What's average?

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This paper reports on the learning of a group of teachers engaged in an action research project to develop critical numeracy classrooms. The teachers initially explored how contexts in the media could be used as bases for activities to encourage student discernment and critical thinking about the appropriate use of the underlying mathematical concepts. As a result they found it is better to embed critical thinking in the design of whole units of study rather than just to treat it as an add-on to the mathematics curriculum. The following suggests a possible unit of study around the topic of average that aims to build students' capacities for critical thinking.

Introduction

The concept of *average* presents several dilemmas for the middle school classroom (see Figure 1). At least three perspectives on the term average are likely to intersect. One relates to the informal understandings that students bring with them based on their social experiences. Another is found in the media reports describing the events taking place in society that students and their teachers encounter every day. Finally, the term *average* encompasses the three technical terms in the mathematics curriculum: *mean*, *median*, and *mode*. That these three perspectives are unlikely to coincide exactly, presents a challenge to teachers wishing to help students develop the ability to think critically and make decisions about averages they meet through the curriculum, and in their daily lives. The challenge includes choosing meaningful and motivating contexts that engage students and involve them in investigating and using the three concepts of average.

Student perceptions of average

Ask a group of middle school students what it means to "be average" and the range of responses shown in Figure 2 is likely (Watson & Moritz, 1999, 2000). It is uncommon for students to respond as a mathematics teacher

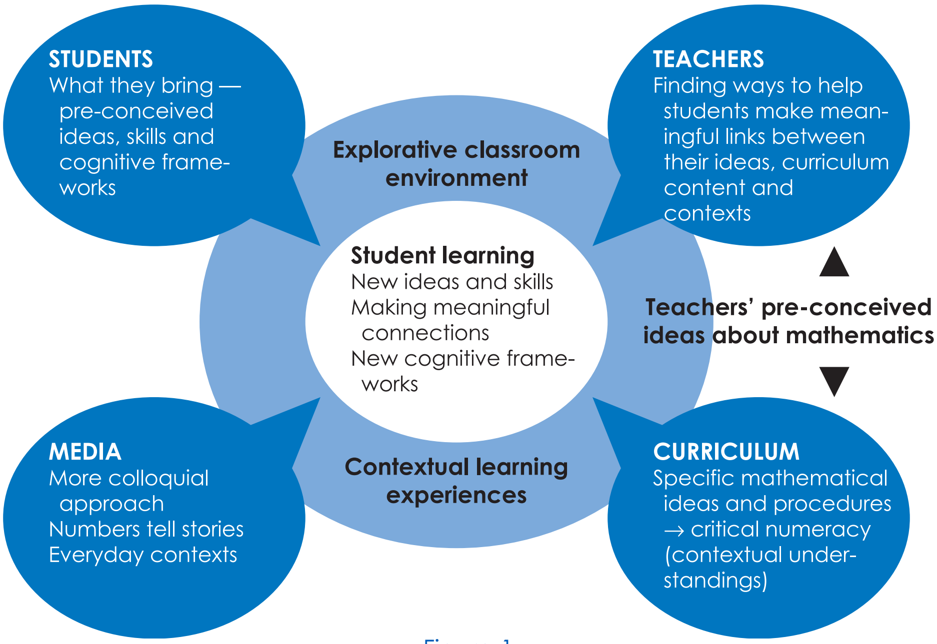


Figure 1

"That you are okay."
"That you are normal."
"That you are the same as everyone else."
"That you are not really good and not really bad but in between."
"That you are the same as most people."
"It would mean you weren't really bad at spelling but you weren't really good at it either. You were half and half."

Figure 2. Student notions of average.

Average Australian job hunter is a married man with kids [Headline].

An average of one in four meals, including snacks, is eaten away from home.

The survey found cats caught 4.67 prey per year.

Each person is now worth an average of $94,100.

The average wage-earner finally can afford to buy the average home after almost two years of mortgage pain.

The median age [in Australia] is now 34.

The average shopper is likely to be a woman aged about 40, employed, married or living with a partner, and likely to have no children living at home, demanding more organically grown fruit and vegetables.

Recent surveys showed that the 60,000 cars that cross the bridge each day carried an average of only 1.2 people.

(extracts from Numeracy in the News: www.mercurynie.com.au)

Figure 3. Media notions of average.

might expect. As a result of rote learning of a formula, the following response is not unusual:

> Find out if you added all the people's scores together, what is the score that was … That's not necessarily the most common is it? … You add the number of people up and you divide it by the score that they got and that gives you the average score.

The responses listed in Figure 2 illustrate the wide range of intuitive ideas that students carry with them about the idea of average. Students use the term in different contexts, and the same student may use several variations of meaning depending on the context. Often these reflect expressions they have heard or read in the media.

Media use of average

The media also present a wide range of meanings for the term average, sometimes in a manner that makes clear what is represented and sometimes not, as shown in Figure 3.

In various ways these statements *represent* the sets that have stories to tell: stories about eating-out, job-hunting, cats' prey, wage earners, age, shoppers, and car occupancy. Although not telling all of the story, they may be good starting points, assuming the reader has the intuition to interpret their meaning appropriately.

Mathematics curriculum

Within the mathematics curriculum, "average" is the general term used to describe the three specific measures of centre: mean, median, and mode. When asked what average means, most mathematics teachers give the definition of the arithmetic mean, based on adding together a group of scores and dividing by the number of scores in the group.

Historically the "three averages," along with graphs such as the column graph, were the first statistical concepts included in the mathematics curriculum. Hence, the averages hold a firm place in the procedural learning of middle school students. If, for no other reason, having the arithmetic mean in the curriculum provides students with abundant practice in addition and division. In the past 20 years, however, an increased emphasis on "chance and data" in the mathematics curriculum has broadened the scope of ideas related to the concept of average.

Average only makes sense in relation to a data set that exhibits variation. Variation is in fact the heart of the Chance and Data curriculum and is displayed in the graphs of data sets that students learn to produce. The *arithmetic mean* is the balance point of the graph if frequencies of data values are plotted on a horizontal axis—balancing out the variation in the data. The *median* is the middle value in the data once they have been ordered from least to greatest value. The *mode* is the most commonly occurring value in the data set; i.e., it would be the highest column in a column graph based on frequencies of individual values or categories.

The links between colloquial and mathematical meanings of average are quite close in that both connotations represent a kind of expectation. How was the TV show? "It was just average," meaning "not so good" in comparison to other TV shows. How did you do on the test? "Just average," meaning "not as good as some others" or "about in the middle of the class". The trick is to make the terms explicit in meaningful ways that will assist students to extend their understanding and make more powerful use of the overall concept.

A new role for teachers?

Teachers face the task of linking the various meanings for average in ways that will facilitate cross-curriculum applications where concepts of centrality and typicality are required to make sense of data in the social, physical, and health sciences. In the past, mathematics teachers may have been satisfied if students could add a set of numerical values and divide by the number of values to find the mean, or if they could order a set of values and identify the middle one (or halfway between the middle two) to find the median. The mode, being the most frequently occurring data value or group was often merely an add-on because it did not require any "calculations" and was often associated with categorical data (such as eye colour). In today's curriculum, however, demands for *numeracy across the curriculum* "to meet the demands of learning, school, home, work, community and civic life" (National Curriculum Board, 2009, p. 5) require these three "averages" to link meaningfully to the content of other curriculum areas, and provide students ways of interpreting statements and claims.

There is now a move from the narrow focus on averages as procedural algorithms that can be tested on simple invented data sets, to seeing averages in the context of the variation present in data sets from which they are derived, and as being only one facet of the story being told by the data set.

This understanding is what adds value to the concept of average in helping to interpret claims that are made in the media, or in helping students make their own claims in reports written in other subject areas. It is not clear, however, that teachers have had opportunities to incorporate these recent transitions in the school curriculum into their practice.

As participants in a single-term action research project, a group of teachers had the opportunity to develop lessons based on these ideas, and to reflect on the experience.

Developing critical thinking using contexts

A key aspect of the action research project was using the media as a source of interesting contexts for students to develop critical numeracy skills. Initially, teachers explored together different media articles, applying critical thinking processes and looking at the potential for activities with their classes. Several teachers used the same articles as starting points for their classes but designed different worksheets and activities, with varying results. In some cases students were more engaged than others, some student groups showed evidence of more critical thinking than others, and some activities showed more potential than others for further exploration.

What was most useful to all teachers involved in the project was the opportunity to explore the reasons behind these differences, helping each other to understand more deeply the issues of asking students to think critically about numeracy and the role of the teacher in mediating this.

One of the key realisations was that rather than occurring in one-off lessons, criticality needs to be developed over time. It is a higher-order skill and students need a good grasp of mathematical terminology, as well as the particular context and its contribution, before they can ask more critical questions about the relationship between the two. Often activities did not succeed because the students did not have time to engage with the terminology or the data – they need time to play with data.

Further, the teachers themselves found that they were not used to asking critical questions such as: "How do I know this is true?" "How might this be positioning me?" "How might this report be used by others?" They found it was important to practise and model such critical thinking with the class, facilitating classroom discussion where students could share alternative perspectives and ideas. Figure 4 captures other teacher insights.

As the project progressed and the teachers developed a better understanding of what critical thinking sounded like, the group developed a model for critical numeracy (see Figure 5) based on the Luke and Freebody (1999) *Four Resource Model*

"We need lots of practice in doing critical thinking ourselves."

"It is important to have a sense of likely student misconceptions from the research - it gives me a sense that my students are normal. But I also need to find out exactly what they are thinking about - help them to articulate their 'alternate' views and explore the merits of them."

"Students may see information coming from particular sources such as a newspaper as authoritative knowledge so it is important to help students see that it is okay to question such knowledge."

"I have come to realise I have been quite a procedural maths teacher - students follow the rules of maths to complete maths worksheets individually. I found I had to adopt a new style of teaching - allowing for discussions or more group work so students could develop conceptual and contextual understandings."

"How can we develop an eye to see the potential of contexts in building critical thinking? What are contexts that we are comfortable with and which we think the students will relate to? How can we use creative activities which enable students to demonstrate their critical thinking?"

Figure 4. Key learnings by teachers.

for Critical Literacy. The aim of such a model is to make the process more transparent for both teachers and students. When new to a context or mathematical idea students need to spend more time *de-coding* and *meaning-making*. However, when the context and mathematical concepts are more familiar, students can move around the four quadrants of the model with greater ease; students can recruit the context to interrogate the appropriateness of the mathematical concepts, or use the mathematical concepts to unpack the implications of the context. Making the different thinking processes visible to students via such a model enables them eventually to bring discernment to their own reading of texts, rather than relying on teacher-scaffolding through worksheets.

"Critical numeracy" classrooms for some of the teachers involved in the project were very different to the ones they had established through teaching mathematics procedurally. Moving to "critical numeracy" classrooms opened up rich dialogue and enabled students to make meaningful connections between contexts of interest and the mathematics, leading to unexpected insights as well as opportunities for creativity. The teachers were often very excited with student thinking and engagement, and felt critical thinking had enhanced their classroom practice.

So, rather than seeing "critical thinking" activities

Critical Numeracy	
De-coding • What are the different ways numbers are used and represented? • What is the terminology being used and what does it mean? • What are the key mathematical concepts? • What are the key mathematical processes and procedures?	**Meaning-Making** • What is the text about? • How does it relate to what I already know? • How can I use what I already know to help me explore further? • How do the mathematical concepts make sense in this context? • How do the mathematical concepts help me understand this context? • What is confusing or misleading? • Are there other possible meanings?
Critical Questioning	
Using • In what ways are the numbers or mathematical concepts in this context significant or useful? • What is the purpose of the text and how does it connect into a bigger picture? • How might this text be used to promote different viewpoints? • What are possible applications and likely impacts? • How would I use this text and what decisions would I make based on it? • In what ways am I now thinking about the issues and the mathematical concepts differently?	**Analysing** • Is it true? — Are the mathematical concepts used appropriately in this text? What is the evidence? Is it based on reasonable assumptions? Is it logical and consistent? Is it researched appropriately? Does it have a reputable source? What do I need to know to be convinced that it is plausible? • Is it fair? — Does it include different views, values, perspectives or types of research? What is missing? Who might be silenced? Where do I look for alternatives? • How does it position me? — What do I think the authors' intentions, values or biases are? What do they want me to believe? How do they use the mathematical concepts or terminology to position me? • Do I believe it?

Figure 5. Critical numeracy model.

as an add-on to the course the teachers were now asking: “How can we use our insights about what it means to develop critical numeracy to inform the way we design mathematical units of study?” “How might we rethink a topic of maths so it becomes a vehicle not just for developing mathematical ideas but also for critical numeracy?” “How can we recruit contexts in meaningful ways?”

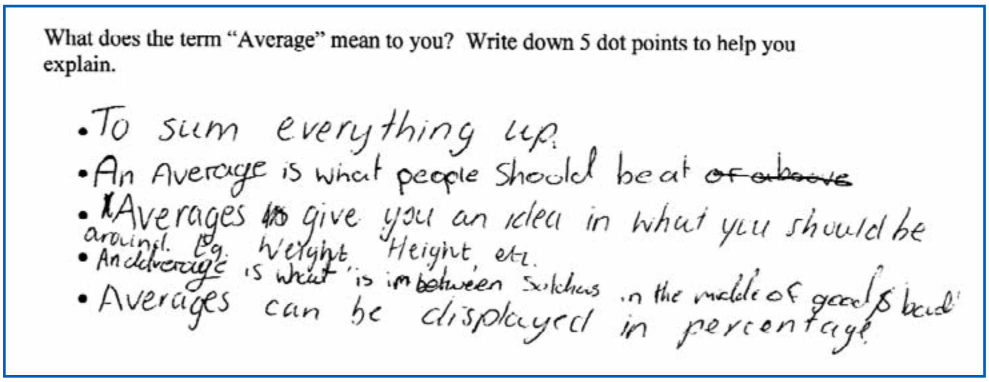
Designing a unit of study to develop mathematical critical thinking — “What does it mean to be average?”

The proposed design of this unit on average pulls together some of the disparate experiences and learning of the teachers involved in the action research to create a coherent whole. Teachers explain how they ran the activities and why, and what they found.

1. Guiding question
“What is average?”

2. Finding out what students know
What are some of the ways that students think about averages both mathematically and as part of everyday language?

> I asked students to list 5 things they knew about average in pairs. Some students could only think of one or two, while other students came up with 5 answers. I was quite surprised about the range of meanings. However now having seen the research on student preconceptions I think my students’ responses were fairly typical.


What does the term “Average” mean to you? Write down 5 dot points to help you explain.

- To sum everything up.
- An Average is what people should be at ~~or above~~
- Averages give you an idea in what you should be around. Eg. Weight, Height, etc.
- An average is what is in between [illegible] in the middle of good & bad
- Averages can be displayed in percentage

Figure 6

3. Introducing terminology (de-coding)
Following the exploration of students’ intuitions it is important to provide definitions of different averages—mean, mode and median. This includes exploring the advantages and disadvantages of using each one.

> After introducing the three definitions of mean, mode and median with students I conducted this activity. Students were each assigned an annual income, ranging from single parent pensioner to billionaire. Through discussion we tried to create a “realistic” cross-section of society, so there was one billionaire, several executives and successful business people, a range of middle income workers and a variety of low income groups.
> The students lined up from poorest to richest and found most of the money was held by a relatively small group at the “wealthy end”. We discussed who

would best be described as an "average" person, the students locating it around the middle of the line. However when the mean income was calculated it was found to be much closer to the "wealthy end" of the line.

The students were able to explain why the actual mean was located away from their predicted location, and concluded that "mean" was not the best descriptor in this case. Some felt "mode" would better describe the "average" person's income, while others preferred to use "median". On investigation, the mode income in this example was found to be slightly higher than the median.

This led to a discussion of whether this was a "fair test", and how data could be collected to make it so. The conclusion was that actual figures for a much larger sample would be needed. They were able to suggest several methods of collection including surveys, and accessing ABS figures.

I found this a very useful exercise as it was an example to which students could relate, and which allowed them to visualise and apply the concepts of summary statistics.

MR AVERAGE The average Tasmanian male	MS AVERAGE The average Tasmanian Female
• 33 years of age • Married • Has at least 1 dependant child • Has a weekly income of $352 • Works 40 hours per week • Has a monthly mortgage of $581 • Does 4 hours, 19 minutes of housework per week • Has 22 hours, 38 minutes of passive leisure activities • Is more likely to attend a sporting event than the cinema, museum or art gallery.	• 34 years of age Married • Has at least 1 dependant child • Has a weekly income of $199 • Works 32 hours per week • Has a monthly mortgage of $581 • Does 17 hours, 9 minutes of housework per week • Has 21 hours, 42 minutes of passive leisure activities • Her favourite participatory sport is aerobics.

Figure 7. "Meet the averages" (Stevenson, 1998).

4. Meet the averages (de-coding and meaning-making)

Using the article "Meet the averages" from *The Mercury* newspaper (Stevenson, 1998) the following questions can be asked of students: Which of these statements do you think comes from mean, mode or median? Why do you think they might be used? How useful are these data? What are you thinking about as you read it? Who do you think might use this?

This approach helped students to develop some criteria for working out which average is being used. The ones with explicit numbers were more likely to be mean, and those that were categories were more likely to be mode. We had problems though wondering whether median was used instead of mean.

5. What's average in my class? (meaning-making and using)

After the initial de-coding and meaning-making discussion, students can now consider the following questions in the context of their own experience. Following on from examining the news article on "Mr and Ms Average" what questions would you ask to find out what is average in your class? Collect data to construct a picture describing who is the average student in your class and write a newspaper article.

I found this activity generated very useful discussion on what makes a good question. For example some students were interested in

Figure 8. What's average in my class?

finding out what cars students had in their families and wanted other students to give the make and model of the cars. We soon realised that this was far too much detail and we needed to categorise it more broadly into just the manufacturer.

We ended up putting our possible questions on the board and critiquing them before students decided on their own questions. The whole class discussion was an important aspect of this exercise. We then used the board to collect all the data from the class so all students could use it. They then had to decide in pairs on whether to analyse the data using mean, mode or median. Based on that, each pair drew their own average classmate and wrote a newspaper article based on what the average student was.

One group found the mean shoe size to be 7.86 and thought this was very funny. They used this number in their article to be deliberately ironical, showing, I felt, a connected sense of what the numbers actually mean.

As an extension for some of my brighter students I asked them could they choose whether to use the mean, mode or median in such a way to make the average class student to be more like them? They took up the challenge, playing around with the numbers. I have begun to recognise how important it is for students to have time to be playful with the numbers and really get inside them.

6. Encouraging critical questioning (using and analysing)

Once students have experienced using the different forms of average they can bring a more critical lens to their use in the media. The following possibilities illustrate various approaches.

- *Whole class discussion*

  The following questions could be starters for a discussion.
  - How useful is it to work out the average of a group of people?
  - Is it useful to have an average as 7.86 foot-size?
  - Who might use these summaries? For example, could the canteen use the information on the favourite food for the class?
  - What is missing that would make the information more useful (e.g., graphs showing variation and more details)?
  - How might the questions about an average person change over time (e.g., what questions might be asked in 1960, 2010 and 2060?)?
  - What are the issues in comparing yourself with an average?
- *Pair analysis*

  If pairs of students in the class have written articles about the "average classmate" these can be shared. Taking an article written by another pair critiques can be written asking: "What is misleading, missing, useful?" "How are the writers trying to position us?" These can be presented to the class for further discussion.
- *Meta-cognition*

  In a debriefing session, rethinking the processes undertaken provides useful meta-cognitive reinforcement. In exploring the notion of average what questions did we ask that helped us to go deeper and deeper? Can we come up with a list of questions for a poster based on the four resource model headings of Decoding, Meaning-making, Using, and Analysing? Which of these are we going to use as we investigate other survey findings?

It is really easy to think you are doing critical thinking because you are having an engaging discussion with your students. It is important to recognise that discussion can centre around "meaning-making" and not get as far as "analysing" or "using". As teachers we have to be deliberate in the questions we ask to encourage students to go deeper.

The temptation is to create scaffolded worksheets or activities for our students that we think will build up their skills; but without encouraging meta-cognition they may not have processes that they can transfer to new contexts. The biggest difficulty I encountered was being able to unpack my own thinking and realising what critical questions I found useful.

7. Independent practice

Students can then use their four resource critical numeracy model poster as a lens in thinking about different news articles featuring average or other mathematical concepts. What do they see, think, wonder? What insights are they making about the issues with surveys? (A new Critical Numeracy website with lessons based around "Numeracy in the News" can be found at www.mercurynie.com.au).

8. Authentic survey/study

Using an issue current to the school, students can now work out how to get information that will help with decision making. Teachers will need to consider ways in which such an activity might be integrated with another curriculum area, and possible additional skills students might need. (Examples include sampling, survey question design, graphing and graphical analysis techniques.) Students could then present findings to the school's decision-making body, with a section on limitations of the study, and invite feedback.

Conclusion

In order to develop critical numeracy classrooms teachers need to teach critical thinking strategies explicitly. The example discussed in this paper suggests how these skills can be developed, gradually building on students' increasing familiarity with the contexts and the mathematical concepts. The teachers in this action research project realised that not every lesson or unit of work could be structured around critical numeracy but there were more possibilities than they first thought. The rewards were also worth the effort.

References

Luke, A. & Freebody, P. (1999). A map of possible practices: Further notes on the four resources model. *Practically Primary, 4*(2), 5–8.

National Curriculum Board. (2009). *Shape of the Australian Curriculum: Mathematics.* Barton, ACT: Commonwealth of Australia.

Stevenson, S. (1998, August 16). Meet the Averages. The Mercury (Hobart, Tasmania), p. 7. Available from Numeracy in the News website (Data Reduction: Meet the averages) http://www.mercurynie.com.au/mathguys/mercury.htm. Retrieved 04/05/2009

Watson, J. M. & Moritz, J. B. (1999). The development of concepts of average. *Focus on Learning Problems in Mathematics, 21*(4), 15–39.

Watson, J.M., & Moritz, J.B. (2000). The longitudinal development of understanding of average. *Mathematical Thinking and Learning, 2*(1&2), 11–50.