THE EFFECTS OF CONSTANT TIME DELAY AND STRATEGIC INSTRUCTION ON STUDENTS WITH LEARNING DISABILITIES' MAINTENANCE AND GENERALIZATION

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The purpose of this series of case studies was to compare the impact of Constant Time Delay and Strategic Instruction on the maintenance and generalization of learning. Four middle school students with learning disabilities were effectively taught two different groups of multiplication facts using Constant Time Delay and Strategic instruction. The researchers measured students' levels of maintenance and generalization after receiving each type of instruction. Maintenance data were collected using 1-minute fluency probes. Generalization data were collected using timed (1-minute) and untimed probes. Strategic Instruction appeared to have a greater impact on the students' maintenance and generalization of multiplication skills. The students' performance and perceptions are discussed in terms of potential implications for the classroom.

According to the National Council of Teachers of Mathematics (2000), mathematics instruction should result in successful problem solving and logical thinking. However, in order to advance to higher levels and solve mathematical problems, it is critical that students be fluent with basic operations (Miller & Mercer, 1993a). Fluency in math computation allows students to allocate their attention to the purpose of the problem instead of computation (Case, 1992; Fleischner & Manheimer, 1997). According to Houchins, Shippen, and Flores (2006), mathematical fluency is the *effortless and automatic ability to perform math operations* (p. 331). Unfortunately, many students with learning disabilities are not fluent in basic mathematical operations, particularly in multiplication (Cawley, Parmar, Yan,, & Miller, 1996).

Two typical instructional methods have been used to teach multiplication fluency to students with learning disabilities. The instructional methods are Constant Time Delay (CTD), a behavioral strategy, (Cybriwsky, & Schuster, 1990; Koscinski, & Gast, 1993; Mattingly & Bott, 1990; Morton & Flynt, 1997) and Strategic Instructional Model (SIM), a cognitive strategy (Mercer, Jordan, & Miller, 1996). Both CTD and SIM have been effective in teaching multiplication facts to students with learning disabilities. Each method is briefly described below.

Constant Time Delay
CTD provides teachers with a systematic, data-based method of using flashcards to teach multiplication facts. The method transfers stimulus control from a controlling prompt (e.g., teacher's verbal prompt) to a natural cue (e.g., multiplication fact). Mattingly and Bott (1990) used CTD to teach multiplication facts to elementary students with learning disabilities, mild intellectual disabilities, and emotional behavioral disorders. As a result of the intervention, all

of the students demonstrated a high rate of correct verbal responding (98.3%). The students maintained their performance up to four weeks after training and generalized their oral performance to pencil-paper tasks consisting of multiplication facts that were written vertically rather than horizontally as presented on time delay cards.

Cybriwsky and Schuster (1990) used CTD to teach fifteen multiplication facts to a ten-year-old student with learning and behavior disabilities. The student demonstrated mastery across teachers, settings, and materials with different orientations, appearance and color. CTD resulted in nearly errorless learning. Koscinski and Gast (1993) replicated Cybriwsky and Schuster's (1990) findings with five elementary students with learning disabilities. The researchers measured generalization using vertical orientation rather than horizontal, reversing factors, and presenting paper-pencil tasks. The procedure was near errorless with a student-error rate of only 4%. The students transferred their multiplication skills to independent paper-pencil tasks, demonstrating generalization to typical classroom tasks.

Strategic Instruction Model

SIM differs from CTD in that there is an emphasis on the student's cognitive processing. The purpose of the SIM is to build independent strategy use and self-regulation through explicit instruction (Lenz, Ellis & Scanlon, 1996). This model has been used to teach math problem solving and computation (Harris, Miller, & Mercer, 1995; Maccini & Ruhl, 2000; Mercer & Miller, 1992b; Miller & Mercer, 1993a; Miller & Mercer, 1993b;). Mercer and Miller (1992a) developed a SIM program based on this model for basic mathematical concepts. In addition to the principles of SIM, one of the main instructional methods used in this program is the concrete-representational-abstract (C-R-A) learning sequence. The C-R-A methodology fosters conceptual understanding of mathematical operations before memorization of facts. The concrete instructional phase involves the use of manipulatives to demonstrate the meaning of multiplication. During the second phase, the teacher and students illustrate the multiplication process by drawing lines to represent the numbers. The third instructional phase is the abstract phase that involves computing facts to automaticity.

Mercer and Miller (1992b) found that the combination of SIM and the C-R-A sequence resulted in accuracy and fluency over time for students with learning disabilities. The results from field studies indicated that students with learning disabilities were able to acquire computational skills across facts, solve word problems, and generalize skills across examiners, settings, and tasks. The students' performance improved from an average of 5 correct facts per minute to 14 correct facts per minute after instruction. The students maintained the same level of accuracy ten days after instruction. Harris, Mercer, and Miller (1995) replicated previous findings with second grade students with learning disabilities in inclusive settings. The students' rate and accuracy in computing multiplication facts increased from a range of 1-4 digits correct per minute during baseline to a range of 11-14 digits per minute. With regard to fluency, the increase for students with disabilities was similar to that of students without disabilities. These students acquired multiplication skills at a developmentally appropriate time.

Research has demonstrated that methods based on behavioral principles such as CTD, and those based on cognitive behavioral principles, such as SIM, are effective in teaching math facts. However, there is no literature regarding a comparison of the two methods. Therefore, the purpose of this series of case studies was to compare CTD and SIM. Specifically, maintenance and generalization of multiplication facts of students with learning disabilities were examined.

Method

Research Design

The purpose of this article is to present a series of case studies which examined the influence of two methods of multiplication fact instruction, CTD and SIM, with regard to maintenance and generalization. Maintenance was defined the number of correct digits per minute written when presented with previously taught multiplication facts. Maintenance was measured 1 week after instruction and 5 weeks after instruction. Generalization was defined as computation of unknown multiplication facts. Generalization was measured in two ways: (a) through a timed (1-minute) probe that included a set of unknown multiplication facts, and (b) through an untimed probe that included a different set of unknown multiplication facts. This article focuses on application of CTD and SIM instruction and their effects on the performance of students with specific learning disabilities in the area of mathematics.

Setting and Participation Requirements
Participants were students with specific learning disabilities attending sixth grade in a major southeastern city. The school was chosen based a recommendation by the district administration. Students were selected based on their area of disability eligibility, the presence of prerequisite math skills, and their knowledge of multiplication. Students whose area of eligibility was listening comprehension were not considered since this would interfere with probe administration and parts of instruction that were presented verbally. In order to participate in the study, students met the following criteria as indicated by Mercer and Miller (1992a) in the *Strategic Math Series*: (a) count to 81, (b) compute addition facts to 18, (c) add single column sums to 81 without regrouping, and (d) copy 40 digits in 1 minute. Students were also selected based on their lack of skill mastery with regard to multiplication facts. That is, students needed a pool of at least thirty unknown multiplication facts. The range of unknown multiplication facts was 30 to 50 multiplication facts. All of the students performed at least three grade levels below their grade placement in reading, math, and writing. The interventions took place during a thirty-minute period designated for remedial instruction. This period was school-wide, not a special education initiative. The primary researcher provided one-on-one instruction with each student for 15-20 minutes per session. The primary researcher was a white female certified in special education, with 7 years of experience. The instructor received professional development in both CTD and SIM through her teacher preparation program and professional development workshops. She implemented both instructional methods successfully with students with disabilities in her previous position as a middle school resource teacher.

Instructional Procedures
SIM was implemented according to Mercer and Miller’s (1992a) *Strategic math series: Multiplication facts 0-8*. CTD was conducted according to a modified version of procedures outlined by Wolery, Ault, and Doyle (1992). All procedures were identical to Wolery et al., with the exception that students were told to write their answer instead of simply providing a verbal response. Since SIM requires writing, this was an attempt to equalize time issues across both methods.

Instructional Materials
Prior to instruction, students completed a multiplication test that included 100 multiplication facts, factors 0-9. The multiplication facts that were missed were grouped into four independent (no duplicated facts such as *6x8* and *8x6*) and balanced groups of facts. To ensure that CTD did not influence SIM, groups used within each type of instruction consisted of facts that were less likely to influence the learning of other facts (for example, knowing *9x5* might lead to solving *9x6* by solving *45+9.*). No strategies were taught within either method that might have encouraged such problem solving, but efforts were made to minimize the chance incidental learning. The four groups of facts were used for the two intervention phases (ten facts for each condition) and the two sets of generalization probes (five facts for each probe). Students’ writing fluency was also measured to ensure adequate motor skills to reach criterion of writing 40 digits per minute with no more than two errors. Given 1 minute, the students copied numbers written on a page.

The materials for CTD consisted of multiplication facts printed vertically in black ink on a 4-by-6 inch index card with print that was approximately 2 inches in height. The order of the flashcards was changed between each trial so that the order of presentation was not predictable. In addition to the flashcards, instructional materials included a multiplication sheet that corresponded to the flashcards. After verbally responding to the flashcards, the students wrote the answer on the multiplication sheet. This modification of CTD materials was to prepare the students for the written probes. The materials for fluency instruction consisted of a sheet containing the target facts, written multiple times in a random order.

During SIM, the materials included a contract in which the teacher and student agreed to work rigorously using strategies to master multiplication facts. The materials for lessons 1-3 were paper plates, three- dimensional objects, and a worksheet with target multiplication facts written vertically and horizontally. The materials for representational phase involved drawings that were pre-printed on the lesson sheet and target multiplication facts written vertically and horizontally. After instruction at the representational level, the DRAW (**D**iscover the sign, **R**ead the problem, **A**nswer, or draw tallies and/or circles and check your answer, **W**rite the answer) was introduced where the materials consisted of a sheet printed with the strategy. The last phase of instruction involved fluency activities and the materials included facts written repeatedly and in a random order. The materials also included a chart used to record student progress after each activity. The probes for CTD and SIM consisted of multiplication facts written repeatedly in random order.

Inter-rater Reliability

Treatment integrity was conducted during 25% of the lessons (Poling, Methot, & LeSage, 1995). Live observations were used to evaluate the instructor's behaviors. A treatment checklist for each intervention was used in order to ensure that procedures were carried out correctly. A teacher was trained in using the treatment integrity checklists through demonstration and practice. When the teacher completed a checklist with 100% accuracy, treatment integrity checks began. Treatment integrity was calculated at 100% for the study.

Treatment Fidelity

Inter-rater reliability was conducted for 25% of all of the multiplication probes administered. A teacher was trained through demonstration and practice in the scoring procedures and reliability checks began after the teacher had scored a probe with 100% accuracy. The primary researcher collected data on a daily basis and scored fluency probes. The trained teacher scored the same multiplication probes. To calculate inter-rater reliability, the total number of agreements between the teacher and the primary researcher were divided by the total number of observations and this answer was multiplied by one hundred (Poling et al., 1995). Inter-rater reliability was calculated for CTD responses, independent written work during SIM, fluency, and mastery probes. Inter-rater reliability was 100% for the study.

Case Study 1: Sam

Sam's background

Sam was a 12 year-old African American male in the 6th grade who qualified for special education services with specific learning disabilities in the areas of mathematics computation, reading decoding, and written expression. Sam also qualified for services through other health impairments with a medical diagnosis of sickle cell anemia which interfered with his educational performance due to frequent illness and fatigue. Sam's cognitive ability was within the average range (WISC-III, FS=89). Sam performed below grade level in the areas of math computation (2:0), reading decoding (K:1), and written expression (2:0). Sam was an outgoing middle school student who verbalized excitement about participating in instruction. Sam reported that remembering his math facts was a problem and this caused difficulties when completing math assignments.

CTD Instruction with Sam

Sam learned the first group of 10 multiplication facts through CTD instruction. Sam expressed enthusiasm for learning multiplication facts. He reported being very pleased with himself when he answered correctly. When Sam was unsure of an answer, he waited for the instructor's prompt. This resulted in repeated practice with correct responses. He reported that he like CTD because instruction was fast-paced. Sam was absent on 4 occasions during instruction and this was not unusual given his medical condition and as reported by his resource teacher. Sam reached the criterion for proficiency (40 correct digits written on a one-minute multiplication probe over three consecutive trials). He reached criterion after 14 probes, ranging from 4 to 46 correct digits per minute. His mean performance for CTD instruction was 26.07 correct digits per minute.

SIM Instruction with Sam

Five weeks after CTD instruction, Sam learned the second group of multiplication facts through SIM instruction. Sam appeared to understand multiplication throughout each phase of instruction, moving to each successive phase without re-teaching. Sam quickly learned and used the DRAW strategy appropriately. Two facts were particularly difficult for Sam, *6x8* and *8x9*. During the representational phase of instruction, when given the problem *6x8*, Sam solved the problem by adding the product of *5x8* to the product of *1x8* to arrive at the answer *48*. He used the same procedures to solve *8x9*. Instruction did not include explanation of the distributive property. Despite his progress during SIM, Sam reported that he did not like this kind of instruction because it took too long. He reported that he would rather learn using CTD because it was faster. During SIM, Sam reached criterion after 19 probes ranging from 8-48 digits per minute. His mean performance for this condition was 29.63 correct digits per minute.

Sam's Performance

Table 1 presents a summary of Sam's and other students' performance. One week after instruction in CTD, a multiplication probe was administered to check maintenance of his skills. Sam wrote 22 correct digits per minute (dpm). Sam reported that he was disappointed that he did not remember more of his facts after taking the first maintenance probe. Another maintenance probe was administered five weeks after CTD instruction ended. Sam wrote 8 correct dpm. Sam remembered 2 of the 10 math facts and wrote correct digits for the same fact and half of the correct digits for another fact. The instructor administered Generalization probes including 5 unknown facts, one timed probe (1-minute) and a probe with no time limit. Sam wrote 0 correct dpm when given the timed probe and 0 dpm when given the probe with no time limit. Sam reported that he was finished with the untimed probe at 2 minutes. Sam reported that he did not know those facts because he had not learned them previously. Next, Sam learned the second group of 10 multiplication facts through SIM instruction. Sam reached the criterion for proficiency (40 correct digits written on a one-minute multiplication probe over three consecutive trials). One week after instruction in SIM, a multiplication probe was administered to check maintenance of his skills. Sam wrote 38 correct digits per minute (dpm). Sam reported that he was pleased with his performance. Another maintenance probe was administered five weeks after SIM instruction ended. Sam wrote 20 correct dpm. Sam wrote most of the digits automatically, without hesitation. He began using the strategy that he had learned during SIM instruction, but ran out of time for this strategy to be useful within the 1-minute time limit The instructor administered Generalization probes including 5 unknown facts, one timed probe (1-minute) and a probe with no time limit. Sam wrote 0 correct dpm when given the timed probe and 42 dpm when given the probe with no time limit. Sam reported that he was finished with the untimed probe at 5 minutes, 20 seconds. Sam used the strategy that he had learned in SIM instruction to complete the unknown fact problems.

Table 1
Performance Summary

CTD	Maintenance 1-week dpm	Maintenance 5-week dpm	Generalization 1-minute dpm	Generalization Untimed Digits/min:sec
Sam	22	8	0	0 / 2:00
Kate	32	16	9	7 / 2:54
Jerry	16	18	4	44 / 2:30
Cindy	36	6	0	0 / 1:15
SIM	Maintenance 1-week dpm	Maintenance 5-week dpm	Generalization 1-minute dpm	Generalization Untimed Digits / min:sec
Sam	38	20	0	42 / 5:20
Kate	40	19	40	50 / 1:45
Jerry	40	33	40	50 / 2:25
Cindy	36	20	4	50 / 5:20

Comparison of Sam's CTD and SIM Performance
Sam learned 2 groups of multiplication facts (10 in each) and reached proficiency with both instructional methods. However, there was a difference in how those groups were maintained over time. Sam's 1-week maintenance performance decreased with both methods, the decrease was significantly lower after CTD (52% lower) than after SIM (21% lower). Sam's 5-week performance decreased again for both methods, but the decrease was lower after CTD (83% lower) than after SIM (58% lower). The difference between Sam's performance on the generalization probes favored SIM instruction. Sam's learning during the CTD condition did not generalize to the unknown facts. Sam did not write any digits correctly for either of the probes. However, Sam's knowledge of the strategy appeared to be useful in generalizing skills to the unknown facts. Sam did not write any correct digits with the timed probe. However, when given unlimited time, Sam wrote 42 dpm. Sam demonstrated his use of the strategy learned in SIM instruction by drawing the problems and solving them correctly.

Case Study 2: Kate
Kate's background
Kate was an 11 year-old African American female in the 6th grade who qualified for special education services with specific learning disabilities in the areas of mathematics reasoning. Kate's cognitive ability was below the average range (C-TONI comp=83). She performed below grade level in the areas of math computation (3:1), reading decoding (3:1), and written expression (2:1). Although Kate qualified for services in the area of mathematics reasoning, current assessment data were not available in that area. Kate appeared friendly and was cooperative throughout the study. She reported that math was her most difficult subject and remembering math facts was difficult. Kate was interested in learning new ways to learn and remember math facts and reported that she was happy to participate.

CTD Instruction with Kate
Kate learned the first group of 10 multiplication facts through CTD instruction. She was present for every scheduled session. Kate's mastery of facts increased steadily over sessions. She demonstrated patience, waiting for the instructor's prompt when she was unsure of an answer. Kate appeared engaged and eager to participate throughout each of the sessions. Kate reached the criterion for proficiency (40 correct digits written on a one-minute multiplication probe over three consecutive trials). Kate reached criterion after 10 probes, ranging from 10 to 49 correct dpm. Her mean performance for this condition was 29.8 correct dpm.

SIM Instruction with Kate
Five weeks after CTD instruction, Kate learned the second group of multiplication facts through SIM instruction. Each instructional phased was completed without the need for re-reaching. Kate appeared to like SIM instruction as evidenced by reports that she thought it

was easier to remember her facts. She liked using the DRAW strategy for facts that she could not quite remember. She also reported that she needed more time to remember some of the math facts and that she liked that there was no time limit during instruction as in CTD instruction. Similar to Sam, Kate discovered an alternate way to solve facts problems that she did not remember. Her grouping of facts also included *8x6* and *8x9*. She had mastered *8x5* and *8x8* earlier in SIM instruction. Rather than using the DRAW strategy, she added the product of *5x8* to the product of *1x8* to arrive at the answer *48*. She used the same procedures to solve *8x9*. This was not a part of the strategic instruction program. During SIM instruction, Kate reached criterion after six probes, ranging from 26 to 46 dpm. Her mean performance for this condition was 36.83 correct dpm.

Kate's Performance
Table 1 (above) presents a summary of Kate's and other students' performance. One week after instruction in CTD, a multiplication probe was administered to check maintenance of her skills. Kate wrote 32 correct dpm. Kate appeared pleased with her performance. Another maintenance probe was administered five weeks after CTD instruction ended. Kate wrote 18 correct dpm. The instructor administered Generalization probes including 5 unknown facts, one timed probe (1-minute) and a probe with no time limit. Kate wrote 9 correct dpm when given the timed probe and 7 dpm when given the probe with no time limit. Kate reported that she was finished with the untimed probe at 2 minutes and 54 seconds. Next, Kate learned the second group of 10 multiplication facts through SIM instruction. She reached the criterion for proficiency (40 correct digits written on a one-minute multiplication probe over three consecutive trials). One week after instruction in SIM, a multiplication probe was administered to check maintenance of her skills. Kate wrote 40 correct dpm. Kate appeared to be satisfied with her performance. Another maintenance probe was administered five weeks after SIM instruction ended. Kate wrote 19 correct dpm. The instructor administered Generalization probes including 5 unknown facts, one timed probe (1-minute) and a probe with no time limit. Kate wrote 40 correct dpm when given the timed probe and 50 dpm when given the probe with no time limit. She reported that he was finished with the untimed probe at 1 minute, 45 seconds. Kate used the strategy that she had learned in SIM instruction to complete the unknown fact problems.

Comparison of Kate's CTD and SIM Performance
Kate learned 2 groups of multiplication facts (10 in each) and reached proficiency with both instructional methods. However, there was a difference in how those groups were maintained over time. Kate's 1-week maintenance performance decreased with both methods, the decrease was significantly lower after CTD (45% lower) than after SIM (13% lower). Her 5-week performance decreased again for both methods, but the decrease was lower after CTD (67% lower) than after SIM (59% lower). The difference between Kate's performance on the generalization probes favored SIM instruction. Her learning during the CTD appeared to generalize to unknown facts to some degree. She wrote 9 correct dpm during the timed probe and 7 correct dpm during the untimed probe. However, Kate's knowledge of the strategy appeared to be more useful in generalizing skills to the unknown facts. She wrote 40 correct dpm with the timed probe and , when given unlimited time, wrote 50 correct digits. Kate demonstrated her use of the strategy learned in SIM instruction by drawing the problems and solving them correctly.

Case Study 3: Jerry
Jerry's background
Jerry was an 11 year-old African American male in the 6th grade who qualified for special education services with specific learning disabilities in the areas of mathematics computation and reading decoding. Jerry's cognitive ability was within the average range (Stanford-Binet, Comp=85). Jerry performed below grade level in the areas of math computation (3:5) and reading decoding (2:5). Jerry was a quiet and reserved middle school student who reported

interest in participating in instruction. Jerry reported having difficulty completing math problems because he did not know all of his multiplication facts.

CTD Instruction with Jerry

Jerry learned the first group of 10 multiplication facts through CTD instruction. Jerry was present for every scheduled session. He began mastering facts immediately after presentation using a zero-second delay. There were only two instances of teacher prompts during presentation with the 4-second delay. Although Jerry mastered facts quickly, he reported that is was hard to remember and that he didn't like that the answers were hidden on the back of the flashcard. Jerry reached the criterion for proficiency (40 correct digits written on a one-minute multiplication probe over three consecutive trials). Jerry reached criterion after five probes, ranging from 16 to 40 correct dpm. His mean performance for this condition was 31.6 correct dpm.

SIM Instruction with Jerry

After 5 weeks of CTD instruction, Jerry began SIM instruction with the second group of facts. He completed each instructional phase and there was no need for re-teaching. Jerry mastered facts quickly and used the DRAW strategy on several probes when he did not automatically remember facts. Jerry reported that he like SIM instruction because it was easier to remember facts. Jerry reached criterion after nine probes, ranging from 18 to 50 dpm. His mean performance for this condition was 30.89 correct dpm.

Jerry's Performance

Table 1 (above) presents a summary of Jerry's and other students' performance. One week after instruction in CTD, a multiplication probe was administered to check maintenance of his skills. Jerry wrote 16 correct dpm. Jerry appeared frustrated with his performance after the probe and reported that he wished he could remember more of his facts after taking the first maintenance probe. Another maintenance probe was administered five weeks after CTD instruction ended. Jerry wrote 18 correct dpm. The instructor administered Generalization probes including 5 unknown facts, one timed probe (1-minute) and a probe with no time limit. Jerry wrote 4 correct dpm when given the timed probe and 44 dpm when given the probe with no time limit. Jerry reported that he was finished with the untimed probe at 2 minutes and 30 seconds. Next, Jerry learned the second group of 10 multiplication facts through SIM instruction. Jerry reached the criterion for proficiency (40 correct digits written on a one-minute multiplication probe over three consecutive trials). One week after instruction in SIM, a multiplication probe was administered to check maintenance of his skills. Jerry wrote 40 correct digits per dpm. Another maintenance probe was administered five weeks after SIM instruction ended. Jerry wrote 33 correct dpm. The instructor administered Generalization probes including 5 unknown facts, one timed probe (1-minute) and a probe with no time limit. Jerry wrote 40 correct dpm when given the timed probe and 50 dpm when given the probe with no time limit. Jerry reported that he was finished with the untimed probe at 2 minutes, 25 seconds. Jerry used the strategy that he had learned in SIM instruction to complete the unknown fact problems.

Comparison of Jerry's CTD and SIM Performance

Jerry learned 2 groups of multiplication facts (10 in each) and reached proficiency with both instructional methods. However, there was a difference in how those groups were maintained over time. Jerry's 1-week maintenance performance decreased with facts learned during CTD (60% decrease), but not after SIM (20% decrease). Jerry's 5-week performance decreased for both methods, but the decrease was lower after CTD (55% lower) than after SIM (34% lower). The difference between Jerry's performance on the generalization probes favored SIM instruction. Jerry wrote 4 correct dpm on the first generalization probe. However, when given the untime probe, Jerry's performance was surprisingly higher with 44 correct digits written in about 2 minutes. Jerry's knowledge of the strategy appeared to be useful in generalizing skills to the unknown facts. Jerry wrote 40 correct dpm with the timed probe. His performance increased to 50 correct digits written in about 2 minutes.

Case Study 4: Cindy

Cindy's background

Cindy was an 11 year-old African American female in the 6th grade who qualified for special education services with specific learning disabilities in the area of mathematics reasoning. Cindy's cognitive ability was below the average range (WISC-III FS=76). She performed below grade level in the areas of math computation (3:2), reading decoding (3:2), and written expression (3:2). There were current data regarding math reasoning although this was her area of eligibility. The resource teachers reported being most concerned about her performance in math. They reported that she had a particularly difficult time learning, demonstrating memory problems to a greater extent than other peers with specific learning disabilities.

CTD Instruction with Cindy

Cindy learned the first group of 10 multiplication facts through CTD instruction. Cindy was present for every scheduled session. Cindy made slow progress at the beginning of instruction. She mastered 2 facts and maintained that level of performance for 7 sessions. Cindy was impatient during instruction as evidenced by quick incorrect responses rather than waiting for the instructor's prompt. Her progress was more sporadic than the other students in that one say she would appear to have mastered a fact one day but not the next. During the last 5 probes before the criterion, there was a large increase in her performance and this increase continued until the criterion was reached. Cindy reached the criterion for proficiency (40 correct digits written on a one-minute multiplication probe over three consecutive trials). Cindy reached criterion after 16 probes ranging from 2 to 44 correct dpm. Her mean performance for this condition was 17.5 correct dpm.

SIM Instruction with Cindy

Five weeks after CTD instruction, Cindy learned the second group of multiplication facts through SIM instruction. Each instructional phased was completed without the need for re-reaching. Cindy did not appear to like SIM instruction. She learned the DRAW strategy without difficulty and used it appropriately when she could not remember a fact. Cindy reported that she liked SIM instruction better than CTD because the DRAW strategy provided her with a plan that she could use to remember facts. Cindy reached criterion after 13 probes, ranging from 0 to 42 dpm. Her mean performance for this condition was 23.69 correct dpm.

Cindy's Performance

Table 1 (above) presents a summary of Cindy's and other students' performance. One week after instruction in CTD, a multiplication probe was administered to check maintenance of her skills. Cindy wrote 36 correct dpm. Cindy appeared pleased with her performance. Another maintenance probe was administered five weeks after CTD instruction ended. Cindy wrote 6 correct dpm. The instructor administered Generalization probes including 5 unknown facts, one timed probe (1-minute) and a probe with no time limit. Cindy wrote 0 correct dpm when given the timed probe and 0 correct digits when given the probe with no time limit. Cindy reported that she was finished with the untimed probe at 1 minutes and 15 seconds. Next, Cindy learned the second group of 10 multiplication facts through SIM instruction. She reached the criterion for proficiency (40 correct digits written on a one-minute multiplication probe over three consecutive trials). One week after instruction in SIM, a multiplication probe was administered to check maintenance of her skills. Cindy wrote 36 correct dpm. Another maintenance probe was administered five weeks after SIM instruction ended. Cindy wrote 20 correct dpm. The instructor administered Generalization probes including 5 unknown facts, one timed probe (1-minute) and a probe with no time limit. Cindy wrote 4 correct dpm when given the timed probe and 50 correct digits when given the probe with no time limit. She reported that he was finished with the untimed probe at 5 minute, 20 seconds. Cindy used the strategy that she had learned in SIM instruction to complete the unknown fact problems.

Comparison of Cindy's CTD and SIM Performance
Cindy learned 2 groups of multiplication facts (10 in each) and reached proficiency with both instructional methods. Although her performance decreased with both methods, her performance was slightly lower with the CTD condition (18% lower) than with the SIM condition (14% lower). Her 5-week performance decreased again for both methods, but the decrease was lower after CTD (86% lower) than after SIM (52% lower). The difference between Cindy's performance on the generalization probes favored SIM instruction. Her learning during the CTD did not appear to generalize to unknown facts, writing 0 correct digits on both timed and untimed probes. However, Cindy's knowledge of the strategy appeared to be more useful in generalizing skills to the unknown facts. She wrote 4 correct dpm with the timed probe and , when given unlimited time, wrote 50 correct digits. Cindy demonstrated her use of the strategy learned in SIM instruction by drawing the problems and solving them correctly.

Discussion
Instruction and Performance
This investigation was designed to examine the differences between CTD and SIM with regard to maintenance and generalization of multiplication facts. Consistent with previous studies, the students maintained their learning (Mattingly & Bott, 1990). However, this study measured maintenance both one and five weeks after instruction rather than three weeks in other studies. This study extended previous SIM and CTD findings with regard to generalization, defining it as computation of untaught multiplication facts. Mattingly and Bott (1990) and Cybrinsky and Schuster (1990) defined generalization as the transfer of verbal identification to written tasks. Koscinski and Gast (1993) defined it as the ability to identify the same previously taught facts using a different orientation and through completion of written multiplication facts. Mercer and Miller (1992b) defined generalization as the ability to use previously taught facts in other mathematical algorithms, across settings, and with different examiners.

This study demonstrated a difference between CTD and SIM with regard to generalization and maintenance. After SIM, two students wrote more correct digits during the timed generalization probe and all of the students wrote more digits during the untimed probe. These increases were significantly higher with increases of 39 and 43 more digits. Three of the four students maintained a higher level of fluency one week after the SIM condition with increases as high as 16 and 24 more dpm. All of the students maintained a higher level of fluency five weeks after the SIM condition, with increases as high as 15 dpm.

Student Background, Behavior, and Perceptions
The students all had similar backgrounds with regard to age, grade, ethnicity, achievement, special education eligibility, and amount of special education services. The students' behaviors were consistent with their performance in that engagement and ease of following procedures was consistent with the number of probes completed before mastery. For example, Cindy had great difficulty during CTD, frequently impulsively responding with incorrect answers rather than waiting for the teacher's prompt. This frequent practice with incorrect responses may have contributed to her rate of progress. There was a surprising finding in the students' perceptions of the CTD and SIM. All of the students but Sam perceived SIM instruction as the better instructional method. This was surprising because Sam performed much better during SIM instruction. He discovered an alternate way to solve unknown multiplication facts through the distributive property. He also used the DRAW strategy during generalization which led to a significant increase in his performance on the untimed probe. Despite his increased performance and demonstration of understanding of multiplication, he perceived CTD as better way to learn because it was fast-paced. This did not appear to affect his performance and it is not known how the perceptions of the other students influenced their performance.

Implications of Findings

The students in this study were older than the students involved in most of the previous research (Mattingly & Bott, 1990; Cybrinsky & Schuster, 1990; Koscinski & Gast, 1993; Harris et al., 1995) and had a history of failure. A concern regarding the students' age might be the amount of instructional time devoted to learning basic multiplication facts rather than other mathematical skills (Cawley, Parmar, Yan, & Miller, 1996). However, ten to fifteen minutes outside of regularly scheduled math instruction for as few as five days produced an increase in proficiency and fluency. It is realistic that middle school teachers could devote this amount of time to instruction (Zigmond & Baker, 1990). Although both interventions were implemented individually, CTD can be used in a group format (Keel & Gast, 1992; Keel, Slaton, & Blackhurst, 2001; Koscinski & Hoy, 1993) and SIM was designed for group instruction (Mercer & Miller, 1992a). Group implementation would be a more efficient use of time.

The most significant implication for the results of this study was the effects of SIM on maintenance and generalization. The students maintained greater levels of fluency on the 5-week maintenance measures. According to cognitive theory, learning occurs through a process involving attention, perception, encoding information for storage, and retrieving previously learned information (Case, 1992; Pressley, 1995). Perhaps manipulating objects and using visual representations increased students' attention and built a more elaborate mental framework for multiplication, leading to more efficient storage and retrieval of information. Other evidence that the students demonstrated greater understanding of multiplication was that they discovered other strategies for solving multiplication problems. During SIM, when given the problem *6x8*, two students added the product of *5x8* to the product of *1x8* to arrive at the answer *48*. Two other students solved *8x9* using the same procedure. Perhaps the manipulation of objects and use of visual representations led to increased understanding and use of the distributive property.

The increased generalization performance might have been due to the students' strategy-use during the SIM condition. It was evident that the students used this strategy during the generalization probes because they translated the numerical problems into pictures on the timed generalization and untimed generalization probes. In addition to maintaining their knowledge of multiplication facts, the students applied their strategic knowledge to another task. The use of the DRAW strategy for unknown multiplication facts took more time, but led to greater accuracy. This may explain the students' increase in accuracy when there was no time limit for completion.

Limitations

The limitations of this study are in the area of external validity, the degree to which the results can be generalized beyond the experimental conditions (Kazdin, 1982). First, the students who participated in this study were similar with respect to grade level, ethnicity, disability, amount of time served in special education, and level of achievement. It is not known whether the same results would be obtained with characteristically different students. A further limitation of this study is the generality of the behavior change agent (Kazdin, 1982). The primary researcher delivered instruction rather than the students' math teacher. This may have increased the treatment integrity, but the results may be less practical or applicable. Also, the likelihood that this type of instruction will continue to be implemented by the students' teacher is probably small since she did not experience the students' progress firsthand. Implementation of these methods by a teacher in a typical classroom situation would strengthen the results. In order to bridge the gap between research and practice, appropriately trained teachers should implement research procedures in a typical classroom situation. Finally, the amount of time employed for the maintenance condition is a limitation. The study was conducted during the second semester of the school year and maintenance measures up to 5 weeks ensured that students finished both conditions by the end of the school year. A longer maintenance condition would have increased validity and in practice, maintenance would be expected to be much longer than five weeks.

Future Research and Conclusion
Further research is needed to investigate how long the treatment effects are maintained. Measurement of maintenance each week for several weeks might provide more practical maintenance information. Also, further research may be needed in order to investigate whether and/or how much practice is needed to maintain the treatment effects over time. Further investigation of generalization is also needed. Although generalization of multiplication skills to untaught facts was measured, this study did not involve programmed instruction for generalization. This is a component of other programs following SIM (Deshler, Schumaker, Harris, & Graham, 1999; Schmidt, Deshler, Schumaker, & Alley, 1989). The generalization measures in this study did not extend to problem solving and real world application of skills as called for by the National Teacher of Mathematics Standards (2000). Future research should involve instruction and measurement in this area.

CTD and SIM have demonstrated improvements in the mathematical performance of students with LD (Cybrinsky & Schuster, 1990; Koscinski & Gast, 1993; Harris et al., 1995; Mattingly & Bott, 1990; Mercer & Miller, 1992b ; Miller & Mercer, 1993a; Miller & Mercer, 1993b). Both teaching strategies are effective, but SIM may have a greater impact on the maintenance and generalization of skills. It is important that teachers know which strategies improve the retention of math skills for students with LD, so that these students have a greater likelihood of academic success.

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