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Early Childhood Teachers' Professional Learning in Early Algebraic Thinking: A Model that Supports New Knowledge and Pedagogy

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The implementation of a new mathematics syllabus in the elementary context is problematic, especially if it contains a new content area. A professional development model, Transformative Teaching in the Early Years Mathematics (TTEYM) was specifically developed to support the implementation of the new Patterns and Algebra strand. The model was grounded in theories related to a community of practice (Lave & Wenger, 1991), a socio-constructivist perspective (Vygotsky,1962/1934) and effective models of professional development (Clarke & Hollingsworth, 2002; Guskey, 1986). This paper focuses on the effectiveness of the model as six Year 1 teachers worked in pairs developing and implementing learning experiences for three differing aspects of the Patterns and Algebra strand. The results indicate the model offered positive professional learning experiences for the six teachers and assisted them in becoming experts in their own right, especially with regard to content and pedagogical knowledge of the Patterns and Algebra strand.

Young students' engagement with early algebraic thinking is becoming more prevalent in the early years' classroom. Recent research presents evidence confirming that young children can think algebraically. It is conjectured that this thinking not only supports more formal algebraic understandings but also offers a 'pay off' for arithmetic in the primary classroom. The Patterns and Algebra strand in the Queensland Syllabus (Queensland Studies Authority, 2004) contains new content that teachers are required to embrace. The themes in the strand reflect two of Kaput's (2006) three core areas of algebra, namely: algebra as a study of structure and systems abstracted from computations and relations; and algebra as a study of functions, relations and joint variation. Early years' teachers commonly hold concerns regarding the teaching of mathematics. The Patterns and Algebra strand requires early years teachers to reconceptualise arithmetic and patterning, drawing out structures in an activity based environment. To support the implementation of this new mathematical knowledge and pedagogy in the early years' classroom, we were invited by Education Queensland to work with a selection of schools. Our aim was to develop appropriate learning activities for the introduction of early algebra concepts. The intention was that these activities would be shared with all teachers across Queensland, a project called *Transformative Teaching in the Early Years Mathematics* Project *(TTEYM).* Our first consideration was with what professional development model would best assist teachers to learn new mathematical knowledge and pedagogical knowledge.

Past research indicated that most professional development models tended to fall into two broad categories, namely, linear models (e.g., Guskey, 1986) or cyclical models (e.g., Clarke & Hollingsworth, 2002; Clarke & Peter, 1993). The common focus of these models was supporting teacher growth and change through changing teachers' beliefs and attitudes. For example, Guskey's (1986) linear model was based on the theory that teachers change their beliefs and attitudes through changing their practice and reflecting on the results. It also articulated that a key catalyst for teacher change is changes in student learning outcomes. Thus the model was based on the premise that staff development leads to changes in teachers' classroom practices, which results in changes in student learning outcomes. It is this third component that impacts on teachers' beliefs and attitudes. This process is summarised in Figure 1.

The most significant changes in teacher attitudes and beliefs occur after teachers begin implementing a new practice successfully and can see changes in students' learning outcomes (Guskey, 1986). Thus, for teacher change to occur teachers need to implement new ideas presented at staff development in their own classrooms and evaluate student learning.

Change also appears to not only be assisted by classroom enactment of ideas but also by teacher reflections on both the teaching process and students' learning. For this to occur effectively Clarke and Peter (1993) and Clarke and Hollingsworth (2002) suggested the process of professional development should be cyclical with multiple entry points and connected pathways. Several spheres of influence are incorporated into this model. The dimensions of Guskey's model were renamed the External Domain (external domain of information or stimulus), the Domain of Practice (professional experimentation), the Domain of Consequences (salient outcomes such as improved student learning), and the Personal Domain (knowledge and beliefs). The distinguishing feature of this model was its non-linear nature. It recognised professional growth as a continuing process of learning.

Since teaching for understanding relies on teachers grappling with complex subject matter and developing learning activities for a diverse range of students, the expertise necessary to make this vision of practice a reality needs to be contextual and responsive to the teachers' own needs and the students with whom they are interacting on a day to day basis. Effective professional development extends beyond mere support for teachers' acquisition of new skills

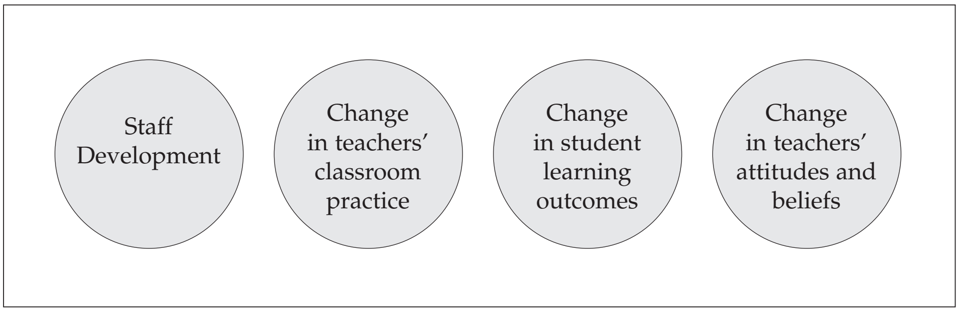

Figure 1. Guskey's model of the process of teacher change.

or knowledge, to provide occasions for teachers to reflect critically on their practice and "to fashion new knowledge and beliefs about content, pedagogy and learners" (Darling-Hammond, 1995). Fundamental to effective professional development is teacher learning.

This paper summarises a model developed and implemented using this focus as its primary concern. Our model builds on previous research by incorporating effective features such as allowing for implementation of ideas in the classroom (Guskey, 1986) and being cyclical with connected pathways, allowing for continual reflection (Clarke & Hollingsworth, 2002). In contrast to previous models, our premise for professional development evolved from a consideration of theories of effective learning and communities of learning, particularly the learning theories of Vygotstky, 1934/1962) and theory related to building a community of practice.

The specific aims were to:

1. develop and implement a professional development model that was grounded in theories of learning;
2. identify particular elements in the model that supported teacher learning; and
3. ascertain how this model assisted teachers to become more knowledgeable about a new mathematical domain.

Theoretical Framework

In the Vygotskian perspective of learning, the Zone of Proximal Development (ZDP) is postulated as one's potential capacity for development through the assistance of a more knowing person (Vygotsky, 1962/1934). This development is predicated by how this more knowing person scaffolds the task at hand. The quality of the interactions, in terms of their appropriateness and intellect, is conjectured to impact on the development of the ZPD. Valsiner (1987) expanded the Vygotskian theory of learning by including the notions of the Zone of Free Movement (what the child is allowed to do by the adult), and the Zone of Promoted Actions (what is being promoted by the adult) with no obligation for the child to accept what is promoted. The Zone of Free Movement (ZFM) represents the environment constraints that limit freedom of action restricting what the child is allowed to do. The Zone of Promoted Actions (ZPA) represents what is being promoted by the adult including the set of activities offered by adults and their orientation towards the promotion of new skills. Traditionally the Vygotskian theory is applied to support effective teacher-student interactions. The abstraction in this paper is its utilisation by a community of learners consisting of an expert (more knowing person) and six teachers (novices with regard to knowledge about patterns and algebra in an early years' setting). In this instance the more knowing person was believed to not only know more terms and facts about the algebraic domain than a novice, but also had developed a conceptualization of how those terms, facts, and concepts fit together into an overall schema (Walker, 1987).

Many researchers have argued that meaning is constructed through discourse and interactions and that the construction of knowledge does not occur in isolation but within a social and cultural context (Bauersfeld, 1992; Sfard, 1998; Wood, Cobb, & Yackel, 1992). Learning is about knowledge construction, is knowledge-dependent, relies on current knowledge and is highly aligned with the situation in which it takes place (Resnick, 1989). The significance of the Vygostkian perspective is that it extinguishes the traditional boundaries between individual effort and social interactions as the individual comes to 'know'. Learning is situated in a community of practice, a community that involves "ways of doing things that are shared to some significant extent among members" (Lave & Wenger, 1991). In this context learning as an acquisition of knowledge is situated in social relationships, a process of social participation. Lave and Wenger (1991) illustrated their theory of learning by proposing differing stages of participation. To master new knowledge and skills new-comers are required to move towards full participation in the socio-cultural practices of the community. Thus, the participating teachers in this professional development were called to move from a position of novice, particularly with regard to their knowledge about a new mathematical domain. They were also invited to become part of a community of practice that was believed to be supportive of this transition.

Teacher knowledge, both content and pedagogical, is viewed as a key contributor to effective teaching (Bobis, Clarke, Clarke, Thomas, Wright, Young-Loveridge, & Gould, 2005). Teachers with more explicit and organised knowledge tend to provide instruction that features conceptual connections, appropriate and varied representations, and active and meaningful student discourse. Alternatively, those with limited knowledge have been found to portray the subject as a collection of static facts, to provide impoverished or inappropriate examples, analogies and or representations, and to emphasise seatwork assignments and/or limited student input as opposed to meaningful dialogue (Stein, Baxter, & Leinhardt, 1990). Many professional development models tend to focus on either content knowledge or pedagogical knowledge (White, Mitchelmore, Branca, & Maxon, 2004), but rarely on both. Thus, the model developed for this project contained specific elements that supported teachers' learning about patterns and algebra, and appropriate pedagogy that supported students' learning about patterns and algebra. It also provided inbuilt structures that encouraged pairs of teachers to share with each other in the development of their construction of learning in a particular focussed area, and for the group of teachers to share their co-construction of learning across different focus areas.

Thus, the premises that drove the selection of the model for professional development for the TTEYM were that: (a) teachers need to collaborate as they develop, implement and share learning activities; and (b) an expert voice assists teachers to reach their learning potential by scaffolding them to deeper understanding of new knowledge. The professional development model chosen for TTEYM contained specific elements that supported teachers' learning about patterns and algebra, with inbuilt structures that encouraged teachers to share with each other in the development of learning activities in the three focus areas, namely, patterns, equivalence and equations, and functions.

The TTEYM Model

The TTEYM professional development model was constructed on the principle that learning is cyclical consisting of four distinct components: Knowing Person; Collaborative Planning; Collaborative Implementation; and Collaborative Sharing. The theory underpinning the development of the model was that the completion of cycles of TTEYM would support a pathway of change guiding the novice learner to become an expert. Figure 2 presents the key components of the professional learning model together with the key focus of each.

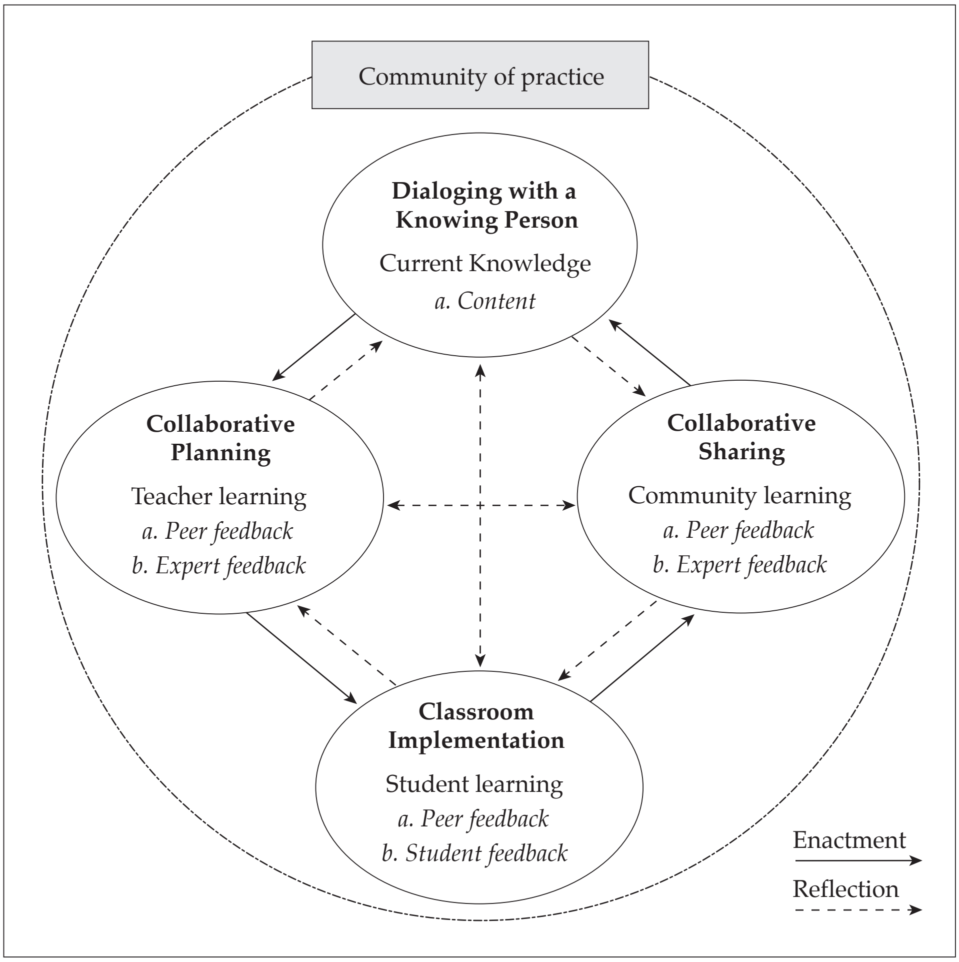

Figure 2. The TTEYM professional development model.

Method

The Mathematical Focus

The patterns and algebra strand consists of three distinct knowledge areas, namely patterns, equivalence and equations, and functions. These particular areas were chosen in the new syllabus because each underpins algebraic thinking in the higher grades.

Patterns. Patterns can be both repeating and growing. Repeating patterns can continue in both directions and can be 'split' into discrete repeats, the beginning stages of repeated addition. Growing patterns are concrete representations of number patterns. The importance of understanding growing patterns is that they lead to the development of an understanding of a variable which is well acknowledged in the literature.

Equivalence and equations. This strand aims to develop understandings of methods, symbol systems and language associated with balancing and solving equations. The focus in the early years is on equals as representing equivalent situations rather than indicating a pointer to the answer, a computational understanding of equals.

Functions. This strand develops an understanding of mathematics as consistent change and the inverse is represented as reversing the consistent change. Past research has indicated that many students complete primary school without the understanding that addition and subtraction are related operations. This strand attempts to explicitly support these connections by representing addition as change and subtraction as reversing this change or vice versa.

The Pedagogical Focus

Teachers were challenged to introduce the key concepts in each focus area using unmeasured quantities. The advantage of unmeasured quantities is that numbers are not required to investigate ideas such as equivalence and non-equivalence or generalisations such as $a = c + d$ then $c + d = a$ (Davydov, 1975). These can be explored by using concrete models such as streamers of differing lengths. In fact Davydov believes that the use of numbers can act as detractors from the focus on the underlying structure of mathematics. The introduction of numbers tends to invoke a propensity to compute. In numberless contexts it is believed that young students can investigate, conjecture and generalise about the 'big' ideas of mathematics, focus on processes rather than products, and develop relevant language and representations before they even formally commence number. For example, for the content area of functional thinking, in the first cycle the two teachers chose activities that did not involve numbers but focused on developing an understanding that functions are rules that we follow (Warren, Benson, & Green , 2007). These rules involve consistent change that could be reversed. One such rule is "make it larger". In the activity implemented in one of the classrooms students fed toys (e.g., a small blue teddy bear) into a function machine and a larger toy (e.g., a large blue teddy bear) emerged. They also reversed this rule and discussed how the inverse of "make it larger" is "make it smaller". In the second cycle TTEYM teachers mapped this thinking onto the operation "add 2" and its inverse "subtract 2".

Selection of the Participants

In an endeavour to explore each of the three content areas in some depth Education Queensland was asked to select three schools for the project. We also requested that each school provide two Year 1 teachers who were willing to work collaboratively to develop learning experiences for implementation in their classrooms. Thus, a total of six teachers participated in the project, two from each of three different schools. These teachers were also aware that their learning would be fully supported by an 'expert' in the area and the outcomes of the project would be posted on the web to share with other Year 1 teachers. Each pair of teachers was asked to choose one of the three content areas.

Implementation of the TTEYM Professional Development Model

The Professional development consisted of two cycles. In the first cycle teachers planned and implemented four lessons focussing on the key ideas in their chosen content area in numberless contexts. In the second cycle teachers then 'plotted' these understandings onto numbered contexts. At the completion of the two cycles all teachers presented at a forum attended by key curriculum personnel. The whole process was completed within a six month period.

For this research the initial entry to the model was through the Knowing Person component (the expert in both subject matter and pedagogical knowledge). In the beginning phases teachers were considered learners with the expert guiding and challenging the teachers as they constructed new knowledge and practices (Borko, 2004). The particular focus in this initial learning phase was on sharing the content knowledge, pedagogy, and the ability to identify activities and materials that encapsulate the knowledge and learning. The aim was to allow teachers to explore their knowledge and pedagogy of the patterns and algebra strand in an early years' context. Thus the decision was made by the expert that it was simply not enough to 'talk' about knowledge and pedagogy; there was a need to demonstrate how this might 'look' in the classroom context. Thus, a demonstration of the interaction between knowledge and pedagogy occurred in each teacher's classroom. The expert constructed and implemented learning experiences that were believed to be relevant for young children and that reach the heart of the knowledge contained in a particular strand. For example, one learning experience focussed on distinguishing growing patterns from repeating patterns using concrete materials in an investigative approach. The aim of this activity was to begin to expose the expert's Zone of Promoted Actions to the participating teachers.

In the next element of the model teachers worked in pairs, with the assistance of the expert, for Collaborative Planning of learning experiences in their chosen content area. Email contact between the expert and the teachers was maintained throughout this process with ongoing discussion about the choices of activities and their appropriateness. The participating teachers then implemented these ideas in their classrooms (Classroom Implementaton). All lessons were videotaped.

The final component of TTEYM, Collaborative Sharing, consisted of each pair of teachers sharing with the community of practice (the other teachers and the expert). Video excerpts of the lessons that they had implemented in their classroom assisted these discussions. The focus was on delineating the particular content and pedagogical knowledge on which they focused in their classrooms and sharing visual examples of student learning. As a result of this sharing and reflection, adjustments were made to the learning activities that each pair had developed. Thus, the model not only offered opportunities for teachers to learn but also allowed them to share their learning with other teachers and thus take on the role of 'experts'. The Zone of Free Movement was exemplified by allowing teachers to undertake their own planning bounded by particular broad content and pedagogical goals, such as developing hands on activities that supported the development of mathematical thinking within their chosen content area.

At the completion of the two cycles the teachers presented their learning activities to a large forum of personnel from Education Queensland, and their activities were published on the web.

Data Sources and Analysis

To investigate the professional learning of teachers and students, data sources included: (a) field notes; (b) videotape recordings of all lessons; (c) videotape recordings of the professional learning days; (d) interviews at the end of each cycle with a representative sample of six children from each school; and (e) two teacher interviews, one at the conclusion of the second cycle of the TTEYM and one eighteen months later. The data sources selected for this paper were the interviews, field notes, and video tapes of the professional learning days, the three data sets that specifically relate to ascertaining the effectiveness of TTEYM and identifying particular elements that supported teachers' learning.

For the first interview teachers were presented with a discussion stimulus that summarised key components of implementation of TTEYM (see Figure 3). The stimulus provided an initial starting point for sharing reflections on the process. Teachers were asked to share their learning experiences, commenting on particular elements that supported their learning.

The interviews were open-ended, allowing the respondents to direct the conversation with the interviewer while prompting in-depth reflections at appropriate phases. The interviews were conducted by a third party, thus maximising the reliability of the comments proffered by the participants. In the first interview participants were asked to comment on the particular components of the model, namely the role of the expert, the use of pairing in the school situation, and the opportunity to share with other teachers who participated in TTEYM. After transcription each interview was returned to each teacher for confirmation that the response was an accurate record of the interview. Each

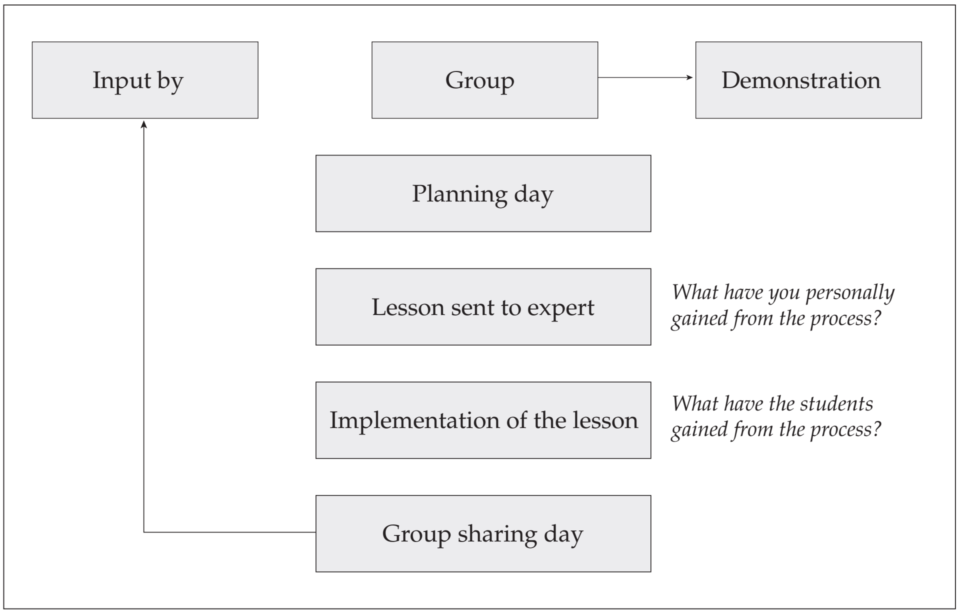

Figure 3. Stimulus for Interview 1.

transcribed interview was analysed using grounded methods. Incidental comments concerning their growth in knowledge and pedagogy and understanding of student learning were evident throughout their reflections.

Transcripts of the semi-structured interviews were analysed in two steps. Dick (2000) described the process of grounded theory as an emergent process with specific steps undertaken in the research situation. The first step comprised note-taking, coding and categorising, memoing, sorting, writing and constantly comparing data. The second step included a member check by another person as a validation process. The analysis was performed by the author and the member check was conducted by the person who interviewed the six teachers. Three core categories (categories with high frequency of mention) emerged from the data. These were mathematical knowledge, mathematical thinking, and personal confidence. Each had subcategories, categories that contributed to core categories, commonly referred to in the literature as the properties of the core category. The next section summarises the categories and subcategories with illustrative supporting quotations from participating teachers.

Findings

Key Components of Teachers' Learning

In the upcoming sections, the findings are presented by the categories and sub-categories that emerged from data analysis, along with examples from the teacher interviews.

Mathematical knowledge

Expert input: The two areas in which teachers gained insights in the initial conversations with the 'expert' were mathematics content knowledge, and sequencing knowledge.

> Teacher B: Very knowledgeable, great to get the information, challenged my understanding and made me think deeply. It showed me a lot easier way to simplify the maths, what it was about.

> Teacher D: I could see what I was doing in Year 1 was beneficial to Year 7 … and in the high school.

Demonstration lesson: This element contributed to the teachers' knowledge about mathematical language, the types of materials and activities that promote students' thinking, and pedagogical knowledge.

> Teacher F: Just listening to the terminology she used really helped us.

> Teacher A: I was pleased to see that she used everyday items and not elaborate aids. It was great to see my class 'thinking', very insightful.

> Teacher C: It was nice to see the hands-on approach.

Working in pairs: This also contributed to their increased content and pedagogical knowledge about patterns and algebra.

> Teacher A: Both of us had strengths and weaknesses. We balanced out well. Filled in the gaps.

Ongoing feedback: This element helped teachers to 'fine tune' their mathematical knowledge.

> Teacher B: She was able to correct my ideas, the way we structured the worksheet, so valuable.

All of the above components assisted pairs of teachers to deepen the knowledge they had about their particular focus area.

Sharing day: In the sharing day teacher pairs shared their new knowledge with the other four teachers. One area that all of the participants valued was the opportunity to learn from each other. In the area of mathematical knowledge they gained new content knowledge and knowledge about activities and pedagogy.

> Teacher F: The extra knowledge you gain [about maths] amazing. I learnt about sequencing, patterning and equations. It gave me access to new knowledge in my language and my vocabulary.

> Teacher E: So many new ideas. I couldn't wait to do these with my children.

Thus, knowledge construction was impacted by all four components of TTEYM, with each contributing in different ways. It seemed each acted as a context in which to draw attention to particular aspects of an overall understanding and knowledge about the patterns and algebra strand.

Mathematical thinking

Expert input: Teachers talked about how the initial conversations not only challenged them to think mathematically but also gave them greater understanding of students' mathematical thinking.

> Teacher D: It changed the way I think in the classroom.

> Teacher C: I had no idea of what this was. I know the students weren't thinking mathematically but I wasn't sure why.

Ongoing expert input: This component also supported the development of the teachers' mathematical thinking.

> Teacher A: It continually challenged me to think more deeply. We need this sort of feedback and ongoing discussion.

From the data it appears that the main component that encouraged them to think mathematically was the interaction with the expert.

Personal confidence

Demonstration lesson: This component contributed to each teacher's own personal confidence in his/her participation in the project. It also contributed to the relationship between the expert and the participants, giving the expert credibility in their eyes.

> Teacher A: Made us realise that she wasn't expecting these perfect perfect lessons. Be yourself, relax.

> Teacher B: She could actually connect with the children and her ideas worked.

Working in pairs: This also appeared to greatly contribute to the teachers' confidence to plan learning activities for their classrooms.

> Teacher C: I couldn't have done this by myself. The planning was very useful together.

Sharing days: These days affirmed the teachers' ability to teach mathematics in their own classrooms.

> Teacher E: It was great to see we all have the same challenges, problems. See other ways to work in the classroom.

Confidence about teaching mathematics seems to be underpinned by opportunities to compare one's own teaching with the teaching of others, be it an expert or other teachers.

TTEYM Model

Teachers were also asked to share some overall comments about the model and the benefits it provided for student learning.

> Teacher D: Maths knowledge gained ... very positive a great learning experience.

> Teacher A: I can now see why I didn't understand it and where the teaching was going wrong for me. I am thinking more deeply about the maths. Mathematical thinking for the first time in 32 years, sad isn't it.

> Teacher C. Each time I walked away feeling very excited about the ideas.

Participation in TTEYM was seen as a very beneficial learning experience, supporting these teachers to think more deeply about mathematics, and exciting them about learning mathematics.

While student learning was not an explicit component of TTEYM it is subsumed in the implementation component. In the interviews teachers contended that the experience supported students as independent learners, and encouraged the students to develop a deeper engagement with mathematical concepts.

> Teacher A: Even the weaker ones can go and get the counters … act it out, use concrete aids to find out what they don't know.
>
> Teacher C: They have gained a deeper understanding of number. They can visualise a lot more because they visualise it going through the [function] machine. Made them think outside the square.

Elements of the model that supported teacher learning. The interviews indicated that the main focus of the teachers' comments was on learning, especially in terms of the knowledge gained concerning mathematics and learning mathematics. The role of the expert within this dialogue thus appeared two fold; first in terms of assisting these teachers to access new knowledge and second in supporting these teachers to become experts in their own right. The learning appeared to occur as a consequence of four interactions: the first between the expert and the teachers; the second between the pairs of teachers; the third between the teachers and their students; and the fourth between all six teachers when they gathered to share their learning. Interestingly, the main reflection these teachers had with regard to TTEYM was couched in the language of learning and knowledge rather than in the language of change and growth, common themes that currently appear in the literature of professional development (Clarke & Peter, 1993; Guskey, 1986).

How the model assisted teachers to become more knowledgeable. The model provided another unexpected outcome – the teachers developing their own expertise and 'standing' as experts or knowing people. It appeared that this began to occur as each pair of teachers shared their new knowledge with the four other teachers during the sharing days. From an analysis of the videotapes each gradually adopted the role of 'teacher' with the remaining four acting as 'learners'. This was as a direct result of each pair being allocated different content areas to explore. After the second round of the learning cycle, they appeared more confident and could openly discuss the mathematics that they and their students had learnt. It can thus be conjectured that not only does the model offer positive professional learning for teachers but also opportunities for these teachers to develop into experts in their own right.

Eighteen Months Later

Eighteen months after the completion of the project we contacted the teachers and asked them to share what long term impact, if any, their participation in TTEYM had had on their engagement with mathematics. Four of the six teachers partook in this interview. The interview focussed on questions relating to the three core categories that had emerged in the previous interviews, namely mathematical knowledge, mathematical thinking, and personal confidence. The participants were also invited to comment on elements of professional development that they felt were important to classroom teachers.

Mathematical thinking. Analysis of the follow-up interviews revealed the increased mathematical thinking presently occurring in two of the classrooms was directly linked to the teachers' increased knowledge and confidence gained from participation in TTEYM.

> Teacher E: I think more deeply about mathematics now. I am much more confident and comfortable to allow children to explore what they think.

The most profound change occurred with Teacher C. Her body language at the beginning of TTEYM indicated her strong dislike for mathematics. She physically sat so that her body was turned away from the discussion. As indicated by her comments previously she exhibited the most substantial change during participation in TTEYM. This change has been sustained 18 months after contact had ceased. She shared these comments in the follow-up interview:

> Teacher C: The whole room is a babble of mathematics. Investigative approach. I now have a board with about 12 different strategies and encourage children to work out the answer. They know the first answer is not always right. Reflection and sharing of other children's responses to problems is now very very important. Maths is now as important as literacy in my classroom.

Mathematical knowledge. All three teachers believed they had gained a deeper knowledge about mathematics. Two continued to deepen this knowledge by engaging in further research and forging links with secondary mathematics teachers. They all shared many explicit examples that clearly showed a deeper knowledge about mathematics and students' mathematical learning. The following excerpt exemplifies a typical example of the discourse that now occurs in their classrooms.

> Teacher B: One child came in and said I know what 5 groups of 2 are. That is how many weekends before our excursion. So let's work it out … One child said it was 7. Another one said no it's not you are using the wrong process you are using addition we don't add [5 and 2] you need to make 5 lots of 2 and then add it up. [the child then proceeded to take out counters and act this out].

Personal confidence. The growth in personal confidence of all four teachers was evident in the post interview. One teacher's comments, inparticular, reflected the feelings of all participants:

> Teacher C: I have more enthusiasm for mathematics and more confidence in my ability in mathematics. I am more relaxed about ideas and go with children's understanding. There is more than one way to get the answer. I am seen as an expert in my school [in mathematics] and now help others. Three of us have presented at two conferences.

Professional development. The teachers were also invited to comment on what they felt were key elements in good professional development. Their comments ranged from "having at the heart of it ways of encouraging children's thinking" to "it has to be practical, a balance between theory and practice". All three agreed that participating in TTEYM had had a substantial impact on their whole perspective of mathematics. As the greatest change occurred with Teacher C, it is only fitting that she has the last say:

> Teacher C: Nothing [else] has ever changed how I have been teaching mathematics for 34 years. It is the only model that has worked for me. The other PD I have got these odd ideas and the little gems that I have used in the classroom but nothing has changed me as much as this. Now I have totally changed.

Conclusions and Implications

The Transformative Teaching Early Years Maths (TTEYM) Project proved to be an effective conduit for teacher learning not only in their specific focus area but also in other focus areas of the pattern and algebra strand. The literature offers insights into why it was successful. First, the model was firmly based in the Vygostkian theories of learning, where participants collaboratively shared (in a nonthreatening environment) what worked and what did not work. As suggested by Vygotsky (1934/1962) the inclusion of the expert in these conversations was crucial. The particular aspect that contributed to its success was what Resnick (1989) refers to as current knowledge, and knowledge that is highly aligned with the situation in which it takes place. A continual focus on learning, both teacher and student, resulted in the development of mathematical knowledge and mathematical thinking for both. The results also illustrate the stages of participation through which teachers progressed. They moved from being under-confident novices to experts in their chosen content areas. It could be conjectured that full participation in the community of learners (Lave & Wagner, 1991) is also representative of 'confidence and expertise'.

An interesting dimension that emerged from this experience was that as teachers came to an understanding of the mathematical knowledge, not only did their confidence in mathematics increase but so also did their willingness to experiment in the classroom and allow students' conversations to play a more directive role in classroom interplay. This conjecture extends the research of Bobis et al. (2005) in that as teachers become more explicit and organised in their mathematical knowledge they tend to provide instruction that features conceptual connections, appropriate and varied representations, and active and meaningful student discourse. In this research these elements appeared to grow simultaneously.

TTEYM possesses many of the characteristics identified by Darling-Hammond (1995) that support effective professional development. First, the model itself demonstrated the process of learning, and was grounded in inquiry, reflection and experimentation. Second, it was collaborative, involving the sharing of knowledge connected to and derived from student work, an important element of knowledge construction (Lave & Wegner, 1991). It also allowed for elements of reflection and included classroom implementation with a focus on student learning (Guskey, 1986). One cannot undervalue the cyclical nature of the model. As Clarke and Peter (1993) claimed, professional development should be cyclical, with many points of entry. There was a marked difference between teachers' understanding of and enthusiasm for mathematics as they proceeded through the second loop of the cycle. The role that the expert played throughout these cycles needs further investigation. The results of this study suggest that this role was a crucial catalyst for learning, or as Vygotsky (1934/1962) suggested a catalyst for reaching one's developmental potential.

More importantly, TTEYM provided a model for teachers to employ in their own classrooms. Its long term effect appeared to be in the establishment of classrooms where thinking was generated through inquiry and justification. For Teacher C this was a substantial transition. The post interview showed that two teachers, Teacher C and Teacher B had metamorphosed from tentative teachers in mathematics to experts willing to share their knowledge within their school and the wider educational community. It is conjectured that all elements in the model played significant roles in these transformations, especially the opportunity to collaboratively plan, implement and reflect on their own and their students' learning. The issue of readiness of teachers for change needs further investigation. Teacher C appeared to be the most reluctant participating teacher at the beginning of the professional learning but experienced the most substantial change. A characteristic that this teacher possessed was a firm understanding of the pedagogy appropriate for an early years setting. It would appear that once she had access to the mathematics content knowledge and its contribution to higher levels of mathematics' understanding she became extremely inventive and engaged.

In conclusion, TTEYM proved effective in assisting teachers to implement new curriculum that contained unfamiliar mathematics content knowledge and pedagogy. From the data it appears that its effectiveness was independent of the particular content knowledge being introduced. Its main strengths lay in the interactions between the 'expert' and the teachers, and the teachers themselves as they co-constructed new knowledge. Thus the model offers implications for the introduction of both mathematics and other curriculum areas within early years' classrooms. Key elements include opportunities for: (a) inquiry, reflection and trial implementation; (b) sharing student and teacher learning; and (c) conversations and interactions with an expert knowledgeable about the content area and the pedagogy needed to implement the content within the context of the classroom.

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