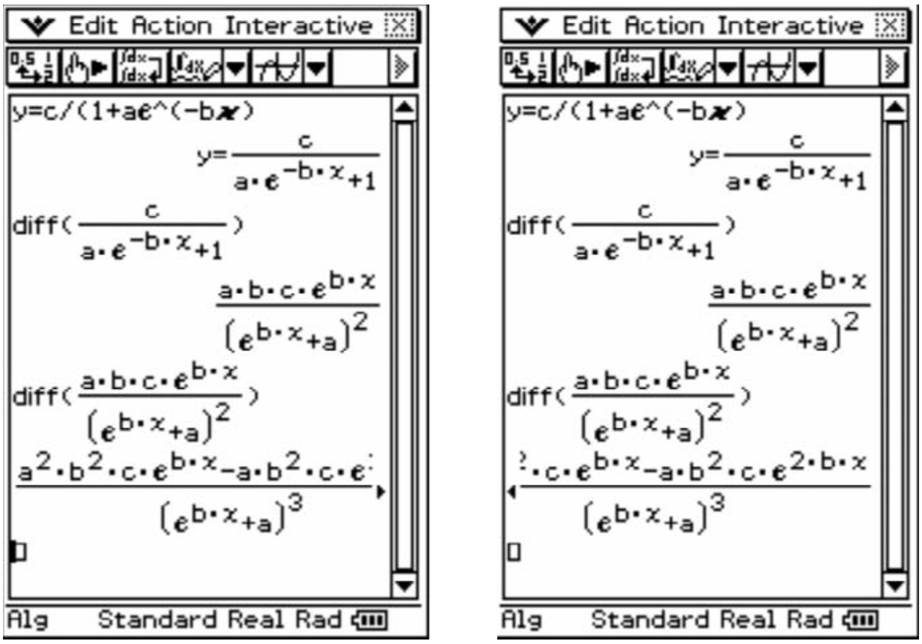

Figure 7

Now all we have to do is solve the equation

$$\frac{a^2b^2c\ e^{bx} - ab^2c\ e^{2bx}}{\left(e^{bx}+a\right)^3} = 0$$

Using the ClassPad we enter:

$$\text{solve}\left(\frac{a^2b^2ce^{bx}-ab^2ce^{2bx}}{(e^{bx}+a)^3}\right)=0$$

Note: To save typing, most of this can be dragged in from the previous line using the stylus.

We then obtain the elegantly simple result,

$$x = \frac{\ln a}{b}$$

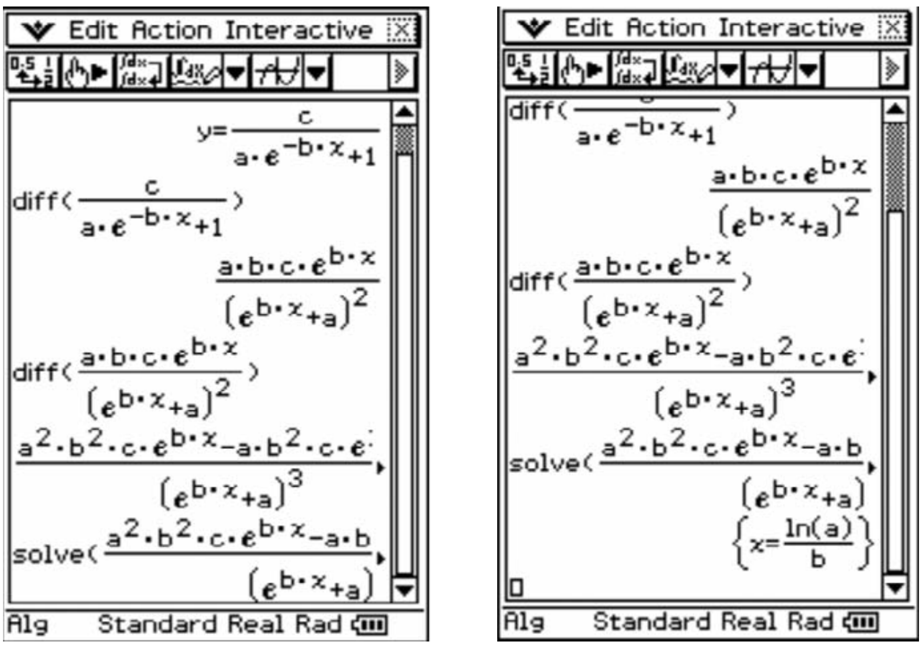

Figure 8

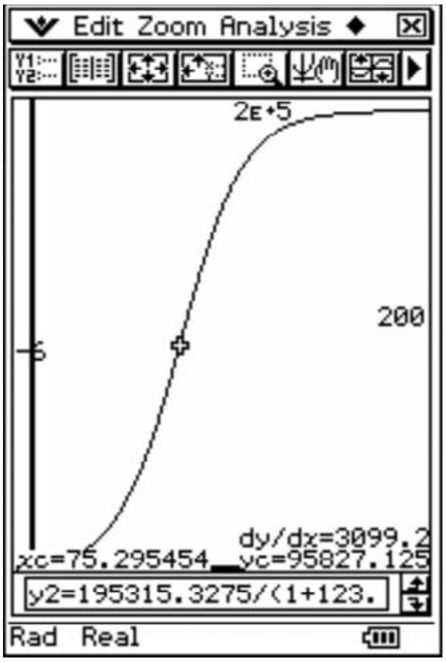

Figure 9

Applying this result to the swine flu data gives,

$$x = \frac{\ln(123.77)}{0.0635} = 75.9$$

To check this "black box" result we could perform the algebra by hand. Alternatively, we could check the result graphically using the graph of our swine flu model. As we trace along the graph, we can see that the growth rate, represented by dy/dx, reaches a maximum when x is approximately equal to 75.3.

All of this suggests that the growth of swine flu peaked around the 76th day after 23 April, i.e., 8 July. At best, this is an optimistic result, given the aforementioned assumptions and limitations. It has been an interesting mathematical exercise, however, and in the process we may have gained some insight into the nature of the spread of disease—but if anyone could find exact models for the growth of diseases such as swine flu and avian flu then, let's face it, pigs might fly!

References

A Context for Calculus: http://math.smith.edu/Local/cicchap1/node1.html

Casio Education Australia: http://www.casio.edu.shriro.com.au/

Texas Instruments Activity Exchange: The Spread of Disease: http://education.ti.com/educationportal/activityexchange/activity_detail.do?cid=us&activityid=4245

World Health Organisation: http://www.who.int/csr/disease/swineflu/en/

From Helen Prochazka's

Scrapbook

When some natural fact strikes us as strange, it means that we are looking at it from the wrong point of view. If we find the universe mysterious, it is because we have some idea about what the universe might be like, and are then surprised to find it is something different. The fault lies with our original idea, not with the universe.

W. W. Sawyer