# Supporting "out-of-field" teachers of secondary mathematics

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Principals are very aware that there is a great deal of competition in attracting qualified teachers of mathematics to their school since there are just not enough qualified mathematics teachers out there. Recent studies of the qualifications of teachers in Australian schools have confirmed that there is a crisis in the supply of qualified teachers of mathematics for secondary schooling (Australian Council of Deans of Science, 2006; Harris & Jensz, 2006; McKenzie, Kos, Walker & Hong, 2008). In Australian secondary schools, significant numbers of teachers of junior secondary mathematics (almost 50%) as well as teachers of (usually less advanced) senior mathematics subjects (32%) do not hold the recommended tertiary mathematics qualifications for teaching secondary mathematics (McKenzie et al., 2008). Furthermore, many of these teachers will not have completed pre-service training in mathematics teaching.

The outlook is not good. There are falling numbers of students studying tertiary mathematics and the subsequent shrinking of mathematics faculty in Australian universities (ACDS, 2006). Also, enrolments in each Australian state in an advanced mathematics subject in the final year of secondary school, the prerequisite subject for entry to tertiary mathematics, have shown a continuous decline since the turn of the century (Forgasz, 2006). To address these trends the Commonwealth Government has reduced tertiary fees for undergraduate mathematics and science students. However, this program will take at least four years to realise any significant increase in qualified mathematics teachers. Some education systems have put programs in place to increase the supply of teachers in fields of shortage (for example the Career Change Program in Victoria) and the Commonwealth Government has announced a program to attract highly qualified mathematics and science graduates into a school-based preservice program. However, principals realise that they will need to continue to rely on "out-of-field" mathematics teachers in their schools since these initiative will take some time to change the availability of qualified specialist teachers.

Fortunately, Australian governments are beginning to realise the extent of the problem. In response to this disturbing situation, the Council of Australian Governments made the following recommendation in its *National Numeracy Report* (2008):

That in recognition of the likely continued reliance in the medium term on teachers teaching secondary mathematics 'out of field,' systems develop strategies to support such teachers to improve the depth and extent of their mathematical and pedagogical content knowledge. (Council of Australian Government, Recommendation 14)

Whilst this recommendation is welcome, it is long overdue. In the medium term, what can schools do?

In response to this situation the mathematics education and mathematics staff of Victoria University have designed and delivered two professional learning programs for "out-of-field" teachers. One was initiated by a secondary school in a regional city for "out-of-field" teachers in their cluster and focussed on preparing these teachers to teach senior secondary mathematics (Victorian Certificate of Education Mathematics Professional Learning Program [VCEM PLP]); the second was developed in collaboration with the Western Metropolitan Region staff of the Department of Education and Early Childhood Development (DEECD) for "out-of-field" teachers in four schools in the western region of Melbourne. The aim of the second program was to enhance mathematical and pedagogical content knowledge for the teaching of junior secondary mathematics (Junior Secondary Mathematics Professional Learning Program [JSM PLP]), and hence the focus was on the development of multiplicative thinking and proportional reasoning and the connections with many concepts in junior secondary mathematics (Goos, Stillman & Vale, 2007; Seimon, Virgona & Cornielle, 2001; Shield & Dole, 2008). In each case the schools and DEECD were supported by a funding grant from the Australian Government Quality Teaching Project (AGQTP).

In this paper the first of these professional learning programs is described, together with the role of the schools and their mathematics staff, and the outcomes for teachers who participated in this program. To understand the purpose and goals of the program what is meant by pedagogical content knowledge (PCK) and mathematics knowledge for teaching (MKT) is described, and reasons suggested as to why this knowledge is important for effective teaching of mathematics.

### PCK, MKT and effective teachers

The importance of teachers' mathematical content knowledge is recognised as critical for improving students' mathematical learning in recently reported Australasian studies and literature reviews (Timperley, Wilson, Barrah & Fung, 2007; White, Mitchelmore, Branc & Maxon, 2004; Council of Australian Governments, 2008). Indeed the Australian Association of Mathematics Teachers (AAMT) recognises this importance in its *Standards for Excellence in Teaching Mathematics in Australian Schools*:

Excellent teachers of mathematics have a sound, coherent knowledge of mathematics appropriate to the student level they teach, and which is situated in their knowledge and understanding of the broader mathematics curriculum. They understand how mathematics is represented and communicated, and why mathematics is taught. They are confident and competent users of mathematics who understand connections with mathematics, between mathematics and other subject areas, and how mathematics is related to society. (AAMT, 2002)

Hill, Rowan and Ball (2005) claimed that mathematics knowledge for teaching (MKT) was a specialised mathematics knowledge that is "the math-

ematical knowledge used to carry out the work of teaching mathematics" (p. 373). Hence, as implied in the AAMT statement, pedagogical knowledge in the context of mathematics teaching and learning is part of MKT. Much earlier, Shulman (1987) defined pedagogical content knowledge (PCK) as "the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (p. 8). Ma (1999), who focussed on primary teachers' knowledge of fundamental mathematics, demonstrated the strong relationship between profound understanding of fundamental mathematics and pedagogical content knowledge. For her, profound understanding is more than procedural and conceptual knowledge; it is "an understanding that is deep, broad, and thorough" (p. 120), where depth means being able to connect a topic with "more conceptually powerful ideas of the subject" (p. 121), breadth as being able to "connect it with those of similar or less conceptual power" (p. 121) and "thoroughness is the capacity to connect all topics" (p. 124).

Research has shown that it is the knowledge of mathematical connections and MKT that is strongly related to effective teaching of mathematics (Askew, 2008; Hill, Rowan & Ball, 2005) rather than the level of mathematics qualification alone (Askew, 2008; Stacey, 2008). Agreeing with the importance of these elements of teachers' knowledge in practice, Watson and Mason (2007) explained that effective teachers are able to "act in the moment through having pertinent possibilities come to mind" (p. 209).

### "Out-of-field" professional learning programs

Being at home in the "work of teaching mathematics" and being able to "act in the moment" were goals for the two programs we designed specifically for "out-of-field" teachers of junior secondary mathematics. While the specific content of these programs are different, they followed the same structure. These two programs differ from other contemporary practice-based models of professional learning which typically focus on implementing new curriculum or changing teaching approaches and methods. In these two professional learning programs, teachers became students of mathematics. The majority of the time spent in workshops was on solving mathematics problems. Pedagogical content knowledge was not ignored. Rather it was directly related to the teachers' experience of doing mathematics in the program, drew on their general pedagogical knowledge as practicing teachers and was enhanced by classroom and school-based inquiry.

### VCEM PLP

The Deputy Principal of a regional secondary school wished to implement a professional learning program. The school was faced with the prospect of having too few teachers with the knowledge of senior secondary mathematics to be able to provide a full range of senior mathematics options for the students in the near future. The school decided to prepare some of the out-of-field teachers of junior secondary mathematics to teach advanced senior mathematics. Victoria University was approached to design a professional learning program for teachers in the school and in other schools in the region.

The VCEM PLP involved seminars as well as school-based self-directed

inquiry and portfolio development. The program was conducted over a school year for eleven teachers from five schools in the regional city in 2007. Teachers attended three-hour fortnightly seminars conducted in the afternoons during school terms. The 21 seminars were conducted in a school classroom, partly during school working hours and partly in teachers' own time. The focus of the program was on the mathematics content of VCE Mathematics Methods and VCE Further Mathematics and we included both mathematics tasks and tasks that focussed on pedagogical content knowledge (professional learning tasks) during the seminars. The mathematics tasks were typical of those used in senior secondary mathematics and included investigations, problem solving, mathematical modelling, proof and exercises from various sources (for example, Barnes, 1991, CASCAT project; Mason, Graham & Johnston-Wilder, 2005; RITEMATHS), including textbooks for VCE mathematics. The teachers were provided with CAS calculators and learned to use these tools to aid their mathematical learning and pedagogical content knowledge. The professional learning tasks were related to the authentic work of teachers such as reviewing mathematics tasks, designing problems and analysing students' responses to problems. We were also able to use their experience of doing mathematics in the program to discuss students' thinking and misconceptions.

The sequence of topics for the seminars followed the sequence normally used by teachers of these Year 12 subjects. Hence, we included seminars with tasks about the formal assessment tasks of the VCE subjects at roughly the same time that VCE teachers were designing and assessing students with these tasks. Experienced senior secondary mathematics teachers also conducted a few sessions in the program. Their sessions focussed on curriculum knowledge, long-term planning for teaching and assessment, and strategies and resources for teaching and assessing VCE mathematics.

The practice-based component of the program occurred in the teachers' schools between seminars. We encouraged participants to establish a mentor relationship with an experienced teacher of senior secondary mathematics to support their school-based self-directed inquiry. We recommended that they negotiate with their colleagues to observe and/or team-teach Year 11 or 12 mathematics lessons, observe students doing mathematics (in lessons or by tutoring students), reflect on observations, analyse student work, research and critique teaching and assessment resources and materials, and to participate in the moderation processes of student assessment for these subjects.

In practice the school-based inquiry varied for the participants. Three of the teachers in the program were mentored by an experienced VCE mathematics teacher in their school. They discussed mathematics problems, teaching resources and learning programs used in the classes at their school. One of these teachers also participated in moderation of Year 11 and Year 12 school-based assessment. Two other teachers were mentored by an experienced teacher at another school and one of these teachers continued to tutor Year 11 students at her mentor's school. Two other teachers from one school volunteered to take "extras" together for a Year 11 class on a couple of occasions. The teachers in this program developed a portfolio of their self-directed inquiry and presented selected artefacts to the group in the final session.

### **Evaluation**

To evaluate these programs participating teachers completed a questionnaire at the beginning and at the end of the program, field notes on each of the seminars were kept, and samples from teachers' portfolios gathered. Ten months after the completed of the program, interviews with eight of the participants of the VCEM PLP were also conducted. Outcomes in terms of teachers' MKT for participants in the VCEM PLP have been reported elsewhere (Vale & McAndrew, 2008). These findings are here summarised, and changes in the participating teachers' professional situation and identity briefly reported.

Teachers' professional situation and identity

The teaching experience of the ten teachers who completed the VCEM PLP ranged enormously; three of the teachers were in their second year of teaching while two had more than 15 years' experience. They had all taught junior secondary mathematics for at least one year and four had taught mathematics for more than five years and two (one in their first year) had taught a non-advanced Year 11 VCE mathematics subject (General Mathematics).

Not surprisingly they appreciated and enjoyed the networking and were still in contact with each other and continuing to share their experiences of teaching mathematics six months later. Of particular importance was the enjoyment in doing mathematics and the growth in their confidence with mathematics that these teachers experienced during the program. Two teachers were particularly positive claiming that they rediscovered their passion and confidence:

I hadn't done maths for many years. At uni I [was] sort of put off maths, I heard a lecture that was for super brain and so I went from maths being, you know, I really loved [maths] in Year 12 to, you know, losing confidence. I got my confidence back.

It's triggered I suppose, the passion to challenge myself more and more again... it will be just from that whole thing of challenging my understanding again. Then you just pipelined us to keep doing more and more in that. That's the way I have always been since I was a kid.

Since all participants had worked collegially with their mathematics teacher colleagues none reported major changes in their relationships yet each one, in their own way, felt more at ease or more involved in the activities of the mathematics faculty. Individual teachers reported that they understood and participated more in curriculum discussions or sensed that their senior school mathematics colleagues were more supportive of their teaching or curriculum programs in junior secondary mathematics.

There were no major changes to their mathematics teaching load yet all saw teaching advanced mathematics in Year 11 and 12, with ongoing support and mentoring from colleagues, as achievable and desirable within the next few years. Two teachers were given the opportunity of teaching a Year 11 General Mathematics class following the program and two others were teaching a Year 10 mathematics class for the first time. Two teachers took on new leadership positions in the school (Curriculum Coordinator and VCAL coordinator) and believed that their involvement in the program had enabled them to develop knowledge useful in these roles.

### Mathematics knowledge for teaching (MKT)

Data concerning teachers' MKT collected during the VCEM PLP have been analysed and reported in some detail elsewhere (Vale & McAndrew, 2008). While the aim to develop teachers' knowledge of mathematics needed for teaching senior secondary mathematics, it was discovered that focusing on senior secondary mathematics had deepened and broadened their understanding of junior secondary mathematics content and pedagogy. This was particularly evident through the connections that teachers made between mathematical concepts, the use and understanding of multiple representations, deconstruction of content into key components, understanding of students' misconceptions, and an appreciation of the inadequacy of procedural and instructional thinking.

In the follow-up interviews teachers' referred to the deepening of their understanding of junior secondary mathematics. When providing examples of mathematics knowledge that they had developed through the program they commonly cited mathematics concepts that with which they were familiar prior to taking the course and teaching in their mathematics classes in 2008. This is perhaps understandable since none were teaching the algebra, calculus or statistics content that featured in the program.

However, all teachers discussed these examples from the perspective of having a better understanding of more advanced mathematical concepts. At least five of the interviewees claimed to have a more connected understanding of mathematics with implications for their teaching practice. They described adopting a more careful and critical approach to the role of mathematical language (terms, symbols and their meanings); the importance of fluency in fundamental skills such as decimals and fractions, and representation of data; the importance of identifying students' prior knowledge at the beginning of a topic, and revisiting key ideas; and, having a stronger awareness of structure and the implications for understanding. Two teachers described their structural awareness of functions with reference to properties and transformations and one thought that everything we did was connected: "I suppose the main thing that stands out for me is the integration of all the topics that we have learnt ... it is not like separate chapters of knowledge."

### Pedagogical content knowledge (PCK)

The participating teachers believed that they each had become more aware of students' needs or their mathematical thinking. Each described an aspect of practice that paid more attention to students' needs, either by taking more care to find out students' prior knowledge, or by developing approaches that were more engaging for students, especially high achievers and students who needed to be challenged.

I feel confident in what I am doing, that probably comes from having a better background as to where the students are heading with their learning so, instead of just simply knowing where they have come from, I can identify where they are going to with their learning and having that knowledge informs what I am teaching them at times. So the bigger picture has been a really valuable help.

Teachers' new-found confidence had enabled them to be less dependent on the textbook; to use alternative resources and approaches; and, to trust their personal knowledge of mathematics to design or adapt tasks, or develop differentiated learning programs for their students.

### Conclusion

The VCEM PLP was successful in affirming teachers' identity as teachers of secondary mathematics, building their confidence, knowledge and practice and relationships with colleagues, and enabling them to plan a career in mathematics teaching. Critical for the success of this program was the sustained "teachers as learners of mathematics" design of the program. It was found that the school and its mathematics staff can make a great contribution to the success of such programs, and contribute to the enhancement of teachers' pedagogical content knowledge (PCK) and mathematical knowledge for teaching (MKT). In the schools where there was a more clearly defined and ongoing mentoring relationship the teachers were able to gain more practice-based experience of senior secondary mathematics teaching and learning, to put this knowledge into practice in Year 10 or 11 classes, and then use this knowledge to enhance their teaching of junior secondary mathematics. Preliminary analysis of data from the JSM PLP suggests that the role of coaches (or mentors) and sustained collaboration with colleagues have also been important for enhancing PCK.

Of concern are the structural impediments in schools that limit teachers' opportunity to learn from colleagues, to broaden and deepen their mathematics knowledge by observing and engaging with teachers and students located on senior campuses. Schools need to engage in succession planning for the teaching of senior secondary mathematics and to provide professional leadership in junior secondary mathematics. The initiative of the schools involved in these programs is a step in the right direction. It is to be hoped that the success of the program described in this paper will inspire others to develop programs to support other out-of-field teachers of mathematics who are generously teaching mathematics and who are keen to learn and do the best possible for their students.

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# Memorising Pi

Akira Haraguchi, a retired Japanese engineer is known for memorising and reciting digits of pi. On 3 October 2006 he recited 100 000 digits in a public hall. It took 16 hours with a five-minute break every two hours during which Haraguchi ate onigiri rice balls.

On 21 March 2005 Mats Bergsten of Sweden recited pi to 9778 places while juggling three balls.