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Negotiating Roles and Meaning While Learning Mathematics in Interactive Technology-Rich Environments

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Abstract: The authors examined how undergraduate students negotiated roles and developed a shared understanding of mathematics while working together on computer-based modules. The subjects were videotaped while working on these modules and their computer output was simultaneously collected on a separate videotape. Examination of the data tentatively suggests ways pairs of students approach academic work and ways they interact as they process and organize knowledge. This study builds on an exploratory study the authors conducted that generated a set of research questions addressing the nature of learning in interactive technological environments.

Key Words: Mathematics education, Computer algebra systems, Learning styles, Collaborative learning, Interactive learning.

I. Introduction.

In the early 1900's, Edison predicted that motion pictures would make books obsolete and in the 1950's, many mistakenly believed that educational television would revolutionize schooling (Reiser, 2001). Personal computers and the Internet, like TV in the 1950's, have exploded on the educational scene in the last decade with hyperbolic promises and predictions about how they will affect the way we teach and learn. Similarly, collaborative approaches to learning have been endorsed as a means of ensuring deeper and more authentic learning. As teachers many of us are eager to embrace these educational innovations that are touted as holding great promise of energizing our classrooms. Often, however, teachers adopt curricular and instructional changes without carefully evaluating the efficacy and consequences of these new approaches.

This study builds on an exploratory study the authors conducted that generated a set of research questions addressing the nature of learning in interactive technological environments (Bookman and Malone, 2003). In that study, the authors formulated three categories of research questions: (1) what is the role of the university instructor in interactive technology-rich environments? (2) What types of behavior and thinking processes are university students engaged in as they work together in front of the computer? and (3) what opportunities and obstacles are raised by the technology itself? Our objective in the study reported in this paper is to analyze the interactions and social relationships between students as they worked together on computer-based math modules. *The primary question of interest is: What patterns of behavior and social relationships emerge when students learn mathematics collaboratively in technology rich, socially interactive learning situations, and what impact might these patterns of behavior have on the opportunity to learn in these settings?* In the case of these two particular innovations

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mentioned above (collaboration and computer enhanced learning) there has been, over the past decade, a significant amount of research. For example, Barron (2003) and others (Rogoff, Turkanis, and Bartlett, 2001; Roschelle, 1992) have investigated the nature of collaborative learning and conceptual change (see:
http://ctl.sri.com/publications/downloads/ConvergentConceptual.pdf
and http://www.leaonline.com/doi/abs/10.1207/S15327809JLS1203_1?journalCode=jls
Likewise, Inkpen (1997) and others (Shechtman, Roschelle, Haertel, Knudsen, and Tatar 2005), have examined the shared use of the computer as a pedagogical tool (see:
http://www.futureofchildren.org/information2826/information_show.htm?doc_id=69809
and http://www.cs.sfu.ca/people/Faculty/inkpen/publications.html.

However, much of this work on the collaborative use of technology as a pedagogical approach has focused on younger K-8 students. Less empirical research has been published concerning the way undergraduate students learn college-level mathematics interactively using computers and Internet based problem solving environments. Thus, the focus of our study was to examine how *undergraduate* students go about negotiating roles and developing a shared understanding of mathematics while working together on computers solving Internet based problem modules.

The study reported in this paper utilized a grounded theory approach (Glaser and Strauss, 1967) to generate several conceptual categories based on observations of students' work. Glaser and Strauss (1967) describe *grounded theory* as "the discovery of theory from data systematically obtained from social research." Grounded theory is more of an inductive approach as contrasted with "theory generated by logical deduction from *a priori* assumptions." The first step in this process is to examine the data with the purpose of establishing categories and/or constructs while minimizing the effect of to preconceptions, preexisting theories or prejudices. For these reasons, this methodology is particularly appropriate for the kind of exploratory research described in this paper.

Our interpretation of the "grounded" approach to theory development is that it occurs in roughly four stages:

1. Start with the data and observations and see what research questions and categories emerge.
2. Refine and reformulate specific categories with the goal of generating hypotheses.
3. Design and implement new and more focused data collections whose purpose is to eliminate and refine hypotheses and move to an emerging theory. By emerging theory we mean the accumulation of evidence that supports certain hypotheses and the organization of those hypotheses into a coherent framework that may explain the phenomenon being studied. Formulate theories that can be empirically investigated.
4. During this four stage process the specific research tools get refined (e.g. coding schemes get developed, tested and revised) so that eventually empirical studies can be designed and implemented to test the theory.

Our previous work examining collaborative student learning in web-based environments addressed the *first* stage of this process. In this paper, we describe our work on the *second* stage, refining and reformulating specific categories. Our objective was to analyze the interactions and social relationships between students as they worked together on computer-based math modules. At this stage of this research agenda, any analysis or conclusions must be tentative, preliminary, and subject to revision and further data collection. Our purpose in this paper is not to propose a comprehensive theory but, instead, to identify and clarify issues and suggest hypotheses that we believe will lead researchers into the third stage of the development of a grounded theory of learning mathematics in technology rich environment. We hope that the small steps we take in this paper will contribute to moving this endeavor forward.

II. Background and Literature Review.

Mathematics educators largely agree that students in mathematics classes should: investigate meaningful contextualized applications of math; utilize technology to solve problems; work together cooperatively; and engage in collaborative discourse to use language to communicate mathematical ideas (Zemelman, Daniels and Hyde, 1998; National Research Council, 1991). Many of the projects that grew out of the calculus reform movement of the 1990’s also reflect these goals (McCallum, 2000).

Duke University’s Connected Curriculum Project (CCP) was developed in this educational and historical context. “The Connected Curriculum Project is a coordinated effort to create interactive learning environments for a wide range of mathematics and mathematically-based applications. Our materials combine the flexibility and connectivity of the Web with the power of computer algebra systems. These materials may be used by groups of learners as an integrated part of a course or by individuals as independent projects or supplements to classroom discussions.” (http://www.math.duke.edu/education/ccp/aboutccp.html). The research discussed in this study examines students working on CCP modules.

The uses of computers and technology in support of learning mathematics have been well documented in the research literature. As early as the 1970s, researchers such as Papert (1980) were studying the ways computers might foster greater understanding of mathematics. By the early 1980s, Kelman and associates (1983) had completed extensive studies describing the potential role of computer technology in mathematics education. There was great hope that computers would make possible new approaches to teaching and leaning mathematics.

Expectations have been high that computers and technology would have significant effects on instructional practices and learning outcomes. However, as educational historians such as Cuban (1986, 2001) have indicated, the expectations for technology have typically been significantly greater than the actual outcomes. Pea (1987) provided a historical perspective on the transformational roles played by computers and advanced technology in mathematics education. Pea indicated that although the computer has the potential to serve as a mediational tool for promoting dialogue and communication on mathematical problem solving, computers have rarely been used to facilitate this function explicitly. (p. 105)

More than fifteen years have passed since Pea and Cuban first questioned how well educators have incorporated computers and technology into their instructional practices. Today, very few critics would question that significant progress has been made in discovering meaningful ways that computers can foster the learning (Aleven, Stahl, Schworm, Fischer, and Wallace, 2003; Farrell, 1996). For example, Becker and Riel (2000) reported that when computers are integrated into constructivist instructional approaches, computers could become effective tools for improving students’ learning. Ellis (2000) argued that “technology is changing the way calculus is taught and learned, as well as the topics presented and the interactions in and out of the classroom” (p.67). Dubinsky, Matthews, and Schwingendorf (2001) indicated that the thoughtful use of technology can be very beneficial to student learning. Goos, Galbraith, Renshaw, and Geiger (2003) found that technology can serve as a “discourse tool” which is useful in mediating class discussions and changing the ways teachers and students interact with each other and with learning tasks.

Along with the research on technology and mathematics, an equally rich research literature exists that examines the use of interactive social contexts and cooperative learning in mathematics education (Dubinsky, Mathews, and Reynolds, 1997). Much of the research in this area is based on the foundational work of cognitive psychologist Lev Vygotsky (1978) who maintained that the development of higher level thinking in mathematics is rooted in social interactions. More current theorists such as Noddings (1990) and Schoenfeld (1985) have contributed to this understanding of mathematics as a social activity. These researchers, as well as others, have demonstrated how dialogue and structured social interaction among mathematics learners can be helpful in fostering mathematical thinking and conceptual change.

For example, in their edited book, *Cooperative Learning in Undergraduate Mathematics*, Rogers, Reynolds, Davidson, and Thomas (2001) examined issues surrounding the use of dialogue and student-to-student collaboration in college mathematics classrooms. They concluded that "mathematics problems are particularly well suited for group discussions because they have solutions that can be logically demonstrated." (p. 3). These researchers pointed out that a meta-analysis (Springer, Stanne, and Donovan, 1999) of studies involving college mathematics students indicated that cooperative learning has significant positive effects on achievement and attitudes among undergraduates learning mathematics.

Van Zee (2000) used audio-tapes and video-tapes to examine the nature of student-to-student discourse in a science and mathematics education seminar. Van Zee interpreted the dialogue among what she termed “collaborative sense making” students in the seminar to determine instances of inquiry learning, student questioning, and collaborative sense making. For instance, Van Zee examined the specific questions students asked each other about a particular issue having to do with the phases of the moon. The framework for analysis of the students’ conversations in the classroom “was based on a negotiation metaphor” (p. 119) that identified instances of students helping other students make their meanings clear. Van Zee concluded that both “students and teachers can build principled knowledge through joint talk and action.” (p. 137).

In recent years the phrase “social negotiation of meaning” has appeared more frequently in the research literature. Woolfolk (1995) defined “social negotiation” as an “aspect of the learning process that relies on collaboration with others and respect for different perspectives.” (p. 482). Woolfolk indicated that when the American Psychological Association Task Force on Psychology in Education published its twelve “Learner Centered Psychological Principles” that the ninth principle stated that “learning is facilitated by social interaction and communication with others in flexible, diverse, and adaptive instructional settings,” (p. 480). Woolfolk noted that that the notion that students develop higher mental processes through collaborative discourse and the social negotiation of meaning is rooted in the work of Vygotsky and is an underlying principle of constructivist approaches to teaching.

Alexander and Murphy (1998) noted that, “Learning is as much a socially shared undertaking as it is an individually constructed enterprise. One of the most powerful observations that has emerged in the psychological literature in the past several years … is the recognition that learning is continuously and markedly shaped by the social context in which it occurs” (p. 41). They quote Resnick (1991) who argued that:

> Recent theories of *situated cognition* are challenging views that the social and the cognitive can be studied independently, arguing that the social context in which cognitive

> activity takes place is an integral part of that activity, not just the surrounding context for it … every cognitive act must be viewed as a specific response to a specific set of circumstances. Only by understanding the circumstances and the participants' construal of the situation can a valid interpretation of the cognitive activity be made (p. 4).

As we will argue in this paper, these insights are relevant to developing a framework for understanding learning in the collaborative interactive technology-rich environments explored in this study.

This emphasis on social interaction has given rise to a significant body of research that examines various aspects of cooperative and collaborative learning in mathematics. For many years, organizations such as the Network for Cooperative Learning in Higher Education have disseminated research on both cooperative and active-learning (http://www.csudh.edu/SOE/cl_network/default.htm). As Cooper and Robinson (1998) pointed out, the evidence for the effectiveness of cooperative learning in science and math instruction is strong: "Perhaps the most compelling evidence regarding the power of small-group instruction in SMET (Science, Mathematics, Engineering, and Technology) disciplines comes from a recent evaluation of over 340 NSF project directors. They were asked to evaluate which of 13 possible innovations in undergraduate teaching were central to effective teaching. Students working in teams was ranked highest of the thirteen."

Our review of the research on collaborative learning in mathematics also revealed a number of researchers specifically investigating the interaction of technology and collaborative learning. For example, Roschelle, J., Pea, R., Hoadley, C., Gordin, D., and Means, B. (2001) investigated the ways computers can be used to improve learning in the classroom in light of four fundamental characteristics of effective learning: active engagement, participation in groups, frequent interaction and feedback, and connections to real-world contexts. The researchers indicated that although some critics maintain that the computer fosters asocial behavior, the use of computers to facilitate educational collaboration is increasing dramatically. Roschelle et al noted that: "Reports from researchers and teachers suggest that students who participate in computer-connected learning networks show increased motivation, a deeper understanding of concepts, and an increased willingness to tackle difficult questions."

Another sign of rising interest in the interaction of collaborative and technology is a recent issue of the journal *Educational Psychologist* (Volume 40, Number 4, Fall 2005) which was focused entirely on ways computers can be used as metacognitive tools for enhancing learning. Included in this journal is a study by White and Fredericksen on the development of self-regulatory skills among fifth graders working collaboratively with computers. The researchers used videos of students working together in classrooms, as well as interviews with students and teachers, as a basis of their analysis. They concluded that the collaborative use of technology is not only highly engaging, but leads to the development of metacognitive knowledge and skills necessary for collaborative inquiry and reflective learning.

Other researchers have also examined the use of instructional approaches that effectively combine both technology and collaborative learning. For example, Edelson, Pea, and Gomez (1996) argued that "math and science reforms of the 1960's that were most successful were not just those that emphasized the active nature of the learning through manipulatives and hands-on inquiry, but also those that provided opportunities for students to talk while they were engaged in learning, interacting about what they were learning, what they believed, and what they had difficulty understanding." (p. 152). These researchers developed the Learning Through Collaborative Visualization Project (CoVis) for high school science classrooms. The CoVis

Project utilized computers to engage students in open-ended scientific investigations; students worked collaboratively with other students, with teachers, and with scientists. The researchers indicated that: "social interactions enhance the learning that students achieve through the transformative process of communication" (p.162). The researchers suggested that technology has the potential to enhance social interaction and serve as a mediational tool. The researchers concluded that teachers must begin "to take advantage of these sorts of new technologies to provide their students with opportunities for active learning and meaningful social interaction about scientific subjects" (p.162).

Despite this compelling evidence, our review of the research on collaborative learning of mathematics in technology rich environments yielded little research that focused more specifically on the styles of interacting or learning styles that paired undergraduates establish as they work together cooperatively. Although Inkpen, McGrenere, Booth, and Klawe (1997) examined interaction styles in educational computer environments, their focus was chiefly on "computer mouse interactions." And, while Ross and Lukow (2004) investigated the predictive value of individual learning styles for integrating technology into the curriculum, they focused primarily on the attitudes and learning styles of individual children.

Terms such as learning styles and cognitive styles have long been used by educators and psychologists to describe the different ways that individual learners approach tasks-- their preferences and approaches to doing academic work, as well as their preferred ways of processing and organizing information. Dunn, Dunn, and Price (1984) developed an instrument to measure the learning styles of students, including students' preference for visual versus auditory instruction, working alone versus with others, and psychological inclinations such as working reflectively versus impulsively. While the idea that students bring established learning styles to individual learning tasks seems widely accepted, few studies have examined the interaction of learning styles and collaborative learning. A question of interest in this study was: Do pairs of students who work together to solve a mathematics learning task establish a "collaborative learning style"?

In reviewing the literature on the uses of computers in mathematics education, we discovered that the phrase "mediational tool" was used by researchers to communicate the notion of the computer as intermediary between the mathematical concepts and the learner. At times the term mediational tool has been used explicitly by researchers such as Pea (1987), and by Goos, Galbraith, Renshaw, and Geiger (2003) who indicated, "little consideration has been given to the pedagogical implications of technology as a mediator of mathematics learning" (p.1).

Often, however, this notion of the computer acting as a "go-between" or a mediator is implied. These references in the literature to the computer as a mediational tool describe the potential of the computer to mediate the process of learning by bridging the gap between the learner's current understanding and the new concepts being taught. For the purposes of this study, we defined mediation of learning to mean the process of promoting learning by providing to the learner a tool to assist in making connections between new concepts and existing schema.

With this in mind, the primary objective of our research was to examine the behaviors, interactions, and conversations between students who were using computers to learn mathematics. Two questions of interest to us were: To what degree does the computer "mediate" or foster conversations and social interactions having to do with learning mathematics. And, what patterns of behaviors and interactions emerge?

III. Methodological Issues.

As mentioned above, the study reported in this paper builds on an earlier, preliminary analysis of videotapes of students using Connected Curriculum Project (CCP) modules (Bookman and Malone, in press). This is consistent with our view of how grounded theory is developed. The first paper focused on identifying categories and questions generated by the data; in this second paper, we reexamine the data focusing on a particular category (in this case, interactions and social relationships between students as they worked together on computer-based math modules). The subjects and the data collected were, therefore, the same as in our earlier study.

The subjects were college students at a highly selective research university taking a mathematics course (at a level beyond calculus). The subjects were each paid $25 for volunteering to be videotaped for the purposes of this study. The research procedures and consent forms were approved by the University's Institutional Review Board. Their participation in the study consisted of working through one of the CCP modules with a partner. The students working together were videotaped and their computer output was simultaneously collected on a separate videotape. Each session was 1-2 hours in length and data were collected from a total of 10 pairs of students. The students were familiar with the format of the modules and *MAPLE* (the computer algebra system) having used them for several weeks in their mathematics course work. For all but one pair of the students, the particular module used in the study was a requirement for a course in which they were enrolled. The subjects had been in class together and, in most of the cases, had previously been lab partners with the person they had been paired with in this study.

The data were gathered in an office (rather than a computer lab) so that the videotaping could be done more effectively. For most of each session, one of the investigators was present in the room, serving the same role that the instructor would serve in the computer lab. Pencil, paper, and a computer with *MAPLE* were on the table, as well as a video camera to record their work and a scan converter connected to a VCR and television to record their computer output. Videotapes of ten pairs of students were collected. We chose vignettes from five of the ten pairs of subjects whose behaviors most clearly illustrated or typified the categories generated**.** Because the cost of transcribing all these conversations was prohibitive, we identified vignettes in these tapes that seemed particularly interesting and transcribed those vignettes, leaving out extraneous verbiage that did not convey any added meaning. In our second study, we revisited these tapes focusing on the social interactions between the subjects. Because of this particular focus, we transcribed these vignettes more exactly and in greater detail, also adding in descriptions of the nonverbal behavior that was observable.

A unique aspect of the current study was that the subjects were videotaped working together and, simultaneously, their computer output was recorded on videotape. Using these simultaneous video recordings as a method of investigating student learning in computer labs has not, to our knowledge, been reported in the research literature. This methodology provided the researchers with an opportunity to closely examine and document student behavior. Viewing both tapes simultaneously was necessary because it is not possible to understand the students' dialogue and interactions without seeing both what the subjects were seeing and what they were working on. Examples of these tapes can be seen at the links below (Note that camera1.rm is paired with computer1.rm):

http://www.math.duke.edu/~bookman/Camera1.rm
http://www.math.duke.edu/~bookman/Computer1.rm

http://www.math.duke.edu/~bookman/Camera2.rm
http://www.math.duke.edu/~bookman/Computer2.rm

Using this methodology, we focused on one of our research categories, the role of social interaction and collaborative discourse in computer-based mathematics instruction. We began by re-watching the tapes, paying particular attention to social interactions. We catalogued the social interactions and behaviors and then reorganized by clumping and condensing these behaviors in order to determine “which phenomena share sufficient similarities that they can be considered instances of the same concept” (Gall, Borg, and Gall 1996, p. 564*)*. This iterative and recursive process required frequent reformulation of the categories which required frequent re-examination of the tapes. Our goal was to extract categories from the data that were coherent, self-contained, sufficiently general, and recognizable and we believe that the process of cataloguing, clumping, condensing and reexamination of the tapes allowed us to make significant progress towards that goal. This is consistent with Romberg’s (1992) method of clinical observations where “the details of what one observes shift from predetermined categories to new categories, depending upon initial observations” (p. 49). It is also consistent with the principle of grounded theory that one generates conceptual categories from evidence and that the categories that “emerged from the data are constantly being selectively reformulated by them. The categories, therefore, will fit the data, be understood both to sociologists and to laymen who are knowledgeable in the area, and make the theory usable for theoretical advance as well as for practical application” (Glaser and Strauss, 1967, p. 249).

To place our analysis of the data in a context, we describe below the three CCP modules on which our subjects worked. In one module, The Equiangular Spiral, http://www.math.duke.edu/education/ccp/materials/mvcalc/equiang/index.html , students examine properties of the chambered nautilus to learn about equiangular spirals in general. The lab also provides an opportunity for students to review polar coordinates. The students are given a picture of a chambered nautilus, superimposed on a polar grid, and asked to show that the radius is an exponential function of the angle theta. They also develop the mathematical basis for why these spirals are called equiangular. In another module, Rotation Matrices, http://www.math.duke.edu/education/ccp/materials/linalg/rotation/index.html, the students learn how to use matrices to represent rotations in the plane, and rotations in space about one of the axes. They learn about the relationship between multiple rotations and matrix multiplication and about determinants and inverses of rotation matrices. In the last module, Linear Correlation and Regression, (http://www.math.duke.edu/education/modules2/materials/test/test/) students examine scatter plots to learn about correlation and lines of best fit. They also examine the difference between correlation and causation.

IV. Analysis of the Data.

Our observations of students working on these three CCP modules provided evidence that suggests different ways students interact while learning math in a technology rich environment. For example, one of the most apparent and recurring observations was that the students focused their attention almost entirely on the computer that served as both a mediator and object of their communication. They conversed with each other by pointing to objects on the computer screen and did so while rarely looking at each other. In addition to these more easily recognized behaviors, an analysis of students’ work revealed more complex interactions that will be described below. Two thematic categories emerged: (1) establishment of roles and (2) social negotiation of meaning. We use the term "social negotiation of meaning" as described above by Woolfolk (1995) and we examined three roles – who controlled the mouse and keyboard, who made decisions concerning the direction and pace of their work, and who served as checker or verifier. These roles were not assigned but were established by the subjects as they proceeded through the assigned modules. Below we describe several vignettes that are exemplars of the categories that emerged from the data.

A. Establishment of roles.

Observations of students working collaboratively in front of the computer revealed that some students explicitly decided who would be responsible for what task, while others arrived at these decisions less consciously and without discussion. For example, the following vignette illustrates how a pair of students verbalized and established who would control the keyboard and mouse. The names used here are pseudonyms; the real names of the subjects were not used.

Alex: Here, why don't you type dude? *(Looks at Neil while speaking to him)*
Neil: Are you sure? *(Looks back at Alex and raised his eyebrows questioning Alex's decision. At this point, Alex stands up and begins switching seats)*
Alex: Yeah, yeah. *(mumbles something inaudible)(Neil begins to take the chair in front of the computer)*
Neil: I thought you wanted to type. *(Sitting and readjusting the keyboard)*
Alex: You're better with commands. *(getting seated)*
Investigator: So what's the deal? Does he usually…*(Neil begins to shake his head in disagreement)*
Alex: Uh, he uh, he did it before because he knew MAPLE *(Alex looks at the investigator in the room while addressing him)* and, I kind of took the last couple.
Neil: We take turns. (*says this while still looking at the screen*)
Alex: Yeah, it's his turn anyway.

In another vignette, Hope and Amit explicitly discussed role assignments. This conversation occurred just after they sat down to begin working.

Hope: Here. Do you want to use the keyboard or mouse or do you care? *(it looks as if she might be pushing the keyboard or mouse closer to him as she asks her question.)*
Amit: You can have both of them and be happy. *(They both laugh at this)*
Two minutes later Amit takes the mouse while Hope is writing at the board. He passes it back to her as she sits down.
Hope: No, go ahead, it doesn't matter.
Amit: *Pushes the keyboard towards her.* No, go ahead.

This interaction was the first of many times that Amit grabbed the mouse or keyboard when Hope was away from her position, but he seemed to relinquish it when she returned. From his knowledge of her, Amit sensed (correctly) that Hope wanted to control the mouse and keyboard and was just being polite in offering it to him. This vignette illustrates a more subtle way of establishing the control of the keyboard and mouse than in the case of Neil and Alex.

Whereas Neil and Alex comfortably and naturally made this decision, there was more tension in how Hope and Amit decided on their role assignments.

In some cases, students who had worked together prior to the videotaped session had already established roles in advance. For example, Kevin, a math major, and Carl, an electrical engineering major, had been lab partners for most of the semester prior to the day when their work was videotaped. They had been working on the module for several minutes when the investigator asked:

> **Investigator:** So you guys have a routine down yet, working together?
> **Kevin:** *(Shrugs and turns to the investigator)* We take turns typing, although Carl types more.
> **Carl:** Yeah, I get along better with the computer; he gets along better with the math. (*Carl finally looks up to acknowledge the investigator, who he is talking to.)*

Sometimes, as in the case of Andy and Larry, no discussion of establishing roles took place. Andy just sat down and took charge of the keyboard and the mouse. These vignettes describe only four instances, on a continuum from explicit to unspoken, about how decisions were made concerning who controlled keyboard and the mouse. These data don't provide an explanation of how these roles were formed; a future study that includes follow-up interviews with the subjects might provide some insight on this question. As we will discuss later, we believe that these observations are consistent with other behaviors of these pairs and might lead to some categorization of the different ways pairs of students work together.

After the pairs of students established who controlled the keyboard and who controlled the mouse, they began to work on the module. At certain critical points in the problem solving process, the students had to establish roles having to do with making decisions about how next to proceed. At these transition moments, these decisions were sometimes jointly made and sometimes made by one individual.

For example, the following vignette illustrates how Carl and Kevin made a decision about who would control navigation. In this situation, Kevin needed to assert himself in order to get Carl to slow down so he could get his question answered:

> **Investigator:** Do you know what standard deviation means?
> **Carl:** Yes.
> **Kevin:** I kind of know intuitively what it means, *(he the looks to the investigator)* is there a good definition?
> **Investigator:** Yeah, there is. You can click on the link.
> **Kevin:** Do you want to click on that? *(Kevin points to the link with the eraser end of his pencil. However, Carl ignores Kevin and continues typing in the answer with which he has been working. At this point, Kevin takes control of the mouse.)*
> **Kevin:** I am just going to click on that and see what standard deviation means.
> **Carl:** Okay. *(Carl sits back in his chair and yawns while he waits for Kevin to read the definition of standard deviation. Kevin finishes reading the link on standard deviation and closes the window but is unsure how to use the computer to retrieve the module, so he relinquishes control of the mouse again to allow Carl to re-open the window.)*

This exchange was typical of their division of labor as when Carl was explaining why he was at the keyboard and he said, “I get along better with the computer; he gets along better with the math.”

In another case, even though Jim did all the typing, his partner Mary directed the decisions for the computing process.

> **Mary:** See, this *(points to paper)* and this *(points to a new spot)* are what you want to see. Natural log of r, so if you type it in like this it should work. *(pointing to the piece of paper that they had been given by the investigator).*
> **Jim:** Take the zip and rewrite l and r?
> **Mary:** Yeah.
> **Jim:** Okay. Oh I see.

This was one of many examples we observed where, in some of the pairs, the person controlling the keyboard and the mouse was not the person in control of some of the decisions about how next to proceed.

In almost complete contrast, Larry made numerous suggestions to his partner Andy who usually ignored him.

> **Larry:** I am surprised that you can’t just get it [the computer] to find that for you. *(both Andy and Larry look at each other and then Andy turns back to work on the problem).* Just set up a function now that iterates from like 1 to… *(Andy turns and looks at Larry again. Andy is smiling and silently laughing)*
> **Andy:** No. *(Andy continues smiling, but he does not ever look away from the problem, and his language is very curt.)*
> **Larry:** 10. *(Larry is still looking at Andy)*
> **Andy:** No. *(Andy continues not to acknowledge Larry)*
> **Larry:** by 0.001
> **Andy:** No. *(looks at Larry this time when he responds, but continues to use the curt tone).*
> **Larry:** And return the one.
> **Andy:** No, we are not doing that. *(Shakes his head no as he turns back to the computer again)*

These vignettes demonstrate some of the ways in which decisions were made and how roles were established, varying from shared to unilateral. In the case of Andy and Larry, Andy controlled the mouse and keyboard and also directed the decision making process. This left Larry no role to play and feeling like and outsider or observer of the learning process. In the case of Kevin and Carl, although Carl controlled the keyboard, Kevin insisted on making critical decisions when he felt he needed to. If the person controlling the mouse and keyboard was not the key decision maker, a pattern of advice and consent by the keyboarder often emerged. As seen above, this was the case with Mary and Jim. What we saw in our data was that, in the well functioning pairs, the person not at the keyboard had an equal or greater share of the decision making.

In some cases, the subjects also established roles concerning who would take primary responsibility for mathematical thinking. For example, in the case of Carl and Kevin, Kevin assumed primary responsibility for that role, as was clear when Carl said, "I get along better with the computer; he gets along better with the math." In fact, throughout the module, Kevin (a math major) almost always did the pencil and paper algebraic computation and other mathematical thinking. Carl (an electrical engineering major) worked equally hard on the technical (e.g., syntax) aspects of the problem. Although there was a clear division of labor for who was leading in a particular task, they each took responsibility to understand what the other was contributing. As we have pointed out this is consistent with their collaborative style.

A variation of this sharing of the mathematical thinking is seen in the work of Mary and Jim (Bookman and Malone, in press). The following vignette can be viewed by using the following links to Realplayer files: http://www.math.duke.edu/~bookman/Camera1.rm and http://www.math.duke.edu/~bookman/Computer1.rm. After a few minutes of trying to remember how to get Maple to compute derivatives (they needed to find the derivative of $x=r_0\ e^{k\theta} \cos\theta$ with respect to θ), Mary gave up and said, "We can just do it by hand." She began to do the calculation on paper, but Jim said, "I'm trying to remember how Maple works." After about a minute, Mary completed the calculation by hand and then said:

Mary: Okay.
Jim: Shut up. *(said in friendly and jocular manner).*
Mary: *(laughs)* Here, it's just the product rule.
Jim: Yeah. It would be nice if Maple will do it for us.
Mary: It will.
Jim: Yeah. I want it to do it.
A minute later, working together, they got MAPLE to do the calculation.
Mary: You see, it's exactly what I did.
Jim: Yeah, but your way is stupid.
Mary: But it was quicker.

The instructions then asked them to divide $dy/d\theta$ by $dx/d\theta$ to get a formula for dy/dx. Although this computation would have been quite difficult to do by hand, they were now (because they had figured out the correct syntax) able to use *Maple* to do this computation in a couple of seconds. The instructions then directed them to evaluate an even more complicated expression that reduced to $1/k$. Jim said, "Wow. I want to work this out on paper. I don't believe that." Here, although their roles are reversed, with Jim advocating use of pencil and paper, they shared the responsibility for making the mathematical decisions. One might think that the person in control of the keyboard and mouse controlled the pace and direction of their work as well as their mathematical thinking. But our analysis of the data, as illustrated above, indicated that these responsibilities were shared more frequently than we had originally anticipated.

Another role, sometimes taken on by the student not at the keyboard, was that of checker/verifier. In two of the five cases, the pair of students worked closely together where the non-keyboard person monitored their work, acting as the checker/verifier. For example, while Mary and Jim were trying to figure out the best fit line for the data points, they had the following conversation:

Jim: What do you think the formula is? It is going from 18-70.
Mary: What is?
Jim: The data points. It looks like it is doubling for every gap of 2.

Mary: No, not doubling. It is more like multiplying by 1.5.
Jim: Should we say 1.5 then?
Mary: Yeah, I think somewhere between 4/3 to 1½ relationship.
Jim types this in.

Notice that Mary, who was not at keyboard, was instrumental in checking and verifying that the work is correct. Again, this is consistent with what we saw in pairs that worked well together.

In another example, Alex and Neil demonstrated similar behavior of establishing roles of checker and verifier.

Neil: All right. What do the pictures say to us about data with correlation coefficients near +1 or –1? *(Neil reads the problem out loud, while both he and Alex read the problem off the computer monitor.)*
Alex: They fit. *(He speaks while still looking at the computer screen).*
Neil: All of the… *(moves his fingers in almost a snapping motion as he tries to say points)*
Alex: Yeah, the tightness of fit. *(Neil types this into their answer sheet)*
Alex: Of 1 is perfectly linear *(Neil types this in as Alex says it, at the same time shrugging his shoulders and raising the corner of his mouth to the statement that most are 0.99)*, most of them are like .99 or something. *(Neil nods his head left to right, but his facial expression seems to be frustration that he has typed something into the worksheet incorrectly)*
Neil: Is this fine? *(Turns wrists palms up, like a mini-shrugging motion to question Alex).* No this right here.
Alex: That's fine. *(nods his head in agreement)*
Neil: Approaching? *(Turns his one hand upward in a questioning gesture and nods his head while raising his shoulders)*
Alex: Yeah that's good.
Neil: What correlation…*(he and Alex both are intent on reading the screen)* Oh wait, correlation of 0. *(begins typing again)*
Alex: There's no relationship between the lines. *(rolls his eyes up, as if he is thinking about what he is saying. Neil types it in, still looking at screen)*

Alex's short, quick comments indicated to Neil that he was in agreement and that they could proceed. We noticed that each of the pairs of students developed its own style for checking work. The checking and verifying by the non-keyboarder appeared to help these students focus on the learning situation.

Conversely, when the student not at the keyboard was not the checker, we saw, in the case of Andy and Larry that their work proceeded badly. Andy and Larry had trouble establishing roles and determining how their work would be checked and verified. Failure to establish these roles often led to a breakdown in the learning process. For example, Andy's refusal to listen to Larry's suggestions resulted in Larry being less focused on their work and Andy going off in wrong directions. In fact, for several minutes, Larry, who later turned out to be right, suggested to Andy that they must be on the wrong track. Several times Larry said things like, "That can't be right." and was totally ignored. These difficulties communicating with each other clearly impeded their efforts to solve the math problems presented in the module.

An alternative hypothesis is that the lack of understanding determined what we observed in the collaborative styles. We feel, though, that this data provides some tentative evidence for the opposite view – that collaborative style affected the student's progress through the assignments. As will be seen in a vignette to be discussed later, Larry did understand that something was wrong but couldn't get Andy to listen to him. And we observed that Mary and Jim's cooperation helped them overcome difficulties in their understanding.

B. Negotiating Meaning.

The establishing of well defined roles is closely related to issues concerning the negotiation of meaning, which was the second thematic category that emerged from the analysis of data. We turn next to examining situations in which students had to negotiate meaning and understanding through dialogue, writing, and non-verbal communication. By "negotiating meaning" we refer to the process of collaborative discourse in which students take turns putting their understanding of a problem into their own words by challenging and building on the ideas their partner has expressed. We include non-verbal communication because analysis of the data indicated that students frequently pointed to the computer screen and made other gestures in an effort to communicate their understandings.

For example, the following vignette illustrates how a pair of students collaborated to construct a shared understanding of linear regression. In the linear regression module, the subjects were asked, "Given scatter plots of Test 2 scores versus Test 1 scores and Test 2 scores versus Test 3 scores, if a student scored an 82 on Test 1, what do you predict he or she would score on Test 2? If the student scored an 82 on Test 2, what would you predict for his or her score on Test 3? Which prediction do you expect to be more accurate? Why?" As they were trying to construct a written response to this question (the CCP modules typically require students to express their understanding in writing), the following dialogue occurred between Carl and Kevin:

> **Carl:** So…(*They are beginning to answer a problem about predicting a students' test scores from the data).*
> **Kevin:** Test 2 and Test 3 you can't fit because they are not related. *(Carl continues to type out their joint written response)* Therefore, any prediction for Test 2 would be more accurate than Test 3.
> **Carl:** Based on Test 1.
> **Kevin:** Yeah. *(Both are looking at the screen and then they read part of the problem)*
> **Carl:** Just say you can only make a judgment on positive and negative association if there is some sort of linear association.
> **Kevin:** If you can more easily fit a line though the data and it is easier to make predictions and find a relationship.
> **Carl:** Do we need to explain here?
> **Kevin:** In the second scatter plot, there is no line that is going to fit nicely, therefore it is hard to come up with a relationship and make predictions. I guess it wouldn't have to be a line, but since we are talking about linear stuff, focus on the line. *(Makes a suggestion)* An informed guess.

Carl: Form a guess. *(repeats Kevin's words back and continues to type out their written response)*
Kevin: *(makes another suggestion as to wording that could be used in the written explanation used)* I would say, in other words, it is hard to guess a student's test score.

Kevin offered a tentative provisional understanding that Carl responded to. They went back and forth until they were comfortable that they shared an understanding of this particular concept. This form of discourse provides students the opportunity to articulate their understanding and to seek agreement on meaning. Vygotsky (1978) maintained that peers can work together to co-construct knowledge as they provide cognitive scaffolding for one another. Kevin and Carl appear to be providing cognitive scaffolding for each other as they actively negotiate their written solution of the problem.

Another example of social negotiation of meaning is seen in the following dialogue between Mary and Jim (a dialogue we used earlier in this paper to illustrate a different point). The two students were trying to find a functional model for some data points that appear to be growing exponentially.

Jim: What do you think the formula is? It is going from 18-70.
Mary: What is?
Jim: The data points. It looks like it is doubling for every gap of 2.
Mary: No, not doubling. It is more like multiplying by 1.5.
Jim: Should we say 1.5 then?
Mary: Yeah, I think somewhere between 4/3 to 1½ relationship. *(Jim types this in and as he types he appears to be thinking deeply about it).*
Jim: That's not what it does. This does not look correct.

Jim and Mary then reconsidered their response and jointly tried to make sense of the problem. It is not until later, and after the investigator intervened with some advice, that Mary and Jim solved the problem. However, this short dialogue exemplifies the type of exchanges that were quite typical among the pairs of students who appeared to be working together effectively. These dialogues are typified by a bantering quality, with short sentences and polite interruptions. We refer to this kind of dialogue as "cognitive bantering." These types of discussions, of course, are typical of discussions in other, less technology intensive cooperative learning environments. We expect, however that technology (perhaps particularly in the area of mathematics) may be able to increase the likelihood of these opportunities for collaborative and active learning because the screen is a physical object that focuses their attention, and the objects on the screen can be changed quickly to respond to inputs from the users. For example, as in the case of Jim and Mary above, the computer algebra system provides immediate feedback to their hypothesis that 1.5 is the correct parameter.

As discussed earlier we analyzed videotapes of five pairs of students working on three different CCP modules. The five pairs were chosen from the ten pairs of students videotaped because highly the behaviors of these 5 pairs of subjects most clearly illustrated or typified the categories generated. Of the three CCP modules, the linear regression module produced particularly highly interactive conversations. This module included the java applet Guessing Correlations, http://www.stat.uiuc.edu/~stat100/java/GCApplet/GCAppletFrame.html. The applet shows students four scatter plots and gives them four correlation coefficients; their task is to match the correlation coefficients with the scatter plots. If their matching is correct, they get a "point" and can continue to see how many matchings they can get correct. This particular applet seemed to capture the students' interest and appeared to increase the intensity of the discussion. The following is an excerpt of Neil and Alex working on this applet:

> **Alex:** The other one is pretty weak though *(Neil points to the screen).* It looks like, uh, yeah it looks like, no, it's not that bad though. Maybe, uh, maybe use 0.3.
> **Neil:** Point 3? *(Raises his eye brows while asking this in a unbelieving tone)*
> **Alex:** I mean. Yeah, I said point 3. *(Neil looks over at him like he cannot believe what his partner is saying; he raises his eyebrows and scrunches his nose)*
> **Neil:** It's not point 3; it's not nearly that bad. *(Neil looks over at Alex and points to the graph on the screen)*
> **Alex:** No it's better than that, that's why I'm saying it's point 3. *(Alex points to a lower figure on the monitor)*
> **Neil:** Oh that's a negative? What are you talking about, that's 0.7. *(and points to the figure that he had originally pointed to again, and turns toward Alex waiting until he replies)*
> **Alex:** That's point seven *(motions with pen to scatter plot on screen)*, that's not point 7. *(motions with pen to scatter plot on screen)*
> **Neil:** No that's .96 *(Neil points to the graph)*
> **Alex:** Oh really. Here go down. *(Pointing toward the scrollbar, indicating that he wants Neil to scroll downward)* So, what do you want to say then?
> **Neil:** Huh?
> **Alex:** You want to guess like .7?
> **Neil:** I would say like, I would say .75. Or .95 and .75.
> **Alex:** All right that sounds good. *(Shakes his head in agreement)*
> **Neil:** Oh, and they are both positive, right?

This transcript of the conversation does not fully reflect the high level of engagement and the intensity of Neil and Alex's interaction. The Java applet (perhaps because of the game-like nature of the applet or the immediate feedback it provided in response to the students' predictions) appeared to engage the students deeply in a conversation about the mathematical problem. The applet uses variable and positive reinforcement methods from behaviorism yet also provides an opportunity for students to support each other as they develop and test their understanding. This combination of Skinnerian ideas of reinforcement together with Vygotskyian notions of social learning seemed to have a powerful effect on students. The students' natural and seamless dialogue is another example of what we refer to as cognitive bantering as each student took turns offering a provisional answer and then waited for the response of the partner. In this particular case, the immediate and visual feedback provided by the applet appeared to make this productive dialogue more likely to occur than it would have with a pencil and paper task. The computer appeared to serve as a mediational tool for fostering the students' thinking aloud activities.

This same scatter plot applet also appeared to bring about a clear change in the ways other pairs of students interacted when they began work on the applet. For example, Carl and Kevin appeared to have different goals and priorities throughout most of their work session. As we discussed earlier, Carl preferred being responsible for understanding the software and Kevin preferred being responsible for the mathematical understanding. However, when Carl and Kevin began working on the scatter plot applet, their dialogue became more focused on the mathematical problem as seen below:

> **Carl:** I would say this one is going to be 0.94.
> **Kevin:** There are two that are positive. That one *(points with his pen)* is a closer correlation than that one *(uses his pen to point again)* so, yeah A would be 0.94 and D would be 0.47. The other ones are going to be harder to predict because they are both negative.
> **Carl:** I think this one is better than that one. What do you think?
> **Kevin:** Yeah. *(Steven takes his pencil to the screen to try to make slope predictions)* What I am looking at here is that they are almost evenly distributed on either side of the lines. (*uses his finger to point to this on the screen)* What do you think?
> **Carl:** I like this more because …*(Carl proceeds to give his reasoning)*

For this activity, they seemed to equally share the responsibility for understanding the mathematical concepts and for the proper use of the computer.

In analyzing the videotapes we noted that during these moments when the pairs of students were engaged with each other's thinking and with the mathematical problem they were attempting to solve, the students seemed “in-synch.” Aspects of being “in-synch” include active listening, asking each other questions, and feeling comfortable challenging each other in a constructive way; these are also behaviors often associated with meaningful learning. These moments in which students appeared to be “in-synch” also seemed to be the moments when the most learning was taking place.

On the other hand, the fact that students worked together using a computer to solve interesting math problems did not always result in “in-synch” collaborative learning. Not all of the dialogues were productive. Even when working on the scatter plot applet, our most dysfunctional pair, Andy and Larry, had difficulty working effectively with each other. Andy rarely took Larry seriously and this lack of respect seemed to contribute to unproductive dialogues even when Larry was offering ideas that would have helped.

> **Larry:** I guess just Test 1, Test 2.
> **Andy:** Nope, no, we are not plotting tests against each other, we want to plot…*(squints his eyes, like he is thinking…)*
> **Larry:** Yeah we were, weren’t we? *(Still looking at the monitor)*
> **Andy:** No. *(shakes his head left to right in a short motion and carries tone of annoyance in his voice)*
> **Larry:** I thought we were plotting the data in Test 1 against the data in Test 2.
> **Andy:** No, um, we’re plotting Test 1, Test 2 *(points to the screen)* so we want to do those against, just like, *(uses hand gesture, turning palm upright)* the one so that each number represents a 1.
> **Larry:** Oh wait, you mean we are just putting the plots of both of them on the same graph and not actually plotting them against each other?
> **Andy:** Yes. I think that’s the idea. *(Mumbles, and nods his head at the same time)*
> **Larry:** Okay.

> **Andy:** *(mutters)* I don't know if this is going to work. (*Mutters under his breath as he enters the numbers and smiles)* This is going to be good though. Ready? *(mumbles to himself)* We just need to try this plot.

Even though Andy and Larry are engaged in a dialogue, it would be difficult to characterize the dialogue as "collaborative discourse" in the sense that this term is typically used. Andy's inability to actively listen to Larry impeded the collaborative learning process. Very little "meaning" is being negotiated because the two students do not establish a shared understanding of what the problem is asking them to do. Andy seemed convinced that his approach was correct (even though he was completely off track) so the qualities of provisionalism and negotiation that we saw in the other pairs of students were not present in this case.

In each of the videotapes we observed instances of students negotiating the meaning of the mathematical problems they are confronted with. In most cases, we observed that the students' efforts to construct a shared understanding reached a high level of engagement and thought. This was particularly true when the CCP modules contained a feedback loop and required the students to actively make predictions and hypotheses.

V. Discussion.

Our analysis of the five pairs of students working on three CCP modules provided insights into the behaviors of undergraduate students learning math in a collaborative technology rich environment. In this study, we've focused on how students negotiate roles and meaning while learning in these environments. Below we summarize the analysis of our observations and offer some tentative conclusions. Some observations are evident from the sample of vignettes discussed above, while others are based on the many hours of videotape that could not be summarized in a few vignettes.

One observation that is repeatedly supported by the videotape data is that the computer plays a significant role in the collaborative learning process. In this study, the computer served the dual role as a mediator between the two students as well as the object of their communication. The students rarely looked at each other while conversing and working together; their eyes were almost always focused on the computer screen. The students pointed to the screen to demonstrate ideas or to make a point. We concluded that the medium of the computer appeared to be more of a "player" in the learning process than a textbook might be.

We also concluded that working in pairs in front of a single computer necessitates that students establish certain roles, such as control of the mouse and the keyboard. Our observations indicate that these roles may not always be discussed explicitly, even when pairs of students work together effectively. Contrary to what many might think, the student in control of the keyboard and mouse did not necessarily control the direction of the learning and mathematical work. The student not burdened with the keyboard and mouse often took on more responsibility for the mathematical thinking, such as assuming the role of verifier. Furthermore, as was seen in the case of Amit and Hope, where control of the mouse changed hands in subtle ways, these roles can be fluid and interchangeable. The only pair of students we observed that experienced significant difficulty in negotiating meaning and developing an understanding of the problem (Andy and Larry) was the pair in which one partner took all the responsibilities and acted in a unilateral fashion.

Much of what we observed confirmed or was consistent with the work of other researchers who have examined the role of technology in interactive learning environments. For example, our observations corroborated the claims of Van Zee (2000), discussed earlier in this paper, that social negotiation of meaning and collaborative sense making appear to help students build conceptual understanding. Analysis of our data supports Alexander and Murphy's (1998) assertion that "learning is as much a socially shared undertaking as it is an individually constructed enterprise" (p. 41). This study provided support for Edelson's et al. (1996) claim that technology can serve as a mediational tool to enhance social interaction and learning. In addition, our findings are consistent with those of Goos et al. (2003), who indicated that technology can facilitate collaborative inquiry through eliciting conversation and discussion among students.

In addition to supporting the findings of prior research, this study also provides evidence for a hypothesis that extends the work of other researchers. Our observations lead us to hypothesize that when pairs of student are placed in a collaborative learning situation, pairs of students often establish recognizable ways of interacting and learning together. We will call this pattern of behaviors a "collaborative learning style." An individual learning style is a preference and approach to doing academic work and a preferred way of processing and organizing knowledge. In contrast, a collaborative learning style refers to the way a pair of students approaches academic work and the ways they interact as they process and organize knowledge. When we used the term collaborative learning style we do not necessarily mean a fixed entity that is immutable and uncontrollable. Collaborative learning styles could change as partners change, as tasks change and other conditions change; collaborative learning styles are probably situational states as opposed to characteristic traits. But, at this stage, these are still open questions and our purpose here is to suggest that these collaborative learning styles may exist and are an object worthy of further study. The impact that these collaborative learning styles have on student behavior in academic situations could have implications for developing a broader theory of how students learn.

This hypothesis was developed in a way consistent with the notion of grounded theory methodology - starting with the data and observations, then seeing what research questions and categories emerged, refining and reformulating those categories, and generating hypotheses. As we focused our observations on the social aspects of student learning in collaborative technology-rich learning environments and as we catalogued, clumped, condensed and reexamined the data, this hypothesis of the existence of collaborative learning styles emerged.

Our observations have led us to tentatively hypothesize the existence of three distinct collaborative learning styles: (1) in-sync or congruent, (2) parallel, and (3) orthogonal. In-sync pairs have shared goals and many of the characteristics we list below that are typical of productive partners (Neil and Alex and Mary and Jim would represent pairs of students with an in-sync learning style). A parallel collaborative learning style is manifested by compatible but different goals, division of labor, and mutual respect (as in the case of Carl and Kevin). Pairs that have an orthogonal collaborative learning style display a lack of mutual respect, differing goals, and the absence of shared responsibility (as in the case of Andy and Larry). These collaborative learning styles may not be fixed. For example, even though Carl and Kevin exhibited a parallel collaborative learning style during most of their work session, when they were using the Guessing Correlations applet they exhibited an in-sync style.

Our observations lead us to conclude that students that exhibit in-sync collaborative learning styles are more likely to become deeply engaged in mathematical problem solving in technologically rich environments. Pairs of students we classify as in-sync tended:

(a) to feel comfortable interrupting and challenging each other. There existed a shared understanding that challenges were productive and appropriate. We call this back and forth, give and take conversation, "cognitive bantering."
(b) to share humor and exhibit an intellectual playfulness.
(c) to respect their partners.
(d) to feel comfortable thinking aloud.
(e) to actively listen to each other in order to understand their partner's point, often rephrasing their partner's ideas in their own words.
(f) to communicate nonverbally (such as pointing and facial expressions) in order to build and demonstrate shared understanding.
(g) to make predictions and hypotheses.
(h) to offer provisional ideas as opposed to definitive responses.

The results of this research have theoretical and practical implications for teaching and learning mathematics. Much of the variation we observed in the ways that students went about solving mathematical problems, establishing roles, and negotiating meaning can be explained by examining collaborative learning styles. These collaborative learning styles, which may go unrecognized by instructors, may determine to some extent the success students experience as they engage in interactive learning activities in technology rich environments. In order to better understand under what conditions the use of technology and socially interactive, inquiry-based approaches to learning mathematics lead to student understanding, this concept of collaborative learning styles needs to be further examined. An understanding of the role of collaborative learning styles may have important implications for classroom practice. In particular, teachers need to develop an awareness of what kinds of instructional materials (e.g., java applets like "Guessing Correlations") are more likely to foster in-sync collaborative learning styles.

VI. Limitations.

As we've stated in our initial analysis of these data (Bookman and Malone, 2003), "In interpreting these data, it is important to realize that these students were talented students doing mathematics at a level beyond calculus and using specific software in a laboratory setting. It is not our purpose here to generalize these results to a larger population, but to use these observations to suggest areas for future study. It is also important to note that each entering class of students brings more familiarity, more comfort, and more sophistication with using educational technology. It is not clear which problems faced by the subjects in this study will likely be problems for students several years from now."

Another shortcoming is the lack of triangulation. The only source of data was the videotapes and the observations made during the videotaping by the investigators. Pre and post interviewing of the subjects and collecting other sources of data, such as students' written work, would have been helpful in documenting and cross-checking conclusions. Since these data were collected outside the classroom, issues concerning the classroom environment – the pedagogical, affective and physical environment – were not addressed. Neither were gender and cultural differences addressed. These are all certainly important areas for future study.

VII. Future research.

Many of the questions and issues raised by the current and previous study on interactive technology rich learning are relevant to active learning environments in general. Throughout our work on this research project, we found ourselves asking whether a particular instance of behavior was unique to a computer-based learning environment or more relevant to all active learning situations. A next step in this line of research would be to investigate the differences between students working in an active learning environment using only pencil, paper and hand-held calculators and students working in a technology-rich active learning environment.

Because we realize that the limited number of subjects and observations limits our ability to generate these results, another next step would be to develop and test a coding scheme for analyzing the kind of videotapes we've collected of students working together, where the coding scheme and categories grow from the ground up as a theory emerges. This would allow for a more efficient and reliable collection of data so that larger samples could be studied resulting in more replicable and more generalizable research. Verifying the existence of collaborative learning styles, categorizing them, and placing these styles into a larger theory of collaborative learning is a potentially rich area for future research. Researchers will then need to investigate the interaction among collaborative learning styles, the tools available to the student (e.g. computer algebra systems) and prior mathematical experiences and knowledge and how these factors impact learning and affect achievement.

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References

Aleven, V., Stahl, E., Schworm, S., Fischer, F., and Wallace, R. (2003). Help seeking and help design in interactive learning environments. ***Review of Educational Research***, *73*, 277-320.

Alexander, P.A. and Murphy, P.K. (1998). The research base for APA's learner-centered psychological principles. In N. Lambert and B. McCombs (Eds.), How students learn (pp. 25 - 60). Washington, DC: American Psychological Association.

Archer, J. (1998). The link to higher scores. Education Week on the Web. Retrieved March 15, 2003 from http://www.edweek.org/sreports/tc98/ets/ets-n.htm.

Azmitia, M. and Crowley, K. (2002). Reflections on the social context of scientific understanding. In Crowley, K.; Schunn, C. D.; and Okada, T. (Eds.), Designing for Science: Implications from Professional, Instructional and Everyday Science, Mawah, NJ: Lawrence Erlbaum Associates.

Barron, Brigid (2003). When Smart Groups Fail. ***Journal of the Learning Sciences*** 12:3, 307-359.

Becker, H and Riel, M (2000). Teacher professional engagement and constructivist-compatible computer use. teaching, learning, and computing: 1998 National Survey. Report #7. Irvine, CA: Center for Research on Information Technology and Organizations.

Bookman, J. and Malone, D. (2003). The Nature of Learning in Interactive Technological Environments - A Proposal for a Research Agenda Based on Grounded Theory, edited by edited by Selden, A., Dubinsky, E., Harel, G., and Hitt, F., Research in Collegiate Mathematics Education V., Providence, RI: American Mathematical Society.

Cooper, J. and Robinson, P. (1998). Small group instruction in science, mathematics, and engineering, and technology: A discipline status report and a teaching agenda for the future. ***Journal of College Science Teaching***, *27* (6), 383-8.

Cordes, C. (1998, January 16). As educators rush to embrace technology: A coterie of skeptics seeks to be heard. The Chronicle of Higher Education, p.A25.

Cuban, L. (1986). Teachers and machines: The Classroom Use of Technology Since 1920. New York: Teacher College Press.

Cuban, L. (2001). Oversold and underused: Computers in the classroom. Cambridge: Harvard University Press.

Dubinsky, E., Mathews, D., and Reynolds, B. (Eds.). (1997). Readings in Cooperative Learning for Undergraduate Mathematics. Washington, D.C.: The Mathematical Association of America.

Dubinsky, E., and Schwingendorf, K. (2001). Calculus, Concepts, Computers, and Cooperative Learning (C4L): The Purdue Calculus Reform Project-Constructivist Learning Theory in Practice. Retrieved March 11, 2003 from http://www.math.purdue.edu/~ccc/.

Dunn, R., Dunn, K. and Price, G. (1984). Learning Style Inventory. Lawrence, KS: Price Systems.

Edelson, D., Pea, R., and Gomez, L. (1996). Constructivism in the Collaboratory. In B.G. Wilson (Ed.), Constructivist learning environments: Case studies in instructional designs. (pp. 151-164). Englewood Cliffs, NJ: Educational Technology Publications.

Ellis, W. (2000). Technology and calculus. In S. Ganter (Ed.), Calculus renewal: Issues for undergraduate mathematics education in the next decade. (pp. 7-18). Reston, V.A.: Monograph 4 of the National Council of Teachers of Mathematics.

Farrell, A. (1996). Roles and behaviors in technology-integrated pre-calculus classrooms. ***Journal of Mathematical Behavior***, *15*, 35-53.

Forman, E., and Ansell, E. (2002). Orchestrating the multiple voices and inscriptions of a mathematics classroom. In A. Sfard, and K. McClain (Guest editors). Perspectives on the role of designed artifacts in mathematics learning. ***The Journal of the Learning Sciences***, 11 (2and3), 251-274.

Gall, M.D., Borg, W.R., and Gall, J.P. (1996). Educational Research: An Introduction. (Sixth Ed.). White Plains, N.Y.: Longman.

Glaser, B.G. and Strauss, A.L. (1967). The discovery of grounded theory. Chicago: Aldine Publishing.

Goos, A., Galbraith, P., Renshaw, P., and Geiger, V. (2003). Perspectives on technology mediated learning in secondary school mathematics classrooms. ***The Journal of Mathematical Behavior***, *22*, 73-89.

Inkpen, K., McGrenere, J., Booth, K.S., and Klawe, M. (1997). Turn-Taking Protocols for Mouse-Driven Collaborative Environments. Proceedings of Graphics Interface '97, Kelowna, BC, May 1997, pp. 138-145.
http://www.cs.sfu.ca/people/Faculty/inkpen/Papers/GI97/gi_97.html

Kelman, P., Bardige, A., Choate, J., Hanify, G., Richards, J., Roberts, N., Walters, J., and Tornrose, M. (1983). Computers in teaching mathematics. Reading, MA: Addison-Wesley.

McCallum, W. (2000). The goals of the calculus course. In S. Ganter (Ed.) Calculus renewal: Issues for undergraduate mathematics education in the next decade (pp. 11-22). New York, NY: Plenum Press.

National Research Council. (1991). Moving beyond myths. Washington, DC: National Academy Press.

Noddings, N. (1990). Constructivism in mathematics education. In R. Davis, C. Maher., and N. Noddings (Eds.), Constructivist views on the teaching and learning of mathematics (pp. 7-18). Reston, V.A.: Monograph 4 of the National Council of Teachers of Mathematics.

Papert, S. (1980). Mindstorms: Children, computers, and powerful ideas. New York: Basic Books.

Pea, R.D. (1987). Cognitive technologies for mathematics education. In A. Schoenfeld (Ed.), Cognitive science and mathematics education (pp. 89-122). Hillsdale, NJ: Lawrence Erlbaum Associates.

Reiser, R.A. (2001). A history of instructional design and technology part one: A history of instructional media. ***Educational Technology Research and Development***, *49*, 53-64.

Resnick, L.B. (1991). Shared cognition. In. L.B. Resnick, J.M. Levine, and S.D. Teasley (Eds.), Perspectives on socially shared cognition (pp.1-20). Washington, DC: American Psychological Association.

Rogers, E., Reynolds, B., Davidson, N., and Thomas, A. (2001). Cooperative learning in undergraduate mathematics: Issues that matter and strategies that work. The Mathematical Association of America.

Rogoff, Barbara, Turkanis, C.G. and Bartlett, L. (2001). Learning Together: Children and Adults in a School Community New York: Oxford University Press.

Romberg, T.A. (1992). Perspectives on scholarship and research methods. In D.A.Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp.49-64). New York: Macmillan.

Roschelle, J. (1992). Learning by collaborating: Convergent conceptual change. ***The Journal of the Learning Sciences***, 2(3), 235-276.
http://ctl.sri.com/publications/downloads/ConvergentConceptual.pdf

Roschelle, J., Pea, R., Hoadley, C., Gordin, D., and Means, B. (2001); "Changing how and what children learn in school with collaborative cognitive technologies," in M. Shields (Ed.), ***The Future of Children (Issue on Children and Computer Technology*, Los Altos, CA), David and Lucille Packard Foundation**, 10(2), 76-101.

Roschelle,J., Tatar, D.,and Kaput, J. (2005). Getting to scale with innovations that deeply restructure how students come to know mathematics. Unpublished draft manuscript at

Ross, C. M., and Lukow, J. 2004. "Are learning styles a good predictor for integrating instructional technology into a leisure studies curriculum?" ***Journal of the Scholarship of Teaching and Learning***, 42, 1, 42-53.

Schoenfeld, A.H. (1985). Mathematical problem solving. New York: Academic Press.
Shechtman, N., Roschelle, J., Haertel, G., Knudsen, J., and Tatar, D. (2005). Measuring student learning gains in conceptual mathematics when scaling a technological intervention for middle school mathematics. Paper presented at the American Educational Research Association Annual Meeting, Montreal, Canada.
http://ctl.sri.com/publications/downloads/SimCalc_Assessment_AERA_2005.pdf

Springer, L., Stanne, M., and Donovan, S. (1999). Effects of small group learning on undergraduates in science mathematics engineering and technology: A meta-analysis. ***Review of Educational Research***, 69, 21-51.

White, B. and Frederiksen, J. ((2005). A theoretical framework and approach for fostering metacognitive development. ***Educational Psychologist*** Volume 40, Number 4, Fall 2005, 211-224.

Woolfolk, A. (1995). Educational psychology. Boston: Allyn and Bacon.

Van Zee, E.H. (2000). Analysis of a student generated inquiry discussion. ***International Journal of Science Education***, 22, 115-142.

Vygotsky, L.S. (1978). Mind in society: The development of mental process. Cambridge, MA: Harvard University Press.

Zemelman, S., Daniels, H., and Hyde, A. (1998). Best practice: New standards for teaching and learning in America's schools. Portsmouth, NH: Heinemann.