# Mathematics Teachers' Topic-Specific Pedagogical Content Knowledge in the Context of Teaching $a^0$, 0! and $a \div 0$

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**Abstract**

The aim of this study is to explore high-school school mathematics teachers' topic-specific pedagogical content knowledge. First, 639 high-school students were asked to give explanations about "$a^0 = 1$, $0! = 1$" and "$a \div 0$" where $a \neq 0$. Weak explanations by the students led to a detailed research on teachers. Fifty-eight high school mathematics teachers in Northern Cyprus were the participants. They were asked to write how they teach the above topics to high school students. The researcher determined some categories and themes from the written explanations of the teachers using inductive content analysis. Then, three pre-service mathematics teachers had a four-hour training on deductive content analysis. The pre-service teachers went over the written explanations independently and used the pre-determined categories and themes to code the explanations in a deductive way. The results indicate ($\chi^2$) that experienced teachers propose more conceptually based instructional strategies than novice teachers. The results also indicate that strategies proposed by all the participants were mainly procedural, fostering memorization. Hence, giving teachers the opportunity to view and teach mathematics in a more constructivist way and more emphasis on topic-specific pedagogical content knowledge in teacher training programs are recommended.

**Key Words**

Mathematics Teachers, Topic-Specific Pedagogical Content Knowledge, Teaching Zero.

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Rapid developments in science and technology demand for better educated students who can solve problems creatively, learn how to learn, and think critically. For these reasons, in the last two decades a number of studies have attempted to define the nature and the components of teacher knowledge that are necessary especially for mathematics teaching in line with the above skills (Adey & Shayer, 1994; Halpern, 1992; Hill, Ball & Schilling, 2008; Neubrand, 2008; Nickerson, Perkins, & Smith, 1985; Schoenfeld, 1992; Sternberg, 1994; Swartz & Parks, 1994; Tishman, Perkins, & Jay, 1995; Zohar, 2004). Since many research findings emphasize the effects of teachers' knowledge on students' achievement (Brickhouse, 1990; Clark & Peterson, 1986; Hanushek, 1971; Nespor, 1987; Neubrand, 2008; Strauss & Sawyer, 1986; Tobin & Fraser, 1989; Wilson & Floden, 2003) and students' weak knowledge in zero concept (Ball, 1990; Henry, 1969; Ma, 1999; Reys, 1974; Tsamir, Sheffer, & Tirosh, 2000; Quinn, Lamberg & Perrin, 2008), in this study, we aimed to explore the topic-specific pedagogical content knowledge of mathematics teachers in the context of teaching $a^0$, 0! and $a \div 0$. Many research findings about the differences of novice and experienced teachers' pedagogical content knowledge (Ball & Bass, 2000; Cooney & Wiegel, 2003; Çakmak, 1999; Hill et al., 2008; Neubrand, 2008) led us to emphasize experience as a categorical variable in this study. For this reason, answers were sought for the following questions:

1) What are the instructional strategies proposed by mathematics teachers for teaching of $a^0 = 1$?

2) Are the instructional strategies proposed for teaching of $a^0 = 1$, teaching experience dependent?

3) What are the instructional strategies proposed by mathematics teachers for teaching of $0! = 1$?

4) Are the instructional strategies proposed for teaching of $0! = 1$, teaching experience dependent?

5) What are the instructional strategies proposed by mathematics teachers for teaching of $a \div 0$?

6) Are the instructional strategies proposed for teaching of $a \div 0$, teaching experience dependent?

## Pedagogical Content Knowledge and Topic-Specific Mathematics Pedagogical Content Knowledge

In the last two decades, a great deal of studies have been attempted to investigate the nature and the components of teacher knowledge that is necessary for teaching mathematics. Many researchers specify that actual teaching practice might differ from the knowledge, which a teacher has acquired in the formal education (Ball & Bass, 2000; Cooney & Wiegel, 2003; Çakmak, 1999; Hill et al., 2008; Neubrand, 2008). Therefore, in this study in-service mathematics teachers were concerned. An initial characterization of teacher knowledge comes from Shulman's work (Shulman, 1986, 1987) who divided teacher content knowledge into three categories which are the *subject matter content knowledge*, the *pedagogical content knowledge* and the *curricular knowledge*. Although content knowledge is crucial, many research findings revealed that effective mathematics teaching depends mainly on the richness of a teacher's pedagogical content knowledge (Bolyard & Packenham, 2008; Fawns & Nance, 1993; Fenstermacher, 1986; Sanders & Morris, 2000). For this reason, in this study, we focused on the pedagogical content knowledge. Pedagogical content knowledge can be viewed as a set of special attributes that helps a teacher transfer the knowledge of content to others. It includes those special attributes a teacher possessed that helped him/her guide a student to understand the content in a manner which is personally meaningful (Geddis, 1993; Nakiboğlu & Karakoç, 2005; Shulman, 1987; Türnüklü, 2005). In other words, pedagogical content knowledge includes an awareness of the ways of conceptualizing subject matter for teaching (Shulman, 1986, 1987). Although these definitions and categories are not specific to mathematics teaching, many researchers in mathematics education have used these as a framework. Ball and Bass (2000) used the term *mathematics knowledge for teaching* to capture the complex relationship between mathematics content knowledge and teaching. They also distinguished this knowledge into two key elements: "common" knowledge of mathematics that any well educated adult should have and "specialized" mathematical knowledge that only mathematics teachers need to know (Ball, Hill, & Bass, 2005).

Although there are many research results about in-service mathematics teachers' pedagogical content knowledge, the number of studies focusing on mathematics pedagogical content knowledge of secondary school teachers is small (Baturo & Nason, 1996; Burton, Daane, & Giesen, 2008; Rowland, Huckstep, & Thwaites, 2004, 2005; Türnüklü, 2005). For example, Even and Tirosh (1995) studied teachers' pedagogical content knowledge on function concept. An, Kulm and Wu (2004) considered a network of pedagogical content knowledge and investigated the differences in teachers' pedagogical content knowledge among middle school mathematics teachers in China and the United States. Chinnappan and Lawson (2005) studied secondary school teachers' pedagogical content knowledge in the context of geometry. However, more studies are needed to examine high-school mathematics teachers' pedagogical content knowledge especially in topic-specific context (Hill et al., 2008; Potari, Zachariades, Christou, Kyriazis & Pitta-Padazi, 2007). Mason and Spence (1999) stated that it was vital to know how to act at a specific topic in mathematics teaching. In fact, they emphasized topic-specific pedagogical content knowledge in mathematics education. For this reason, in this research high-school mathematics teachers' topic-specific pedagogical content knowledge, which is considered as a sub-dimension of pedagogical content knowledge, is explored (Magnusson, Krajcik & Borko, 1999; Nakiboğlu & Karakoç, 2005; Tamir, 1988). Veal and Makinster (1999) considered pedagogical content knowledge as a combination of (1) knowledge of the students, (2) knowledge of content, and (3) knowledge of instructional strategies. Hence, in this study topic-specific pedagogical content knowledge in the context of knowledge of instructional strategies is considered.

## History of Zero and Its Place in Mathematics

Many researchers have put forward that zero was first used in India by the Hindus (Boyer, 1968; Davis & Hersch, 1981; Kaplan, 1999; Ore, 1988; Pogliani, Randić, & Trinajstić, 1998; Seife, 2000). The earliest record about *zero* is in A.D. 595 and the earliest record of a system with zero is from A.D. 876 (Pogliani et al., 1998). The Hindu mathematician, astronomer, and poet Brahmagupta (A.D. 588-660) is credited with the introduction of zero into arithmetic (Pogliani et al., 1998). Wells (1997) stated that zero was taken over from Hindus by the Arabs and transferred into Europe. Although Hindus contributed a lot to the introduction of zero into mathematics, the Italian mathematician Leonardo Pisano was the first scientist who represented zero as an oval figure. Leonardo Pisano, in his book *Liber abaci*, introduced the Hindu-Arabic number system into European mathematics. Pisano described the nine figures of the Indians together with the sign 0, which is in Arabic called *as-sifr* (Davis & Hersch, 1981). The Arabic word *as-sifr* stands for *empty* or *vacuum*. Some researchers stated that the Medieval Latin word *zephirum* is evolved from *as-sifr* and then from the word *zephirum* its variants *cipher* and *zero* are derived (Eves, 1983; Suryanarayan, 1996; Vorob'Ev, 1961).

Although scientists, teachers and students started to work with zero many years ago, research studies have shown that conceptualization of zero and applications with zero are still very big problems especially for students (Ball, 1990; Henry, 1969; Ma, 1999; Quinn et al., 2008; Reys, 1974; Tsamir et al., 2000). Distinctive characteristics of zero compared to other numbers demand careful planning for teaching it. We might meet many students asking questions like "is zero a number?", "if it is a number, is it even or odd?", "does it mean empty?" and "does it mean nothing?" It is easy to choose the easiest answer and say that it represents empty or nothing, but when you go deeper it can be realized that it is not so easy to understand zero. For example, a horizontal line has slope zero, and for some people this means that there is no slope. On the other hand, a road having no slope, not being a hill, would be referred as having no slope by many people; however no slope in mathematics means a vertical line, a line for which slope does not exist. These examples reveal that both teaching and understanding zero is very challenging. Difficulty in understanding zero is not only a problem for students. For example, many research studies have shown that teachers' knowledge about the meaning of $a \div 0$ especially at the conceptual level is weak (Arsham, 2008; Pogliani et al., 1998; Quinn et al., 2008). In this study, it is aimed to explore high-school mathematics teachers' knowledge in the context of teaching *a0 and 0!* Lack of research in exploring teachers' knowledge especially in the context of teaching $a^0$ and *0!* makes this study important.

## Method

### Model

In this study, qualitative methods were used partially, beside quantitative methods in analyzing the data. A group of high-school mathematics teachers were asked to write how they teach $a^0$, 0! and $a \div 0$ to high school students. Teachers' written explanations were studied through inductive and deductive content analysis (Strauss & Corbin, 1990; Yıldırım & Şimşek, 2008) and coded as themes/approaches. In order to test the differences amongst the percentages, $\chi^2$ test was used. Cramer $\phi$ (Hinton, 1996) was used to measure the effect sizes (0.2: small, 0.3: medium and 0.5: large). The significance level used throughout the study was .05.

## Participants

Fifty-eight high school mathematics teachers from four regions, namely Girne, Magosa, Lefkoşa and Güzelyurt, in the Turkish Republic of Northern Cyprus were the participants of this study. The teaching experience of the participants varied from 1 to 17 years ($M$ = 5.9, $SD$ = 4.65). The participants who had 1-4 years of teaching experience were considered as *novice* (n = 33) and the participants who had 5-17 years of teaching experience were considered *experienced* teachers (n = 25). In determining the experienced and novice teachers, the Teacher Training Authority of the Ministry of Education was consulted.

## Collection of Data

The instrument used in this study, which was developed by the researcher, is composed of 3 questions as *(1) How do you teach* $a^0 = 1$ *to high school students? When* $a \neq 0$, *(2) How do you teach* $0! = 1$ *to high school students?* and *(3) How do you teach* $a \div 0$ *to high school students? When* $a \neq 0$. The teachers were asked to write one of their best instructional strategies for teaching of $a^0$, $0!$ and $a \div 0$. Two mathematics educators and 2 experienced mathematics teachers were asked to judge the content validity of the test. All the experts concluded that the content and format of the items were consistent with the definition of the variables and the study participants. After the instrument was administered to 65 mathematics teachers, in order to find themes/approaches in the written explanations the researcher has chosen the explanations of 15 teachers randomly and used inductive content analysis to determine the categories and themes (Strauss and Corbin 1990). Then, three pre-service mathematics teachers had a four-hour training on deductive content analysis. The pre-service mathematics teachers went over the written explanations independently and used the predetermined categories and themes to code the explanations of the teachers in a deductive way. Inter-rater consistency coefficient (Shavelson & Webb, 1991) of the pre-service teachers (*Kappa Coefficient)* was .93 (Lantz & Nebenzahl, 1996). At the first stage, if a theme/approach was shared by at least two of the pre-service mathematics teachers, the theme/approach was taken into consideration and coded, otherwise as a second stage the pre-service mathematics teachers and the researcher of this study all sat for a consensus on the conflicting themes/approaches. Generated themes/approaches and related examples can be seen in Table 1. At the second stage the themes/approaches were also divided into two categories as *conceptual/qualitative reasoning based* (algebraic approach, pattern approach, and limit) and *procedural/fostering memorization based* (it is a rule approach, undetermined approach, no division by zero approach).

**Table 1.**
*Mathematics Teachers' Themes/Approaches and Sample Explanations Related to $a^0 = 1$, $0! = 1$ and $a \div 0$*

| Main Category | Sub-Category | Theme/Approach | Sample Teacher Explanation |
|---|---|---|---|
| $a^0 = 1$ | Procedural/ Fostering Memorization Based | It is a Rule Approach | *It is OK to say "this is a rule" in teaching $a^0 = 1$.* |
| | Conceptual/ Qualitative Reasoning Based | Algebraic Approach | *Example 1: We know that $(a)^2 \div (a)^2 = 1$. On the other hand $(a)^2 \div (a)^2 = (a)^2 x (a)^{-2}$.* *Since $(a)^2 x (a)^{-2} = (a)^0$ then $(a)^0 = 1$.* *Example 2: We know that $(a)^n x (a)^m = (a)^{n+m}$. Then $(a)^n x (a)^0 = (a)^n$, So $(a)^0 = 1$.* |
| | | Pattern Approach | *Let's look at the following pattern;* *$(3)^4=81$... $(3)^3=27$... $(3)^2=9$... $(3)^1=3$... $(3)^0=?$* *As you see each time we divide by 3 to get the next. Then if we divide 3 by 3, we get $(3)^0=1$.* |

| | | | |
|---|---|---|---|
| 0! = 1 | Procedural/ Fostering Memorization Based | No Answer | – |
| | | It is a Rule Approach | *In teaching mathematics it is not always possible to prove everything. We can say this is a rule to be memorized.* |
| | Conceptual/ Qualitative Reasoning Based | Pattern Approach | *It is obvious that 3!=6, 2!=2 and 1!=1. We divided by 3 and 2 respectively. When we divide 1! by 1 then 0! should be "1".* |
| | | Algebraic Approach | *4!=4x3x2x1 and we know that n! = n x (n−1)x(n−2)....3x2x1. Later on (n!) / (n−1)! = n. If we substitute "n" by "1" we get 1!/0! = 1. Finally it is obvious that 0! = 1.* |
| a ÷ 0 | Procedural/ Fostering Memorization Based | No Answer | – |
| | | It is a Rule Approach | *When we divide a number by 0, we get infinity and this is a rule.* |
| | | Undetermined Approach | *Dividing a nonzero number by zero is undetermined. This is a basic rule in mathematics.* |
| | | No Division by Zero Approach | *It is not possible to divide by zero.* |
| | Conceptual/ Qualitative Reasoning Based | Pattern Approach | *Let us each time divide "1" by very small numbers; 1/1=1...1 /0.1=10...1/0.01=100.......... ..............1/0.000001= 1000 000. As you can see when the denominator gets smaller the quotient becomes larger. Then in the long run we'll get infinity.* |
| | | Limit Approach | *If we use the limit approach it is clear that the result is infinity.* |

## Procedures

Many research findings have indicated the effects of teachers' pedagogical content knowledge on students' achievement (Brickhouse, 1990; Clark & Peterson, 1986; Hanushek, 1971; Nespor, 1987; Neubrand, 2008; Strauss & Sawyer, 1986; Tobin & Fraser, 1989; Wilson & Floden, 2003). On the other hand, many students believe that zero "*has no meaning*" and "*causes* *some problems in doing mathematics*" (Ball, 1990; Henry, 1969; Ma, 1999; Quinn et al., 2008; Reys, 1974; Tsamir et al., 2000). So in order to show the importance of this research, first, 639 randomly selected high-school students were asked to answer the questions; "(1) *Why does* $a^0 = 1$*? When* $a \neq 0$, (2) *Why does* $0! = 1$*? and* (3) *What is* $a \div 0$*? When* $a \neq 0$*.*" Mostly procedural and memorizing-oriented explanations of students (see Table 2) raised a need to conduct this study. Later, in order to administer the instrument, 65 mathematics teachers were randomly selected from 4 regions, namely Lefkoşa, Girne, Magosa, Güzelyurt in the Turkish Republic of Northern Cyprus. From each region, 3 schools were selected randomly beforehand. After that, the teachers were asked to answer the following questions; *(1) How do you teach* $a^0 = 1$ *to high school students?, (2) How do you teach* $0! = 1$ *to high school students?* and *(3) How do you teach* $a \div 0$ *to high school students?* The teachers were asked to submit their written explanation to the school principals in two days. After two days, 7 teachers did not submit their written explanations, so 58 teachers were taken into consideration in this study. The Content analysis of the written explanations, completed by 3 pre-service mathematics teachers, took almost a month.

## Data Analysis

Both qualitative and quantitative methods were used in analyzing the data of this research. In order to find themes/approaches in the written explanations of the mathematics teachers both inductive and deductive content analysis were used. In determining if there were significant differences in the percentages of the themes and if these differences were experience dependent, $\chi^2$ procedures were used. In order to understand the strength of the relationship between the variables and the magnitude of the differences, *Cramer* $\phi$ effect size measures were used. Hinton (1996) characterized $\phi = 0.2$ as a small effect size, $\phi = 0.3$ as a medium effect size, and $\phi = 0.5$ as a large effect size. The level of significance used throughout the study was .05.

**Table 2.**
*$\chi^2$ Analysis on Students' Themes/Approaches*

| Equation/ Operation | Theme/Approach | Percentage | $\chi^2$ | *df* | *p* | $\phi$ |
|---|---|---|---|---|---|---|
| $a^0 = 1$ | It is a Rule Approach | 83 | 274.74 | 1 | .001 | 0.66 |
| | No Answer | 17 | | | | |

↳

| | | | | | | |
|---|---|---|---|---|---|---|
| 0! = 1 | It is a Rule Approach | 43 | 304.85 | 2 | .001 | 0.69 |
| | No Answer | 55 | | | | |
| | Algebraic Approach | 02 | | | | |
| a ÷ 0 | It is a Rule Approach | 16 | 050.98 | 3 | .001 | 0.28 |
| | No Answer | 34 | | | | |
| | Undetermined Approach | 29 | | | | |
| | No Division by Zero Approach | 21 | | | | |

## Results

### Results Related to "$a^0 = 1$"

As seen in Table 3, in general, *it is a rule* was the most frequently observed approach in the explanations of mathematics teachers for teaching of "$a^0 = 1$". Nearly 76% of the teachers used this approach. Fourty-four percent of the experienced teachers and all of the novice teachers used this approach. Thirty-six percent of the experienced teachers and none of the novice teachers used the *pattern* approach. Although it was beyond the parameters of this research, 3 of the experienced teachers gave explanations for teaching of negative powers like "$(2)^{-1} = ½$". One of those teachers stated that he also discovered the rule for the negative powers (see Fig. 1). Twenty percent of the experienced teachers and none of the novice teachers used the *algebraic* approach. Briefly, the approaches suggested by the mathematics teachers for teaching of "$a^0 = 1$" were distributed in the following manner; (1) it is a rule approach (75.9%), (2) pattern approach (15.5%) and algebraic approach (8.6%). Large effect size ($\phi = 0.65$) shows that the differences amongst the percentages are large.

**Table 3.**
*$\chi^2$ Test Results About the Themes/Approaches*

| Equation/ Operation | Theme/ Approach | Group[a] | Percentage[b] | $\chi^2$ | *df* | *p* | $\phi$ |
|---|---|---|---|---|---|---|---|
| $a^0 = 1$ | It is a Rule Approach | A | 44 *(19)* | 24.36 | 2 | .000 | 0.65 |
| | | B | 100 *(56.9)* | | | | |
| | Pattern Approach | A | 36 *(15.5)* | | | | |
| | | B | - | | | | |
| | Algebraic Approach | A | 20 *(8.6)* | | | | |
| | | B | - | | | | |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 0! = 1 | No Answer | A | - | 11.97 | 3 | .008 | 0.45 |
| | | B | 15.2 *(8.6)* | | | | |
| | It is a Rule Approach | A | 44 *(19)* | | | | |
| | | B | 48.5 *(27.6)* | | | | |
| | Pattern Approach | A | 32 *(13.8)* | | | | |
| | | B | 3 *(1.7)* | | | | |
| | Algebraic Approach | A | 24 *(10.3)* | | | | |
| | | B | 33.3 *(19)* | | | | |
| a ÷ 0 | No Answer | A | - | 11.97 | 5 | .005 | 0.53 |
| | | B | 15.2 *(8.6)* | | | | |
| | It is a Rule Approach | A | 20 *(8.6)* | | | | |
| | | B | - | | | | |
| | Pattern Approach | A | 12 *(5.2)* | | | | |
| | | B | - | | | | |
| | Undetermined Approach | A | 48 *(20.7)* | | | | |
| | | B | 72.7 *(41.4)* | | | | |
| | No Division by Zero Approach | A | 12 *(5.2)* | | | | |
| | | B | 9.1 *(5.2)* | | | | |
| | Limit Approach | A | 8 *(3.4)* | | | | |
| | | B | 3 *(1.7)* | | | | |

[a] *Letter "A" under the heading* **Group** *stands for the experienced teachers group and the letter "B" stands for the novice teachers group.*

[b] *Regular text under the heading* **Percentage** *stand for the percentages within experience and italic text stand for the total percentages.*

## Results Related to "0! = 1"

As seen in Table 3, nine percent of all the participants gave no explanation for teaching of "0! = 1". This group of teachers were all from the novice teachers group. On the other hand, nearly 47% of all the participants suggested it is a rule approach for teaching of "0! = 1". As in Table 3, there is a small difference between experienced (44%) and novice teachers (48.5%) in terms of the percentages of suggesting the it is a rule approach. However, the difference between the experienced (13.8%) and novice teachers (1.7%) groups is large in terms of the percentages of suggesting the pattern approach.

Original Explanation Translation of the Original Explanation

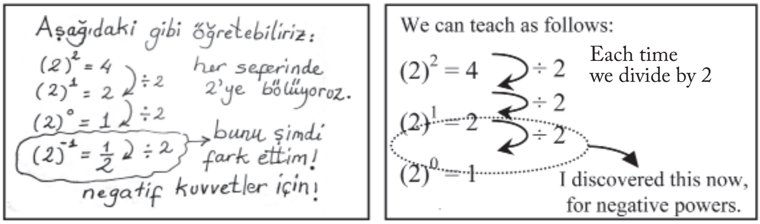


**Figure 1**

*Explanation of an experienced teacher for teaching of "$a^0 = 1$"*

The difference between the experienced teachers group (24%) and the novice teachers group (33.3%), in terms of the percentages of suggesting the algebraic approach, is greater compared to it is a rule approach. The difference is favoring the novice teachers group. Briefly, the approaches suggested by the mathematics teachers for teaching of "0! = 1" were distributed in the following manner; (1) it is a rule approach (46.6%), (2) algebraic approach (29.3%), (3) pattern approach (15.5%) and (4) no answer approach (8.6%). Large effect size ($\phi = 0.45$) shows that the differences amongst the percentages are large.

## Results Related to "a ÷ 0"

As seen in Table 3, once again nearly 9% of all participants gave no explanation for teaching of "a ÷ 0". This group of teachers are all from the novice teachers (15.2%) group. On the other hand 48% of experienced and 72.7% of novice teachers suggested the *undetermined* approach for teaching of "a ÷ 0". Nearly 10% of all the participants suggested the *no division by zero* approach. Twelve percent of the experienced and 9.1% of the novice teachers suggested this approach. Twelve percent of the experienced teachers and none of the novice teachers suggested the pattern approach for teaching of "a ÷ 0". Similarly, 20% of the experienced teachers and none of the novice teachers suggested the it is a rule approach for teaching of "a ÷ 0". The *limit* approach is the least (5.1%) suggested approach. Eight percent of the experienced teachers and 3% of the novice teachers suggested the limit approach for teaching of "a ÷ 0" . Approaches suggested by the mathematics teachers for teaching of "0! = 1" were distributed in the following manner; (1)

undetermined approach (62.1%), (2) no division by zero approach (10.4%), (3) it is a rule approach-no answer (8.6%), (4) pattern approach (5.2%) and (5) limit approach (%5.1). Large effect size ($\phi$ = 0.53) shows that the differences amongst the percentages are large.

## Discussion

The results revealed that the explanations of the mathematics teachers for teaching of $a^0$, *0!* and $a \div 0$ were generally procedural which can promote rote memorization rather than conceptual understanding. It is also observed that the explanations were usually weak in qualitative reasoning. For example, in this study 75.9% of the mathematics teachers suggested instructional strategies which promote procedural understanding and rote memorization and only 24.1% of the mathematics teachers suggested instructional strategies which promote conceptual understanding and qualitative reasoning for teaching of "$a^0 = 1$". Similar findings were observed for teaching of "0! = 1" and "a ÷ 0". Surprisingly, some of the mathematics teachers (novice mathematics teachers) could not even give any explanation for teaching of "0! = 1" and "a ÷ 0". Munby, Russel & Martin (2001) stated novice teachers lack of content and pedagogical content knowledge as the main reason for this. So, teacher training should be reconsidered in the light of these findings. For example, Fennema and Franke (1992) had revealed that mathematics teachers need a good basic conceptual understanding of the subject matter (mathematics) and the pedagogical knowledge (mathematics knowledge) to shift their practice from "telling" to promoting student thinking.

In this study the results also revealed that the experienced teachers suggested more conceptually and qualitative reasoning based instructional strategies for teaching of the mathematical cases than the novice teachers. Similarly, many researchers (e.g., Clarke, 1995; Lampert, 1988; Ma, 1999; Spence, 1996) revealed an unfamiliarity of new teachers in the secondary schools with the content of mathematics and the processes of concept building that affected students' mathematics education. New (novice) mathematics teachers' perception of their weak pedagogical content knowledge may lead them to shift their practice from conceptual to procedural/traditional which they feel safe (Hitchinson, 1996). So, teaching experience may be considered as an important element in the development of pedagogical content knowledge

In this study none of the novice mathematics teachers could give an explanation for teaching of "$a^0 = 1$". This might be an important indicator of their weak Content Knowledge (CK) beside weak Pedagogical Content Knowledge (PCK). In line with this finding, Neubrand (2008) and Çakmak (1999) stated that PCK was positively influenced by a sound CK. They also stated that teachers should have very strong CK. For teaching of "$a^0 = 1$" 36% of the experienced teachers suggested the pattern and 20% of the experienced teachers suggested the algebraic approaches. Although the approaches suggested by the experienced teachers were mainly conceptual, 44% of these teachers suggested it is a rule approach. This shows that experienced teachers may also need conceptually oriented in-service training. None of the explanations of the novice teachers for teaching of "$a^0 = 1$" were in line with algebraic or pattern approach. Since the algebraic or pattern approaches can be considered as more conceptual, it can be concluded that the novice teachers in this study have mostly a procedural way of thinking. This, again, reminds us the importance of training programs and practices based on constructivist perspectives in pre-service teacher education (Hill et al., 2008; Ma, 1999; Neubrand, 2006).

Although the difference was not so large, the novice teachers suggested more algebraic approaches than experienced teachers for teaching of "0! = 1". In most mathematics teacher training programs usually the factorial concept is taught in an algebraic way. The novice teachers in this study might still be under the influences of their university education. Since the algebraic approach for teaching of "0! = 1" is more formula oriented (Hiebert, 1986) this might be a reason for the novice teachers to prefer this approach for teaching of "0! = 1". On the other hand, most probably more practical teaching experience of the experienced teachers (32%) led them to suggest the pattern approach more than the novice teachers (3%) suggested for teaching of "0! = 1". High percentages of suggesting the it is a rule approach for teaching of "0! = 1" by the experienced (44%) and novice teachers (48.5%) revealed that both of the groups had a weak PCK in this topic. Nearly 15.2% of the novice teachers gave no explanation for teaching of "0! = 1". This might be another reason of their weaker PCK compared to the experienced teachers. The difference between the percentages of the algebraic approach suggested by the experienced (24%) and novice (33.32%) teachers for teaching of "0! = 1" surprisingly favored the novice teacher. Informal interviews with some mathematics teachers in this study revealed that experienced teachers preferred the pattern approach for teaching of "0! = 1" for its simplicity and practical nature.

The difference between the percentages of the undetermined approach suggested by the experienced (48%) and novice teachers (72.7%) was large. Although it is not wrong, the inclination of the novice teachers to the undetermined approach, most probably showed their rule-oriented thinking for teaching of "a ÷ 0". The routine approaches like no division by zero and it is a rule were preferred by both the experienced and novice teachers for teaching of "a ÷ 0". This revealed that both of the groups had a weak PCK based on procedural understanding in this topic, too. Similar findings were observed by Ball (1990), Eisenhart et al. (1993) and Quinn et al. (2008). The researchers asked a group of pre-service and in-service mathematics teachers to explain "7 ÷ 0" and many of the teachers gave explanations which were conceptually weak. In this study a few mathematics teachers suggested the limit approach for teaching of "a ÷ 0".

The experienced mathematics teachers, in this study, suggested more conceptually based instructional strategies than the novice mathematics teachers for teaching of all 3 mathematical cases. Nearly average 27% of the experienced teachers suggested the pattern approach but only average 1% of the novice teachers suggested this approach for teaching of the 3 mathematical cases. Since the pattern approach is the most qualitative and practical approach, this can be considered as an advantage for the experienced teachers. In spite of this, generally the results revealed that both of the groups had weak PCK fostering mostly procedural understanding and rote memorization. Hence, educators at all levels should not focus on strategies which lead students' reasoning to be along the lines of *my teacher told me so* (Henry, 1969; Reys, 1974).

To summarize, the following recommendations can be offered for researchers, teachers, pre-service teachers, teacher trainers and curriculum experts in light of the findings and current practice.

1) Pre-service and in-service teachers should have the opportunity to view and teach mathematics in a more constructivist way.

2) The pattern approach should mostly be considered in teaching $a^0$, 0! and a ÷ 0.

3) In teacher training programs experienced teachers can be used as mentors.
4) The linkages between content knowledge and pedagogical content knowledge should be considered in all teacher training programs.
5) Topic-specific pedagogical content knowledge should be emphasized more than general pedagogical content knowledge in teacher training programs.
6) Conceptually oriented in-service teacher training programs should be considered in a continuous and systematic way.
7) Qualitative research should be conducted focusing on these and other aspects of mathematics.

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