2008/2009, Vol. 10, 4–14 *MathematicsTeacher Education and Development*

Mathematical Communications: Elementary Pre-service Teachers' E-mail Exchanges with Sixth Grade Students

Zelha Tunç-Pekkan
The University of Pittsburgh

Beatriz S. D'Ambrosio
Miami University

This study examines the communication between pre-service teachers and sixth grade students in a project in which email was used for communication as students learned about fractions and were supported by the pre-service teachers. Specifically, the study investigated how the pre-service teachers applied their mathematical knowledge to understand children's mathematical responses and challenged those responses and mathematical ideas. Each of the 27 pre-service teachers sent 7 to 24 reply e-mails to their sixth grade partners. Those e-mails were examined and a two-way continuum emerged in terms of how they used the voice of the discipline and their disposition to the child's voice in their response e-mails.

In the *Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics (NCTM) (NCTM, 2000) emphasizes the role of mathematical communication, the use of technology, and students' productions of mathematical representations for all grades when learning mathematics. As teacher educators, we asked, "What kind of setting can teacher educators provide to pre-service teachers in which they could use technology and mathematics in their communications, and discuss mathematical representations in a content course?" We thought an authentic e-mail exchange activity between pre-service teachers and sixth grade students about fraction related problems could be an important teaching tool for these purposes. Pre-service teachers' mathematical e-mail exchanges would give them an extraordinary chance to focus on students' mathematical communications before they actually started teaching. In addition, it would be beneficial to us, teacher educators, to know those pre-service teachers in a different context in which sixth grade children are the initiators of the mathematical communications and pre-service teachers are the respondents.

Theoretical Framework

The mathematical content knowledge of pre-service teachers might affect their future instruction when they choose tasks, when they lead mathematical discussions, and when they assess their students' knowledge (Simon, 1995). Several studies have provided clarity as to how the content knowledge of teachers impacts their practice (Hill & Ball, 2004; Ma, 1999).

There are few studies investigating pre-service teachers' professional growth when they experience children's thinking in content courses. Ambrose and Clement (2003) documented pre-service teachers' learning experiences while taking their first content course and at the same time observing an elementary mathematics classroom. This content course was organized in such a way that the course could be thought of as a methods course (since observation of the mentor teacher's teaching was emphasized) even though the authors' stated focus was not about teaching skills. The pre-service teachers were not asked to interact with the elementary school children about their mathematical thinking in detail but mainly were required to observe the mathematics taught in the elementary classroom. The authors indicate their observations were only for the pre-service teachers' own benefit of learning the mathematical content better. Ambrose and Clement (2003) inferred from the pre-service teachers' written reflections and interviews that the mathematics in the college level content course and the observed mathematics in the elementary schools did not always complement each other.

In our content course we focused on parallel mathematical topics, for example fractions, which both pre-service teachers and sixth graders were studying at the same time. The pre-service teachers did not observe the teaching within the sixth grade classroom; instead they focused exclusively on the sixth graders' mathematical understanding provided in their e-mail explanations of the solutions of problems. The pre-service teachers used their own mathematical knowledge to make sense of the children's understanding.

Another study investigated the ways children's thinking can be incorporated into pre-service teachers' first content course (Phillip, Thanheiser, & Clement, 2002). The teacher educators showed interview clips on how to teach elementary school children and assess their knowledge; then they required pre-service teachers to tutor or interview the elementary school children. The course explicitly emphasized what elementary school children know and how pre-service teachers can teach them better. Even though this course is not called a methods course, the activities were very similar to the activities in a methods course. The pre-service teachers had to be practitioners (teachers), but not the ones involved in thinking about the students' solution in depth, when they interacted with their elementary school students. They needed to ask questions on the spot and be prompt when interacting with the children. This role is a huge responsibility for the pre-service teachers who are taking their first content course in a teacher education program.

In our content course, the children's thinking was often the focus of the course and also of the e-mailing project. The purpose of the e-mailing project was to create a disposition for pre-service teachers to listen to and to interact with the children by using their own content knowledge. D'Ambrosio (2004) indicates that the way (pre-service) teachers listen to their children is very important for their growth in constructivist teaching. She defines three different levels of listening as indicated by Davis (1996): *evaluative listening, interpretive listening*, and *hermeneutic listening*. D'Ambrosio (2004) claims that hermeneutic listening is the most compatible type of listening with constructivist teaching since the teacher needs to incorporate *the voice of the discipline* (her mathematical content knowledge, rules and logic of mathematics), *the voice of the children* (her ideas of how children learn certain mathematical ideas), and *her inner voice* (pedagogical

Content knowledge), "the tools that she [the teacher] believes best illustrates the concepts, and her understanding of the nuances involved in the learning of the content" (D'Ambrosio, 2004, p. 137)). We wanted to see how pre-service teachers were constructing the voice of the children using the voice of the discipline and how flexibly they used their voice of the discipline as they interacted with the students when writing e-mails.

Method

Participants

The participants were 27 elementary pre-service teachers from a non-traditional mid-western university for a semester. They were generally in an education program as a second career. Twenty-five of them were females. Students in this six-hour content course expected to be admitted to the elementary education program after finishing the course, but for communication purposes we will still call them "pre-service elementary teachers."

The Course

In this class, the "Learning via Problem Solving" method was used (Masingila, Lester, & Raymond, 2002). Students were given problems to solve in small groups before any instruction or discussion occurred. The problems were challenging for all group members and required a collaborative effort for the group to reach a solution. All group members contributed to the collaborative effort. In this setting, it became important for pre-service teachers to "understand" other people's solutions and to rely more on their own mathematical abilities, as well as to know the importance of building new mathematical knowledge through their own efforts (e.g. asking questions for their own understanding in group work). While pre-service teachers were getting used to reflecting on their thinking in their small group discussions, we required daily mathematics journals. In these journals, we wanted them to reflect deeply on the main mathematical problems, which were discussed in the class sessions. Our aim was to build habits of reflective thinking while improving their mathematical content knowledge.

The assessment in this class was very comprehensive. In addition to mathematics journals, we assigned homework problems related to the mathematical content studied during the week. The pre-service teachers had tests, which also emphasized group work. They did group projects and presentations at the end of the semester. They also wrote a paper about children's thinking based on the experiences in the e-mail discussion project as a requirement of the course.

The Project

In the e-mailing project, every pre-service teacher was paired with at least one sixth grade student who sent e-mails about one problem they had solved in their mathematics classroom every week. There were also times that solutions were faxed, for example when they used drawings of geometric shapes in their solutions. Our pre-service teachers were familiar with the problems because they had solved them by themselves as an assignment before they responded to the sixth graders. These responses were about the children's mathematical approaches and the pre-service teachers' own approaches to the same problems.

Pre-service teachers posted their first message in which they introduced themselves to their assigned sixth grade partners early in the semester. In return, the sixth grade students also introduced themselves via e-mail before a face-to-face meeting. During this project, pre-service teachers visited the middle school two times, when we started (to say "hello") and when we finished at the end of the semester (to say "bye"). On each occasion they played a mathematical game. Between these meetings, pre-service teachers and sixth grade students used a technological support service for classroom instruction for communicating via e-mail.

Each of the 27 pre-service teachers sent 7 to 24 reply e-mails to their sixth grade partners. Those e-mails concerned four mathematical problems related to fractions and algebra emphasizing reasoning with quantities.

Findings

Content analysis of the e-mails was used to understand pre-service teachers' content knowledge (the voice of the discipline) indirectly, and how they use this knowledge when building up a mathematical voice of the child in their interactions. According to Fraenkel and Wallen (2000):

> Content analysis is a technique that enables researchers to study human behavior in an indirect way, through an analysis of their communications. A person's or group's conscious and unconscious beliefs, attitudes, values, and ideas often are revealed in their communications. Analysis of such communications (newspaper editorials, graffiti, musical compositions, magazine articles, advertisements, films, etc.) can tell us a great deal about how human beings live. (p. 469)

Analyzing the e-mailing task and learning a great deal about pre service teachers' content knowledge, what D'Ambrosio calls the voice of the discipline, and how it is used to tune into the voice of the children is similar to Fraenkel and Wallen's (2000) "indirect way" of studying human behavior.

To illustrate our study, we will use one fraction related problem and corresponding e-mail exchanges. These sample data will allow us to talk about how we can conceptualize pre-service teachers tuning into the voice of the children, and what type of mathematical knowledge is needed to understand and carry out mathematical conversations with the children. We use the children's mathematics evident in the e-mails as our starting point and analyze the pre-service teachers' use of mathematics in their e-mail responses.

The first problem was posted by the sixth grade teacher as follows:

> Three brave, but not very bright, treasure hunters recovered a small box of Spanish doubloons aboard a sunken ship. They took the coins back to their camp-

> site. Since it was late, they decided to go to sleep and divide the treasure the next day. One of the treasure hunters, fearing the others didn't understand mathematics well enough to give out fair shares, took one third of the coins in the middle of the night and fled into the darkness. Later that night, another treasure hunter awoke and saw that some of the coins were missing. The treasure hunter took one third of the remaining coins and fled into the darkness. The third treasure hunter awoke and was surprised to see the others gone and many of the coins were missing. Trusting that the others left a fair share, the third treasure hunter took the remaining coins and walked away whistling happily. Which of the treasure hunters ended up with the greatest share of doubloons?

The following e-mail exchanges are examples of how pre-service teachers' mathematical knowledge is differentiated in their e-mail responses. By examining these e-mails, we also created possible mathematical explanations for the pre-service teachers' particular ways of interacting with their sixth grade partners. The categorization of pre-service teachers' e-mails is inevitable if we consider how they use their mathematical knowledge and whether they use it in the service of hearing the voice of the child. Three categories emerged from the data: (1) voice of the children and voice of the discipline; (2) voice of the discipline; and (3) the lost voices.

Category 1: Voice of the children and voice of the discipline

The pre-service teachers' responses in this category were able to discuss or introduce different solutions to the problem, or create related examples in their response, pushing the child to think further and thus demonstrating that they themselves had thought more deeply about the problem. Unfortunately, there were not many response e-mails in this group for this problem, as only four pre-service teachers' responses were inspiring enough that they demonstrated they heard the voice of the child and used the voice of the discipline at the same time.

The following emails provide an example of a pre-service teacher, Charlotte, who is aware of the voice of the discipline and who hears the voice of the child.

> *Sixth grader's e-mail (Charlotte's partner):*
>
> Now let's get down to business with the Fair Share problem. I 1st set the treasure # at 1 because it would give me the fractions. I subtracted 1/3 because that is what the 1st guy took. Next, I took away 1/6 for the 2nd guy, and I don't know why now, and that gave me 3/6 or 1/2 for the last guy.
>
> I am confident that my fi[r]st answer is correct, but my 2nd and 3rd aren't. I know that a third of a third is a nineth. That would make 1/3 of 2/3, 2/9. You multiply because that is what of means. 2/3-2/9=4/9. Those are the only parts I had trouble with. Either way the 3rd guy comes out ahead. Ask if you have any questions.
>
> *Charlotte's response (e-mail 1):*
>
> You ended up with all of the shares being divided ok and yes the last guy ended up with more than the other two. Can you think why after the first guy took 1/3 leaving 2/3 you would say the second guy took 1/6? There were 2 shares

> remaining equal to 2/3 of the total. The second guy took only 1/3 of the 2/3. It might help if you look at the whole in fractions that are divisible by 3, 6 and 9.
>
> Let me know if this is too confusing or if it helped.

There was no response from the child, but Charlotte wrote another e-mail.

> *Charlotte's response (e-mail 2):*
>
> Okay let's finish 'Fair Share' first. I didn't mean to confuse you with the 3, 6, 9. Here's my thinking on the problem. We have a whole that guy 1 divides into three and then takes 1/3. Guy 2 splits the 2/3's remaining into thirds and takes 1/3 of it. How can we divide the 2/3 equally into 3/3? If I split the 2/3 into sixths there would only be 4 to split up because 2 of those sixths were taken by the first guy. I can't split 4/6 into 3 groups equally. Now I decide to split the 2/3 into ninths. Since I know the first guy took 1/3 of the ninths that is equal to 3/9. There are 6 ninths remaining to split into 3 groups (2 per group) of 2/9 each. Guy 2 took 1/3 equal to 2/9, leaving guy 3 with 4/9. This sounds more complicated than it should. I am often not very good at explaining things so don't worry about telling me you don't understand what I'm saying!!

The child was struggling with the idea that there was something wrong with her thinking on the shares of the second and third treasure hunter. Charlotte was asking a simple but powerful question to open it, "Can you think why after the first guy took 1/3 leaving 2/3 you would say the second guy took 1/6?" Charlotte was not restricting the child's thinking into procedure by her question (e.g., how did you get one sixth, multiply or divide?); her question was very open. Unlike the other pre-service teachers, Charlotte also had an interesting thinking process on the Fair Share Problem about having thirds, sixths, and ninths. Her explanation to decide what to use for dividing the whole (into 3, 6, or 9) was related to the amount of the treasure that was left for the second and third treasure hunters, and their shares.

Another use of the voice of the discipline was to make students think about any number, variable, and finally generalizations in their solutions. These responses acknowledged the child's solution and focused on encouraging the student to make generalizations for this problem and challenged his/her thinking. This is exemplified in the following example of the email response from Denise (pre-service teacher) to her sixth grade partner.

> *Sixth grader's e-mail (Denise's partner):*
>
> The third man would have gotten the most coins. Say there were 33 coins to begin with. If the first man took 1/3, there would be 22 coins left. The first man got 11 coins. If from that 22 coins, the second man took 1/3, he would have taken around seven. There would be 15 coins left. The third man took all 15, making him have the most coins.
>
> *Denise's response e-mail:*
>
> I do believe that your answer to the fair share problem is correct. I did the problem last night and I figured out that the last guy would get the most coins also. No matter how many coins they started out with, the last person would always get the most. Do you know why that is?

Denise was looking at the problem by considering that the amount (the number of the coins) in the whole did not matter as long as the fraction parts were the same for each case (e.g., the first treasure hunter always took a third of the treasure or the second treasure hunter always took a third of the remaining treasure). Denise had the ability to generalize, in which case she did not depend on a certain number of coins. This shows different understanding; she went beyond the procedures that were valid for just one occasion (e.g., 33 coins in the treasure) and she was generalizing the result conceptually.

Category 2: Voice of the discipline

The e-mail responses in this category showed the pre-service teachers accepted the child's thinking but did not try to understand child-constructed mathematics deeply. They sometimes did not recognize the child's explanation or solution as a legitimate solution since the child was not using procedures that the pre-service teachers were used to. At those times, they asked procedural questions, since they were lost in the child's explanation, or they were not sure of their results after comparing them with the child's results. They were only using the voice of the discipline, but unfortunately in an inflexible way which prevented them from listening and coping with the children's mathematics. This can be seen in the following example involving Rose and her sixth grade partner.

> *Sixth grader's e-mail (Rose's partner):*
>
> The paragraph says the first person takes 1/3 of the treasure. The second person got one sixth. I know this because he took 1/3 of what was left. There was 2/3 left 1/3 of 2/3 is 1/6. The last person got the most because he got 1/2. I know he got 1/2 because I converted 1/3 into 6ths so I could add 1/6 and 2/6 which equals 3/6. 6/6 - 3/6= 3/6. Which when reduced equals 1/2.
>
> *Rose's response e-mail:*
>
> When you said you took 1/3 of 2/3 and got 1/6, were you multiplying or dividing? How do you know this? I was just curious why you thought that way. We are also going over fractions in class right now, and it has been a long time since I have worked with fractions, and by reading your steps to solving problems I have started to remember them again.

Rose posed a simple question to follow how the child got a sixth when he took a third of two-thirds of the treasure, but thinking procedurally, she asked a procedural question "were you multiplying or dividing?" Rose's question showed that she was not thinking algebraically because either way when one multiplies or divides 1/3 with 2/3, the result can't be 1/6. Rose's questions might show that she realized that the child got 1/6 of the treasure for the second person, so the child got an unexpected result for the third person depending on what he found for the second person. However, Rose's intent to know about used operations was neither related to analyzing the child's thinking, nor related to the meaning of operations that the child used while getting 1/6 of the treasure as his answer. Rose's e-mail implied that memorizing and remembering what mathematical operations to use are more important than understanding the mathematical reasoning when finding the result.

Another sixth grader solved this problem in a similar way to how Rose's partner solved it and received different answers for the second, and third treasure hunter. Interestingly, those sixth graders were answering the problem correctly (who got most?), but when they were using fractions and trying to be specific in their results for the treasure hunters, their solutions did not match with most of the other sixth graders and conventional fractional procedures. However, in the following exchange between Alex and her sixth grade partner, Alex did not even realize the child's work needed to be questioned further mathematically. This situation gives evidence about the pre-service teacher's inadequate fractional knowledge which prevents her from being in a productive interaction with the sixth grader to gain insight into the sixth grader's operational thinking with the fractions.

> *Sixth grader's e-mail (Alex's partner):*
>
> I noticed that the second guy was the stupid one, because if the dubloons were to be even, #2 would have taken 1/2. Instead, he took 1/3, accidentally leaving 2/3 left for 2 more people, in which there was only one person, so getting this from the original number, the 1st guy got 1/3, the stupid one got 1/6, and the lucky guy got 1/2.
>
> *Alex's (pre-service teacher) response e-mail:*
>
> I think your reasoning on the fair share problem was awesome, although I really had to think about the problem. When I read your reply, I was like, "Oh, yeah, that makes sense." That second one really wasn't very smart.

Alex thought the child's way was "awesome"; what was awesome in this solution for Alex? Maybe finding the correct result of who got most of the treasure in four sentences was "awesome". How can we be sure about this child's thinking without asking what she means by saying "if the dubloons were to be even, #2 would have taken 1/2" or further deepening the child's thinking by asking how the child was comparing the second share to the first one's share and the third one's share? Another important point is that this child found unexpected results for the problem; the child said "the stupid [the second treasure hunter] one got 1/6, and the lucky guy got 1/2". Alex didn't ask her sixth grade partner about how she got 1/6 for the second treasure hunter or 1/2 for the third treasure hunter or what those fractions were part of. Alex had a positive disposition to the child's work. However, this disposition is not enough to be a good mathematical partner or a teacher.

Other e-mail exchanges indicate that some pre-service teachers did not hear the voice of the child and did not even realize that they were asking questions the children already answered. Perhaps, they did not understand the original thinking of the child since they did not have flexibility in related mathematical knowledge or they looked for certain answers but could not realize that the children's answers were not the conventional results. However, it was the thinking behind those results that mattered for these creative sixth graders.

There were 12 e-mail responses of pre-service teachers placed in Category 2. These responses showed that the majority of the pre-service teachers attended to the voice of the discipline in a way that forced the pre-service teacher to forget about the individual child's mathematics. It might also be these pre-service teachers had a certain mathematical thinking about the problems and could not accept any other mathematical explanations the children might have given. They did not challenge those child initiated mathematical explanations, and perhaps these pre-service teachers had a hard time understanding them.

Category 3: The lost voices

Some pre-service teachers avoided responding to their partners for this problem. Eleven out of 27 pre-service teachers were placed in this category. Five of them didn't respond for the other two fraction problems either. Five of them skipped this problem, perhaps because they didn't have enough mathematical knowledge to use for the interaction with their sixth grade partner, or they were not comfortable using writing as the communication tool for mathematics or they were too busy at that particular time of the semester. One of those 11 pre-service teachers was writing about everything but the mathematics in this problem. So, this situation raises a question about the pre-service teachers' professionalism since they wanted to serve their future students, yet they avoided responding mathematically to their motivated sixth grade partners. What are the reasons behind this avoidance? We primarily believe that these pre-service teachers were not confident that they could understand what their sixth grade partners could do mathematically and about the possible ways they could interact with their partners using mathematics.

Discussion

A tentative two-way continuum array emerged from the analysis of e-mail exchanges about the four mathematical problems in the study. Pre-service teachers are continuously placed in this two-way continuum array with respect to the evidence of their mathematical knowledge and their disposition to the voice of the children. The use of this two-way continuum array helps us realize that each pre-service teacher's response, although distinguishable from the others, is not precise (see Figure 1). For example, the response e-mails were different in how the pre-service teachers' own explanations were given for the solutions, how the sixth grade partners' explanations were interpreted, and how the sixth graders were challenged. However, the buildup of these e-mails in this two-way continuum array made it easy to see the general picture of the pre-service teachers' mathematical understanding (the three categories) as well as their disposition to the voice of the children's work.

Three regions of this continuum array describe the relationships of pre-service teachers' knowledge and their disposition (openness) to children's thinking. For example, if the pre-service teachers were open to listening to, discussing and valuing the child's solutions and explanations for the four

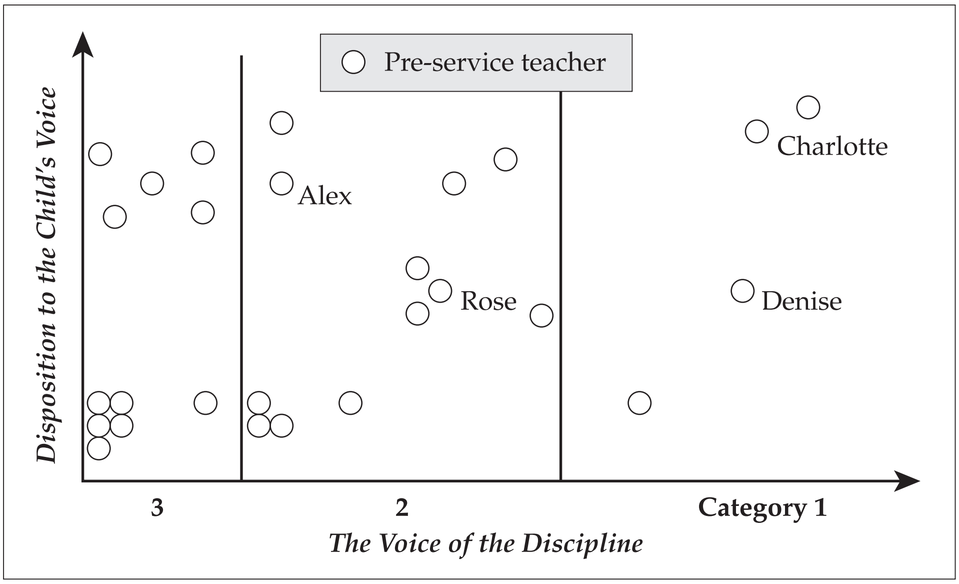

Figure 1. Two way continuum.

problems, they were placed on the upper part of the vertical axis labeled as *disposition to the child's voice* (away from the intersection of two lines), and that meant their disposition was positive. The pre-service teachers in the upper right corner of the continuum have constructed a flexible voice of the discipline and also the disposition to listen to the children positively. They are the type of teachers that have the potential to be "the most wanted teachers"; because they have enough mathematical knowledge for listening to and strengthening the child's understanding of mathematics. On the other hand, the pre-service teachers in Categories 2 and 3 (the areas between the vertical lines) need to improve their content knowledge. While some of them in those two categories are in a more advantageous situation because of their positive disposition to children's mathematics (the ones on the upper part of the vertical axis), their mathematical knowledge needs attention.

There are still questions to be answered about pre-service teacher's content knowledge and its relationship to their disposition to the voice of the children:

1) To what extent can we differentiate among the pre-service teachers who are in Category 1 (assuming they have the voice of the discipline in a flexible way), if we are concerned about what is necessary for hearing the mathematical voice of the children?
2) Can the pre-service teachers who do not show a positive disposition to children's voices (in Category 2) have flexibility in the voice of the discipline?
3) Does having the disposition to children's (mathematical) voices put some of these pre-service teachers in a better position to work with teacher educators than the pre-service teachers who just have the voice

of the discipline? What kind of experiential situations do the teacher educators need to provide to students in these categories so that they can improve themselves and hopefully move to Category 1?

Besides having classroom assignments, non-traditional tasks, like an e-mail project, in content courses can help pre-service teachers have more opportunities to communicate mathematics, learn more about children's mathematical thinking, and gain experiences in asking mathematical questions to children before they undertake the methods courses or field experiences. These types of settings that include children as initiators might also help teacher educators know their pre-service teachers better. Unlike adult communications in mathematics education classrooms, communicating with children requires different dispositions towards mathematics itself and towards the children, and a different way of using one's mathematical knowledge.

References

Ambrose, R., & Vincent, C. (2003). *Blending mathematics learning with an early field experience: What do prospective elementary school teachers learn?* Paper presented at the Annual Meeting of American Educational Research Association, Chicago, IL.

D'Ambrosio, B. (2004). Preparing teachers to teach mathematics within a constructivist framework: The importance of listening to children. In T. Watanabe, & D. R. Thompson (Eds.), *The work of mathematics teacher educators,* (AMTE Monograph Vol. 1, pp. 135-150). San Diego, CA: Association of Mathematics Teacher Educators.

Davis, B. (1996). *Teaching mathematics: Toward a sound alternative*. New York: Garland.

Hill, H., & Ball, D. (2004). Learning mathematics for teaching: Results from California's Mathematics Professional Development Institutes. *Journal for Research in Mathematics Education, 35*(5), pp. 330-351.

Fraenkel, J., Wallen, N. (2000). *How to design and evaluate research in education* (4th edition). New York: The McGraw-Hill.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Ma, L. (1999). *Knowing and teaching elementary mathematics*. Mahwah, NJ: Erlbaum.

Masingila, J., Lester, K., & Raymond, A. (2002). *Mathematics for elementary teachers via problem solving: Instructor manual.* Upper Saddle River, NJ: Prentice-Hall.

Phillip, R., Thanheiser, E., & Clement, L. (2002). The role of a children's mathematical thinking experience in the preparation of prospective elementary school teachers. *International Journal of Educational Reform, 27*(2), 195-213.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*, 115-144.

Authors

Zelha Tunç-Pekkan, School of Education, University of Pittsburgh, Pittsburgh, PA, USA 15260. Email: zelha@pitt.edu.au

Beatriz S. D'Ambrosio, Miami University, Oxford, OH, USA.
Email: dambrobs@muohio.edu