Using NAPLAN items
to develop students' thinking skills and build confidence

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National testing programs provide challenges and opportunities for mathematics teachers. One challenge is to focus on the diverse learning needs of students while preparing them for national testing early in the school year. An opportunity arises if we use test items to assist students who have difficulty reading and interpreting mathematical text, to further develop students' thinking skills, and to analyse common errors and misconceptions, frequently presented as alternative solutions in multiple-choice items. One approach to "teaching to the test" is to use NAPLAN items as discussion starters so that students develop number sense, adopt new problem-solving strategies, and build confidence and resilience.

Background information

In Australia, the debate surrounding mathematics and numeracy achievement has been similar to that experienced elsewhere. There is a growing recognition of the need for greater proficiency and that early intervention provides the best chance of success for children at risk of failure. Until recently, each state and territory in Australia collected student achievement data for the federal government. Concern about the proportion of students not meeting the minimum national benchmark standards (Curriculum Corporation, 2000) has continued with large investments by governments to address the needs of students at risk.

To better monitor student achievement across Australia the *National Assessment Program in Literacy and Numeracy* (NAPLAN) was introduced in 2008. The same tests in literacy and numeracy are now administered throughout Australia to all students in Years 3, 5, 7 and 9. In Years 7 and 9 students complete two 32-item test papers for numeracy, one without the use of a calculator. Testing early in the school year provides diagnostic information to teachers about their students' performance in mathematics topics common to all states and territories as outlined in the Statements of

Learning for Mathematics (Curriculum Corporation, 2006). The results from the assessments are reported in several forms including individual student reports to parents, school and aggregate reports. For more information about NAPLAN see the website: www.naplan.edu.au.

The school reports enable teachers to analyse the results for each year group to determine which items appear to be understood and which are problematic. In addition, school data can be compared to the Australian student data. The information is useful to assist in addressing common errors and misconceptions as well as to aid planning and programming of future learning.

Whether we approve of a national testing regime or not, this level of accountability to the federal government is in place for the foreseeable future with pressure on school principals and teachers to improve results. While the information may be useful after the results are released, teachers of Years 3, 5, 7 and 9 are experiencing increased pressure early in the school year to prepare students for the test. Principals, school systems personnel and parents are scrutinising the results to determine whether schools and their teachers are "measuring up." They want students to be well prepared and to achieve above minimum standards.

Rather than abandon good pedagogical practices and have students individually practise released test items under timed conditions, NAPLAN provides an opportunity to use quality-teaching approaches for test preparation. In this paper, strategies are presented to assist reading and interpreting mathematical text, to promote thinking strategies and number sense, and to raise awareness of common errors and misconceptions. Examples from NAPLAN tests are used for illustrative purposes with reference to relevant research.

Reading and interpreting test items

Teachers may assume that incorrect answers are the result of errors in applying mathematical procedures or lack of understanding. However, many students experience difficulty reading and comprehending test items before they begin to apply mathematical skills and processes. Presenting questions in context adds considerably to the information students need to read and interpret. Many items on NAPLAN require careful reading — Figure 1 provides one example from the sample items for Year 9 available on the NAPLAN website (http://www.naplan.edu.au).

> Jane, Meg and Oliver scored 1400 points altogether on a computer game.
> Jane has 50 more points than Meg and Meg has 4 times as many points as Oliver.
> How many points does Oliver have?

Figure 1. From Year 9 Numeracy calculator-allowed sample test (www.naplan.edu.au/verve/_resources/NAP08_num_y9_calc_web_v10.pdf).

To identify errors in answering word problems, Newman (1983) interviewed and analysed students' solution attempts. From this, she developed Newman's Error Analysis of five levels of difficulty (Table 1). Most errors occurred in the second and third levels of "comprehending" and "transforming" the text into an appropriate mathematical strategy. Since Newman's research, the results have been replicated in other studies with Clements and Ellerton (1992) reporting 70% of students having difficulty at some stage in the first three levels. If teachers continue to prepare students for NAPLAN by providing practise

Table 1. Levels in Newman's Error Analysis.

1.	Reading the question	Reading
2.	Comprehending what is read	Comprehending
3.	Transforming the words into an appropriate mathematical strategy	Transforming
4.	Applying the mathematical process skills	Processing
5.	Encoding the answer into an acceptable form	Encoding

using de-contextualised questions requiring the application of a procedure, they will have missed an opportunity to support the development of skills associated with reading and comprehending mathematical text.

By translating each of the levels from Table 1 into a question for students, teachers are able to determine the first level of difficulty (White, 2005). A possible set of questions is:

- Read the question to me. If you don't know a word, tell me.
- In your own words, tell me what the question has asked you to do.
- Now tell me what method you will use to find your answer (and work out your answer on your paper.)
- Go over each step in your working and tell me what you were thinking.
- What is the answer to the question? Are you able to use another strategy to check this answer is correct?

Using this protocol in an interview situation provides valuable information about individual students although it can be adapted for whole class discussion or students could use the protocol in pairs.

Mathematical text is lexically dense so each word needs to be read carefully and analysed for meaning. It may be a challenge to convince students to do this if they are anxious or worried about completing the test in the allocated time. However, to interpret and make meaning of mathematical text, it is usually necessary to focus on prepositions as well as the order of words and their relation to one another (Dawe, 1995; MacGregor & Moore, 1991). One approach to developing knowledge of the structure of mathematical text is to have students write in mathematics lessons using "impromptu writing prompts" (Miller, 1992), create their own word problems (English, 1997), or develop self-constructed test items (Clarke, 1997). This writer's experience of using self-constructed items is that students enjoyed creating challenging questions particularly when it is suggested they might be used to compile a class test paper. Additional suggestions for reading and writing tasks are described in Wood's (1992) article.

Using thinking strategies

Teachers frequently witness instances in which students write down the first answer they obtain, or record an answer that appears on a calculator without considering whether it makes sense. Encouraging students to think about questions and apply reasoning about number to evaluate answers can be a challenge. One way to support the development of students' thinking strategies is to use test items that focus on mental computation, estimation and number sense (McIntosh, Reys & Reys, 1997). McIntosh et al. (1997, p. 322) describe number sense as: "a person's general understanding of numbers and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful and efficient strategies for dealing with numbers and operations."

Many NAPLAN items require an understanding of operations and place value as well as knowledge about rational number (e.g., Figure 2).

> What is the missing number?
> 15.0 × 0.4 = ____?____
>
> A) 60 B) 6 C) 0.6 D) 0.06

Figure 2. From Year 9 Numeracy non-calculator sample test (www.naplan.edu.au/verve/_resources/NAP08_num_y9_calc_web_v10.pdf).

> Hugo's electricity bill was $180 last month. This month it is $135.
> What percentage decrease is this?
>
> A) 25% B) 33% C) 45% D) 55%

Figure 3. From 2008 Year 7 Numeracy calculator allowed test (www.naplan.edu.au/verve/_resources/nap08_num_y7_calc_netversion.pdf)

> 5427 ÷ 9 = _?_
> A) 63 B) 603 C) 630 D) 6003

Figure 4. From 2008 Year 5 Numeracy test (www.naplan.edu.au/verve/_resources/NAP08_num_y5_netversion.pdf).

While students are frequently reluctant to estimate, this is an important first step. Options in multiple-choice items may be eliminated after considering whether the solutions are reasonable. After students have estimated the answer, pose questions such as the following:

- What strategies could you use to check the solution?
- What would the question need to be to obtain each of the alternative answers?
- What happens when you multiply a whole number by a number less than one?

Many errors occur in items about percentages (see Figure 3). Multiple-choice items typically include common errors and misconceptions as alternative solutions. Pointing this out to students highlights the need for then to take more time to think about their solutions and to consider ways to verify their approach. Questions for discussion about the item in Figure 3 could be:

- How much is the decrease in electricity bill?
- Is the decrease less than or greater than 50% of the first electricity bill?
- What are the fraction equivalents for 25% and 33%? How much would the discount be for each of these?

After estimating the answer for the NAPLAN item in Figure 4, students should be able to eliminate several options. Teacher and students can discuss the errors students would have to make to obtain the incorrect alternatives offered. Further discussion about errors and misconceptions is presented in the next section of this paper.

Errors and misconceptions

Students' errors in mathematics may be caused by many factors. Poor comprehension, language difficulties, anxiety, rushing and carelessness can all lead to errors in completing tasks (Anderson, 1996). However systematic errors are usually a consequence of misconceptions. Ryan and

Williams (2007) surveyed 15 000 students aged 4 to 15 years in the United Kingdom to identify errors and misconceptions in key mathematical topics. While there were errors due to "slips" and other uncertainties, the researchers identified four main development errors caused from "modelling," "prototyping," "overgeneralising," and "process-product linking." The researchers argue these types of errors "should be valued by learners and teachers alike" (p. 13) since they provide focused learning opportunities. A summary of each type is presented here with examples of NAPLAN items that could be used as a stimulus for class discussion.

Modelling

The first error type involves students attempting to connect school mathematics questions to their experiences in the "real" world. If questions are de-contextualised and abstract, students may attempt to relate them to an everyday context and make an incorrect response. For example, when students are confronted with the question 6 ÷ 1/2 and they write 3, they are probably thinking "divide 6 in half" rather than "divide 6 by a half." When teachers rephrase the question as "how many halves are there in 6?" students can usually answer correctly.

Teachers frequently use models to support students' learning but all models have limitations, as do the models used by children to make sense of school mathematics. Ryan and Williams (2007) suggest errors in students' use of models (or contexts, or metaphors) provide learning opportunities when identified by teachers. For the example provided above, teachers are encouraged to ask students:

- What did you think about when you read and attempted the question?
- How would you rephrase the question to help someone who could not do it?
- Rewrite this question so that the answer is 3.

Prototyping

The second error type arises when students use "typical examples" frequently presented to them by teachers or in textbooks. Two examples are presented here for shape identification and reading scales.

Figure 5 includes one prototypical shape as the first alternative that could lead to students nominating this alternative rather than checking the number of sides. Teachers are encouraged to present shapes in a range of orientations as well as present counterexamples for student discussion.

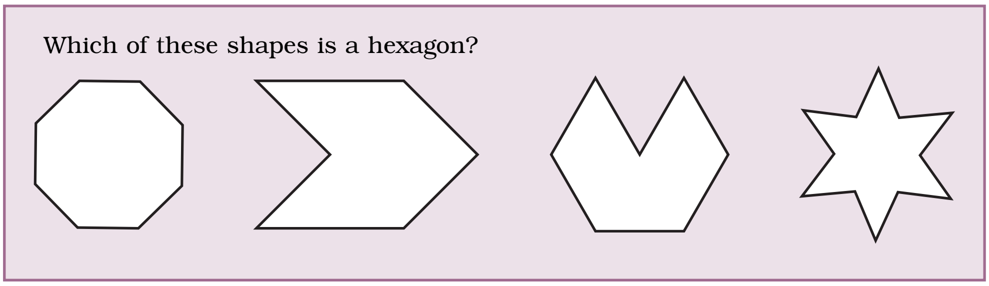
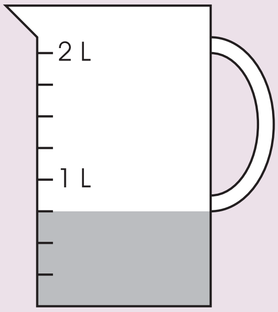


Figure 5. From 2008 Year 5 Numeracy test (http://www.naplan.edu.au/verve/_resources/NAP08_num_y5_netversion.pdf).

This jug has some milk in it.



If Eve adds an extra 500 mL of milk to the jug, how many millilitres (mL) of milk will then be in the jug?

Figure 6. From 2008 Year 7 Numeracy non-calculator test (www.naplan.edu.au/verve/_resources/nap08_num_y7_calc_netversion.pdf).

What is the answer to 6.6 ÷ 0.3?

A) 0.022 B) 0.22 C) 2.2 D) 22

Figure 7. From 2008 Year 7 Numeracy non-calculator test (http://www.naplan.edu.au/verve/_resources/nap08_num_y7_calc_netversion.pdf).

The second example arises when students practise reading scales marked in units, tens or multiples of ten more often than other types of scales such as those with divisions of 2, 250 or 0.25. For the NAPLAN example in Figure 6, many students misread the scale.

Overgeneralising

Ryan and Williams (2007, p. 23) use the term "intelligent overgeneralisation" to refer to students' predisposition to create rules based on experiences. At a particular stage in number learning, children are inclined to use the generalisation "multiplication makes bigger" since this fits the types of questions they are doing. However, this generalisation breaks down when numbers less than one are introduced. Other common generalisations requiring discussion include:

- division makes smaller;
- division is necessarily of a bigger number by a smaller number;
- longer numbers are always greater in value.

Figure 7 presents a typical test item where students could achieve the correct answer from estimation. However, if they are relying on the "division makes smaller" generalisation, they will not choose the correct alternative.

Process-product

The final error type involves understanding the connection between a mathematical process and the product, which is the outcome of the process. For example, 3 + 5 = ? is interpreted as carrying out the process of adding 3 and 5. However the number sentence 3 + 5 = 8 is a number sentence requiring an understanding that "3 + 5 is the same as 8" or "8 is the same as 3 + 5" (Ryan & Williams, 2007, p. 25). Other examples of this error involve students interpreting travel graphs as pictures of walking up or down a hill rather than as indicating a relationship between distance and time.

Cautionary ending

The focus of this paper is the use of NAPLAN items to develop students' competence in reading mathematical text, to promote thinking strategies including estimation, and to evaluate alternative solutions for errors and misconceptions. Showing students test items and discussing strategies for thinking about questions and responses promotes student confidence and resilience, and enables a greater sense of student control over their learning

(Martin, 2003). This paper is not advocating national testing as the most desirable approach to assessing students' knowledge, skills and understanding. Teachers best carry out assessment as they talk to and observe their students — see the AAMT position paper *The Practice of Assessing Mathematics Learning* (AAMT, 2008). For teachers who feel the pressure to prepare their students for the tests, I am recommending the use of NAPLAN items as discussion starters to promote thinking.

References

Australian Association of Mathematics Teachers [AAMT] (2008). *Position paper on the practice of assessing mathematics learning*. Adelaide, SA: AAMT. Accessed 14 April 2009 from http://www.aamt.edu.au/Documentation/Statements.

Anderson, J. (1996). Where did I go wrong? Teachers' and students' strategies to correct misconceptions in secondary mathematics classrooms. In H. Forgasz, T. Jones, G. Leder, K. Maguire & C. Pearn (Eds), *Mathematics: Making connections* (Proceedings of the 33rd Annual Conference of MAV, pp. 1–11). Brunswick, Vic.: MAV.

Clarke, D. (1997). *Constructive assessment in mathematics: Practical steps for classroom teachers*. Berkeley, CA: Key Curriculum Press.

Clements, M. A. & Ellerton, N. (1992). Overemphasising process skills in school mathematics: Newman analysis data from five countries. In W. Geeslin & K. Graham (Eds), *Proceedings of the Sixteenth International Conference on the Psychology of Mathematics Education, Vol. 1* (pp. 145–152). Durham, New Hampshire: IGPME.

Curriculum Corporation (2000). *Numeracy benchmarks Years 3, 5 & 7*. Carlton, Vic.: Curriculum Corporation.

Curriculum Corporation (2006). *Statements of learning for mathematics*. Carlton, Vic: Curriculum Corporation.

Dawe, L. (1995). Language and culture in the teaching and learning of mathematics. In L. Grimison & J. Pegg (Eds), *Teaching secondary school mathematics* (pp. 230–245). Sydney, NSW: Harcourt-Brace.

English, L. D. (1997). Promoting a problem-posing classroom. *Teaching Children Mathematics, 4*(3), 172–179.

MacGregor, M. & Moore, R. (1991). *Teaching mathematics in the multicultural classroom*. Melbourne: The University of Melbourne.

Martin, A. J. (2003). *How to motivate your child for school and beyond*. Sydney: Bantam.

McIntosh, A., Reys, R. E. & Reys, B. J. (1997). Mental computation in the middle grades: The importance of thinking strategies. *Mathematics Teaching in the Middle School, 2*(5), 322–327.

Miller, L.D. (1992). Constructing pedagogical content knowledge from students' writing in secondary mathematics. *Mathematics Education Research Journal, 3*(1), 30–44.

Newman, A. (1983). *The Newman language of mathematics kit*. Sydney, NSW: Harcourt, Brace & Jovanovich.

Ryan, J. & Williams, J. (2007). *Children's mathematics 4–15: Learning from errors and misconceptions*. Maidenhead, UK: Open University Press.

White, A. (2005). Active mathematics in classrooms: Finding out why children make mistakes — And doing something to help them. *Square One, 15*(4), 15–19.

Wood, K. D. (1992). Fostering collaborative reading and writing experiences in mathematics. *Journal of Reading, 36*(2), 96–103.

From Helen Prochazka's

Scrapbook

In short, math matters — a lot more than most people think. We have to make life-and-death decisions based on what numbers tell us. We cannot afford to remain dumb about mathematical ideas simply because we hated them in high school — any more than we can remain dumb about computers, or AIDS. Mathematics is essential, not peripheral, knowledge.

K. C. Cole in her book "The Universe and the Teacup: The Mathematics of Truth and Beauty" (1998)