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Assessing the Impact of Standards-based Curricula: Investigating Students' Epistemological Conceptions of Mathematics

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Since the advent of the NCTM *Standards* (1989), mathematics educators have been faced with the challenge of assessing the impact of *Standards*-based (or "reform") curricula. Research on the impact of *Standards*-based curricula has predominantly focused on student achievement; here we consider an alternative: Students' epistemological conceptions of mathematics. 297 participants were administered a Likert-scale survey instrument, the Conceptions of Mathematics Inventory. Of these, 163 had not experienced *Standards*-based curricula, while the rest had used a *Standards*-based curriculum for over three years. Our results indicate that students at the *Standards*-based site expressed more sophisticated epistemological conceptions of mathematics than those of the students from the non-*Standards*-based site. We interpret this result to suggest that implementation of *Standards*-based curricula may be having an effect on students' epistemological conceptions of mathematics.

Since the advent of the National Council of Teachers of Mathematics' (NCTM) *Standards* (1989, 2000), mathematics educators have been faced with the challenge of assessing the impact of *Standards*-based (or "reform") curricula on the students who used them. (We use the term "*Standards*-based" and "reform" interchangeably to refer to curricula developed with support from the National Science Foundation to achieve the vision of the 1989 NCTM *Standards*.) Despite the complexity of this endeavor, there is a pressing need to evaluate what effect curricula emerging from *Standards* documents have had on the teaching and learning of mathematics. In prior research efforts evaluating reform curricula, *impact* has been defined and operationalized in several ways. Some evaluations of reform have defined impact broadly, to include a range of factors such as differences or changes in attitudes, beliefs, and achievement (e.g., Wood & Sellers, 1997). However, more recently, given the political climate of "what works," research on the impact of curricula has exclusively focused on student achievement (e.g., Senk & Thompson, 2003).

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While acknowledging that improvement in student achievement is one important way to evaluate the impact of reform curricula, in this study we examine the impact of reform curricula on students' *epistemological conceptions* of mathematics. We present an analysis of students' epistemological conceptions of mathematics in two different curricular settings to study the impact of *Standards*-based curricula.

Students' Epistemological Conceptions of Mathematics

One's beliefs and assumptions about the nature of knowledge and knowing establish a psychological context for learning. They color how the learner views a school subject and the process of coming to know in that subject matter. We refer to these ideas and assumptions as *epistemological conceptions*, which we define as students' relatively unexamined beliefs and assumptions about the nature of knowledge and knowing that exist at varying levels of sophistication and commitment. Students' epistemological conceptions of mathematics are suggested to be an important part of students' experiences when learning and doing mathematics (Gfeller, 1999); they establish a psychological context for what it means to know and do mathematics (Schoenfeld, 1992). As epistemological conceptions are central to how students experience the learning process, they may be the single most important construct in educational research (Pajares, 1992).

Epistemological conceptions are a challenging construct to study, for at least three reasons. First, there are varying labels and conceptualizations used across the field of mathematics education and educational research more generally to refer to this construct. What we call *epistemological conceptions*, others refer to as *epistemological beliefs* (Hofer & Pintrich, 1997; Schommer & Walker, 1995) or *epistemological stances* (diSessa, Elby, & Hammer, 2002). We choose to use the term *conceptions* rather than *beliefs* because typically *beliefs* refers to students' assumptions about knowledge and knowing in addition to a range of other constructs (e.g., self-efficacy beliefs); thus, we use *conceptions* to refer to the subset of beliefs that address epistemological issues. Similarly, we feel that *epistemological beliefs* is not an appropriate label, as this label has been used to refer to trait-like, relatively permanent ways of thinking. Further, another inappropriate term for our purposes is *epistemological stances*, which refers to either highly contextual ways of knowing (diSessa et al., 2002) or broader, paradigmatic orientations toward knowing (Schwandt, 2000). Our choice of the term epistemological conceptions indicates the domain of the construct, which focuses on participants' ideas about the nature of knowledge and knowing, as well as an acknowledgement that these conceptions, as reported on a survey instrument, may exist at varying levels of commitment.

A second challenge inherent in studying students' epistemological conceptions concerns whether conceptions are considered to be domain-specific (localized to the school subject of mathematics) or domain general (pertaining to a range of subject matters). In support of the latter view, Schommer and Walker (1995) found that students held a range of conceptions at consistently sophisticated levels across the domains of both mathematics and social studies, including conceptions of knowledge as less certain or simple, learning as a not quick process, and one's ability as not being fixed. However, challenges to this work have been presented by Buehl, Alexander and Murphy (2002), who found evidence supporting the domain-specificity of certain conceptions (e.g., knowledge utility or value – mathematics is more related to other areas than history) when survey items were worded in reference to disciplines.

Finally, the potentially wide range of such conceptions further challenges any study of students' epistemological conceptions. Conceptions can be interrelated in one's mental structures in potentially limitless systems (Abelson, 1979). For example, beliefs about oneself as a learner could be related to beliefs about the learning process, which in turn could be related to one's beliefs about mathematics as a domain. Also, beliefs about the self could also be supported by motivational beliefs. Given this wide range, it is unclear where to bound the study of epistemological conceptions or how to select focal conceptions for study.

Despite the challenges of conceptualizing this construct, a careful analysis of students' epistemological conceptions of mathematics is critical, particularly in efforts to evaluate the impact of reform, for at least two reasons. First, epistemological conceptions have the potential to be the most salient or remembered aspect of students' experiences in mathematics. It is our perspective that the conceptual and procedural knowledge of school mathematics tend to weaken over time, while broader conceptions of the nature of knowledge and knowing endure. This stance is supported by Bishop (1996), who states that affective factors "appear to survive longer in people's memories than does conceptual and procedural knowledge, which unless it is regularly used tends to fade." (p. 19). Similarly, McLeod (1992) suggests that conceptions orient students' perceptions, which gives epistemological conceptions a significant role in shaping longer-term memories of mathematics. These remembered conceptions can be considered an indicator of the impact of reform as they are abstracted from students' experiences in the mathematics classroom (Schoenfeld, 1992).

Second, epistemological conceptions are considered to orient learning of mathematics in terms of students' motivation, achievement, and problem solving. For example, with respect to motivation, Stodolsky, Salk, and Glaessner (1991) suggested students' conceptions about the nature of the school subject are related to their learning goals. Cobb (1985) demonstrated this relationship through two case studies of first grade students, illustrating that a student with an ego-involvement learning goal, such as a focus on performance, also held the conception that mathematical procedures were unrelated to one another. In contrast, a student with a task-involvement learning goal, or persistence in learning the material, viewed relations between procedures. Conceptions about the nature of the school subject co-occurred with particular learning goals for these students. Additionally, some conceptions are related to students' academic achievement and problem solving behaviors.

Schoenfeld (1985, 1988) found that students who held a belief that mathematical problems should be able to be solved in 12 minutes or less (quick learning) also exhibited a lack of persistence when working on challenging problems. Quick learning has also been found to be the epistemological conception that was the strongest predictor of high school GPA—the less students believed in quick learning, the higher GPA they earned (Schommer, Calvert, Gariglietti, & Bajaj, 1997). More evidence is needed to explore whether and how additional epistemological conceptions may support or constrain students' problem solving behaviors and achievement. However, as problem solving is often a significant activity in reform settings, exploring which epistemological conceptions are prevalent among students in reform settings could provide some initial support for future studies of relations between conceptions and students' academic behaviors in mathematics.

Students' Conceptions in *Standards*-based Settings

Relations between epistemological conceptions and students' experiences with learning mathematics have been demonstrated empirically, but much of the existing research on epistemological conceptions in mathematics education focuses on teachers' beliefs (e.g., Thompson, 1984; Cooney, Sheally, & Arvold, 1998) or was conducted before the widespread adoption of *Standards*-based curricula (e.g., Schoenfeld, 1988). Of the small number of studies focusing on students' epistemological conceptions of mathematics in reform curricula, we briefly review three recent papers that have particular relevance to our work. These studies demonstrate the potential for comparing epistemological conceptions of students who experience different mathematics curricula.

Boaler (1998, 1999) compared students with similar demographic profiles and found that students' experiences with different mathematics curricula impacted students' epistemological conceptions. Boaler investigated high school students' experiences in two curricular settings in England: one school with open-ended activities and the other with a more traditional textbook approach. She found that students in the open-ended setting were more likely to express enjoyment for doing mathematics and to appreciate thinking for themselves over memorizing. Students in the traditional setting were more passive about their learning, were more likely to have a set view of mathematics as a vast collection of exercises, rules, and equations, and viewed mathematics as relating less to the world than other school subjects.

Extending this line of work with high school students in the United States, Gresalfi, Boaler, and Cobb (2004) examined students' experiences in three high school mathematics programs, one of which was more traditional than the other two, focusing on analyses of students' epistemological conceptions. They determined that students who studied mathematics in a traditional setting expressed passive conceptions about learning mathematics, including seeing mathematics as an external domain of knowledge and not connecting with mathematics in a personal way. In contrast, students from reform classrooms expressed inquiring conceptions about learning mathematics, such that they were likely to use mathematics to ask questions and probe relationships they observed. The authors argued that these conceptions were more related to students' experiences with curriculum than the learning preferences the students brought with them to the classroom.

Hofer (1999) also contrasted students (N = 438) in two different curricular settings, but she compared college undergraduates who experienced different forms of Calculus: one that emphasized active and collaborative learning both in and out of class and primarily focused on word problems, and the other a more traditional approach of lecture and demonstration. According to the author, students registered for calculus without knowing the type of instruction that would be utilized in their respective sections. The students in the non-traditional calculus course were found to have more sophisticated conceptions of mathematics; they were particularly less likely to believe that doing mathematics involves getting a right answer quickly. Achievement was positively correlated with sophistication in mathematical conceptions (as in Schommer et al., 1997), and students with sophisticated conceptions of mathematics were more likely to have mastery orientations to learning mathematics.

As these studies suggest, students who experience different mathematics curricula may develop different epistemological conceptions. The present exploratory study contributes to this growing line of research by examining the epistemological conceptions of students who have experienced different forms of mathematics curricula for extended periods of time. We introduce a forced choice assessment as an alternative to grounded qualitative analysis of conceptions (Boaler, 1998, 1999; Gresalfi et al., 2004). The choice to utilize a Likert-scale instrument was purposeful, rooted in our conceptualization of epistemological conceptions. Since we believe that students' epistemological conceptions exist at a relatively unexamined level, we asked students to offer their opinions with respect to particular statements. Forced choice instruments have been criticized for reflecting the researchers' meaning making over the students', but they also afford a relatively efficient method for collecting data on large numbers of students and allow for testing hypotheses (Schommer, 1998; Schommer et al., 1997; Schommer, Crouse, & Rhodes, 1992; Schommer & Walker, 1995).

Method

We explored the impact of *Standards*-based curricula on 297 students' epistemological conceptions of mathematics. One hundred sixty three students who had not experienced *Standards*-based curricula, had been administered the survey as part of an initial study of the instrument in 1996. An additional 134 students, all of whom had experienced a *Standards*-based textbook series for over three years, were recruited as part of the present study to serve as a comparison group. (The details of these survey administrations are provided below.) Our hypothesis was that students who experienced different mathematics curricula would differ in their epistemological conceptions.

Instrument

Few survey measures have been developed for the purpose of studying secondary students' epistemological conceptions of mathematics. One such measure is the Conceptions of Mathematics Inventory, or CMI (Grouws, 1994). The 56 questions on the CMI ask students whether they agree or disagree with certain statements about what it means to do, learn, and think about mathematics. The survey questions comprise seven scales (Composition of Mathematical Knowledge, Structure of Mathematical Knowledge, Status of Mathematical Knowledge, Doing Mathematics, Validating Ideas in Mathematics, Learning Mathematics, and Usefulness of Mathematics), each of which assesses a different aspect of students' epistemological conceptions toward mathematics. Students respond on a 6-point Likert scale, with "1" expressing strong agreement and "6" expressing strong disagreement. A student who mostly agrees with all questions on the CMI would hold epistemological conceptions consistent with the aims of recent reform documents. Such a student would view mathematics as being composed of a useful, coherent, and dynamic system of concepts and ideas in which learning is accomplished by sense making and authority is found through logical thought. A student who mostly disagrees with statements on the CMI would find mathematics an irrelevant, unchanging collection of isolated facts and procedures, handed down from a book or teacher, which must be memorized. Table 1 describes the seven scales of items on the CMI; see Appendix A for a complete list of the items on the CMI.

Grouws, Howald, and Colangelo (1996) assessed 163 ninth, tenth, and eleventh graders on their conceptions with the CMI between 1995 and 1996. Their original study was not designed to take into account the impact of curricula on students' epistemological conceptions, but the CMI authors recall that the study participants were exclusively selected from non-*Standards*-based mathematics classes in Missouri (Grouws, personal communication, July 13, 2001, October 5, 2001, May 20, 2003).

Table 1

Scales of the CMI

Composition of Mathematical Knowledge. Mathematical knowledge is composed of EITHER concepts, principles, and generalizations OR facts, formulas, and algorithms.
Structure of Mathematical Knowledge. Mathematics is structured EITHER as a coherent system OR a collection of isolated pieces.
Status of Mathematical Knowledge. Mathematics as EITHER a dynamic field OR a static entity.
Doing Mathematics. Doing mathematics is EITHER a process of sense-making OR a process of obtaining results.
Validating Ideas in Mathematics. Validating ideas in mathematics occurs EITHER through logical thought OR via mandate from an outside authority.
Learning Mathematics. Learning mathematics is EITHER a process of constructing and understanding OR a process of memorizing intact knowledge.
Usefulness of Mathematics. Mathematics is viewed as EITHER a useful endeavor OR as a school subject with little value in everyday life or future work.

Although the questions on the CMI may appear to be designed to indicate whether students' conceptions are consistent with the goals of reform, the CMI has never been administered to a large group of students in reform mathematics classes to "validate" its effectiveness. In other words, although many would assume that students with extensive experience in *Standards*-based mathematics would respond to the CMI in a manner different than those with extensive experience in non-*Standards*-based mathematics, we explored this assumption empirically. Grouws et al. provided us with mean score and standard deviations on each scale for each class in their original 1996 study; the original data from that study were not available.

Participants and Data Collection

As mentioned above, all 163 original respondents to the CMI (Grouws et al., 1996) came from non-*Standards*-based backgrounds in mathematics. Although no data were collected about the schools, courses, or instruction experienced by students in Grouws et al. original sample, our multiple and detailed conversations with the authors of the CMI make us confident that no student in this sample had any recent experience with NSF-funded reform-oriented curricula.

As a contrast, we sought to assess a sample of students who had a fairly long-term experience with *Standards*-based mathematics in a well-enacted setting. We recruited 134 9th grade students from a high school in Michigan to complete the CMI, early in their 9th grade year. (This survey administration took place between 2000 and 2001.) All students completed at least three years of reform-oriented instruction (in 6th, 7th, and 8th grades) in a middle school whose curriculum, the Connected Mathematics Project, (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1997), teachers, and pedagogy were quite familiar to us and, we believe, represented an extremely well enacted version of reform. (The lead mathematics instructor at this middle school is a professional development specialist for the Connected Mathematics Project; the Connected Mathematics Project has extensively documented her teaching as exemplary.) Students were administered the CMI in their regular mathematics classes by their teacher.

Note that while the two CMI administrations occurred approximately five years apart, we still conjectured that the curriculum would play a role in distinguishing differences between students' conceptions, as recent research suggests the nature of curricula continues to play a key role in the development of students' conceptions (Gresalfi et al., 2004), with students developing conceptions that are more aligned with inquiring stances toward mathematics in less traditional settings.

Reliability of Instrument

Grouws et al. (1996) do not report the statistical reliability of the CMI in their original study, but Walker (1999), who also used the CMI, reported that the CMI underwent a lengthy process of analysis and revision during its development in order to increase its reliability. Walker (1999) computed Cronbach's alpha reliability values for her CMI data (N = 256 college students), which ranged from a low of 0.45 for the Composition and Doing scales to 0.91 for the Usefulness scale. We computed alpha reliability values using the CMI responses of the 134 students from a reform background; our results are given in Table 2. Our reliability values are comparable to Walker's (1999).

Table 2

Alpha Reliability of the 7 Scales of the CMI, N = 134

Composition	0.2864
Structure	0.5727
Status	0.5184
Doing	0.4341
Validating	0.3603
Learning	0.2603
Usefulness	0.8719

Nunnally (1978) and Litwin (1995) recommend Cronbach's alphas close to or above 0.7 to indicate satisfactory internal consistency of constructs. By this standard, only the Usefulness scale of the CMI can be considered statistically reliable. However, it should be noted that neither Walker (1999) nor Grouws et al. (1996) qualified their results by expressing concern about the reliability of the CMI. We discuss the impact of the low reliability of the CMI scales below.

Results

Comparison of Students' Epistemological Conceptions

We performed a *t*-test on the mean scores of each CMI scale, comparing Grouws et al. original sample of 163 students from a traditional background to our sample of 134 students from a reform background. Our results are shown in Table 3. Recall that all items used a six-point scale, and that the lower the number, the more reform-oriented the response.

Table 3

Participants' Mean Scores and Standard Deviations on CMI Scales

	Grouws and colleagues' 1996 students (N = 163)		Students from a reform background (N = 134)		Means significantly different?
	Mean	SD	Mean	SD	
Composition	3.90	0.9	3.34	0.5	***
Structure	3.69	0.8	2.76	0.7	***
Status	3.69	0.9	2.75	0.7	***
Doing	3.89	0.9	2.65	0.6	***
Validating	3.96	0.9	3.01	0.6	***
Learning	3.73	0.8	3.01	0.5	***
Usefulness	3.50	0.8	2.20	1.1	***

*** $p < .001$

We found that on average, students from a reform background responded reliably differently to the items on the CMI as compared to students from a traditional background. In particular, students' responses in the reform setting were more aligned with reform-oriented ideas on the scales of the CMI than traditional students' responses. The differences on each scale were statistically significant: Composition, $t(295) = 6.508$, $p < .001$; Structure, $t(295) = 10.3181$, $p < .001$; Status, $t(295) = 8.352$, $p < .001$; Doing, $t(295) = 11.754$, $p < .001$; Validating, $t(295) = 10.001$, $p < .001$; Learning, $t(295) = 11.043$, $p < .001$; and Usefulness, $t(295) = 13.933$, $p < .001$.

In addition to testing for significance, the magnitude of the differences between traditional and reform students' responses can be seen by determining effect sizes using Cohen's *d* (Cohen, 1988). A value of Cohen's *d* larger than 0.8 indicates a large effect. Effect sizes are given in Table 4, indicating that there are very large differences between traditional and reform students' responses on all scales.

Table 4

Values of Cohen's d (effect size) on the Comparison of Students' CMI Responses

	Cohen's *d*
Composition	0.67
Structure	1.07
Status	1.05
Doing	1.43
Validating	1.10
Learning	0.89
Usefulness	1.33

The combination of a statistically reliable difference and a very large effect size on all scales leads us to conclude that, despite only moderate alpha reliabilities on most scales, students from a reform background responded differently on the CMI than did students from a traditional background. In other words, our data indicates that after experiencing several years of exemplary instruction in *Standards*-based mathematics curricula, students appear to develop epistemological conceptions of mathematics that are different from those from a more traditional background.

The results from the Usefulness scale are particularly compelling, as this scale was reliable, showed significant differences, and had a very large effect. The Usefulness scale items include, "Students need mathematics for their future work," "Mathematics is a worthwhile subject for students," "Students should expect to have little use for mathematics when they get out of school" (reversed item). The Cohen's *d* effect size of 1.33 for this scale indicates that the mean response of the reform students (3.50) was greater than the 90th percentile of the traditional group's responses (Cohen, 1988). Reform students' conception of the usefulness of mathematics was clearly quite different from that of students from a more traditional background.

Students' responses on the Structure scale are also revealing. Although somewhat less reliable (see Table 2), this scale also showed significant differences and a very large effect. Structure scale items emphasize the inter-relatedness of ideas in mathematics; items included, "Mathematics involves more thinking about relationships ... than working with separate ideas," "Most mathematical ideas are related to one another," and "Mathematics consists of many unrelated topics" (reversed item). Reform students' responses indicated a conception that relationships do exist between mathematical concepts. This is reminiscent of the work of Gresalfi et al. (2004), mentioned above, who found that students from classroom settings in which communication was emphasized were more likely to see the relationships between mathematical concepts and procedures.

Discussion

Our results contribute to the promising line of research demonstrating that *Standards*-based curricula may impact students' epistemological conceptions of mathematics differently than traditional curricula (e.g., Boaler, 1998, 1999; Hofer, 1999; Wood & Sellers, 1997). Students who experienced different forms of mathematics curricula did indeed express different epistemological conceptions of mathematics, with high school students with experience in *Standards*-based curricula holding more sophisticated epistemological conceptions. These results are similar to Hofer's (1999) study of college students. Of particular note is the difference on the Usefulness scale; students in *Standards*-based settings were much more likely to find mathematics to be useful, which is similar to Boaler's (1998, 1999) findings.

Two limitations that temper these results are the lack of observations of the implementation of the curricula and our limited access to the Grouws et al. data set. First, in terms of implementation of the curricula, we acknowledge that we have only second-hand access to the traditional site. As mentioned previously, after communicating with Grouws and his colleagues, we are confident that the students used in their original study (Grouws et al., 1996) came from classrooms using non-*Standards*-based curricula; however, we have no data to confirm this. We recommend that future studies of students' epistemological conceptions be complemented by classroom observations, as suggested by Hofer and Pintrich (1997).

Second, since we did not have full access to Grouws et al. (1996) data set, we were limited in the statistical analyses we could perform. As a result, we interpret the results of some of our scales with caution, given the moderate to low alpha scores for reliability. However, as discussed above, we did find significant differences and also large effect sizes on all scales. Future research could involve more sophisticated analyses (e.g., factor analyses) with the CMI to reassess and improve the scales for increased reliability.

It is our recommendation that efforts to assess the impact of *Standards*-based curricula broaden to include factors beyond student achievement. In order to achieve these ends, researchers should continue efforts to develop and refine instruments to assess large-scale groups of students. In order to further this line of research, researchers could work to extend the sample beyond two schools, making efforts to carefully match the sites, and analyze for alternative moderating variables, such as teaching style or student achievement. The conjecture that students' experiences with *Standards*-based curricula could impact students' dispositions toward mathematics, such as helping them develop more positive attitudes or sophisticated epistemological conceptions of mathematics, is worth further investigation. Teachers are invested in the development of students' perspectives in addition to growth in their achievement; research should reflect this value.

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Appendix A

Items on the CMI
(* Indicates reversed items)

Composition

9. While formulas are important in mathematics, the ideas they represent are more useful.
25. Computation and formulas are only a small part of mathematics.
39. In mathematics there are many problems that can't be solved by following a given set of steps.
51. Essential mathematical knowledge is primarily composed of ideas and concepts.
*1. There is always a rule to follow when solving a mathematical problem.
*17. Mathematicians work with symbol rather than ideas.
*33. Learning computational skills, like addition and multiplication, is more important than learning to solve problems.
*49. The field of mathematics is for the most part made up of procedures and facts.

Structure

13. Often a single mathematical concept will explain the basis for a variety of formulas.
24. Mathematics involves more thinking about relationships among things such as numbers, points, and lines than working with separate ideas.
37. Concepts learned in one mathematics class can help you understand material in the next mathematics class.
50. Most mathematical ideas are related to one another.
*7. Diagrams and graphs have little to do with other things in mathematics like operations and equations.
*19. Finding solutions to one type of mathematics problem cannot help you solve other types of problems.
*31. There is little in common between the different mathematical topics you have studied, like measurement and fractions.
*41. Mathematics consists of many unrelated topics.

Status

11. New mathematics is always being invented.
27. The field of mathematics is always growing and changing.
42. Sometimes when you learn new mathematics, you have to change ideas you have previously learned.
54. Students can make new mathematical discoveries, as well as study mathematicians' discoveries.
*3. When you learn something in mathematics, you know the mathematics learned will always stay the same.
*21. New discoveries are seldom made in mathematics.
*35. When you do an exploration in mathematics, you can only discover something already known.
*44. Mathematics today is the same as it was when your parents were growing up.

Doing

2. Knowing why an answer is correct in mathematics is as important as getting a correct answer.
16. When working mathematics problems, it is important that what you are doing makes sense to you.
32. Understanding the statements a person makes is an important part of mathematics.
56. Solving a problem in mathematics is more a matter of understanding than remembering.
*8. If you cannot solve a mathematics problem quickly, then spending more time on it won't help.
*29. Being able to use formulas well is enough to understand the mathematical concept behind the formula.
*38. If you knew every possible formula, then you could easily solve any mathematical problem.
*48. One can be quite successful at doing mathematics without understanding it.

Validating

10. Justifying the statements a person makes is an important part of mathematics.
26. It is important to convince yourself of the truth of a mathematical statement rather than to rely on the word to others.
40. When two classmates don't agree on an answer, they can usually think through the problem together until they have a reason for what is correct.
52. When one's method of solving a mathematics problem is different from the instructor's method, both methods can be correct.

*5. When two students don't agree on an answer in mathematics, they need to ask the teacher or check the book to see who is correct.
*15. You know something is true in mathematics when it is in a book or an instructor tells you.
*28. You can only find out that an answer to a mathematics problem is wrong when it is different from the book's answer or when the instructor tells you.
*45. In mathematics, the instructor has the answer and it is the student's job to figure it out.

Learning

14. Memorizing formulas and steps is not that helpful for learning how to solve mathematics problems.
22. When learning mathematics, it is helpful to analyze your mistakes.
43. When you learn mathematics, it is essential to compare new ideas to mathematics you already know.
55. Learning mathematics involves more thinking than remembering information.

*4. Learning to do mathematics problems is mostly a matter of memorizing the steps to follow.
*18. Learning mathematics involves memorizing information presented to you.
*30. Asking questions in mathematics class means you didn't listen to the instructor well enough.
*47. You can only learn mathematics when someone shows you how to work a problem.

Usefulness

6. Students need mathematics for their future work.
20. Mathematics is a worthwhile subject for students.
34. Knowing mathematics will help students earn a living.
46. Students will use mathematics in many ways as adults.

*12. Mathematics has very little to do with students' lives.
*23. Taking mathematics is a waste of time for students.
*36. Mathematics will not be important to students in their life's work.
*53. Students should expect to have little use for mathematics when they get out of school.