

# Predicting Turkish Ninth Grade Students' Algebra Performance

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The prediction of students' achievement in algebra in eighth and ninth grades has become a research interest for practical issues of placement. A group of simple, easily accessible variables was used to predict student performance in algebra after completion of eighth grade. The three variables of school type, grade level, and previous year mathematics performance explained 54% of the variance in algebra performance. Furthermore, school type was the dominant predictor of performance, explaining 33% of the total variation in algebra achievement.

Accepted as a gatekeeper, algebra has been a major focus of school mathematics. The basic reason may be the power that algebra has provided for operating with concepts at abstract levels and applying those concepts in concrete situations, beginning with the invention of "symbolic algebra" by Vieta (Kieran, 1992). The place of algebra in school curricula differs around the world (Howson, 1991), as some countries offer integrated or unified mathematics in their schools and others approach mathematics as a unified body of strands such as algebra, arithmetic, and geometry.

In the United States, a differentiated high school mathematics curriculum (i.e., first-year algebra, geometry, and second-year algebra followed by trigonometry and/or precalculus, and calculus), taught in a linear fashion, appears to be the standard format, differing from state to state in terms of mathematical content and scope. Traditionally, algebra has been a part of the college preparatory mathematics curriculum, offered to only a fraction of students because of tracking or choices made by students, teachers, and parents. For many years, it was offered as a single course or two, promoting manipulative skills with which students were expected to master manual techniques for transforming, simplifying, and solving symbolic expressions that were often isolated from a real-life context. However, the reform movement promoted by the National Council of Teachers of Mathematics (NCTM, 1989, 2000) suggests that school mathematics in general, and algebra in particular, be considered an integrated and cohesive body of concepts, each closely related to the other major strands of school mathematics. With the reform

movement, the distinction between pre-algebra, algebra, and other courses offered as a part of a general mathematics curriculum has become less apparent. The development of an informal understanding of algebraic ideas in grades K-8 is now not only acknowledged but also encouraged, with the hope that all students will have a strong foundation in algebra and geometry by the end of eighth grade and a strong desire to pursue higher algebra and mathematics in high school. In many school districts, Algebra I courses, traditionally offered to ninth grade students and a selective few eighth graders, are now available to all students in eighth grade.

The Turkish educational system endorses a vision of integrated mathematics and a national school mathematics curriculum in which all students are offered the same compulsory mathematics courses until the end of eighth grade. It is more or less the same in secondary education (ninth, tenth, and eleventh grades); and mathematics, and thus algebra, is again compulsory for all students. However, the scope of mathematical concepts may differ somewhat for those students who decide to take the university entrance exam with a social sciences emphasis. This decision is made at the beginning of tenth grade, and those students are required to take fewer mathematics courses than the students whose decisions favor an emphasis on science and mathematics. The situation is similar to that of students in the United States who choose to attend magnet schools.

Formal algebraic concepts are presented to all students and generally begin with an introduction of literal expressions and linear equations in seventh grade and continue with more emphasis in the eighth grade. More specifically, the seventh and eighth grade mathematics curricula prepared by the Committee of Teaching and Training in the Turkish Ministry of National Education and offered to all students in those grades have the following aims:

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#### Seventh Grade:

- understanding Mathematical Expressions,
- knowledge of Proposition, Open Proposition, and Equations,
- ability to solve linear equations with one unknown,
- ability to solve linear inequalities with one unknown,
- understanding the coordinates of a point in the Cartesian plane,
- ability to draw graphics.

#### Eighth Grade:

- ability to make operations with literal expressions,
- understanding Binomial Expansion,
- understanding certain identities,
- ability to factorize,
- ability to solve linear equations with one unknown,
- ability to solve linear equations with two unknowns,
- understanding the equation of a line,
- understanding linear inequalities with two unknowns,
- ability to solve linear inequalities with two unknowns.
- understanding basic trigonometric identities.

The following summary of content of mathematics courses in ninth, tenth, and eleventh grades may give an idea about the place of algebra in the secondary mathematics curriculum:

- The ninth grade mathematics curriculum consists of the following concepts and topics: logic; sets; relations, functions, and operations; numbers (natural numbers, integers, modular arithmetic, rational numbers, real numbers, absolute value, exponential numbers, and radical expressions); polynomials; and quadratic equations, inequalities, and functions.
- The tenth grade mathematics curriculum includes the following concepts and topics: trigonometry; complex numbers; logarithms; permutations, combinatorics, and probability; induction; sequences and series.
- The eleventh grade mathematics curriculum consists of the following concepts and topics: functions and advanced graphing (including special functions such as sine functions, absolute value functions, step functions, etc.); limit; continuity; derivations; integration; and linear algebra (matrices and determinants).

The teaching and learning of algebra in a Turkish context is not much different from what is seen in an American context. In the traditional mathematics course setting, students are introduced to the main points of algebra topics; given the rules and formulas involving manual skills for transforming, simplifying, and solving symbolic expressions, followed by a few rote examples; and then the students are expected to solve similar exercises and problems. Although they are widely available and used even in traditional algebra classrooms in the United States, manipulatives like algebra tiles are generally not available and are not used in the teaching and learning of algebra in Turkey. However, other recommendations of the reform movement in mathematics education have become more visible in Turkish schools; and over the last few years, the national K-8 mathematics curriculum has gone through major changes in terms of content and instructional strategies with more student-centered teaching, use of manipulatives, and utilization of technology, particularly calculators. It also requires the earlier introduction of algebraic concepts, starting in K-5 with scale models and the investigation of various number and shape patterns. The new curriculum, with revised teaching/learning tools and materials, is almost ready for implementation, and groups of lead teachers began in-service training in the 2004-2005 school year. The Ministry of National Education has opened a textbook writing competition among publishers for the new K-8 curriculum; and seven textbooks, from among more than 100, will be selected by a committee and then recommended to all schools nationwide for the 2005-2006 school year. The duration of high school will be extended from three to four years, and research for the development of a new 9-12 mathematics curriculum has begun.

Not only whether algebra is offered to all students but also the ages and grade levels at which algebra is introduced differs from country to country (Howson, 1991), as we can see from the Turkish and American examples. Yet, one thing that might be common everywhere is that the road to algebra is never as smooth as one may wish. Algebra is one of the areas in which students have major problems (Booth, 1988; Kieran, 1992), and identifying student difficulties and measuring student achievement have gained importance as a focus of various research studies (Booth, 1984, 1988; English & Halford, 1995; Flexer, 1984; Herscovics, 1989; Kieran, 1989, 1992).

The prediction of students' achievement in algebra in eighth and ninth grades has become a research interest for practical issues of placement (Flexer,

1984). Students gender, intelligence (IQ), socio-economic status, previous mathematics performance, and standardized test scores have often been used as predictors of achievement in algebra or mathematics (Flexer, 1984). However, the usefulness of these predictors is limited because not every country has an educational system that includes the use of standard achievement or intelligence tests. Thus, it is necessary to be able to predict student algebra achievement using an accessible and small number of predictors. The purpose of this study was to predict, at the beginning of ninth grade, student performance in algebra, using the variables of school type, previous year mathematics performance, grade level, and gender.

### Method

#### Sample

The participants in the study were 217 students from two public academic, one private, and one vocational-technical high school in a socio-economically middle-class district of a metropolis in Turkey. This school district was selected after a pilot study had been carried out in the district to develop some of the test instruments for this study, and the schools were selected by a random sampling method (Fraenkel & Wallen, 1996) according to their type: private school (PrSc), public academic school (PuAc) and vocational-technical high school (VoTe)<sup>1</sup>. Two public academic schools were chosen because the number of such schools was greater than the number of the other two types. Two classes from each school, eight in all, were chosen based on availability and permission of the participant schools. According to Green (1991), for a power of .80 (alpha = .05), four predictors and a medium effect size, the desirable sample size is around 85. The sample size in this study was large enough to conduct a regression analysis.

#### Predictor Variables

Four predictor variables were considered in the present study: (i) school, (ii) previous year mathematics performance, (iii) gender, and (iv) grade level. Data for the predictor variables were measured by a short questionnaire administered along with an algebra test used to measure the criterion variable, performance. The collection of predictor variables, along with a description of the method by which they were measured, is given in Table I.

Table I

*Predictor Variables and their Measurements for the Regression Analysis*

Predictor Variable	Measurement
School	1 = PrSc 2 = VoTe 3 = PuAc-1 4 = PuAc-2
Previous year mathematics performance	1 = Minimum to 5 = Maximum
Gender	0 = Male 1 = Female
Grade level	1 = Preparatory 2 = Ninth Grade

In order to measure the prior mathematic performance, students were asked to provide the grade they had received in their mathematics course at the end of the previous year. In the Turkish educational system, grading is based on a five point scale corresponding to 1 = F, 2 = D, 3 = C, 4 = B, and 5 = A. Thus, as an ordered variable, previous year mathematics performance was scored by integer scaling, ranging from 1 to 5. The scoring for the two dichotomous variables was 0–1 for gender and 1–2 for grade levels as shown in Table I.

#### Criterion Variable

To measure student performance, I developed an Algebra Test (AlTe) and used the first version in a pilot study. Subsequently, some items in the test were rearranged and revised, and in its final form, the test consisted of 25 open-ended items covering 7th and 8th grade algebra topics. While developing the test, I took the new mathematics curricula for seventh and eighth grades, developed by the Education and Training Board of the Turkish Ministry of National Education (1991), into consideration. The algebra topics included on the test were generalization, factorization, literal terms, transformation, first order linear equations (with one and two unknowns), first order linear inequalities (with two unknowns), and word problems. Three of the items had been used in previous research and reported in the literature: two items used in the Concepts in Secondary Mathematics and Science (CSMS) project by Küchemann (1981) and one item used by Whitman (1976) were used with permission. The alpha reliability of the test (Cronbach, 1951) was .76. Sample items and some information asked in AlTe are included in Appendix B.

AlTe was administered during the fall semester by the mathematics teachers of the participating students because the students might not have taken the test seriously if they thought it came from outside the school and would not affect their grades. The teachers were informed about the test and how to administer it before they gave it to the students in their classrooms. The students were given no time restriction for completion of the test and turned in their answer sheets when they felt that they were finished with everything they wanted to do. On the average, the testing time was a class period, which was 45 minutes.

### Results

The data analysis was carried out in two steps: preliminary data analysis and multiple regression analysis. The means (M), standard deviations (SD), and correlations among all variables for students in the

study are presented in Table II. The male students, the ninth graders, the students having a grade of 3 in the previous year's mathematics course, and the students attending a public academic school were dominant profiles in the sample. The students' low performance on the Algebra Test is apparent, and the large standard deviation is an indicator of high variance in the sample.

The correlations among the four predictors ranged from  $r = -0.489$  to  $r = 0.503$ . Among those, school type was moderately correlated with the students' previous year mathematics performance and grade level. Correlations between each predictor and student performance on the algebra test performance ranged from  $r = -0.549$  to  $r = 0.589$ . In particular, scores were moderately correlated with school type and students' previous year mathematics performance.

Table II

*Descriptive Data for and Intercorrelations between Predictor and Dependent Variables*

Variable	1	2	3	4	M	SD
School					2.75	1.03
Students' previous year mathematics performance	-.489** (.000)				2.89	1.19
Gender	.083 (.232)	.097 (.162)			.37	.48
Grade levels	.503** (.000)	-.191** (.006)	.167** (.016)		1.83	.38
Algebra Test	-.549** (.000)	.589** (.000)	.033 (.635)	.034 (.625)	15.49	13.33

\*\* Value is statistically significant with the corresponding p-value shown in italics inside the brackets.

To assess the relative contribution of each predictor variable as to the variation of the prediction of performance, an analysis was carried out by deleting each predictor from the analysis. The results of this process are summarized in Table III. The results show that school type contributed more to the prediction of algebra performance than did any of the other three predictors. For these data, there was not much difference in the relative importance of the students' previous year mathematics performance and their grade levels, and the students' gender contributed the least to explaining the variations in the prediction. In fact, the R-squared value was almost unaffected when gender was removed from the regression equation.

Table III

*Relative Contribution of Predictors to Accuracy of Prediction.*

Predictor deleted	$R_{adj}^2$	Rank
School	0.35	1
Students' previous year mathematics performance	0.42	2.5
Grade levels	0.44	2.5
Gender	0.54	4

Although the number of predictors was small, I conducted an all subset analysis (Pedhazur, 1997, p. 212), with results summarized in Table IV. According to this analysis, school type alone was the best predictor and explained 33% of the variance in algebra performance. School and previous year math performance explained 44% percent of the variation; 54%, when grade level was included.

Table IV

All-possible-subsets analyses for regression

Subset size	Predictors in best subset	$R^2_{adj}$
1	School	0.33
2	School, Previous Math Grade	0.44
3	School, Previous Math Grade, Grade Level	0.54

Finding that school was a fairly good predictor of algebra performance, one may wonder which schools

did better. Figure 1 shows a boxplot display of algebra scores by schools. Figure 2 gives the same information by previous grade in mathematics.

According to Figure 1, students in the private school (PrSc) did relatively better than their counterparts in the public academic schools (PrSc-1 & PuAc-2) and the vocational technical school (VoTe). Although the students in PuAc-1 seemed as competent as the students in VoTe, the performance of students in PuAc-2 seemed very poor compared to the other three schools. Figure 2 shows that only the public academic schools (PuAc-1 & PuAc-2) had students with a mathematics score of not passing (i.e., "1") from the previous year, which means that those students had repeated the ninth grade because they had failed. None of the students in PuAc-2 had a "5" as their previous year's mathematics grade. It is also interesting that students in PuAc-2 showed a horizontal line of means across all the previous year's grades. PrSc had a lower mean only for those students having a "2" as their previous year's mathematics performance.

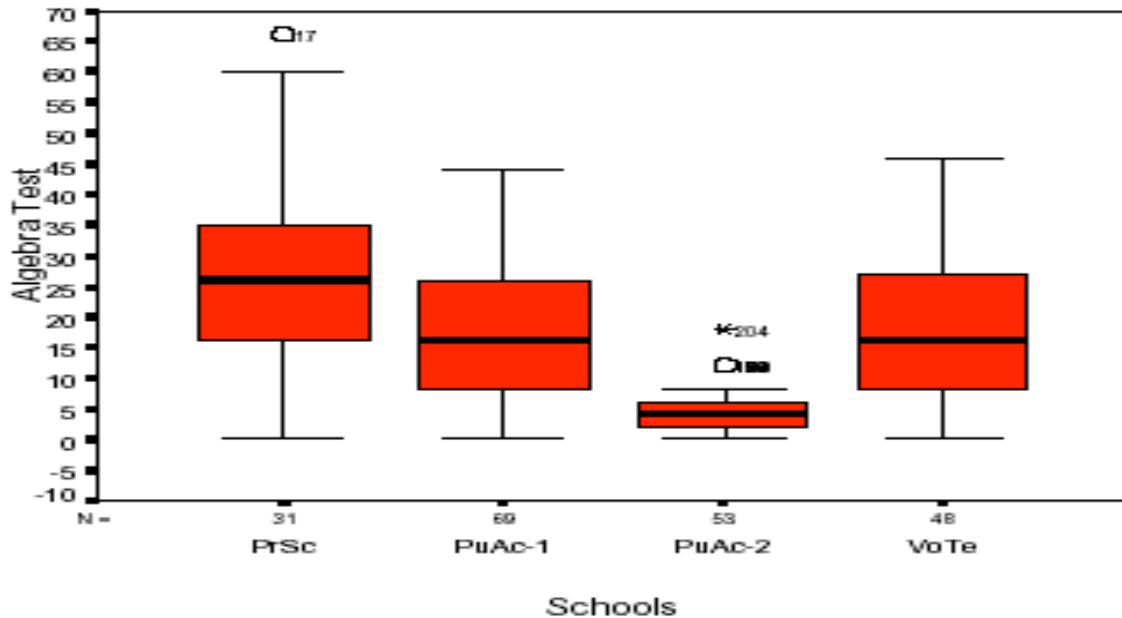


Figure 1. Boxplot display of algebra scores to school type.

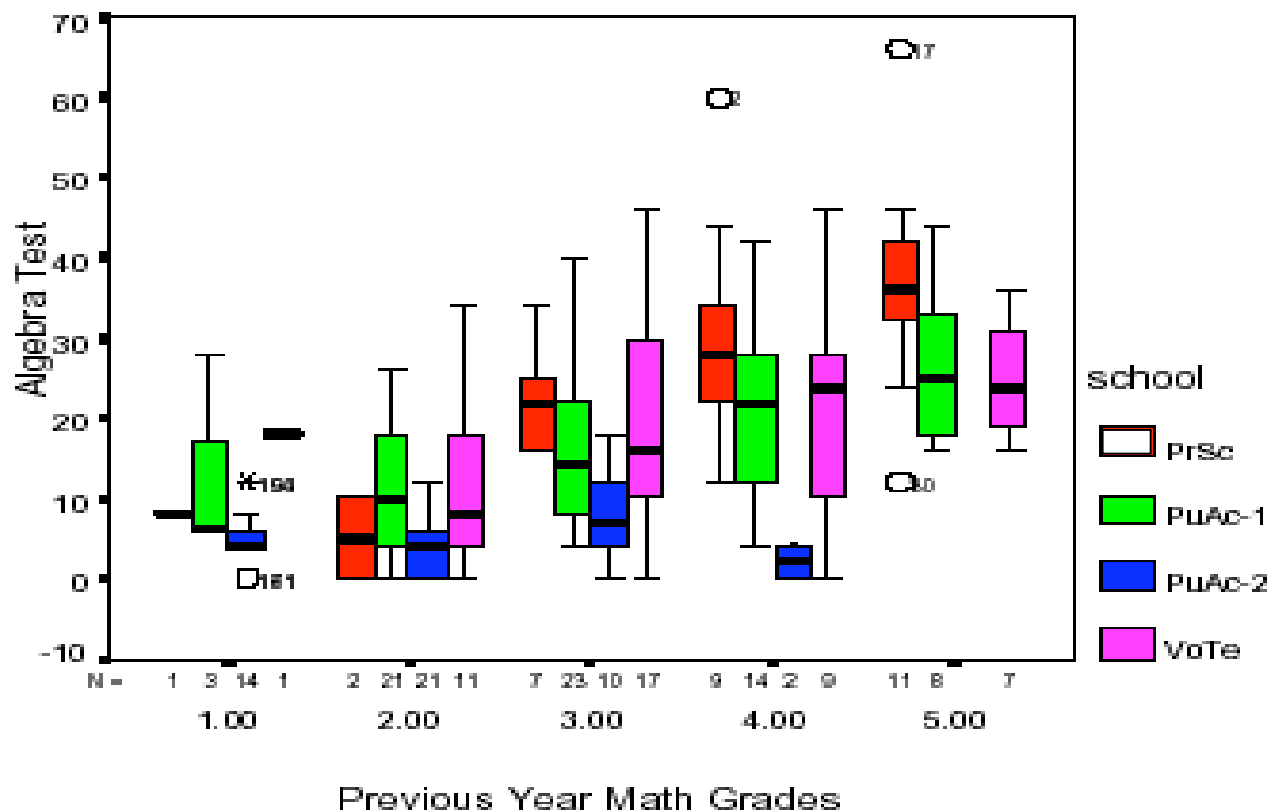


Figure 2. Boxplot display of algebra scores by school within previous year math grades.

### Discussion

The major finding of this study was that a large portion of the variance in student algebra performance at the beginning of ninth grade could be explained, not by sophisticated time and resource consuming variables, but by easily measured variables. Three variables: school type, grade level, and previous year mathematics performance explained 54% of the variance in algebra performance. This result is consistent with previously reported research predicting algebra achievement using more sophisticated variables (see Flexer, 1984; Hanna, Bligh, Lenke & Orleans, 1969). It was also found that gender was not a good predictor of algebra performance. Although this result is different from what Flexer (1984) found, it is consistent with the findings of large scale international studies concerning the effect of gender on mathematics achievement.

Although females have been, historically, considered educationally disadvantaged, large scale studies reveal that the gender gap has been greatly reduced in formal education. According to Lubienski,

McGraw, and Strutchens (2004), similarities rather than differences exist between males and females in overall NAEP results. Even though it was statistically significant at the eighth and twelfth grade levels, achievement differences between males and females were less noticeable when compared with differences related to race/ethnicity and socio-economic status. On the international level, the International Association for the Evaluation of Educational Achievement (IEA) Third International Mathematics and Science Study (TIMSS) revealed that in most countries, girls and boys had approximately the same mean mathematics achievement at seventh and eighth grades (Beaton et. al., 1996). The differences in achievement that did exist in some countries tended to favor boys rather than girls; however, about three-fourths of the differences were not statistically significant, indicating that in most countries, gender differences in mathematics achievement are generally small or negligible in the middle years of schooling. Similarly, findings from TIMSS-Repeat revealed that the gender differences in mathematics achievement were negligible at the eighth grade level (Mullis, Martin, Gonzalez, & Chrostowski,

2004). While girls significantly outperformed boys in a few countries, the opposite was observed in a few others, including the United States. In a more recent international study, Programme for International Student Assessment (PISA) revealed that gender differences favoring boys exist in mathematics achievement in some participating countries, but the differences tend to be small (OECD, 2004).

Even though the students in this study were at the same age levels, the effect of grade levels (i.e., “preparatory” and “9th grade”) were significant. This is probably because of the fact that students in 9th grade were introduced to algebra concepts (for example, functions) that differed from the ones they had encountered in the middle school algebra curriculum and the students had more experience in manipulating variables and literal expressions (in the traditional sense), whereas students in preparatory grade were usually given intensive instruction in a foreign language. However, I also observed that students in both grade levels did equally well in solving linear equations with a single unknown, which is required of all students in both seventh and eighth grade as a part of the curriculum.

Perhaps the most interesting result of this study was that school type was found to be the dominant predictor of students’ performance. Alone, it explained 33% of the variance in algebra performance. The order of students’ performance from highest to least was PrSc, VoTe, PuAc-1, and PuAc-2. This result raises important questions about the reasons for the variance or gaps in students’ performances in different school types. One simple explanation of the difference might be the higher quality of instruction, or education in general, in private schools. Because of the tuition and other private resources, these schools are able to hire experienced teachers and have greater flexibility and willingness (in both teacher and administration) to use alternative methods and materials in instruction. There is teacher and school accountability to the parents who pay for their children’s education, as the parents want to know their money is well spent. This is in contrast to the situation in public secondary schools, where the education is free, parent-school relations are weak, and accountability is virtually nonexistent.

Socio-economic levels of the students in the private and public schools are also noticeably different. Parental education levels for the students in private schools tend to be higher, their attention to their children's education is more pronounced, and their methods for dealing with academic problems are

usually different, too. These factors tend to contribute to students’ motivation in a positive sense.

Both the private high school and the vocational high school have higher expectations and criteria for the students who apply to those schools after primary education, as is evident from the distribution of the students’ previous year mathematics score/grades. For example, approximately 80% of the students in PuAc-2 had previous year mathematics scores lower than “3”, which indicates that most of the students at that public school were average or below average. While this helps us to understand why school and previous year math performance explained 44% percent of the variation, such observations are perhaps indicative of the differences in the general characteristics of the students (for example, socio-economic status, mathematics achievement they carry from primary school to high school, etc.) we might find/observe in students of different types of schools. This larger framework of characteristics is perhaps the place to look for variables in the students’ algebra performances; and variables such as parental and teacher influence, teaching methods, and how students learn algebra in different school settings should be studied in detail. As is often the case, this study, which started with one question, has led to many others for future research.

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<sup>1</sup>. Brief information on the Turkish educational system is accessible in Appendix A. For a complete description of the statistical methods used, please see Appendix (C).

## Appendix A: Turkish Educational System and School Types

The Turkish educational system consists of pre-primary education (Pre-K), primary education (K1-8), secondary education (9-11) and higher education (universities and two year colleges). For all levels of primary and secondary education, there are special education schools serving handicapped (exceptional) children. In Turkey, children ranging in age from 6 to 14 must attend primary school, which is free for all students.

Secondary education encompasses three years in the general high schools (public high schools, science schools, Anatolian high schools, etc.) or four years in the vocational and technical high schools (technical education schools, Anatolian technical education schools, religious education schools, commercial and tourism education schools). General high schools do not prepare students for a specific profession but rather prepare them for higher education. Vocational high schools train qualified individuals for various professions, and also prepare students for higher education. Anatolian high schools, private secondary schools, and some vocational and technical high schools require one year of English (German or French in a few cases) study before the students can continue their three year normal secondary schooling. Although mathematics courses are required in preparatory school, the weekly hours are fewer than in a regular ninth grade. In the sample of this study, there were students in preparatory class in the private high school and the vocational technical high school. However, the ninth grade students in the sample did not go through the preparatory school program.

At the end of eighth grade, on a single date towards summer, students take a non-compulsory centralized/standardized test called the High School Entrance Exam (or shortly LGS in Turkish). Usually the result of this test determines the types of schools students will attend in for their secondary education. Science schools, Anatolian high schools, and most of the (Anatolian) technical education schools accept students with the prerequisite scores on the LGS. The acceptance policies of public high schools and private secondary schools vary. The higher the school's perceived quality and reputation (and achievement), the more selective and difficult the acceptance process is. Public schools usually have no criteria for acceptance other than a primary school diploma. However, some public schools have a special division, Super High School, for students who are above average or high achievers. Students are accepted based on their LGS score and their primary school graduation GPA. Acceptance policies for the private schools vary, but a student's primary school graduation GPA and LGS score, if it is available, again have deterministic roles. Furthermore, since private schools usually have expensive tuitions, socio-economic status is the single most important consideration for attending those schools. There might be scholarships for a select few students.

The private school in the sample of this study was founded by the Foundation of National Education, which has several similar intuitions throughout the big cities of the country. They accept students based on previous academic performance, primary school GPA, and ability to pay the tuition and other expenses. The two public high schools in the study were located in a socio-economically low to mid-level area of the district. They accept students with no particular criteria, except possibly a requirement that students live nearby. The vocational technical high school in the sample had a mixed group of students accepted based on their LGS score and then considered primary school GPA if they had space to accommodate more students.



## Appendix B: Sample Items and information asked in the Algebra Test

### Information

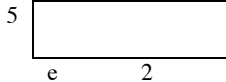
- Age
- Gender
- Date of birth
- Previous year's mathematics grade.

### Items

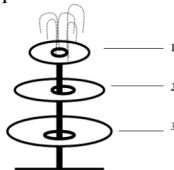
- Simplify  $5(p+q) - 2(p+q)$
- When is the following inequality true? Always / Never / Sometimes

$$\circ L + M + N = L + P + N$$

- What is the area of this rectangle?



- Simplify  $\sqrt{\frac{a+x}{b+x}}$
- Simplify  $\frac{1}{3x} + \frac{2}{x}$
- Factorize the following expression  $(x-1)^2 + 3(x-1)$
- Solve for  $x$ :  $\frac{3}{2x+1} = \frac{5}{x}$
- Rewrite/Expand the following expression  $(3x+2)^2$  so that there are no parentheses.
- What is the value of  $a$  in the following equality?  $69 - \frac{96}{7-a} = 37$
- If  $-\frac{a}{2} = \frac{-b}{3} = \frac{-c}{4}$ , list  $a, b, c$  from smaller to bigger.
- If  $0 \leq x \leq 2$  and  $3 \leq y \leq 5$ , what is the least value of  $x - y$ ?
- If  $a^2 < a$  and  $ab > b$ , would it be true that  $b < 0$ ? Why?
- If  $(x-5)^2 = (x-7)^2$ , what is  $x$ ?
- In the sum  $A = 2a + b + 3c$ , how much  $A$  will increase if  $a$  is increased by 3,  $b$  is increased by 5, and  $c$  is increased by 8?
- Rearrange the following expression for  $k$  so that you can find what  $k$  is:  $s = v - \frac{hk}{a}$
- What is the common solution for the following system of equation:  $3b + 2a = 13$  and  $2b - 3a = 0$ .
- A ball bounces one-third of the height it is fallen. If it bounces 8 cm after it falls three times, what is the original height it falls the first time?
- As shown in the following figure, the first pool in the fountain is filled by the water directly from the fountain and the rest two pools are filled by the water that drops from the one on the top after each is filled out. The volumes of the pools 1, 2, and 3 are  $v, 2v,$  and  $3v$  respectively and the first pool fills out in 2 hours. If the fountain is left open for 10 hours, what percent of the third pools is filled with water? Why?



## Appendix C: Statistical Information

The school variable has four categories consisting of VoTe, PuAc-1, PuAc-2, and PrSc, and scaled by creating three dummy variables. More precisely, PrSc, VoTe and Pac-1 were coded as 1 0 0 0, 0 1 0 0, and 0 0 1 0 consecutively in the data matrix. Here, it should be noted that the 0 0 0 row along those three columns indicates the coding for PuAc-2.

Analyses reported here had been done by using Summarize (descriptive), Regression (linear), Correlations, and Missing Value Analysis commands in SPSS 8.0 and by using the Proc Reg (best subset analysis) command in SAS 6.12.

### Preliminary Data Analysis

The data were first examined for missing values. The examination revealed that 17 subjects (about 8% of the sample) had one or two missing values. Those were determined by using the *regression imputation* technique (Roth, 1994). Before proceeding with a regression analysis, I searched for potential outliers using standardized residuals (Pedhazur, 1997, p. 44) and judged that nine students had Y scores and X-vector scores that were extensively distant from the Y-mean and X-vector of means. After those outliers were removed, the final data matrix was  $208 \times 5$ .

I accepted certain assumptions about the data matrix before the analysis. First, I assumed that any row vector of scores was independent of all other score vectors. Second, a normal probability plot was used to assess multivariate normality. For these data, the plot was virtually linear, indicating that the condition of normality was satisfied (Figures 3 and 4).

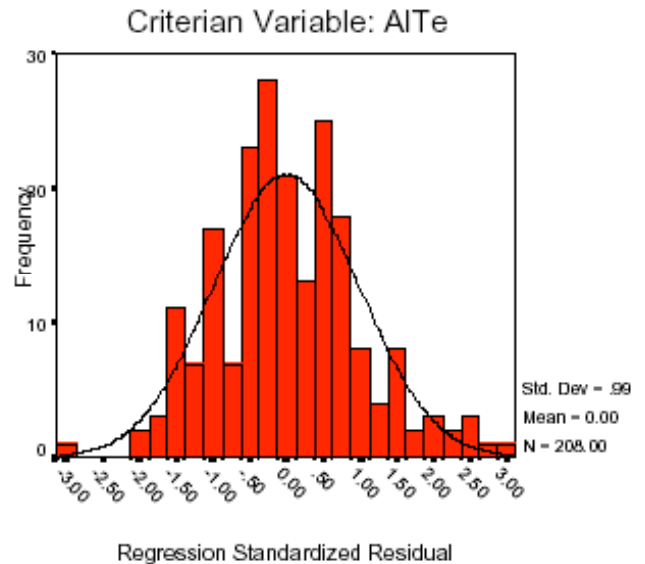


Figure 3. Histogram of standardized residuals for predictors of algebra performance

Normal P-P Plot of Regression Standardized Residuals  
Criterion Variable: AITe

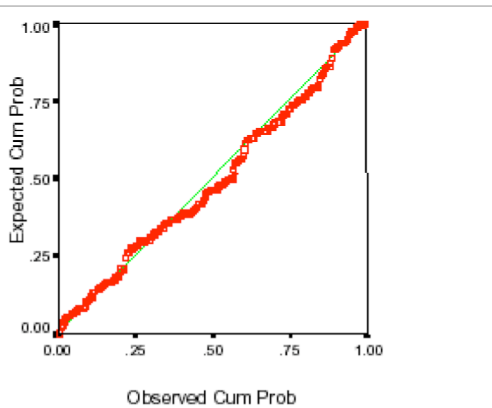


Figure 4. Normal cumulative probability plot of observed versus predicted standardized residuals for predictors of algebra performance.

The homogeneity of the variance in the criterion variable (i.e., Algebra Test) across the range of X-variable scores was assessed by examining the residual plot in Figure 5. Because of the elliptical shape of the plot, the homogeneity condition can be considered satisfied.

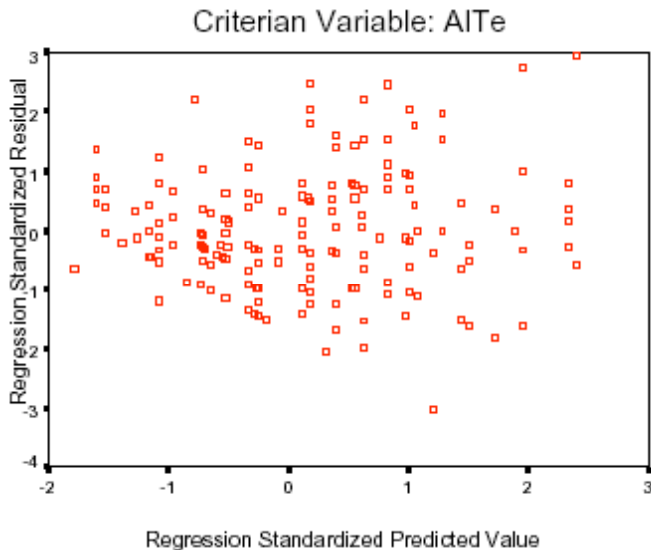


Figure 5. Residual plot ( $Y'$  vs.  $Y-Y'$ ) for predictors of algebra performance.

With the data assumptions met, I proceeded to determine the weights (b values) in the optimal linear composite of the six X variables (note that the school variable was converted into three dummy variables).

$$\hat{Y} = b_0 + \sum_{i=1}^6 b_i X_i$$

Additional analyses were carried out to determine if some analysis units (students) were having an extreme influence on the results. Examining the Cook D index value (Pedhazur, 1997, p. 51), I concluded that no student had undue influence on the bias of the weights. Similarly, I concluded that no student score vector exhibited undue influence on the precision of weight estimates, since there were no extreme covariance ratio index values. On the other hand, examining DFBETA index values (Pedhazur, 1997, p. 52), I found student score vectors that unduly influenced the b weights (considered estimates of population counterparts).

### Multiple Regression and Supplementary Analyses

The regression equation for data was as follows:

$$Y' = -30.823 - .735(\text{Gender}) + 13.42(\text{GradeLevel}) + 4.401(\text{MathPerformance}) + 21.267(\text{PrSc}) + 14.514(\text{VoTe}) + 8.170(\text{PuAc-1}),$$

with nonstandardized weights and,

$$Y' = -.027(\text{Gender}) + .382(\text{GradeLevel}) + .391(\text{MathPerformance}) + .577(\text{PrSc}) + .46(\text{VoTe}) + .289(\text{PuAc-1}),$$

with standardized weights.

Note that to estimate scores for students in PuAc-2, one needs to enter 0 in place of PrSc, VoTe and PuAc-1 since it is coded as all zeros along three dummy variables.

To determine how well the resulting equation fit the data for the 208 students, the simple correlation between Y and  $Y'$  (i.e., R) was calculated. I found  $R^2 \approx .55$ . For inferential purposes, one proceeds if this value is significantly larger than the value that would result by chance (Huberty, 1994). That is, is the  $R^2$  value of .55 significantly greater than the chance value of  $p/(N-1) = 4/(208-1) = .02$ ? The statistical test yielded statistics for this analysis of  $F(8,201) = 24.08$ ,  $p = .0001$ . Thus, I concluded that, overall, the predictions of Y using the four predictors were significantly better than chance.

To assess how well the derived equation would fit data based on new samples, the adjusted R-squared,  $R_{adj}^2$ , was calculated by the formula  $R_{adj}^2 = 1 - \frac{N+p}{N-p}(1-R^2)$  (see Huberty & Petoskey, 1999) and it has been found  $R_{adj}^2 = .52$ . The effect size (i.e., how much better than chance) question was addressed by considering the difference  $R_{adj}^2 - p/(N-1)$ , which was approximately .49 for these data. In general, it seems that the prediction rule is highly usable.