The Strategies of Using the Generalizing Patterns of the Primary School 5th Grade Students

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Abstract

The main purpose of this study is to determine the strategies of using the generalizing patterns of the primary fifth grade students. The practice of this research is conducted on twelve students, which have high, middle and low success levels. Task-based interviews and students journals are used as the tools for data collection. For the analysis of the data, a classification method including "data reduction", "data display" and "drawing conclusion and verification" are used. At the end of the research, it is seen that the visual and numerical approaches are adopted in the generalization of patterns and the visual approach is made easy for generalization, as well. In generally, the present strategies in the generalizing of patterns are also taken into account of near or far generalizing. The recursive strategies are used in the near generalizing. However, the explicit strategies are determined in using far generalizing.

Key Words

Elementary, Mathematics Education, Pattern, Generalization.

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A systematic combination of geometric shapes, sounds, symbols, or actions is defined as a pattern (Souviney, 1994). According to Guerrero and Rivera (2002), a pattern is the rule between the elements of a series of mathematical objects which are constructed. It is defined by Olkun and Toluk-Uçar (2006) as a system of repetitious and orderly arranged objects or shapes. Also, Papic and Mulligan (2005) defined a pattern as a spatial or numerical regularity. According to structures and presentation styles, patterns can be combined in two groups as repeating and changing (Olkun &Yeşildere, 2007). A pattern is a key concept for understanding of mathematical knowledge and concepts. Pattern studies are the basis for understanding of the system and logic of mathematics and the observing of mathematical relationships (Burns, 2000). Mathematical expedition and number sense for children have been developed by patterns (Reys, Suydam, Lindquist, & Smith, 1998). Especially, a pattern is an essential element of mathematical development for young children and also a central construction of mathematical inquiry (Waters, 2004). The mathematical knowledge and skills of young children develop with the process as counting, comparing, classifying, measuring, representing, estimating, and symbolizing. But patterns form a basis to build these mathematical efficiencies (Fox, 2005). Pattern activities performed at kindergarten level have important roles for forming of the basis of algebra. In other words, the studies with patterns and the relationships between patterns are a prerequisite and a basis for developing. In the beginning, the introduction to algebra with patterns enables the difficulties for formal algebra (Resnick, Cauzinille-Marmeche ve Mathieu, 1987 cited in Threlfall, 1999; Orton & Orton, 1999; Zazkis & Liljedahl, 2002; Orton & Orton, 1994; Herbert & Brown, 1997). So, it is necessary to have previous experiences with patterns for developing algebraic thinking and concepts. Generalization, which is the means of communication and the tool of thinking, is the basic for the development of mathematical knowledge and the center of mathematical activities. National Council of Teacher of Mathematics (NCTM; 2000) standards call for generalization as one of the main goals of mathematical instruction. A pattern is an essential step for the formation of generalization. It can be seen that the generalization is a basic structure of algebra and patterns are the basic structure of generalization. Jones (1993 cited in Hangreaves, Shorrocks & Threlfall, 1998) implies that the generalization is the principle of algebra and the search of pattern is the first step for generalization. Also, Kaput (1999) defines algebra as "formation of patterns and constraints and the generalization." According to Kieran (1989 cited in Radford, 2006), the generalization of a pattern as a route to algebra rests on the idea of a natural correspondence between algebraic thinking and generalizing. A pattern generalizing is a major factor for developing the algebraic thinking of children-with particular importance for the development of the concepts of variable and function (Lesley, & Freiman, 2004).

There are abundant number of research studies on the shape or number patterns in the literature (e.g., Carraher, Martinez & Schliemann, 2008; Amit & Neria, 2007, 2008; Lan Ma, 2007; Warren, Cooper & Lamb, 2006; Becker & Rivera, 2005, 2006; Krebs, 2005; Lannin, 2005; Ley, 2005; Looney, 2004; Orton, Orton & Roper 1999; Hargreaves, Shorrocks & Threlfall, 1999; Sasman, Linchevski & Olivier, 1999; Garcia-Cruz & Martinon, 1998; Garcia-Cruz & Martinon, 1997; Stacey, 1989). These studies are used for the determining process of the strategies and approaches. It is seen that the two approaches (i.e., visual and numerical) are the first used in generalization of these patterns when patterns are given with shape representation (Becker, & Rivera, 2005, 2006; Garcia-Cruz, & Martinon, 1997; Krebs, 2005; Lan Ma, 2007; Orton, Orton, & Roper, 1999; Stacey, 1989). Also, three main generalization strategies are defined in research studies are: *(i)* Recursive approach-the using of previous term, *(ii)* Explicit approach-searching for the functional relationship (Amit, & Neria, 2007, 2008; Carraher, Martinez, & Schliemann, 2008; Lannin, 2005; Ley, 2005; Looney, 2004; Sasman, Linchevski, & Olivier, 1999; Stacey, 1989; Warren, Cooper, & Lamb, 2006), and *(iii)* Whole-object making incorrect proportional reasoning, using the ratio $f(x)=ax$, when the relation is $f(x)=ax+b$ ($b\neq 0$) (Stacey, 1989). In Turkey, no research on the generalization of patterns has been found for primary grade levels. Threfore, the aim of this study is to determine the thinking process and generalization patterns of fifth grade students.

Purpose

The main purpose of this study is to determine the strategies, which are used for the generalization of patterns. In this scope, according to the success levels of students, the following research questions are addressed:

1. How do students find the rule of the linear and quadratic shape patterns?
2. How do students extend to the near and far step to pattern?

Method

The study was conducted on twelve primary fifth grade students who had different mathematics success levels (i.e., high, middle, low). Selecting the participants for the study was chieved using purposeful sampling techniques, so the criterion sampling technique was used (Yıldırım, & Şimşek, 2005, p. 112). One criterion was the level of class which is the primary fifth grade; another was the students who had three-different success levels.

Data Collection Process

The data were collected through "task-based interviews" and "student journals." The task-based interview is a technique purposed by Piaget to study the form of knowledge structures and reasoning processes (Clement, 2000). The task-based interview gives the important clues concerning the nature of students' thinking (Ginsburg, 1981). After determining the aims for the research, the task-based interview, which includes the pattern questions, were prepared. Linear and quadratic shape patterns took part in the task. Then, the task-based interview questions which determine the thinking process for resolution of students' interview task were arranged.

The prepared task-based interview, task and questions were presented to two-experts. The tasks were taken on the piloted group reassembly similar features. The study was conducted at a primary school in Eskişehir, and the interviews were video recorded. Adequate time was given for the students to do find their solutions, and the total interview time was arranged for approximately 35-50 minutes was arranged for total interview time (Hunting, 1997). At the end of each interview, the students are given extra time to write their feelings and thoughts about the task in their journals.

Data Analysis Process

Before analyzing the data, the data which were obtained from the task-based interviews, were transcribed and checked by another field expert (Kvale, 1996). For the analysis, the classification method which includes "data reduction", "data display," and "drawing conclusion and verification" has been used (Miles, & Huberman, 1994, pp. 10-12). In the data analysis process, three fields independently coded the data. For the reliability of coding, the agreement percentage suggested by Miles and Huberman (1994), was used as following;

Reliability=number of agreements/(total number of agreements)+(disagreements)

And the result was found to be 91%. The researcher and the two field experts independently studied the data. The common data were categorized under several sub-themes which constitutes themes. The themes are concepts which are discovered from the research data (Bogdan & Biklen, 1998; Meriam, 1998). Themes can be formed from the expressions of participants as well as formed by the researcher (Patton, 1990). Themes were developed by sub-themes, sub-themes, categories in sub-themes and relationships between them. Hereby, the data were displayed. The existing themes and relationships between themes were evaluated and compared under the research questions. The findings were supported by the questions elicited from students' journals.

Results

Finding the Rule of Pattern and Its Investigation

As seen in Figure 1, when they found the rule at linear and quadratic shape patterns, students adopted two approaches as "visual" and "numerical." In the visual approach, students focused on the structure of the shape and in this context, they used the strategies in recursive and explicit strategies. The majority of the students obtained the latter shape from the previous shape in both two patterns within recursive strategies. According to the explicit strategies, some students discovered a functional relationship based on the structure of the shape in linear shape patterns, and others found a functional relationship in quadratic shape patterns. However, the number of students finding the relationships in linear shape pattern was higher. Thus, we have an impression that the structure of given shape pattern was effective for findings a functional relationships. Half of the students who had high-success levels found the functional relationships in each two patterns, the rest of the high-success level students and half of middle-success level students, found functional relationships in a pattern and all of low-success level students, only found the functional relationships in linear shape pattern. In the use of this strategy, students who had low success levels were more successful than middle-success level students. So, we can think that the visual perceptions of low-success level students are more powerful than middle-success level students.

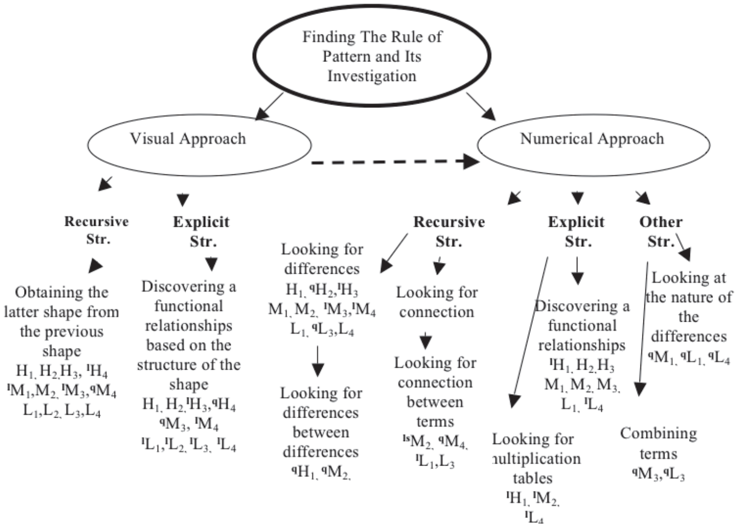

Figure 1: Strategies That Used in Finding the Rule at Linear and Quadratic Shape Patterns

First of all, the students using the numerical approach transformed the shape pattern to the number pattern. In this correlation, students used the strategies, which take part under the recursive, explicit and other strategies. Most students, who adopted the recursive strategies, found the difference between the terms at each pattern, but each student, found the difference of distinction at quadratic pattern. In scope of this research, some students at the middle and low level success used the "looking for connection" strategy. Actually, students using this strategy implied the looking for difference strategy between terms in alternative form because students looking for a different rule and want to form a connection between the latter and previous terms. In the scope of numerical approach, some students also found a functional relationship in both two patterns. These students generalized the terms of finding number pattern with step numbers and they expressed them verbally.

Verbally expressing the step numbers of patterns was effective for generalization. A functional relationship was found by each student who had high and low success levels at linear number pattern. Whereas, others found two patterns. So, in linear shape patterns, a functional relationship was found by many students in comparison to quadratic shape patterns. However, the number of students who found a functional relationship was smaller than the number of students using the looking for difference strategy at recursive strategies.

Extending To a Near and Far Step of Pattern

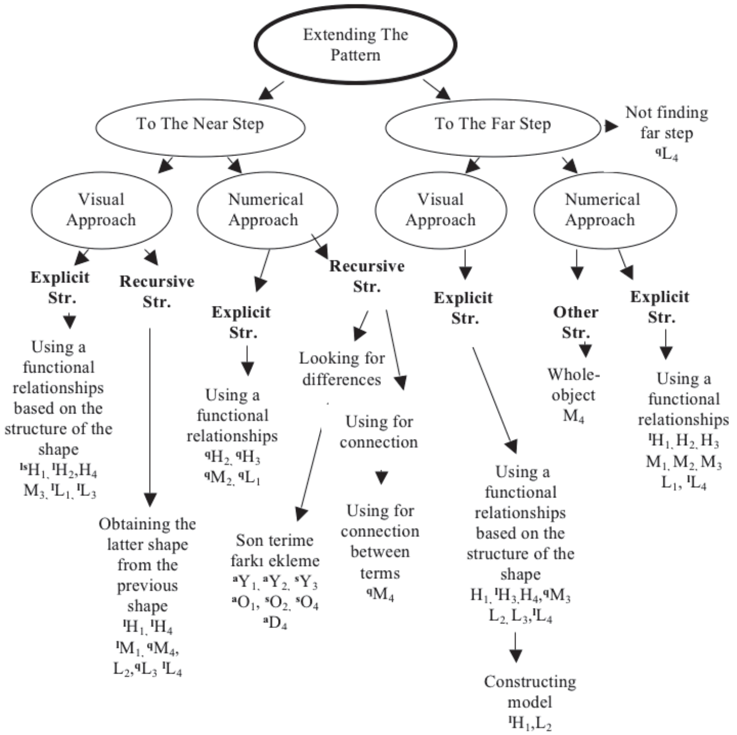

Figure 2: Strategies That Used in Extending to a Near and Far Step of Linear and Quadratic Shape Patterns

As seen in Figure 2, the numerical approach was adopted by fewer students when the patterns were extended to near step. In scope of this approach, some students used the functional relationship, but others used mostly looking for the difference between terms. One the other hand, in general, students who had high and middle-success levels adopted the visual and numerical approaches at each patterns, whereas students who had low-success level usually preferred the visual approach. When the linear or quadratic shape patterns were extended to the far step, the explicit strategies were used in scope of the visual approach. The explicit and other strategies were used in the numerical approach. In these patterns, the majority of the students got the number of shape in the fiftieth step with a functional relationship. But, a middle success level student was considerably coerced when he/she extended the pattern to the far step because he/she focused on the recursive strategies in each two patterns, as well. In this context, he/she carried to the entire extension strategies within other strategies and arrived to a false result. On the other hand, a low-success level student extended the linear shape pattern to the far step using a functional relationship under the numerical approach. However, he/she did not extend the quadratic shape pattern to the far step. In general, high-success level students adopted the visual and numerical approaches, but middle-success level students adopted the numerical approach, and low-success level students adopted the visual approach.

Discussion

In many research findings, when the rules of the linear and quadratic shape patterns were determined by students, the structural properties of the shape at step of patterns were investigated. And the students adopted the visual approach for the formation of shapes and the numerical approach concerning shapes in each step (Becker, & Rivera, 2005, 2006; Garcia-Cruz, & Martinon, 1997; Krebs, 2005; Lan Ma, 2007; Orton, & Orton, 1999; Orton, Orton, & Roper 1999; Stacey, 1989). Moreover, the majority of these students adopted the visual and numerical approaches, but some students adopted only the visual approach (Moses, 1982; Noss, Healy, & Hoyles, 1997; Stacey, 1989 cited in Barbosa, Palhares, & Vale, 2007). It is seen that there is a relationship between the success levels of students and the use of the visual and numerical approaches. Usually, high-success level students adopt both approaches; middle-success level students adopt rather the numerical approach, and low-success level students adopt the visual approach. In getting results, it is seen that the visual approach and the structural properties of patterns simplified the generalization, as well. Students used the recursive, explicit, and the generalization strategies, which define by this research and were used by some of the students under the visual and numerical approaches. The strategies within generalization of patterns were usually considered as the near and far generalizations. However, when the rule of pattern was defined as correct in continuity of patterns this rule was not considered or used as faulty (Orton, & Orton, 1994; Sasman et al., 1999; Stacey, 1989). Also, within far generalizations the focus was on the explicit strategies, and within the near generalizations the focus was on the recursive strategies, which involves the use of previous terms (Stacey, 1989).

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