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A DECISION MODEL FOR STEADY-STATE CHOICE IN CONCURRENT CHAINS

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Grace and McLean (2006) proposed a decision model for acquisition of choice in concurrent chains which assumes that after reinforcement in a terminal link, subjects make a discrimination whether the preceding reinforcer delay was short or long relative to a criterion. Their model was subsequently extended by Christensen and Grace (2008, 2009a, 2009b) to include effects of initial- and terminal-link duration on choice. We show that an expression for steady-state responding can be derived from the decision model, which enables a model for choice that provides an account of archival data that is equal or superior to the contextual choice model (Grace, 1994) and hyperbolic value-added model (Mazur, 2001) in terms of goodness of fit, parsimony, and parameter invariance. The success of the steady-state decision model validates the strategy of understanding acquisition phenomena as a bridge toward explaining choice at the molar level.

Key words: choice, concurrent chains, acquisition, conditioned reinforcement, decision model, contextual choice model, hyperbolic value-added model, key peck, pigeons

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The concurrent-chains procedure is commonly used to study choice between reinforcement outcomes signaled by distinctive stimuli. In a typical version of this procedure, pigeons peck at two lighted response keys during the choice phase or initial links. Concurrent variable-interval (VI) schedules operate during the initial links, and provide access to one of two mutually exclusive terminal-link schedules, which are usually signaled by distinctive stimuli. Responding during the terminal links produces access to food, after which the initial links are reinstated.

Most prior research has used steady-state designs in which the same pair of terminal-link schedules is maintained until responding has stabilized. The terminal-link schedules are varied across conditions. Response allocation during the initial links is interpreted as a measure of the relative value of the terminal-link stimuli as conditioned reinforcers, and the primary challenge has been to describe how response allocation depends on the initial and terminal-link schedules. Various models for concurrent chains have been proposed, including delay reduction theory (DRT; Fantino, 1969), the contextual choice model (CCM; Grace, 1994) and the hyperbolic value-added model (Mazur, 2001), and have been shown to account for a substantial percentage of variance in response allocation.

Although differing in specific details, these models are related to the matching law and share the assumption that choice in the initial links depends on the relative value of the terminal-link stimuli. However, there are reasons to question whether this assumption—which Grace (2002) termed the *value hypothesis*— can be sustained. For example, Grace and Nevin (1999) used a procedure in which the terminal links included no-food trials similar to the peak procedure (Roberts, 1981) so that temporal control of responding in the terminal links could be studied. In their study, pigeons were trained with fixed-interval (FI) 40-s and 20-s terminal links. After 25 sessions, initial-link response allocation strongly favored the alternative leading to the FI 20 s, and the location of peak responding on no-food trials was approximately equal to the schedule

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duration. Next, the pigeons received 25 sessions of training in which only the terminal-link stimuli were presented and the FI 40 s was changed to FI 10 s. The initial links were replaced by an intertrial interval during which the keys were dark, so that technically the procedure was a multiple peak procedure. The location of peak responding adapted rapidly to the new schedule values, and was maintained over the course of the 25 sessions. The pigeons were then returned to concurrent chains. The key result was that although responding on no-food trials showed that pigeons continued to time the 20-s and 10-s delays accurately, initial-link choice favored the alternative that led to the FI 20-s schedule and required many sessions to switch preference. Thus, there was a dissociation between choice and timing such that pigeons responded more in the initial links for the alternative that they ''knew'', based on their terminal-link responding, was associated with the longer delay. These data are difficult to reconcile with the view that initial-link responding reflects the relative value of the terminal-link stimuli, or results from sampling of memory distributions associated with each alternative (Gibbon, Church, Fairhurst & Kacelnik, 1988; Gallistel & Gibbon, 2000).

The challenge that results like Grace and Nevin's (1999) pose for traditional models for choice based on matching and conditioned reinforcement suggests that rather than attempting to understand choice from a ''top down'' perspective, that is, by developing a quantitative model that can describe steady-state allocation at the molar level, it might be worthwhile to try a ''bottom up'' approach. Specifically, studying how response allocation changes when the terminal-link schedules are altered—that is, the dynamics of choice—may lead to a model that not only accounts for changes in response allocation, but makes accurate steady-state predictions as well. In the present article, we show that the decision model proposed by Grace and McLean (2006) and Christensen and Grace (2008) to account for acquisition phenomena leads to an expression for the effects of terminal-link schedules on steady-state choice. In brief, the model assumes that when food is obtained in a terminal link, subjects make a discrimination—a decision—about whether the delay from terminal-link onset to food was relatively short or long. The tendency to respond to the corresponding initial link increases or decreases if the delay is judged short or long, respectively. Response allocation at steady state reflects the cumulative effect of these discriminations. The expression for the effects of terminal-link schedules plays the same role as conditioned reinforcement in the models of Grace (1994) and Mazur (2001), leading to a model that can describe the archival data with a comparable degree of accuracy. We first review the background studies for the decision model proposed by Grace and McLean and extended by Christensen and Grace (2008, 2009a, 2009b). We then note an additional assumption for the model to provide a realistic account of choice, and derive an expression for the effects of terminal-link schedules on steady-state responding. Finally, we apply the resulting model to the same archival data sets used by Grace and Mazur, and compare performance of the various models.

Acquisition of Choice in Concurrent Chains

Hunter and Davison (1985) pioneered the use of a pseudorandom binary series (PRBS) to study the acquisition of choice behavior. In their experiment, the alternative associated with the richer reinforcement rate in concurrent VI VI schedules changed unpredictably across sessions according to a PRBS. The PRBS ensured that whichever alternative was richer for a given session was random, and could not be predicted on the basis of prior sessions. Hunter and Davison showed that pigeons' response allocation adjusted rapidly to changes in the reinforcer ratio. Subsequently Schofield and Davison (1997) used a lagged multiple regression analysis to show after sufficient training with the PRBS procedure, response allocation in a given session depended on the relative reinforcement rate in that session and with little evidence of control from prior sessions (see also Davison & McCarthy, 1988).

Grace, Bragason and McLean (2003) investigated whether pigeons' response allocation in concurrent chains could track unpredictable changes in terminal-link reinforcement delays across sessions using a similar PRBS design. In their Experiment 1, the left terminal link was always FI 8 s and the right terminal link was either FI 4 s or FI 16 s, as determined by a 31-step PRBS across sessions. Multiple regression analyses confirmed that after two exposures to the series (62 sessions), initial-link response allocation depended on the terminal-link delays arranged in the current session and with negligible influence from previous sessions. Results showed that sensitivity to reinforcer immediacy increased through the first half of the session, and remained approximately constant thereafter.

In Experiment 2, Grace et al. (2003) tested whether arranging a unique FI schedule value for the right alternative in each session, rather than selecting either FI 4 s or FI 16 s, would disrupt the acquisition of choice. The pigeons were the same as those from Experiment 1, and training began immediately after that experiment was completed. Surprisingly, Grace et al. found that sensitivity to reinforcer immediacy was approximately the same as the level reached in Experiment 1 and did not change systematically over the course of training (60 sessions). They concluded that whatever response strategy the pigeons had learned in Experiment 1 was not disrupted by the use of different delays for the right terminal link in each session in Experiment 2.

Grace and McLean (2006) noted that these results—particularly the lack of disruption in Experiment 2—were potentially problematic for models of choice based on conditioned reinforcement. If response allocation depended on the learned value of the terminal-link stimuli, then it should have been easier for choice to adjust in Experiment 1, where the right terminal link changed between two values, than in Experiment 2, where the right terminal link was sampled from a potentially infinite population of values. Thus, Grace and McLean conducted an experiment to compare response allocation in two conditions: A *minimum-variation* condition which was identical to that used by Grace et al. (2003) in Experiment 1; and a *maximum-variation* condition in which a different terminal-link FI value was arranged for both alternatives in every session, with the location of the shorter FI determined by a 31-step PRBS. They reasoned that if choice depended on the learned value of the terminal-link stimuli, then pigeons should show greater sensitivity to immediacy in the minimum-variation condition. However, Grace and McLean found that there was no systematic difference in sensitivity between the minimum- and maximum-variation conditions.

Analysis of data from the maximum-variation condition showed two distinct patterns of results: In some cases, scatterplots of log initial-link response allocation as a function of the terminal-link log immediacy ratio were approximately linear, consistent with generalized matching. However, in other cases the scatterplots showed a nonlinear pattern, in which response allocation fell into one of two clusters depending on whether the left or right alternative was favored. Overall, the relationship appeared to be sigmoidal, with a greater difference between the clusters than within them (see their Figure 4). Grace and McLean (2006) suggested that a process akin to categorical discrimination may have influenced responding—that is, the pigeons learned to respond more in each session to whichever alternative led to the shorter terminal link delay, but how much that delay was shorter than the alternative had little influence over choice.

Grace and McLean (2006) proposed a model which could account for the different patterns of results in the maximum-variation condition. They assumed that after reinforcement in a terminal link, pigeons made a decision about whether the preceding reinforcement delay was short or long relative to a criterion. If the delay is judged short, response strength for the associated initial link increases, whereas if the delay is judged long, then response strength decreases. Response allocation is then predicted by the ratio of the response strengths. Changes in response strength are made according to a linear-operator rule (with parallel equations for left and right alternatives):

$$\Delta r_{n+1} = \alpha(r_{max} - r_n) \text{ with probability } p \text{ and}$$
$$-\alpha(r_n - r_{min}) \text{ with probability } 1 - p\text{, so that} \quad (1)$$
$$\Delta r_{n+1} = p\,\alpha(r_{max} - r_n) + (1-p)(-\alpha)(r_n - r_{min}).$$

According to Equation 1, Δr_{n+1}(change in expected response strength for cycle n+1) is a function of response strength on the previous cycle (r_n), and an additive or subtractive term, depending on whether the delay on cycle n was judged as short or long, respectively. If the previous delay was judged as short (with probability p), the response strength increases by a proportion (α) of the difference between the maximum response strength ($r_{\max}$) and current response strength, whereas if the delay was judged as long (with probability $1 - p$), response strength decreases by a proportion of the difference between the current and minimum response strength ($r_{\min}$).

The model assumes that all delays are scaled logarithmically, and computes the probability of a ''short'' decision as the area under a normal distribution to the right of the previous delay, log D. The mean of the distribution is the average of the log delays across both alternatives, and is referred to as the criterion (log C). The standard deviation (σ) is a parameter in the model, which determines the accuracy with which delays are judged as short or long. Specifically, the probability, p, that a delay, log D, is judged short is $1 - \Phi(\log D, \log C, \sigma)$, where Φ is the cumulative normal distribution with mean = log C and standard deviation = σ evaluated at log D. Grace and McLean (2006) showed that when σ was relatively high, classification decisions were less accurate and response allocation was approximately a linear function of the log immediacy ratio (i.e., generalized matching). However, when σ was relatively small, decisions were more accurate and response allocation was a nonlinear (sigmoidal) function of the log immediacy ratio. Grace and McLean fitted the model to the results from individual subjects and showed that it accounted for the major features of the data.

Christensen and Grace (2008) noted that a major limitation of the decision model was that it was unable to account for effects of overall initial- and terminal-link duration, which are well established in the literature (Berg & Grace, 2006; Grace & Bragason, 2004). For example, Fantino (1969) showed that preference between a constant pair of terminal links became less extreme when the initial-link schedules increased. An effect of overall terminal-link duration on preference was first reported by MacEwen (1972), who found that preference for the shorter of two terminal-link schedules in a constant ratio increased as their overall duration increased. The initial- and terminal-link effects were influential in the development of delay-reduction theory (see Fantino, Preston & Dunn, 1993; Fantino & Romanowich, 2007, for reviews) and are predicted by other steady-state models for choice (Grace, 1994, 2004; Mazur, 2001).

Christensen and Grace (2008) proposed that the criterion in the decision model could be calculated as the average of all interstimulus intervals correlated with reinforcement. Thus both intervals between initial-link onset and terminal-link entry, as well as between terminal-link onset and reinforcement, were included. The rationale was that when making decisions about whether delays were short or long, pigeons did not clearly discriminate between initial- and terminal-link intervals. They showed that when the criterion was computed in this way, the decision model predicted the initial-link effect. As initial-link duration increased, the criterion increased and thus p increased for both terminal links. However, they showed that p increased more slowly for the shorter schedule, producing an attenuated preference. Moreover, the model also predicted that preference would decrease for very short initial-link durations, which was confirmed in an experiment.

Christensen and Grace (2009a) showed that the decision model also predicted the terminal-link effect when both initial- and terminal-link delays contributed to the criterion. They showed that when terminal-link duration was increased, the criterion also increased but less than proportionally. Consequently the predicted preference (which is determined by the ratio of p for the left and right terminal links) increased. They reported an experiment using a rapid-acquisition design that confirmed both the increased sensitivity to reinforcer immediacy predicted by the terminal-link effect, but also the less-than-proportional increase in the criterion.

Christensen and Grace (2009b) made two further additions to the decision model. They included a linear-operator term to account for changes in response strength across sessions, and proposed an exponentially weighted moving average (EWMA; Killeen, 1981) for updating the criterion:

$$\log C_{N+1} = \beta(\log D_N) + (1-\beta)\log C_N. \quad (2)$$

The criterion is assumed to be updated after every transition between stimuli (i.e., initial link to terminal link, and terminal-link to reinforcement or at terminal-link entry and after reinforcement might be better). In Equation 2, log C_N and log C_{N-1} are the criterion values after stimulus transitions N and N-1, respectively, log D_N is the Nth stimulus–transition interval, and β indicates how much weight is given to the most recent interval.

Here we propose one further change in the decision model. Because all of the previous studies which have tested the decision model have used FI terminal links, there was no variability in the delay to reinforcement associated with a given terminal link in each session. However, when VI schedules are used, it may be more difficult for subjects to discriminate whether the delay just experienced was short or long. As response strength is updated after reinforcement delivery (according to Equation 1), the decision must be made retrospectively, and when delays for a particular alternative are variable within session, subjects' memory for the just-experienced delay may be influenced to some extent by previous delays in the session. Thus we will assume that the subjects' memory for just-experienced delay can be calculated as a EWMA of the history of delays for that alternative. Separate EWMAs are calculated for each of the terminal links. For simplicity, we will also assume that all VI schedules use exponentially distributed intervals (Fleshler & Hoffman, 1962).

This addition to the model has two major consequences. First, it allows the model to predict preference for variability, that is for VI xs over FI xs. The reason is that although the arithmetic mean delay may be equal for VI x and FI x, the average of the log delays (i.e., the geometric mean) will be lower for VI x. Second, because the geometric mean of a VI distribution is less that of an FI distribution (which equals the arithmetic mean), the model will predict less extreme preference when the terminal links are both VI schedules (e.g., VI x VI y) compared to corresponding FI schedules (FI x FI y). Thus the model predicts more extreme preference with FI FI terminal links for the same reason that it predicts the terminal-link effect: When the overall duration of the terminal links increases, the criterion increases but less than proportionally.

Steady-State Decision Model

We can now derive the steady-state predictions for the decision model. Given sustained exposure to the same terminal links, Equation 1 predicts that response strength for each initial link will reach an asymptotic value. The asymptote may be obtained by setting α = 1 in Equation 1 and simplifying the resulting expression to yield:

$$r_{\infty} = p(r_{\max}) + (1-p)(r_{\min}). \tag{3}$$

Equation 3 states that the asymptotic response strength is a weighted average of the maximum and minimum response strengths ($r_{\max}$ and $r_{\min}$) depending on the probability that a delay associated with the corresponding terminal link is judged short (p). In all subsequent analyses, $r_{\max}$ and $r_{\min}$ were set equal to 1 and .01, respectively, similar to our previous applications of the decision model (Christensen & Grace, 2008, 2009a, 2009b). Predicted response allocation is then given by the ratio of the asymptotic response strengths:

$$\frac{B_L}{B_R} = \frac{r_{\infty L}}{r_{\infty \mathrm{R}}} = \frac{p_L r_{\max} + (1-p_L) r_{\min}}{p_R r_{\max} + (1-p_R) r_{\min}}, \tag{4}$$

where L and R represent calculations for the left and right alternatives, respectively. Equation 4 is an expression for the effects of sustained training with a given pair of terminal-link schedules on initial link choice. The probability that a delay (log D) is judged short relative to the criterion (log C) is computed as the probability that a random sample from a normal distribution with mean equal to log C and standard deviation σ is more than log D:

$$p = 1 - \Phi(\log D, \log C, \sigma), \tag{5}$$

where Φ is the cumulative normal distribution evaluated at log D.

For each terminal link, log D is calculated as the log of the geometric mean reinforcement delay. For FI schedules, log D = the log of the schedule value. For VI schedules, the intervals were randomized from a set of 12 intervals based on an exponential progression (Fleshler & Hoffman, 1962); log D was the log geometric mean of this distribution. For log C, we assumed that the distribution of times spent in the initial link could be approximated by a 12-interval exponential progression with a mean equal to the average time spent in the initial link. Log C was then calculated as (log D_I + log D_I + log D_L + log D_R) / 4, where log D_I is the log geometric mean of the initial-link intervals.

Archival Data Analyses

Next we compare the ability of the decision model to account for results from steady-state concurrent-chains experiments with that of two previous models: the contextual choice model (CCM; Grace, 1994), and the hyperbolic value-added model (HVA; Mazur, 2001). Specific details of CCM and HVA are presented in the articles cited and we will not repeat them here. However, we note that both models are based on the generalized matching law (Baum, 1974) and take the following form:

$\frac{B_L}{B_R} = b\left(\frac{R_L}{R_R}\right)^a \left(\frac{V_L}{V_R}\right)$, or in logarithmic terms,

$$\log\frac{B_L}{B_R} = \log b + a\log\left(\frac{R_L}{R_R}\right) + \log\left(\frac{V_L}{V_R}\right), \quad (6)$$

where B_L, B_R are initial-link responses, R_L and R_R are the rates of entering the terminal links, and V_L, V_R are the values of the terminal links, a is a sensitivity parameter and b is bias. According to Equation 6, initial-link response allocation matches the relative frequency of conditioned reinforcement (i.e., terminal-link entry) provided by the choice alternatives with sensitivity a and bias b, and with a concatenated term (additive in the logarithmic version) that represents the effects of relative terminal-link value. For the decision model, the value ratio in Equation 6 is replaced with Equation 4 (response strength ratio). Thus, like CCM and HVA, the decision model assumes that effects of terminal-link schedules on choice are additive with the effects of the relative frequency of entering the terminal links (cf. Fantino & Romanowich, 2007).

The decision model was fitted to the same archival data sets analyzed by Grace (1994) and Mazur (2001). In addition, CCM and HVA were fitted. The archival data were composed of 19 concurrent-chains studies published before 1994 and based on the following criteria: (a) minimum of four data points for each subject, (b) time-based terminal-link schedules, either FI or VI; and (c) equal terminal-link reinforcer magnitudes. In addition to these criteria, we also omitted several conditions with unequal initial-link schedules in which one of the initial links was VI 0 s, from Fantino and Davison (1983; 1 of 56 conditions) and Davison (1983; 5 of 61 conditions). Because all models made identical predictions for Squires and Fantino (1971; unequal initial links; equal terminal links), this data set was omitted from the analyses. Overall, a total of 1463 individual-subject data points from 18 studies were analyzed[1]. For all studies, response allocation was scaled as the log initial-link response ratio. Thus, a logarithmic version of each model was fitted to the data.

For all models, parameters were estimated that maximized the variance accounted for using Microsoft Excel Solver. For the decision model, there were two parameters fitted to all data sets in which terminal-link entry rates were equal (log b and σ), and three for data sets in which rates were unequal (log b, σ, and a). For CCM and HVA, we first used the same number of parameters as the decision model, that is, either two or three depending on whether terminal-link entry rates were equal (CCM: log b, a_1, and a_2; HVA: log b, a_i, a_t). However, both models contain an additional parameter (k) which was used by both Grace (1994) and Mazur (2001) to provide an adequate fit to studies with uncued terminal links. Here we used the following rule: Both HVA and CCM were initially fitted to the data without letting the k parameter vary. If the variance accounted for was less than 80%, then the model was refitted while allowing k to vary. If the variance accounted for improved by more than 5%, then the fit with the k parameter was used, otherwise the original fit was retained. Thus in all cases, the decision model had the same number or fewer parameters as CCM and HVA.

HVA predicts exclusive preference when one terminal link or the other does not signal an increase in reinforcement value. In this case, because exclusive preference is not possible to achieve on a logarithmic scale, we used a maximum predicted response ratio of 100:1 (or 1:100). This ensured that HVA would have the same maximum predicted preference as the decision model given our choice for r_{max} and r_{min}.

[1] The 18 studies included in the archival analysis were: Alsop & Davison, 1988; Chung & Herrnstein 1967; Davison 1976, 1983, 1988; Davison & Temple 1973; Duncan & Fantino, 1970; Dunn & Fantino, 1982; Fantino, 1969; Fantino & Davison, 1983; Fantino & Royalty, 1987; Gentry & Marr, 1980; Killeen, 1970; MacEwen, 1972; Omino & Ito, 1993; Preston & Fantino, 1991; Wardlaw & Davison, 1974; Williams & Fantino, 1978.

Table 1 shows details of the model fits. Averaged across 18 studies (which included 87 data sets and 1463 data points), the variance accounted for (VAC) by the decision model (DM), HVA and CCM was 88.3%, 84.5%, and 87.6%, respectively. The corresponding medians were 90.4%, 85.5%, and 88.1%. Across the studies, the minimum and maximum VAC were: DM, 73% (Gentry & Marr) and 99% (Duncan & Fantino, 1970); HVA, 63% (Gentry & Marr, 1980) and 94% (Davison, 1976); and CCM, 76% (Fantino & Royalty, 1987) and 97% (Davison, 1976). This shows that all three models provided a reasonably accurate description of the data. Notably, the DM required fewer fitted parameters (n = 202) compared to HVA (n = 223) and CCM (n = 228), even while it accounted for slightly more variance.

We conducted a residual analysis to determine whether there were systematic deviations of the data from predictions of each model (Sutton, Grace, McLean & Baum, 2008). Figure 1 plots residual scores (obtained–predicted) pooled across data sets as a function of the predicted values for each model. Because the models incorporate bias in structurally the same way (i.e., as an additive term), estimates of log b were subtracted from the predicted values prior to the residual analysis. As Sutton et al. noted, removing variance in bias across studies should result in a more sensitive test of systematic trends in the residuals.

Figure 1 shows that there appears to be a similar systematic trend in the residuals of each model: For strongly negative predicted values, the residuals tend to be greater than zero, then decrease below zero as the predicted value increases, then increase as the predicted values become positive, and then finally decrease and become less than zero for strongly positive predicted values. This trend was confirmed by results of polynomial regressions. In these analyses, we regressed the residuals against the predicted values (bias free) and their cube, and tested the significance of the linear and cubic components. Note that quadratic components (i.e., the square of the bias-free predicted values) were excluded because this function (U shape or inverted U shape) is not invariant under admissible transformations of the response ratio, in which left/right or right/left is arbitrary (see Sutton et al., 2008).

Table 2 shows the beta coefficients for the linear and cubic components, and the R^2 value, for the polynomial regressions. Results showed that, for each model, the cubic component was significantly negative, and the linear component was significantly positive. These coefficients confirm that the pattern described above was statistically significant for each model. The R^2 value was lowest for the DM and highest for CCM, with HVA in the middle. However, it is notable that the pattern was identical in all cases, indicating that each model failed to account fully for the data in a similar way.

Some insight about how the data deviated from the models' predictions is provided by Figure 2, which shows the results of the polynomial regressions in terms of an obtained versus predicted scatterplot. If the residuals showed no systematic pattern, the obtained data would fall exactly on the solid major diagonal (i.e., obtained = predicted). However, the curved functions are based on the polynomial regressions, and show that the obtained data deviated from the models' predictions in a similar way. As expected from the regression coefficients in Table 2, the strength of the pattern was strongest for HVA and weakest for the DM, with CCM in the middle.

Finally, we conducted an analysis to determine whether the models' sensitivity parameters were invariant with respect to whether VI or FI terminal links were used. Grace (1994) reported that for CCM, sensitivity to relative terminal-link immediacy was greater for FI than VI terminal links. Thus the studies were separated into two groups depending on whether terminal links were both FI (n = 12) or both VI (n = 6)[2]. For CCM, sensitivity values (a_2) were significantly greater for FI terminal links (M = 1.79) than VI terminal links (M = 0.81), $t(85) = 3.60$, $p < .001$. There was an opposite trend for HVA: a_t was greater for VI terminal links (M = 0.86) than FI terminal

[2] The studies with FI terminal links were: Chung & Herrnstein, 1967; Davison 1976, 1983, 1988; Davison & Temple 1973; Duncan & Fantino, 1970; Gentry & Marr, 1980; Killeen, 1970; MacEwen, 1972; Omino & Ito, 1993; Wardlaw & Davison, 1974; and Williams & Fantino, 1978. The studies with VI terminal links were: Alsop & Davison, 1988; Dunn & Fantino, 1982; Fantino, 1969; Fantino & Davison, 1983; Fantino & Royalty, 1987; and Preston & Fantino, 1991.

Table 1

For the archival studies listed, average estimated parameter values, variance accounted for (VAC), number of data sets per study, number of data points per study, and number of parameters fitted per study for the decision model (DM), hyperbolic value-added model (HVA; Mazur, 2001), and contextual choice model (CCM; Grace, 1994).

	Decision Model (DM)							
	Parameters							
Archival Study	**log *b***	***a***	**σ**	**VAC**		**#Data Sets**	***n***	**#Params**
Alsop & Davison 1988	0.11	0.79	0.52	0.88		6	156	18
Chung & Herrnstein 1967	0.18	1.00	0.22	0.88		6	54	12
Davison 1976	0.24	0.43	0.17	0.97		1	20	3
Davison 1983	0.02	0.76	0.38	0.82		6	314	18
Davison 1988	0.04	1.00	0.46	0.92		6	135	12
Davison & Temple 1973	0.00	1.00	0.32	0.90		8	156	16
Duncan & Fantino 1970	0.08	1.00	0.10	0.99		2	12	4
Dunn & Fantino 1982	0.17	1.00	0.17	0.77		4	24	8
Fantino 1969	−0.40	1.00	0.15	0.91		4	16	8
Fantino & Davison 1983	−0.05	0.16	0.24	0.90		6	330	18
Fantino & Royalty 1987	0.14	1.00	0.28	0.77		6	42	12
Gentry & Marr 1980	0.03	1.00	0.56	0.73		4	36	8
Killeen 1970	−0.13	1.00	0.20	0.96		4	16	8
MacEwen 1972	0.17	1.00	0.19	0.97		4	16	8
Omino & Ito 1993	0.09	1.00	0.31	0.91		6	27	12
Preston & Fantino 1991	0.06	0.62	0.29	0.79		9	65	27
Wardlaw & Davison 1974	0.11	1.00	0.24	0.92		1	20	2
Williams & Fantino 1978	0.27	1.00	0.12	0.91		4	24	8
Average				0.883	**Total**	87	1463	202

	Hyperbolic Value-Added Model (HVA)								
	Parameters								
	log *b*	a_i	a_t	***k***	**VAC**		**#Data Sets**	***n***	**#Params**
Alsop & Davison 1988	0.04	0.79	0.62	0.20	0.88		6	156	18
Chung & Herrnstein 1967	0.11	1.00	0.76	0.21	0.89		6	54	13
Davison 1976	0.26	0.35	1.14	0.20	0.94		1	20	3
Davison 1983	0.05	0.73	0.66	1.83	0.84		6	314	24
Davison 1988	0.04	1.00	0.86	0.20	0.87		6	135	12
Davison & Temple 1973	−0.06	1.00	0.33	0.99	0.81		8	156	20
Duncan & Fantino 1970	0.07	1.00	1.09	0.20	0.83		2	12	4
Dunn & Fantino 1982	0.16	1.00	1.00	0.00	0.93		4	24	12
Fantino 1969	−0.47	1.00	1.07	0.20	0.78		4	16	8
Fantino & Davison 1983	−0.11	0.15	0.73	1.83	0.76		6	330	19
Fantino & Royalty 1987	0.13	1.00	1.14	0.17	0.78		6	42	13
Gentry & Marr 1980	0.02	1.00	0.45	0.07	0.63		4	36	12
Killeen 1970	−0.11	1.00	0.79	0.20	0.93		4	16	8
MacEwen 1972	0.30	1.00	0.88	0.20	0.79		4	16	8
Omino & Ito 1993	0.05	1.00	0.53	0.20	0.91		6	27	12
Preston & Fantino 1991	0.01	0.54	0.75	0.20	0.81		9	65	27
Wardlaw & Davison 1974	0.11	1.00	0.69	0.20	0.91		1	20	2
Williams & Fantino 1978	0.26	1.00	1.33	0.20	0.91		4	24	8
Average					0.845	**Total**	87	1463	223

	Contextual Choice Model (CCM)								
	Parameters								
	log *b*	a_1	a_2	***k***	**VAC**		**#Data Sets**	***n***	**#Params**
Alsop & Davison 1988	0.19	0.81	0.39	1.00	0.88		6	156	18
Chung & Herrnstein 1967	0.17	1.00	2.77	1.24	0.87		6	54	13
Davison 1976	0.27	0.67	3.26	1.00	0.97		1	20	3
Davison 1983	−0.02	0.81	0.97	0.47	0.81		6	314	24
Davison 1988	0.04	1.00	0.42	0.33	0.92		6	135	18

Table 1
(Continued)

Archival Study	Contextual Choice Model (CCM)								
	Parameters								
	log *b*	***a*₁**	***a*₂**	***k***	**VAC**		**#Data Sets**	***n***	**#Params**
Davison & Temple 1973	−0.05	1.00	1.18	0.62	0.86		8	156	20
Duncan & Fantino 1970	0.07	1.00	4.67	1.00	0.95		2	12	4
Dunn & Fantino 1982	0.16	1.00	1.87	1.00	0.96		4	24	8
Fantino 1969	−0.24	1.00	1.47	1.00	0.92		4	16	8
Fantino & Davison 1983	−0.09	0.16	0.83	0.74	0.84		6	330	21
Fantino & Royalty 1987	0.25	1.00	1.00	0.93	0.76		6	42	13
Gentry & Marr 1980	0.02	1.00	0.90	0.47	0.76		4	36	12
Killeen 1970	−0.15	1.00	2.18	0.75	0.94		4	16	9
MacEwen 1972	0.45	1.00	1.31	1.00	0.77		4	16	8
Omino & Ito 1993	0.00	1.00	0.93	1.00	0.95		6	27	12
Preston & Fantino 1991	0.35	0.75	0.18	1.00	0.77		9	65	27
Wardlaw & Davison 1974	0.10	1.00	2.00	1.00	0.88		1	20	2
Williams & Fantino 1978	0.26	1.00	5.18	1.00	0.94		4	24	8
Average					0.876	**Total**	87	1463	228

links (M = 0.72), but the difference failed to reach significance, $t(85) = 1.80$, $p = .07$. For the decision model, the average sensitivity (σ) was nearly equal for FI (M = 0.30) and VI terminal links (M = 0.29), $t(85) = 0.30$, *ns.*

This analysis shows that parameter estimates which measure sensitivity to terminal-link schedules were overall more consistent for the decision model than for CCM and HVA. For CCM and HVA, sensitivity parameters tended to vary depending on the type of terminal-link schedule, whereas for the decision model they did not. This suggests that the decision model performs better than CCM and HVA on the criterion of parameter invariance (Nevin, 1984).

DISCUSSION

The goal of the present study was to determine if the decision model proposed for acquisition of choice in concurrent chains by Grace and McLean (2006) and Christensen and Grace (2008, 2009a) could produce a viable model for steady-state responding, and to compare its accuracy with that of previous models (CCM; Grace, 1994; HVA; Mazur, 2001). To accomplish this, we derived an expression for asymptotic relative response strength (Equation 4), representing the effects of terminal-link schedules on initial-link responding, and used it in the generalized-matching law framework adopted by previous models (Equation 6) as a replacement for relative terminal-link value. The resulting model accounted for slightly more variance in log initial-link responding (88.3%) than CCM and HVA (87.6% and 84.5%, respectively) across a range of archival studies while requiring about 10% fewer free parameters in total. Moreover, the decision model showed no evidence of systematic differences in parameter estimates for studies with VI and FI terminal links, unlike CCM and, to a lesser extent, HVA. We therefore conclude that the decision model, originally developed to explain individual differences in pigeons' responding under dynamic conditions in which terminal links changed unpredictably across sessions (Grace & McLean, 2006) provides an account of molar, steady-state choice that is at least as good as, and arguably perhaps better than, existing models.

However, the decision model, like CCM and HVA, does not provide a complete account of choice. Analysis of residuals found that a similar pattern of systematic deviations was present in the predictions of all three models, which could be characterized as a third-order polynomial with positive linear and negative cubic components. The simplest interpretation of this pattern is that over an approximately 4-$\log_{10}$ unit range, log response allocation is a nonlinear (sigmoidal) function, increasing more rapidly and then less rapidly as preference moves away from indifference, whereas the decision model, HVA, and CCM all predict that the rate of increase in log

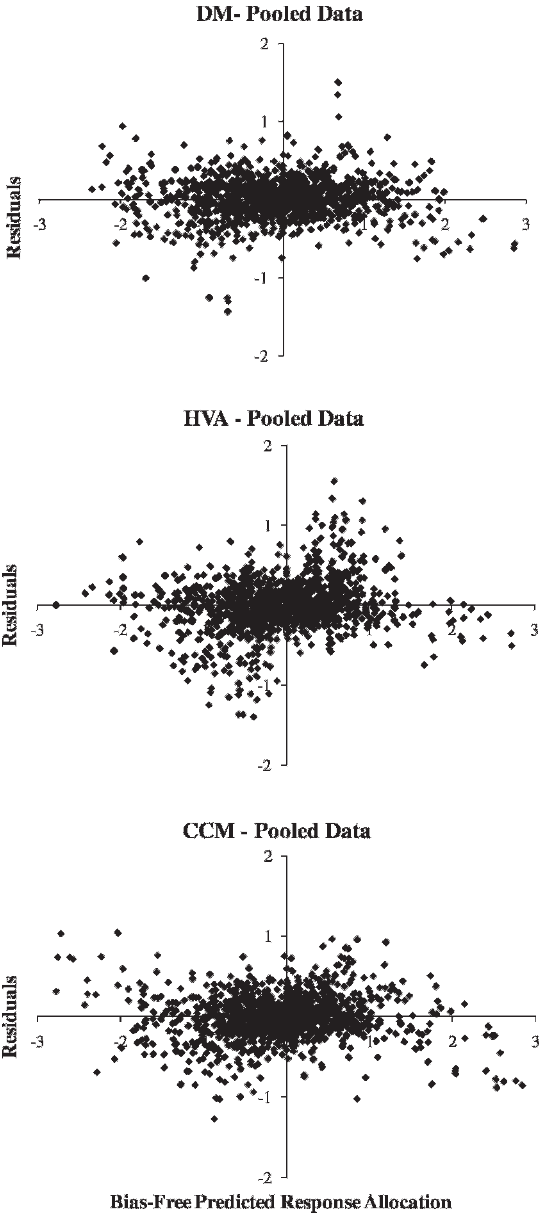

Fig. 1. Residual values (obtained–predicted; n = 1463) plotted as a function of bias-free predicted values for the decision model (DM), CCM and HVA.

response allocation should show less change. This pattern is most clearly apparent in relation to CCM, which predicts that when overall initial- and terminal-link durations are constant, log response allocation is a linear function of the log terminal-link immediacy (i.e., reciprocal of delay) ratio. In contrast to CCM, the decision model predicts that log

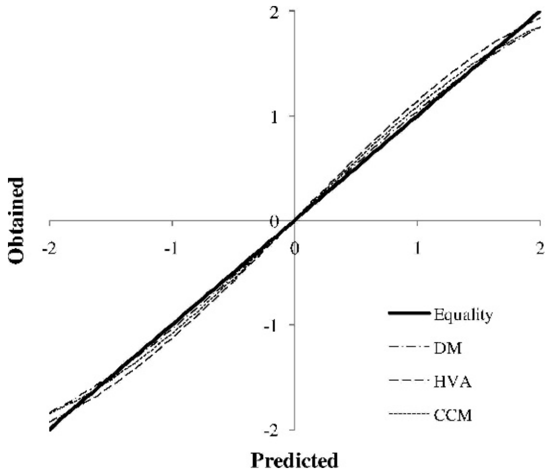

Fig. 2. Obtained versus predicted log response allocation scatterplot showing fitted cubic polynomials based on the regression analysis for each model's residuals. If the residuals bore no systematic relationship to predicted values, the fitted functions would correspond to the major diagonal (dark line), representing obtained = predicted. The curvature in the dashed lines indicates how the obtained data deviated systematically from the predictions of the DM, HVA, and CCM.

response allocation should be a sigmoidal function of the log terminal link immediacy ratio, with the degree of nonlinearity determined by the parameter σ (see Grace & McLean, 2006, Figure 6). However, the residual analysis shows that the decision model fails to capture the full extent of the nonlinearity in the data.

The variance accounted for by CCM and HVA is somewhat lower than that reported by Grace (1994) and Mazur (2001), who found that the models accounted for approximately 90% of the variance in response allocation. This can be attributed to their use of choice proportions rather than log ratios, which impose ceiling and floor effects and thus limit the deviations of obtained from predicted values for relatively extreme preference condi-

Table 2

Results of polynomial-regression analysis of residual scores. Shown are the beta coefficients for the linear and cubic polynomial components and R^2, for the DM, CCM and HVA models.

Model	Linear	Cubic	R^2
DM	0.08***	−0.04***	.042
HVA	0.19***	−0.06***	.081
CCM	0.14***	−0.05***	.103

***$p < .001$

tions. It is notable that when the analyses in the present article were carried out using choice proportions (not reported here), no systematic deviations were found in the residuals of any model. This confirms that log ratios provide a more sensitive assay of response allocation, and should be used instead of choice proportions, particularly when models are fitted.

Unlike CCM and HVA, the decision model was able to account for the more extreme preference observed with FI terminal links but with no systematic change in estimated values of the sensitivity parameter. The reason that the decision model is able to predict less extreme preference with VI terminal links is that the use of an exponentially-weighted average of log delays to compute the criterion yields a lower value when terminal links are VI than FI. Thus the decision model predicts more extreme preference with FI than VI terminal links for the same reason that it predicts an effect of overall terminal-link duration (Christensen & Grace, 2009a): Use of FI terminal links results in a longer criterion delay compared to VI terminal links with the same average reinforcement delay.

The success of the decision model in specifying an expression for steady-state responding validates the strategy of studying acquisition as a means toward explaining molar choice. The rapid acquisition design in which terminal-link schedules change unpredictably across sessions according to a PRBS (Hunter & Davison, 1985) is ideally suited for this purpose, because it yields learning curves within individual sessions. Experiments based on this design can easily generate sufficient data points to distinguish between linear and nonlinear response allocation (Grace & McLean, 2006; Kyonka & Grace, 2007), which is more difficult in steady-state designs because many sessions are required to obtain each data point.

It is also important to note that the incremental modifications to the decision model proposed by Christensen and Grace (2008, 2009a, 2009b) and here do not fundamentally change the structure of the decision model as initially specified by Grace and McLean (2006). For example, Christensen and Grace's (2008) proposal—including the initial-link intervals in the criterion—did not change predictions for Grace and McLean because the prior study did not vary initial-link duration. Similarly, calculating the delay to be judged short or long as a EWMA of previous delays on an alternative—which was necessary here to account for the preference between VI schedules—does not affect the previous application of the decision model to FI schedules. Our strategy has been to make the necessary changes to the decision model in a step-by-step fashion, thus increasing its generality and extending it to a broader range of situations. The alternative approach of defining a complete model at the outset would not have worked, as it would have been unnecessarily complex for the initial applications. Further elaboration of the model will be necessary to extend its scope further, for example to incorporate the effects of reinforcer magnitude and probability.

At a more theoretical level, the decision model provides an alternative to conditioned reinforcement as an explanation for initial-link responding in concurrent chains. According to the traditional view shared by models such as DRT, CCM and HVA, terminal-link stimuli acquire the capacity to reinforce responding through a process akin to Pavlovian conditioning, and consequently response allocation during the initial links reflects the relative conditioned value of the terminal-link stimuli. In contrast, the decision model assumes that differential initial-link responding results from the cumulative effect of making discriminations about terminal-link delays. According to the decision model, what is learned and expressed as response allocation in concurrent chains is the relative propensity to respond in the presence of the initial-link stimuli. Regarding the terminal links, the decision model assumes that subjects learn the reinforcer delays signalled by the stimuli (represented by the EWMA), and so those stimuli can provide discriminative control for terminal-link responding. Thus, the decision model is able to accommodate the results of experiments which have examined temporal control of terminal-link responding (e.g., Grace & Nevin, 1999; Kyonka & Grace, 2007), which are problematic for accounts based on conditioned reinforcement. The dissociation between choice and timing reported by Grace and Nevin occurs because the determiners of responding in the initial- and terminal links are different. Initial-link responding is updated through a retrospective process (i.e., decisions about recent terminal link delays) and requires the initial-link stimuli to be present for the effects of those decisions, in terms of changes in response strength, to be made.

Effects of temporal context on choice—that is, overall initial- and terminal-link duration—are among the most important results in concurrent chains. Previous models have explained these effects in terms of how conditioned reinforcement depends on temporal context (Fantino, 1969; Mazur, 2001), or temporal context modulates the sensitivity of choice to terminal-link value (Grace, 1994). The decision model is different from these accounts because it assumes that temporal context effects result essentially from a confusion of initial- and terminal-link stimuli: Whereas optimal decisions regarding which terminal link had the shorter delay would require comparison with a criterion that depended solely on terminal-link delays, according to the decision model the intervals between initial-link onset and terminal-link entry also contribute to the criterion, and explain why temporal context effects occur. This suggests a testable prediction of the model: Assuming that making initial-link stimuli more discriminable from terminal-link stimuli means that they are less likely to contribute to the criterion, attenuated effects of temporal context should be obtained when initial- and terminal-link stimuli are more discriminable. For example, an experiment might compare the magnitude of the terminal- or initial-link effect in two conditions that differed in terms of whether the initial- and terminal-link stimuli differed in both color and position (e.g., white side keys for the initial links; red or green center keys for the terminal links) or in just whether the alternative key was illuminated (e.g., initial links signalled by left red and right green keys; terminal links signalled by extinguishing the alternative initial link). Stronger effects of temporal context should be obtained in the latter condition, where the initial- and terminal-link stimuli are more similar.

Some evidence from previous studies suggests this prediction may be valid. Grace (1994) found that the parameter k, which scales the effect of temporal context, was only necessary to fit for studies in which the terminal links were uncued; otherwise k was equal to 1. Typically in these studies the terminal links were signalled by blackout (e.g., Chung & Herrnstein, 1967; Gentry & Marr, 1980), and Grace found that for these studies, a better fit was obtained with $k < 1$, indicating a weaker effect of overall terminal-link duration. The distinctiveness of the initial- and terminal-link situations is arguably greater when the initial links are signalled by keylights with a houselight providing general illumination and the terminal links are signalled by blackout (as in the uncued studies cited above), than when the houselight is always illuminated and the only difference between the initial and terminal links is which keys are lighted and their color. The confusability of the initial- and terminal-link situations should be less in the former case, and if this reduces the contribution of the initial-link delays to the criterion the effect of overall terminal-link duration would be reduced, consistent with the data.

The decision model assumes that delays are scaled logarithmically. The reason for this assumption is simplicity: By using log delays, the model is able to use a single parameter (σ) which determines the accuracy with which delays are judged short or long relative to the criterion. This entails that the relative discriminability of a pair of terminal-link delays depends on their ratio and not their absolute values, consistent with Weber's Law. The model is able to predict the well-known deviations from Weber's Law in concurrent chains—the initial- and terminal-link effects—because the initial link delays are included in the computation of the criterion. However it should be noted that a model which assumed a linear scaling of delays could make equivalent predictions, provided that the standard deviation increased proportionally with the criterion (C). For such a model, the coefficient of variation (σ / C) would be the fundamental sensitivity parameter (as in Gibbon, 1977), comparable to σ in the current model.

One of the most well known results in the concurrent-chains literature is preference for variability—that is, for a VI schedule over an FI schedule that provides the same reinforcement rate (Herrnstein, 1964). Although the decision model predicts preference for VI over FI schedules with the same arithmetic mean delay, because of the use of log scaling and the EWMA to update the terminal-link delay it predicts that the VI–FI equivalence value, that is, the FI schedule that should be equally preferred to a VI schedule, should occur at the geometric mean of the intervals comprising the VI. By contrast, most research suggests that the VI–FI equivalence value occurs at the harmonic mean of the VI intervals (Killeen, 1968; Mazur, 1984). A task for the future is to determine whether the decision model is able to provide an adequate account of results of studies on preference for variability, and whether it is able to predict VI–FI equivalence at the harmonic mean.

It is important to note that although the decision model describes the average course of acquisition for a pair of terminal-link schedules, this does not necessarily correspond to the actual change in response allocation that might be observed in any particular session. Like all linear-operator models, the decision model predicts a steady approach towards asymptote. But data from individual sessions rarely show smooth acquisition curves. For example, Grace and McLean (2006) examined data at the level of session twelfths, and found that trajectories within single sessions were highly variable (see their Figure 10). Moreover, there is substantial evidence that abrupt switches in response allocation (i.e., from favoring one alternative to the other) occur within sessions when schedules are changed frequently (Gallistel, Mark, King, & Latham, 2001; Kyonka & Grace, 2007, 2008). Because the decision model computes the probability of a ''short'' decision—not the actual decision that is made—it is limited to describing the average course of acquisition. Whether a modified version of the model can be applied to single sessions (perhaps through simulating real-time decisions) is a task for future research.

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